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POSTER SESSIONS
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POSTER SESSIONS

Valeria Ambrosio (University of Cambridge)
A CONTINUOUS PROOF OF THE EXISTENCE OF THE SLE8 CURVE

Suppose that η is a whole-plane space-filling SLEκ for κ in $(4, 8)$ from infinity to infinity parameterized by Lebesgue measure and normalized so that $\eta(0) = 0$. For each $T > 0$ and κ in $(4, 8)$ we let $\mu_{\kappa,T}$ denote the law of η when restricted to the interval $[0, T]$. We show for each $a, T > 0$ that the family of laws $\mu_{\kappa,T}$ for κ in $[4 + a, 8)$ is compact in the weak topology associated with the space of probability measures on continuous curves $[0, T] \to C$ equipped with the uniform distance. As a direct byproduct of this tightness result, we obtain a new proof of the existence of the SLE8 curve which does not build on the discrete uniform spanning tree scaling limit of Lawler–Schramm–Werner.

[1] arXiv:2203.13805

Simone Baldassarri (University of Florence)
CRITICAL DROPLETS AND SHARP ASYMPTOTICS FOR KAWASAKI DYNAMICS WITH STRONGLY ANISOTROPIC INTERACTIONS

We analyze metastability and nucleation in the context of the Kawasaki dynamics for the two-dimensional Ising lattice gas at very low temperature. Let $\Lambda \subset \mathbb{Z}^2$ be a finite box. Particles perform simple exclusion on Λ, but when they occupy neighboring sites they feel a binding energy $-U_1 < 0$ in the horizontal direction and $-U_2 < 0$ in the vertical one. Along each bond touching the boundary of Λ from the outside to the inside, particles are created with rate $\rho = e^{-\Delta\beta}$, while along each bond from the inside to the outside, particles are annihilated with rate 1, where $\beta > 0$ is the inverse temperature and $\Delta > 0$ is an activity parameter. We consider the parameter regime $U_1 > 2U_2$ also known as the strongly anisotropic regime. We take $\Delta \in (U_1, U_1 + U_2)$, so that the empty (respectively full) configuration is a metastable (respectively stable) configuration. We consider the asymptotic regime corresponding to finite volume in the limit as $\beta \to \infty$. We investigate how the transition from empty to full takes place with particular attention to the critical configurations that asymptotically have to be crossed with probability 1. To this end, we provide a model-independent strategy [1] to identify some unessential saddles (that are not in the union of minimal gates) for the transition from the metastable (or stable) to the stable states and we apply this method to our model. The derivation of some geometrical properties of the saddles allows us to identify the full geometry of the minimal gates and their boundaries for the nucleation in the strongly anisotropic case [2]. Moreover, we derive sharp estimates for the asymptotic transition time for the strongly anisotropic case. This is a joint work with F. R. Nardi.

[1] arXiv:2108.02017, [2] arXiv:2108.03133

Mikhail Basok (École Normale Supérieure)
DOUBLE DIMER MODEL IN TEMPERLEY DOMAINS AND ITS CONVERGENCE TO CLE(4)

The aim of this presentation is to expose the current state of the conjecture asserting that the sequence of random loop ensembles appearing in a simply connected planar domain under sampling the double dimer model in its Temperley discretization converges to CLE(4) measure in the domain. This prediction was originally made based on Kenyon work on convergence of the dimer height function to the Gaussian free field in the same setting. Later on Kenyon introduced a family of observables of the double dimer model, called "topological correlators". Dubedat discovered that the limit of these observables are described by isomonodromic tau functions and proved the convergence; in the same work Dubedat established a relation between isomonodromic tau functions and CLE(4) statistics. In the subsequent work Chelkak and the author used this result of Dubedat to prove the convergence of probabilities of cylindrical events of the double dimer model to a conformal invariant limit. Finally, in the work of Bai and Wan it was proven that this limit coincides with the set of cylindrical probabilities computed for CLE(4). This reduced the aforementioned conjecture to a certain (depending on the topology of interests) a priori regularity statement for the double dimer model.

[1] arXiv:1403.6076, [2] arXiv:1809.00690, [3] arXiv:2106.11088

Rodrigo Bazaes (University of Münster)
STOCHASTIC HOMOGENIZATION OF HAMILTON–JACOBI–BELLMAN EQUATIONS ON CONTINUUM PERCOLATION CLUSTERS

The main goal of this work is to study the behavior as $\varepsilon \to 0$ of the viscosity solution of the Hamilton–Jacobi–Bellman (HJB) equation

$$\begin{cases} \partial_t u_\varepsilon = \frac{\varepsilon}{2}\mathrm{div}\big(a\big(\frac{x}{\varepsilon},\omega\big)\nabla u_\varepsilon\big) + H\big(\frac{x}{\varepsilon}, \nabla u_\varepsilon, \omega\big), & (t,x) \in (0,T) \times \mathcal{C}_\infty(\omega) \\ u_\varepsilon(t,x) = f(x), & (t,x) \in \{0\} \times \mathcal{C}_\infty(\omega) \cup (0,T) \times \partial\mathcal{C}_\infty(\omega) \end{cases}$$

in an almost sure sense. Here, $\mathcal{C}_\infty(\omega)$ represents a realization of the infinite component containing the origin of a continuum percolation cluster, and H is a convex and coercive Hamiltonian in the second variable. Our contribution relies upon the fact that the probability measure $\mathbb{P}_0$ on which the environments live is not stationary under the canonical shifts, and the matrix a is not uniformly elliptic, so that standard methods cannot be applied directly. We combine known techniques from the stationary and ergodic setting with new ones from random walks on percolation clusters to develop a unifying method for this class of problems.

Manan Bhatia (MIT)
ENVIRONMENT SEEN FROM INFINITE GEODESICS IN LIOUVILLE QUANTUM GRAVITY

Liouville Quantum gravity (γ-LQG) is an important model of random metric spaces and is conjectured to be the scaling limit of a large class of discrete random planar maps. As a metric space, LQG has a natural definition of geodesics and these have a rich coalescence structure. Geodesics, being distance minimizing, should intuitively go through regions of low weight and this heuristic leads to the question of describing the environment around a typical point on the geodesic and comparing it to the unconditional environment. In the poster session, we introduce γ-LQG and its properties as a metric space and then discuss some progress in the above-mentioned direction. Based on [1], which is joint work with Riddhipratim Basu and Shirshendu Ganguly.

[1] arXiv:2107.12363

Ahmed Bou-Rabee (University of Chicago)
CONVERGENCE OF THE RANDOM ABELIAN SANDPILE

We prove that Abelian sandpiles with random initial states converge almost surely to deterministic scaling limits. The proof follows the Armstrong–Smart program for stochastic homogenization of uniformly elliptic equations.

[1] arXiv:1909.07849

Sung-Soo Byun (Korea Institute for Advanced Study)
POLE DYNAMICS AND AN INTEGRAL OF MOTION FOR MULTIPLE SLE(0)

We describe the Loewner chains of the real locus of a class of real rational functions whose critical points are on the real line. Our main result is that the poles of the rational function lead to explicit formulas for the dynamical system that governs the driving functions. Our formulas give a simple method for mapping the class of rational functions into solutions to a non-trivial system of quadratic equations, and for directly showing that the curves in the real locus satisfy geometric commutation and have the geodesic multichord property. These results are entirely self-contained and have no reliance on probabilistic objects, but make use of an integral of motion for the Loewner chain that is motivated by ideas from conformal field theory. We also show that the dynamics of the driving functions are a special case of the Calogero–Moser integrable system, restricted to a particular submanifold of phase space carved out by the Lax matrix. Our approach complements a recent result of Peltola and Wang, who showed that the real locus is the deterministic $\kappa \to 0$ limit of the multiple SLE(κ) curves.

[1] arXiv:2011.05714

Baptiste Cerclé (Université Paris-Saclay)
THREE-POINT CORRELATIONS IN THE SL3 TODA THEORY

Toda Conformal Field Theories (CFTs) form a family of 2d CFTs indexed by semisimple and complex Lie algebras. They are natural generalizations of Liouville CFT in that they enjoy an enhanced level of symmetry encoded by W-algebras. These theories can be rigorously defined using a probabilistic framework, which involves correlated Gaussian Multiplicative Chaos measures. With this poster I wish to explain how certain three-point correlation functions, which generalize the celebrated DOZZ formula, can be computed within this probabilistic framework for the $\mathfrak{sl}_3$ Toda Conformal Field Theory and coincide with predictions from the physics literature by Fateev-Litvinov. Along the derivation of this formula I will highlight how a new Brownian path decomposition in Euclidean spaces (generalizing the one-dimensional result of Williams) allows to describe the joint tail expansion of correlated Gaussian Multiplicative Chaos measures and extend the range of values for which the probabilistic definition of the DOZZ formula makes sense.

[1] arXiv:2102.11219, [2] arXiv:2105.01362

William Da Silva (University of Vienna)
GROWTH-FRAGMENTATIONS EMBEDDED IN PLANAR EXCURSIONS

Growth-fragmentation processes are branching processes which model the evolution of a cloud of atoms that may grow and dislocate as time evolves. In a pioneering work, Bertoin, Budd, Curien and Kortchemski describe the branching structure of these particle systems, as well as a particular family obtained in the scaling limit from a Markov peeling process of large random planar maps. In a joint work with Élie Aïdékon (Fudan University), we construct, on a half-plane Brownian excursion, a signed version of the critical growth-fragmentation process revealed by Bertoin, Budd, Curien and Kortchemski. We will then present an extension to stable Lévy processes, and establish the spinal decomposition. Connections to planar maps and Liouville quantum gravity, in particular via the mating-of-trees, will be discussed.

Stefano Decio (Norwegian University of Science and Technology)
ZEROS OF SUMS OF EIGENFUNCTIONS

Eigenfunctions of the Laplace–Beltrami operator are fundamental objects in several areas in analysis and mathematical physics. A guiding principle in the study of their local properties is that they should resemble those of polynomials. In this light, I will discuss oscillations and zeros for linear combinations of Laplace eigenfunctions on Riemannian manifolds. In particular, I will prove that zeros become dense in the manifold if not too many eigenfunctions are summed. Some related open questions will be mentioned, the most promising of which are of probabilistic flavor.

[1] arXiv:2009.10581

Akshunna S. Dogra (Imperial College London)
SOME MATHE-PHYSICAL PERSPECTIVES AND EFFECTIVE THEORIES ON DEEP LEARNING

Deep learning has ushered significant changes in how analysis and modelling in multiple scientific disciplines is pursued. Unsurprisingly, existing insights from many fields have made their way to deep learning as well, informing the methods by which we study and analyze this relatively modern, but increasingly important scientific field. In this work, we will introduce how ideas from Koopman Operator Theory, Renormalization Group theory, and Numerical analysis can individually inform/characterize various aspects of deep learning (model sizes, architectures, optimization, etc). In particular, we will show how various methods of analysing optimization dynamics within the community (like NTK theory) can be gainfully assimilated within Koopman theory. We will also how pruning models via Iterative Magnitude pruning can be viewed from the perspective of Renormalization Theory and what that perspective implies about the so called 'Lottery Ticket Hypothesis'. Time permitting, we will then show how we can combine those insights to build efficient Neural Network Differential Equation solvers.

[1] arXiv:2110.03210, [2] arXiv:2006.02361, [3] arXiv:2007.04433

Paweł Duch (Adam Mickiewicz University in Poznan)
FLOW EQUATION APPROACH TO SINGULAR STOCHASTIC PDES

A novel technique of renormalization of singular stochastic PDEs based on the Wilsonian renormalization group theory and the Polchinski flow equation is presented. The technique is applicable to a large class of semi-linear parabolic and elliptic SPDEs with fractional Laplacian, additive noise and polynomial non-linearity. It covers equations arbitrarily close to criticality and completely avoids algebraic and combinatorial problems arising in different approaches. Based on [1] and [2].

[1] arXiv:2109.11380, [2] arXiv:2201.05031

Marina Ferreira (University of Helsinki)
ASYMPTOTIC LOCALIZATION IN MULTICOMPONENT MASS-CONSERVING COAGULATION EQUATIONS

Smoluchowski's coagulation equation is a nonlocal integrodifferential equation describing the size distribution of particles undergoing coagulation upon binary collision. We consider a generalization where each particle is characterized not by a scalar, but by a d-dimensional composition vector, describing for instance the volume of each chemical component of the particle. We show that for large times the mass of the system concentrates around a line in the d-dimensional composition space. The direction of the line is not encoded in the kernel, but in the initial condition. In this way, localization can be seen as an emergent property of multicomponent systems. Localization allows to reduce the study of multicomponent coagulation equations to the study of one-component equations. In particular, it allows to fully characterize the longtime behaviour of solutions for kernels that are not constant, but are constant along lines.

[1] arXiv:2203.08076

Eugenia Franco (University of Helsinki)
ON THE SELF-SIMILAR BEHAVIOUR OF COAGULATION SYSTEMS WITH INJECTION

Smoluchowski's coagulation equation describes the evolution in time of a system of atmospherical particles, coagulating upon binary collision. In this poster we consider a generalization of the Smoluchowski's coagulation equation: the coagulation equation with a constant influx of particles from the origin. We present a result on the existence of a self-similar solution for this equation and we present its regularity properties. We also present some formal arguments showing that this self-similar solution is expected to describe the longtime asymptotics of the Smoluchowski's coagulation equation with a time independent source of clusters concentrated in small sizes.

[1] arXiv:2106.12421

Vladislav Guskov (KTH Royal Institute of Technology)
LARGE DEVIATIONS PRINCIPLE AND SCHRAMM–LOEWNER EVOLUTION

Schramm–Loewner Evolution (SLE) is proved to satisfy small time Large Deviations Principle (LDP). Depending on how you see SLE curves, e.g. as subsets of the complex plane or as continuous curves, in other words depending on assumed topology, the proof of LDP requires different techniques and cannot be deduces from Schilder's theorem due to lack of continuity of the Loewner transformation. We give an overview of how LDP can be proved when SLE is seen as capacity parametrised curves and the topology is generated by the supremum norm, that is the topology of uniform convergence.

Joscha Henheik (IST Austria)

ADIABATIC THEOREM IN THE THERMODYNAMIC LIMIT WITH A GAP ONLY IN THE BULK

We present a generalized super-adiabatic theorem for extended fermionic systems assuming a spectral gap only in the bulk. More precisely, we assume that the infinite system has a unique ground state and that the corresponding GNS-Hamiltonian has a spectral gap above its eigenvalue zero. The proof follows the approach of constructing super-adiabatic non-equilibrium almost-stationary states (NEASSs) for the infinite system via adiabatic perturbation theory. If time permits, we will discuss the implications of this result for linear response theory at zero temperature. This is joint work with Stefan Teufel.

[1] arXiv:2002.08669, [2] arXiv:2012.15238, [3] arXiv:2012.15239

Liam Hughes (University of Cambridge)

METRIC GLUING OF QUANTUM WEDGES IN γ-LIOUVILLE QUANTUM GRAVITY FOR $\gamma \in (0,2)$

We consider the γ-Liouville quantum gravity (LQG) model for $\gamma \in (0,2)$, introduced by Polyakov as a canonical example of a random surface, and described rigorously by Duplantier and Sheffield as a random area measure on a planar domain D, together with a boundary length measure on ∂D. Both of these are constructed using the Gaussian free field (GFF), an analogue of Brownian motion parametrized by D rather than by (one-dimensional) time. When a certain type of such a quantum surface, called a quantum wedge, is decorated by an appropriate independent Schramm–Loewner evolution (SLE) curve, the wedge is cut into two surfaces which are themselves quantum wedges and are independent of each other. Sheffield showed that this process also works in reverse: given two independent wedges, when one glues their boundaries together according to LQG boundary length and conformally maps the resulting surface to a planar domain, one obtains an SLE-decorated quantum wedge.

A series of works by Miller and Sheffield showed how to construct γ-LQG as a random metric when $\gamma = \sqrt{8/3}$, in which case there is an equivalence between LQG and so-called Brownian surfaces, which are encoded in a natural way by Brownian motion. In the light of these results, Gwynne and Miller showed that the original wedge can also be recovered as a natural metric space quotient of the two resulting wedges.

Subsequent work by Gwynne and Miller constructed the γ-LQG metric for all $\gamma \in (0,2)$, and the present work gives the appropriate generalizations of the metric gluing results for $\gamma \in (0,2)$. Since the equivalence with Brownian surfaces does not hold in this more general setting, we instead use GFF techniques to establish estimates relating distances, areas and boundary lengths which may be of independent interest.

Hongchang Ji (IST Austria)
TRACY-WIDOM LIMIT FOR FREE SUM OF RANDOM MATRICES

We prove that the largest eigenvalue of the sum of two large, unitarily invariant, and Hermitian matrices has the same distribution as that of a Gaussian unitary ensemble. Unitarily invariant matrices are prime examples of asymptotically free random variables, the averaged eigenvalue distribution of their sum converges to so-called free additive convolution. In this poster, we first discuss regularity of generic free convolutions, with some illustrative examples. Then we explain how unitary invariance connects to analytic subordinations and ultimately results in universal behaviors in random matrices.

[1] arXiv:2110.05147

Shamoona Jabeen (UST Bannu, Pakistan)
RIDER HARRIS HAWKS OPTIMIZATION ALGORITHM FOR OPTIMIZED MODULAR COMMUNITY DETECTION IN BIPARTITE NETWORKS

Community structure discovery in complex networks is a common and significant phenomenon that has lately attracted a lot of attention. It not only shows insights about a multifarious network's hierarchical structure, but it also aids in a better understanding of the network's basic activities. The discovery of communities in bipartite networks is investigated in this research. Through node similarity, we introduced a novel technique Rider-Harris Hawks Optimization (RHHO) for community discovery in bipartite networks. RHHO is created by combining the Rider Optimization Algorithm (ROA) with the Harris Hawks Optimization (HHO) algorithms. Furthermore, a novel assessment measure, the h-Tversky Index (h-TI), is presented for assessing node similarity and fitness while taking modularity into account. The purpose of modularity is to measure the quality of a certain network division in order to assess the suggested community detection's accuracy. The Citation networks datasets (containing cit-HepPh and cit-HepTh) and Bipartite network datasets (including Movie lens 100K and American revolution dataset) are analyzed by means of efficiency and modularity. The proposed method demonstrated the maximum fitness of Adamic-Adar(AA) including h-index based link prediction, modified adaptive genetic algorithm (MAGA), genetic algorithms(GA), A Memetic Algorithm for Community Detection in Bipartite Networks(MACD-BN), and Harris Hawks Optimization for 250 iterations using a simulation (HHO).

Aleksandra Korzhenkova (École Polytechnique Fédérale de Lausanne)
THE EXPLORATION PROCESS OF CRITICAL BOLTZMANN PLANAR MAPS DECORATED BY A TRIANGULAR O(N) LOOP MODEL

In this work we investigate pointed (q, g, n)-Boltzmann loop-decorated maps with loops traversing only inner triangular faces. Using peeling exploration Budd (arXiv:1809.02012) modified to this setting we show that its law in the non-generic critical phase can be coded in terms of a random walk confined to the positive integers by a new specific boundary condition. Combining this observation with explicit quantities for the peeling

law we derive the large deviations property for the distribution of the so-called nesting statistic and show that the exploration process possesses exactly the same scaling limit as in the rigid loop model on bipartite maps that is a specific self-similar Markov process introduced in Budd (2018). Besides, we conclude the equivalence of the admissible weight sequences related by the so-called fixed point equation by proving the missing direction in the argument of Borot et al. (2012).

[1] arXiv:2112.11576

Ilya Losev (University of Cambridge)
HOW LONG ARE THE ARMS IN DBM?

We study Diffusion Limited Aggregation (DLA) and its one-parameter generalisation – Dielectric-Breakdown Model (DBM) on $\mathbb{Z}^2$ and $\mathbb{Z}^3$. These stochastic aggregation processes are the models of mineral deposition, electrodeposition, lightnings, bacterial growth, etc. The main problem about these models is finding the description and properties of their scaling limits. Although this problem has attracted a lot of attention from both mathematicians and physicists, very little is known rigorously. Essentially, the only rigorous result is due to Kesten, who demonstrated that DLA clusters do not grow too fast. In particular this means that the dimension of DLA clusters is bounded from below. We prove a growth estimate for DBM, analagous to the famous Kesten's estimate for DLA. We also provide a new proof of the latter. Our approach is based on multifractal analysis of the harmonic measure. This allows us to prove Kesten's theorem without using Beurling's estimate, which was crucial in the original proof.

Sid Maibach (University of Bonn)
FROM THE WEYL ANOMALY FORMULA TO
THE VIRASORO ALGEBRA

In Euclidean two-dimensional conformal field theory, the Weyl anomaly formula gives the factor by which correlation functions change under local rescaling of the metric. The "Liouville action" in this factor can be written as an anti-symmetric pairing on conformally equivalent metrics. Properties of this pairing allow the definition of a real determinant line – a real one-dimensional vector space associated to the surface, which captures the Weyl anomaly. These determinant lines form a vector bundles over moduli spaces of Riemann surfaces. Here I will focus on the moduli space of cylinders, which forms a semigroup under the gluing of cylinders along their boundary components. The determinant lines together with the gluing operation induce a central extension of the diffeomorphism group of the circle. I explicitly compute that the associated Lie algebra is the Virasoro algebra – the algebra of symmetries in conformal field theory. Related topics include modular functors, Virasoro uniformization and Malliavin–Kontsevich–Suhov loop measures.

Hamidreza Maleki Almani (University of Vaasa)

MULTI-MIXED FRACTIONAL BROWNIAN MOTIONS AND ORNSTEIN–UHLENBECK PROCESSES

We study the so-called multi-mixed fractional Brownian motions (mmfBm) and multi-mixed fractional Ornstein–Uhlenbeck (mmfOU) processes. These processes are constructed by mixing by superimposing (infinitely many) independent fractional Brownian motions (fBm) and fractional Ornstein–Uhlenbeck processes (fOU), respectively. We prove their existence as L2 processes and study their path properties, viz. long-range and short-range dependence, Hölder continuity, p-variation, and conditional full support.

Matthew Nicoletti (MIT)
STOCHASTIC SIX VERTEX MODEL AND A (2+1) DIMENSIONAL SURFACE GROWTH PROCESS

We introduce a family of irreversible growth processes which can be seen as Markov chains on discrete height functions defined on the 2-dimensional square lattice. Each height function corresponds to a configuration of the six vertex model on the infinite square lattice, and "irreversible" means that the height function has nonzero average drift. The dynamics arise naturally from the Yang–Baxter equation for the six vertex model, namely from a construction called "bijectivisation". These dynamics preserve the KPZ phase translation invariant Gibbs measures for the stochastic six vertex model, and we compute the current (the average drift) in each KPZ phase pure state with horizontal slope s. Using this, we analyze the hydrodynamic limit of a non-stationary version of the dynamics acting on quarter plane six vertex configurations.

[1] arXiv:2201.12497

Maximilian Nitzschner (Courant Institute, New York University)
SMOOTHNESS OF THE DIFFUSION COEFFICIENTS FOR PARTICLE SYSTEMS IN CONTINUOUS SPACE

We consider a class of particle systems with local interactions in continuous space, which are reversible with respect to the Poisson measures with constant density. A natural quantity of interest capturing the large-scale behavior of particles in this set-up is the bulk diffusion matrix. Recent work by Giunti, Gu and Mourrat has established that finite-volume approximations of this diffusion matrix converge at an algebraic rate. We show that the bulk diffusion matrix is an infinitely differentiable function of the density of particles, and obtain relatively explicit expressions for the derivatives in terms of the corrector, an object which already appeared in the description of the bulk diffusion matrix itself.

[1] arXiv:2112.06123

Izak Oltman (University of California, Berkeley)
A PROBABILISTIC WEYL-LAW FOR BEREZIN–TOEPLITZ OPERATORS

This work generalizes Martin Vogel's paper from 2020 which proves a probabilistic Weyl-law for quantized tori (arXiv:1912.08876). Here I randomly perturb quantizations of smooth functions on a Kähler manifold (Berezin–Toeplitz operators), and prove the spectrum obeys a probabilistic Weyl-law.

Roger Van Peski (MIT)
LOZENGE TILINGS AND THE GAUSSIAN FREE FIELD ON A CYLINDER

I will discuss new results on height fluctuations of random lozenge tilings on an infinite cylinder, which may be analyzed using the periodic Schur process introduced by Borodin, and which exhibit interesting behaviors not present for tilings of simply connected domains. This is joint work with Andrew Ahn and Marianna Russkikh.

[1] arXiv:2105.00551

Matthew Powell (University of California, Irvine)
POSITIVITY OF THE LYAPUNOV EXPONENT FOR QUASIPERIODIC OPERATORS WITH A FINITE-VALUED BACKGROUND

We describe recent and ongoing work regarding Lyapunov exponents for analytic quasiperiodic operators with finite-valued backgrounds. We prove that, for sufficiently large coupling constant, the Lyapunov exponent is positive with a uniform lower bound. Our method is an adaptation of a proof that goes back to Bourgain.

Dr. Naeem Ahmad Pundeer (Jadavpur University)
THE SEMICONFORMAL CURVATURE TENSOR ON RELATIVISTIC SPACETIMES

The semiconformal curvature tensor has been studied for the spacetime of general relativity. It is shown that the energy-momentum tensor with divergence-free semiconformal curvature tensor is of Codazzi type, as well as the energy-momentum tensor of a spacetime having semi-symmetric semiconformal curvature tensor is also semi-symmetric. The semiconformal curvature tensor has also been expressed in terms of different tensors already known in the literature, and the relationship between their divergences has been established.

Andrew Rout (University of Warwick)
A MICROSCOPIC DERIVATION OF GIBBS MEASURES FOR THE 1D FOCUSING CUBIC NONLINEAR SCHRODINGER EQUATION

In this work, we give a microscopic derivation of the Gibbs measure and the associated time-dependent correlation functions for the 1D cubic focusing nonlinear Schrodinger equation (NLS) from many-body quantum theory. These objects were first studied in the constructive quantum field theory literature in the 1970s and later in the nonlinear dispersive PDE literature by Lebowitz, Rose, and Speer and by Bourgain. Previous derivation results for the corresponding defocusing problems were obtained by Lewin, Nam, and Rougerie and by Frohlich, Knowles, Schlein, and the second author. This is the first result that treats the focusing regime, i.e. the regime in which the Hamiltonian is not necessarily positive. Due to the focusing nature of the problem, we need to add a suitable truncation in the rescaled number of particles in the quantum problem and on the mass in the classical problem. Our method is based on the perturbative approach developed in the earlier work of Frohlich, Knowles, Schlein, and the second author. This method has to be adapted to take into account the aforementioned truncation. Our results hold both for the local NLS and for the nonlocal NLS with arbitrary L^p convolution potentials.

Kieran Ryan (University of Vienna)
HEISENBERG MODELS AND SCHUR–WEYL DUALITY

While the classical Heisenberg model was proved to undergo a phase transition in three dimensions some 46 years ago by Fröhlich, Simon and Spencer, the problem of proving the same for the quantum model remains open. Representation theory has in the past 10 years arisen as a tool to approach this and related problems. I will present joint work with Jakob Björnberg and Hjalmar Rosengren in which we give detailed analysis of certain generalisations of the quantum Heisenberg model, on complete and complete bipartite graphs. Our results are exact and comprehensive: we can, for example, give an exact formula for the free energy and certain thermodynamic limits. Our main tool is the representation theoretic framework of Schur–Weyl duality, which connects the classical groups with the symmetric group and Brauer algebra. I will give an introduction to the models we study, Schur–Weyl duality, and our results, and conclude with an outlook towards the model in three dimensions.

[1] arXiv:2201.10209

Miyanawathura Ihala Gamage Suranga Sampath (Wayamba University of Sri Lanka)
STEADY-STATE ANALYSIS OF A SINGLE SERVER QUEUEING SYSTEM SUBJECT TO DIFFERENTIATED VACATIONS WITH PARTIAL VACATION INTERRUPTIONS AND WAITING SERVER ..

Queues or waiting lines are the most common situations in health care management systems such as operation theaters, medical laboratories, etc. In this research, one type of queue that occurs in operation theaters is considered and the model is built up by using the queueing theory. This model is named as the single server queueing system subject to differentiated vacations with partial vacation interruptions and a waiting server. The operation theater is considered a single server queueing system. It is assumed that customer arrivals follow a Poisson process and service time is exponentially distributed. When there are no customers, the system allows the server to go for a vacation (vacation type - I) and returns after completing the vacation have a random time duration. Still, if there are no customers, the server takes another vacation of a shorter duration (vacation type - II). Both types of vacations are exponentially distributed. For example, vacation type - I is setting up the operation theater for the next patients or cleaning and sterilizing the operation theater, and vacation type – II is taking some actual rest by the operation theater personnel. When the server is in the vacation type –II and the system size reaches a predefined threshold value, it is interrupted. For example, an operation theater personnel's break may be interrupted due to some emergency situations in the hospital. Practically, servers wait for some random time duration for the next customers before taking vacations. Therefore, the model is developed by adding the waiting server concept. It is assumed that the waiting time duration is exponentially distributed. For the considered model, the explicit expression for the system size probabilities is obtained at the steady-state. Finally, the time-independent expressions for some important performance measures are given.

Anne Schreuder (University of Cambridge)
LÉVY-DRIVEN LOEWNER EVOLUTIONS

This talk is about the behaviour of Loewner evolutions driven by a Lévy process. Schramm's celebrated version (Schramm–Loewner evolution), driven by standard Brownian motion, has been a great success for describing critical interfaces in statistical physics. Loewner evolutions with other random drivers have been proposed, for instance, as candidates for finding extremal multifractal spectra, and some tree-like growth processes in statistical physics. Questions on how the Loewner trace behaves, e.g., whether it is generated by a (discontinuous) curve, whether it is locally connected, tree-like, or forest-like, have been partially answered in the symmetric alpha-stable case. We consider the case of general Lévy drivers. Joint work with Eveliina Peltola (Bonn and Helsinki).

Jinwoo Sung (University of Chicago)
MINKOWSKI CONTENT OF LIOUVILLE QUANTUM GRAVITY METRIC SPACES

A Liouville quantum gravity (LQG) surface is a "canonical" random two-dimensional surface, initially formulated as a random measure space and later as a random metric space. We show that the LQG volume measure is almost surely a deterministic multiple of the Minkowski content measure for the LQG metric, answering a question of Gwynne and Miller. Our primary tool is the continuum mating-of-trees theory for space-filling SLE.

[1] math.uchicago.edu/~jsung/talks/LQG_Minkowski_Content_Poster.pdf

Gerardo Barrera Vargas (University of Helsinki)
A QUANTITATIVE ESTIMATION OF WASSERSTEIN DISTANCE BETWEEN BROWNIAN MOTION AND THE GOLDSTEIN–KAC TELEGRAPH PROCESS

In this poster, we present a non-asymptotic process level control between the telegraph process and a Brownian motion with suitable diffusivity constant via a Wasserstein distance with quadratic ergodic cost. In addition, non-asymptotic estimates for the corresponding time average p-th moments are explicitly given. The proof relies on coupling techniques such as coin-flip coupling, synchronous coupling and the celebrated Komlós–Major–Tusnády coupling.

[1] arXiv:2201.00422

Cornelia Vogel (University of Tübingen)
TYPICAL MACROSCOPIC LONG-TIME BEHAVIOR FOR RANDOM HAMILTONIANS

We study the long-time behavior of an isolated macroscopic quantum system restricted to an (finite-dimenisonal) energy shell which is described by a random Hamiltonian. Following von Neumann (1929), we take for granted a decomposition of the Hilbert space $\mathcal{H}$ into mutually orthogonal subspaces $\mathcal{H}_\nu$ representing the different macro states and ask, under which conditions it is true that for all μ and ν and most initial wave functions $\psi \in \mathcal{H}_\mu$ the superposition weights $\|P_\nu \psi_t\|^2$ are close to some (time-independent and ψ-independent) values $M_{\mu\nu}$ in the long run ("generalized normal typicality"). One example of a sufficient condition is the eigenstate thermalization hypothesis (ETH), in the version formulated and proved by Cipolloni, Erdös and Schröder (2021) for the Wigner ensemble (i.e., i.i.d. entries). However, we are interested in slightly more realistic Hamiltonians, more precisely, random matrices with a band structure (i.e., entries far away from the main diagonal are small). For such matrices the ETH is not expected to hold. It can be shown without exploiting the randomness of the Hamiltonian that under suitable assumptions only on the dimensions of the subspaces the absolute errors are small (Teufel, Tumulka, V., in preparation). In order to obtain lower bounds for

the $M_{\mu\nu}$, and therefore upper bounds for the relative errors, delocalization properties of the eigenvectors of random matrices are needed. While most delocalization results only yield upper bounds, the no-gaps delocalization proved by Rudelson and Vershynin (2016), which is also available for some classes of random band matrices, enables us to prove the desired lower bounds. (Joint work with Stefan Teufel and Roderich Tumulka)

Baojun Wu (Aix-Marseille University)
CONFORMAL BOOTSTRAP ON THE ANNULUS IN LIOUVILLE CFT

This paper is the first of a series of work on boundary bootstrap in Liouville conformal field theory (CFT), which focuses on the case of the annulus with two boundary insertions lying on different boundaries. In the course of proving the bootstrap formula, we established several properties on the corresponding annulus conformal blocks: 1) we show that they converge everywhere on the spectral line and they are continuous with respect to the spectrum and the primary weights; 2) we relate them to their torus counterparts by rigorously implementing Cardy's doubling trick for boundary CFT; 3) we extend the definition of these conformal blocks to the whole Seiberg bound; 4) when there are two insertion and their weights are γ, we show that the block degenerates and the bootstrap formula settles a conjecture of Martinec; 5) we also extend the bootstrap formula to the one point case. As an application of our bootstrap result, we give an exact formula for the bosonic LQG partition function of the annulus when $\gamma \in (0, 2)$. Our paper serves as a key ingredient in the recent derivation of the random moduli for the Brownian annulus by Ang, Remy and Sun (2022). We also solve several conjectures relate to torus conformal blocks which arise from physics literature.

Weile Weng (Technische Universität Berlin)
QUENCHED FUNCTIONAL CLT FOR RANDOM WALKS IN RANDOM ENVIRONMENTS WITH BOUNDED CYCLE REPRESENTATION

We consider a special type of model for non-reversible random walks in random environments on $\mathbb{Z}^d$ ($d \geq 2$): the probability space for the random environments is invariant and ergodic with respect to spacial shifts; and almost every environment can be represented as a collection of oriented nearest-neighbor cycles with uniformly bounded length, and with weights that only depend on each cycle. In particular, such random environment is doubly stochastic. The model is a generalization of random conductance model, and it is motivated by the quest to investigate quenched functional CLT in non-reversible RWRE without imposing uniform ellipticity condition. In this research, we prove quenched functional CLT only with moment conditions for the speed and inverse speed. This is joint work with Jean-Dominique Deuschel (TU Berlin) and Martin Slowik (University of Mannheim).

[1] www.youtube.com/watch?v=5MgErbeNqTg

Yizheng Yuan (Technische Universität Berlin)

PRECISE REGULARITY OF SLE

Many random processes have fractal behaviour; this can be quantified e.g. by laws of iterated logarithm, modulus of continuity, or variation regularity. For Brownian motion, precise rules are known for all these. I will discuss analogous results for SLE (Schramm-Löwner evolution), a random curve appearing in random conformal geometry and statistical physics. Sharp gauge functions (up to multiplicative constants) can be obtained for certain variants of SLE (namely, two-sided whole-plane SLE and space-filling SLE). In the proof we also obtain sharp estimates on the lower tail of the Minkowski content. This talk is based on ongoing work with Nina Holden.

Yuanyuan Xi (IST Austria)
QUANTITATIVE TRACY–WIDOM LAWS FOR THE LARGEST EIGENVALUE OF GENERALIZED WIGNER MATRICES

We show that the fluctuations of the largest eigenvalue of a generalized Wigner matrix of size N converge to the Tracy–Widom laws at a rate nearly $O(N^{-1/3})$, as N tends to infinity. We allow the variances of matrix entries to be different but comparable. Our result improves the previous rate $O(N^{-2/9})$ obtained by P. Bourgade and the proof relies on a long-time Green function comparison theorem near the edges without any moment matching condition. This is joint work with K. Schnelli.