Quantum jumps and rate operators in open quantum system dynamics

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Non-Markovian Processes and Complex Systems Group

Contents

1. Markovian: Monte Carlo Wave Function

   Dalibard, Castin, Molmer
   PRL 1992

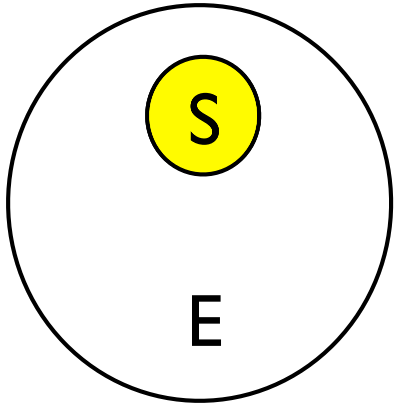
2. Non-Markovian Quantum Jumps

   Piilo, Maniscalco, Härkönen, Suominen:
   PRL 2008

3. Unified framework:
   ROQJ - Rate operator quantum jumps

   Smirne, Caiaffa, Piilo
   PRL 2020
   See also
   Chruscinski, Luoma, Piilo, Smirne
   arXiv:2009.11312

Preliminaries: problem setting



- Any realistic quantum system S is coupled to its environment E
- Master equation description:

$$\frac{d\rho(t)}{dt} = -i[H, \rho_S] + \sum_k \gamma_k(t) \left(A_k \rho_S(t) A_k^\dagger - \frac{1}{2} A_k^\dagger A_k \rho_S(t) - \frac{1}{2} \rho_S A_k^\dagger A_k \right)$$

- Decomposition of the density matrix

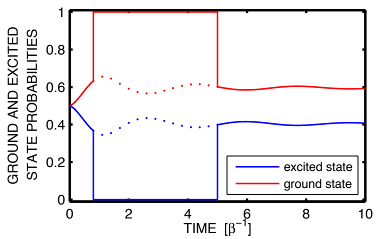
$$\rho(t) = \sum_i P_i(t) |\psi_i(t)\rangle\langle\psi_i(t)|$$

→ Stochastic descriptions

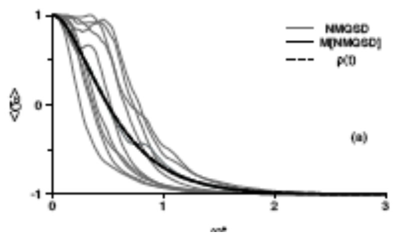
Simple classification of Monte Carlo/stochastic methods

	Markovian	non-Markovian
Jump methods:	MCWF (Dalibard, Castin, Molmer) Quantum Trajectories (Zoller, Carmichael)	Fictitious modes (Imamoglu) Pseudo modes (Garraway) Doubled H-space (Breuer, Petruccione) Triple H-space (Breuer) Non-Markovian Quantum Jump (Piilo et al)
Diffusion methods:	QSD (Diosi, Gisin, Percival...)	Non-Markovian QSD (Strunz, Diosi, Gisin, Yu) Stochastic Schrödinger equations (Barchielli)

Jump



Diffusion



Plus: Wiseman, Gambetta, Budini, Gaspard, Lacroix, Donvil and Muratore-Ginanneschi (not comprehensive list, apologies for any omissions)

Simple classification of Monte Carlo/stochastic methods

This talk

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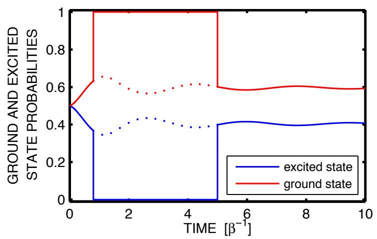
Unified framework missing

Jump



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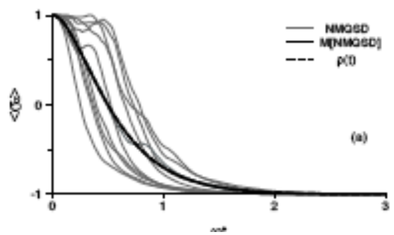

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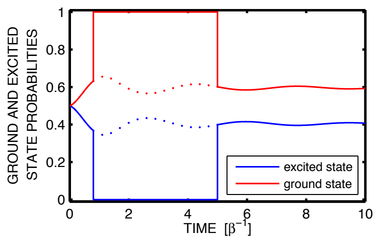
Unified framework missing

Connection missing

Jump



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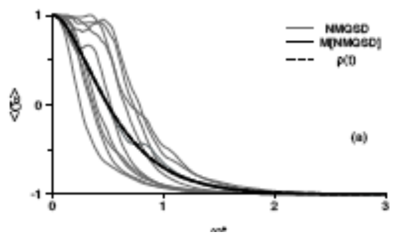


Plus: Wiseman, Gambetta, Budini, Gaspard, Lacroix, Donvil and Muratore-Ginanneschi (not comprehensive list, apologies for any omissions)

For a solution:
see Luoma, Struntz, Piilo

Markovian
Monte Carlo Wave Function method

Suppose now we want to solve the semigroup, Markovian GKSL equation

$$\frac{d\rho(t)}{dt} = -i[H, \rho_S] + \sum_k \gamma_k \left(A_k \rho_S A_k^\dagger - \frac{1}{2} A_k^\dagger A_k \rho_S - \frac{1}{2} \rho_S A_k^\dagger A_k \right)$$

Q: How to solve the master equation?

- Few exact models and analytical solutions
- Can we find the solution by evolving an ensemble of state vectors instead of directly solving the density matrix?

Generally, we can decompose the density matrix as

$$\rho(t) = \sum_i P_i(t) |\psi_i(t)\rangle\langle\psi_i(t)|$$

Basics of stochastic state vector evolution

Monte Carlo wave function method (Markovian)

(Dalibard, Castin, Molmer, PRL 1992)

Ensemble of N state vectors

$$\begin{array}{lllll} \psi_1(t_0) \longrightarrow & \psi_1(t_1) & \ldots\ldots & \psi_1(t_n) \\ \psi_2(t_0) \longrightarrow & \psi_2(t_1) & \ldots\ldots & \psi_2(t_n) \\ \vdots & & & \\ \psi_N(t_0) \longrightarrow & \psi_N(t_1) & \ldots\ldots & \psi_N(t_n) \end{array}$$

Time $\longrightarrow$

At each point of time, density matrix ρ as average of state vectors Ψ_i:

$$\rho(t) = \frac{1}{N} \sum_{i=1}^{N} |\psi_i(t)\rangle\langle\psi_i(t)|$$

The time-evolution of each Ψ_i contains stochastic element due to random quantum jumps.

Jump probability, example

Time-evolution of state vector Ψ_i:

At each point of time: decide if quantum jump happened.

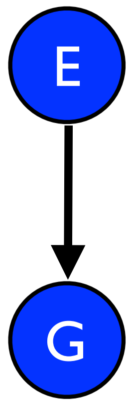
P_j: probability that a quantum jump occurs in a given time interval δt:

$$P_j = \delta t \; \Gamma \; p_e$$

time-step (δt), decay rate (Γ), occupation probability of excited state (p_e)

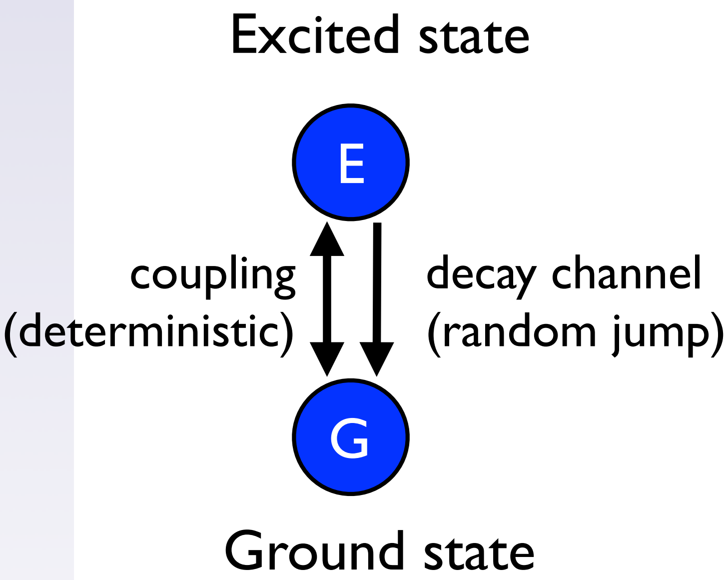
For example: 2-level atom

Probability for atom being transferred from the excited to the ground state and photon emitted.



Quantum jump: Discontinuous stochastic change of the state vector.

Excited state probability P for a driven 2-level atom



$$\frac{d\varrho}{dt} = -i[H,\varrho] + \Gamma\left[\sigma_-\varrho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\varrho\}\right]$$

$$\rho(t) = \frac{1}{N}\sum^{N} |\psi_i(t)\rangle\langle\psi_i(t)|$$

Example: driven 2-state system, Markovian

Quantum jump: Discontinuous stochastic change of the state vector.

Excited state probability P
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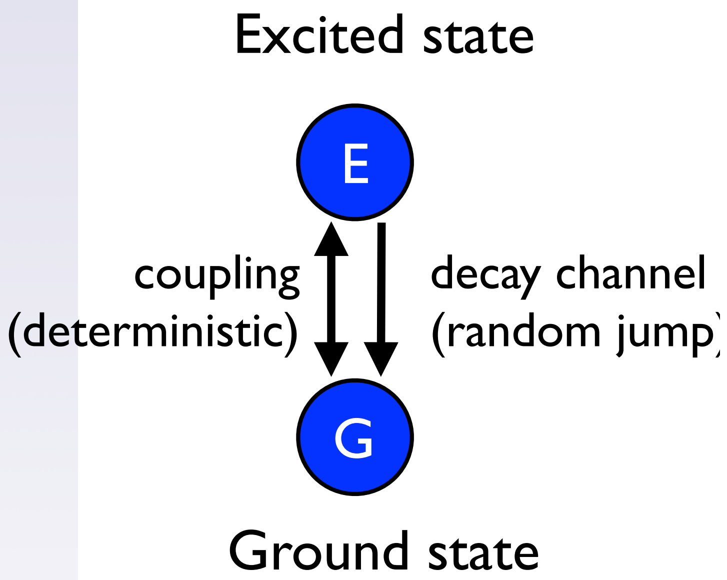
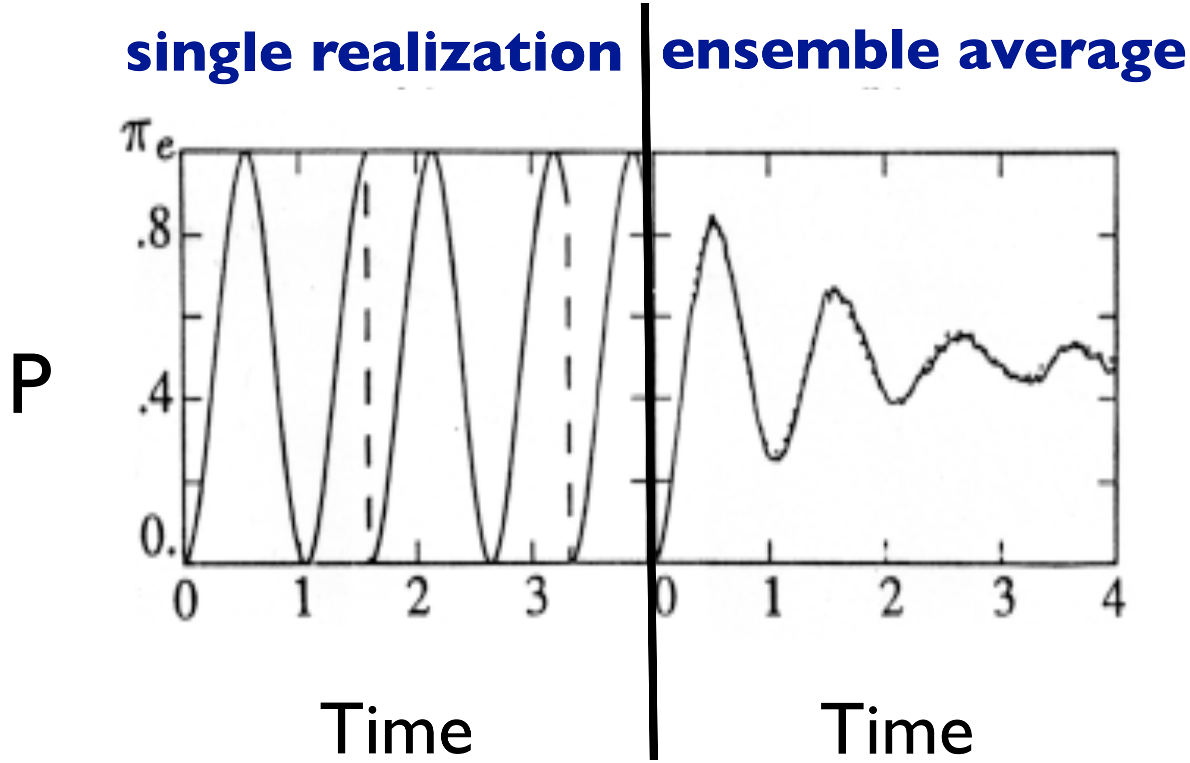


Markovian Monte Carlo



damped Rabi oscillation of the atom

$$\frac{d\varrho}{dt} = -i[H, \varrho] + \Gamma \left[\sigma_- \varrho \sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \varrho\} \right]$$

$$\rho(t) = \frac{1}{N} \sum^{N} |\psi_i(t)\rangle\langle\psi_i(t)|$$

Markovian Monte Carlo wave function method

Master equation to be solved:

$$\frac{d\rho(t)}{dt} = \frac{1}{i\hbar}[H_S,\rho] + \sum_m \Gamma_m C_m \rho C_m^\dagger - \frac{1}{2}\sum_m \Gamma_m \left(C_m^\dagger C_m \rho + \rho C_m^\dagger C_m\right)$$

For each ensemble member ψ:

$$i\hbar \frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle$$

Solve the time dependent Schrödinger equation.

$$H = H_s + H_{dec}$$

Use non-Hermitian Hamiltonian H which includes the decay part H_{dec}.

$$H_{dec} = -\frac{i\hbar}{2}\sum_m \Gamma_m C_m^\dagger C_m$$

Key for non-Hermitian Hamiltonian: Jump operators C_m can be found from the dissipative part of the master equation.

$$\delta p_m = \delta t \Gamma_m \langle \Psi | C_m^\dagger C_m | \Psi \rangle$$

For each channel m the jump probability is given by the time step size, decay rate, and decaying state occupation probability.

Dalibard, Castin, Molmer: PRL 1992

Algorithm:

1. Time evolution over time step δt

2. Generate random number, did jump occur?

No

Yes

3. Renormalize ψ before new time step

$$|\psi_i(t+\delta t)\rangle = \frac{e^{-iH_{\text{eff}}\delta t}|\psi_i(t)\rangle}{\sqrt{1-\delta p}}$$

3. Apply jump operator C_j before new time step

$$|\psi_i(t+\delta t)\rangle = \frac{C_j|\psi_i(t)\rangle}{||C_j|\psi(t)\rangle||}$$

4. Ensemble average over ψ :s gives the density matrix and the expectation value of any operator A

$$\langle A\rangle(t) = \frac{1}{N}\sum_i \langle\psi_i(t)|A|\psi_i(t)\rangle$$

Measurement scheme interpretation

Two-level atom in vacuum

Two-level atom MC evolution by

$$C = \sqrt{\Gamma}|g\rangle\langle e|$$ Jump operator

$$H_{dec} = -\frac{i\hbar\Gamma}{2}|e\rangle\langle e|$$ Non-Hermitian Hamiltonian

$$P = \delta t \Gamma |c_e|^2$$ Jump probability

Total system evolution

Measurement scheme:
continuous measurement of photons in the environment.

$$(c_g|g\rangle + c_e|e\rangle) \otimes |0\rangle \rightarrow$$
$$(c'_g|g\rangle + c'_e|e\rangle) \otimes |0\rangle + \sum_\lambda c_\lambda |g\rangle \otimes |1_\lambda\rangle$$

- Continuous measurement of the environmental state gives conditional pure state realizations for the open system
- The open system evolution is average of these realizations

Equivalence with the master equation:

The state of the ensemble averaged over time step:
(for simplicity here: initial pure state and one decay channel only)

This gives comm. + anticomm. of m.e.

This gives "sandwich" term of the m.e.

$$\overline{\sigma(t+\delta t)} = (1-P)\frac{|\phi(t+\delta t)\rangle\langle\phi(t+\delta t)|}{1-P} + P\frac{C|\Psi(t)\rangle\langle\Psi(t)|C^{\dagger}}{\langle\Psi(t)|C^{\dagger}C|\Psi(t)\rangle}$$

Average

"No-jump" path weight

t-evol. and normalization

"Jump" path weight

Jump and normalization

Keeping in mind two things:

a) the time-evolved state is (1st order in dt, before renormalization):

$$|\phi(t+\delta t)\rangle = \left(1 - \frac{iH_s\delta t}{\hbar} - \frac{\Gamma\delta t}{2}C^{\dagger}C\right)|\Psi(t)\rangle$$

b) the jump probability is:

$$P = \delta t\Gamma\langle\Psi|C^{\dagger}C|\Psi\rangle$$

it is relatively easy to see that the ensemble average corresponds to master equation

$$\frac{d\rho(t)}{dt} = -i[H,\rho_S] + \sum_k \gamma_k \left(A_k\rho_S A_k^{\dagger} - \frac{1}{2}A_k^{\dagger}A_k\rho_S - \frac{1}{2}\rho_S A_k^{\dagger}A_k\right)$$

Questions:

- What happens when the decay rates depend on time?

- What happens when the decay rates turn temporarily negative?

$$\frac{d\rho(t)}{dt} = -i[H, \rho_S] + \sum_k \gamma_k(t) \left(A_k \rho_S(t) A_k^\dagger - \frac{1}{2} A_k^\dagger A_k \rho_S(t) - \frac{1}{2} \rho_S A_k^\dagger A_k \right)$$

2.
Non-Markovian Quantum Jumps

Piilo, Maniscalco, Härkönen, Suominen:
PRL 2008

Markovian vs. non-Markovian evolution (1)

Markovian dynamics:
Decay rate constant in time.

Non-Markovian dynamics:
Decay rate depends on time, obtains temporarily negative values.

Markovian vs. non-Markovian evolution (1)

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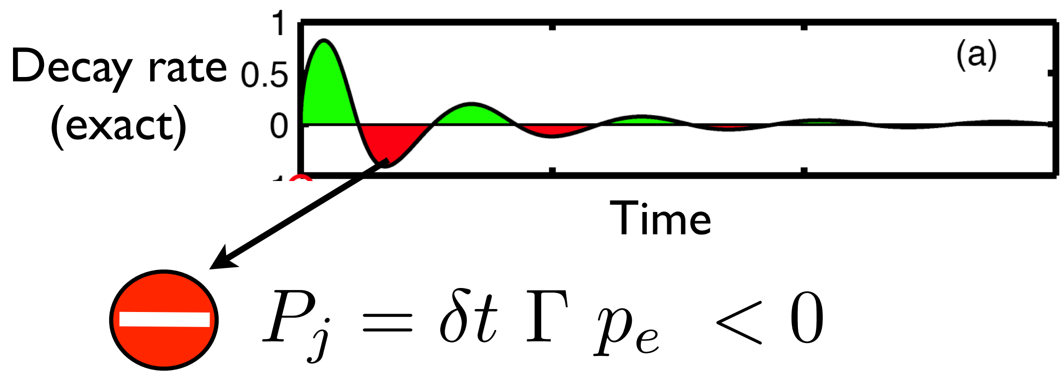
Example: 2-level atom in photonic band gap.



$$P_j = \delta t \; \Gamma \; p_e \; < 0$$

Markovian description of quantum jumps fails, since gives negative jump probability.
For example: negative probability that atom emits a photon.

Non-Markovian master equation

Starting point:

General non-Markovian master equation local-in-time:

$$\frac{d\rho(t)}{dt} = \frac{1}{i\hbar}\left[H_S,\rho\right] + \sum_m \Delta_m(t) C_m \rho C_m^{\dagger} - \frac{1}{2}\sum_m \Delta_m(t)\left(C_m^{\dagger} C_m \rho + \rho C_m^{\dagger} C_m\right)$$

- Jump operators C_m
- Time dependent decay rates $\Delta_m(t)$.
- Decay rates have temporarily negative values.

Non-Markovian master equation

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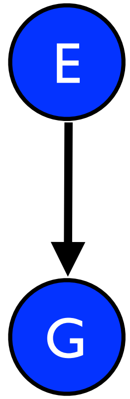
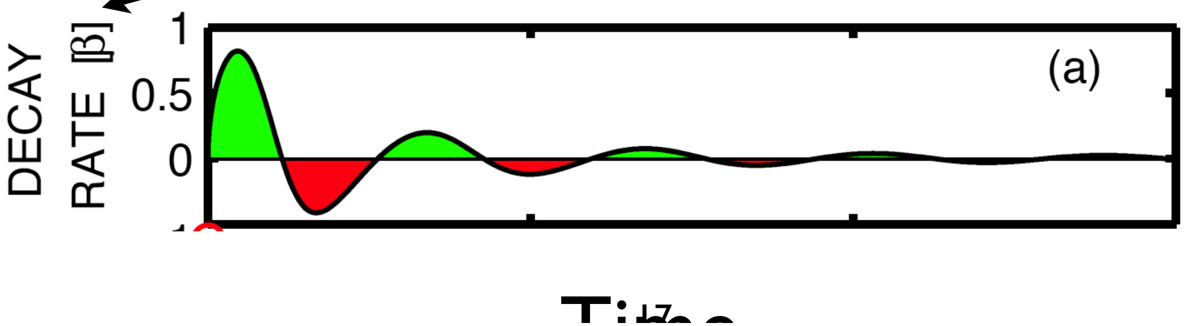


Example: 2-level atom in photonic band gap.

Jump operator C for positive decay: $\sigma_- = |g\rangle\langle e|$

$$\frac{d\rho(t)}{dt} = \frac{1}{i\hbar}[H_S,\rho] + \Gamma(t)|g\rangle\langle e|\rho|e\rangle\langle g| - \frac{1}{2}\Gamma(t)(|e\rangle\langle e|\rho + \rho|e\rangle\langle e|)$$



Non-Markovian quantum jump (NMQJ) method

Quantum jump in negative decay region:
The direction of the jump process reversed

$$|\psi\rangle \rightarrow |\psi'\rangle = \frac{C_m|\psi\rangle}{||C_m|\psi\rangle||}, \quad \Delta_m(t) > 0$$

$$|\psi\rangle \leftarrow |\psi'\rangle = \frac{C_m|\psi\rangle}{||C_m|\psi\rangle||}, \quad \Delta_m(t) < 0$$

Negative rate process creates coherences

Jump probability:

$$P = \frac{N}{N'}\delta t|\Delta_m(t)|\langle\psi|C_m^\dagger C_m|\psi(t)\rangle$$

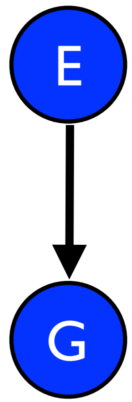
N: number of ensemble members in the target state
N': number of ensemble members in the source state

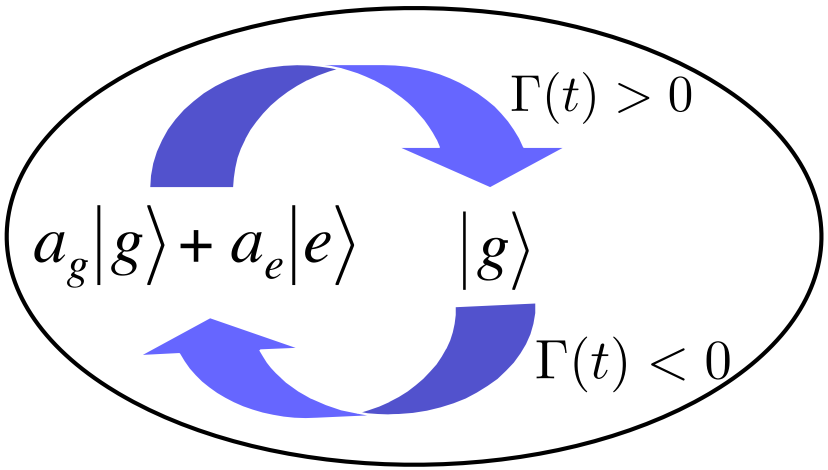
The probability proportional to the target state!

NMQJ example

For example: two-level atom

$$\sigma_- = |g\rangle\langle e|$$





Jump probability: $P = \frac{N_0}{N_g}\delta t|\Gamma(t)|\ |\langle\psi_0|e\rangle|^2$

The essential ingredient of non-Markovian system: memory. Recreation of lost superpositions.

In terms of probability flow in Hilbert space:

Positive rate

$$\rho(t) = \frac{N_0(t)}{N}\left|\Psi_0(t)\right\rangle\left\langle\Psi_0(t)\right| + \sum_i \frac{N_i(t)}{N}\left|\Psi_i(t)\right\rangle\left\langle\Psi_i(t)\right| + \sum_{i,j} \frac{N_{i,j}(t)}{N}\left|\Psi_{i,j}(t)\right\rangle\left\langle\Psi_{i,j}(t)\right| + \ldots$$

No jumps

1 random jump (channel i)

2 random jumps (channels i, j)

Non-Markovian quantum jumps

In terms of probability flow in Hilbert space:

Positive rate

$$\rho(t) = \frac{N_0(t)}{N}\left|\Psi_0(t)\right\rangle\left\langle\Psi_0(t)\right| + \sum_i \frac{N_i(t)}{N}\left|\Psi_i(t)\right\rangle\left\langle\Psi_i(t)\right| + \sum_{i,j}\frac{N_{i,j}(t)}{N}\left|\Psi_{i,j}(t)\right\rangle\left\langle\Psi_{i,j}(t)\right| + \ldots$$

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Negative rate

No jumps

1 random jump (channel i)

2 random jumps (channels i, j)

Memory in the ensemble: no jump realization carries memory of the 1 jump realization; 1 jump realization carries the memory of 2 jumps realization...

Negative rate: earlier occurred random events get undone.

NMQJ: general algorithm

$$\begin{aligned}\frac{d}{dt}\rho &= -i[H(t),\rho] \\ &+ \sum_k \Delta_k^+(t)\left[C_k(t)\rho C_k^\dagger(t) - \frac{1}{2}\left\{C_k^\dagger(t)C_k(t),\rho\right\}\right] \\ &- \sum_l \Delta_l^-(t)\left[C_l(t)\rho C_l^\dagger(t) - \frac{1}{2}\left\{C_l^\dagger(t)C_l(t),\rho\right\}\right]\end{aligned}$$

$$\rho(t) = \sum_\alpha \frac{N_\alpha(t)}{N}|\psi_\alpha(t)\rangle\langle\psi_\alpha(t)|$$ ensemble

Deterministic evolution and positive channel jumps as before...

Negative channel with jumps

$$D^{j-}_{\alpha\to\alpha'}(t) = |\psi_{\alpha'}(t)\rangle\langle\psi_\alpha(t)|$$

where the source state of the jump is

$$|\psi_\alpha(t)\rangle = C_{j_-}(t)|\psi_{\alpha'}(t)\rangle / ||C_{j_-}(t)|\psi_{\alpha'}(t)\rangle||$$

...and jump probability for the corresponding channel

$$P^{j-}_{\alpha\to\alpha'}(t) = \frac{N_{\alpha'}(t)}{N_\alpha(t)}|\Delta_{j_-}(t)|\delta t\langle\psi_{\alpha'}(t)|C^\dagger_{j_-}(t)C_{j_-}(t)|\psi_{\alpha'}(t)\rangle.$$

Basic steps of the proof

Basic idea:
Weighting jump path with jump probability and deterministic path with no-jump probability gives the master equation (as in MCWF)

The ensemble averaged state over dt is

$$\overline{\sigma(t+\delta t)} =$$

$$\frac{N_0(t)}{N}\frac{|\Phi_0(t+\delta t)\rangle\langle\Phi_0(t+\delta t)|}{1+n_0}$$

0 jumps earlier, no jumps to be cancelled

$$+\sum_i \frac{N_i(t)}{N}(1-P_{i\to 0})\frac{|\Phi_i(t+\delta t)\rangle\langle\Phi_i(t+\delta t)|}{1+n_i}$$

1 jump earlier,
does not cancel jump at this time

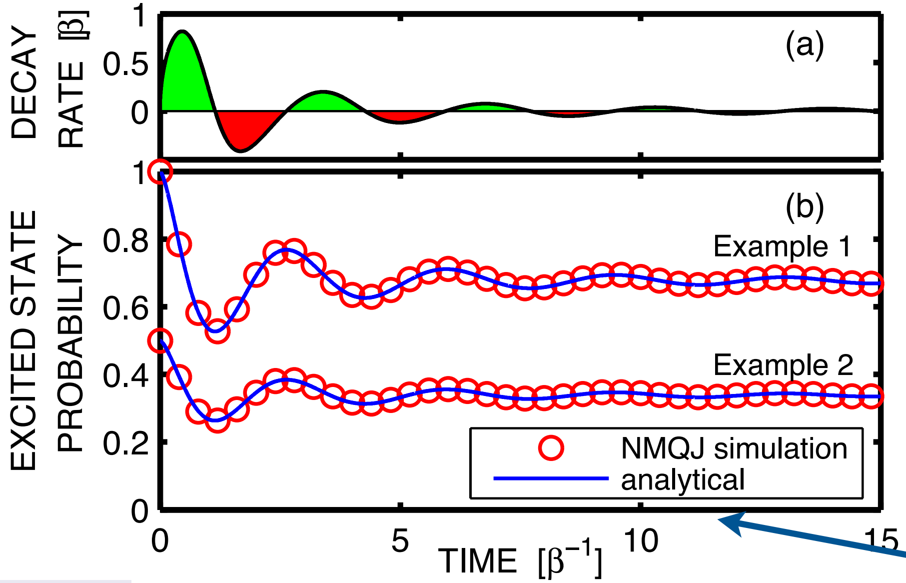
$$+\sum_i \frac{N_i(t)}{N}P_{i\to 0}\frac{D_{i\to 0}|\Phi_i(t+\delta t)\rangle\langle\Phi_i(t+\delta t)|D_{i\to 0}^{\dagger}}{n_{i\to 0}}+\dots$$

1 jump earlier, cancels jump

Here, other quantities are similar as in original MCWF except:
P's: jump probabilities
D's: jump operators

By plugging in the appropriate quantities gives the match with the master equation !

Example: 2-level atom in photonic band gap



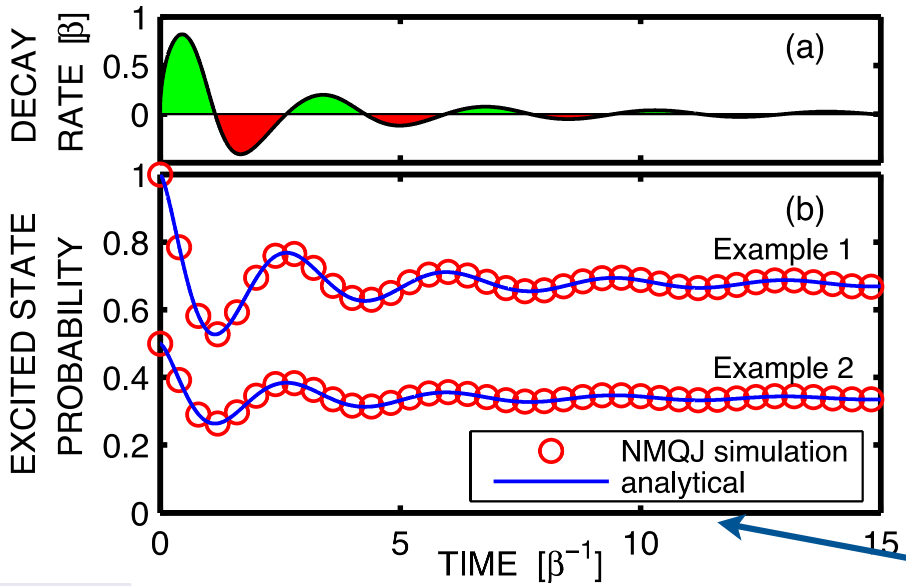
The simulation and exact results match.

Typical features of photonic band gap:

- Population trapping
- Atom-photon bound state.

Density matrix: average over the ensemble

Example: 2-level atom in photonic band gap



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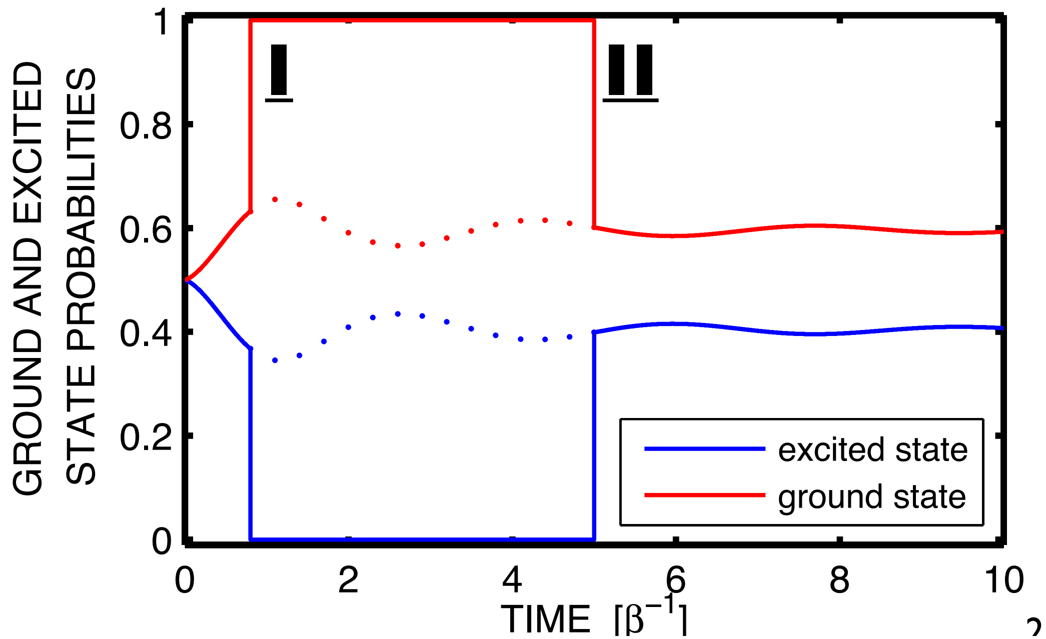
Density matrix: average over the ensemble

Single state vector history



Example of one state vector history:

I: Quantum jump at positive decay region destroys the superposition.

II: Due to memory, non-Markovian jump recreates the superposition.

Piilo, Maniscalco, Härkönen, Suominen: PRL 2008

Some applications of NMQJ…

- Rebentrost, Chakraborty, Aspuru-Guzik:
  "Non-Markovian quantum jump in excitonic energy transfer"
  The Journal of Chemical Physics 2009

- Ai, Fan, Jin, Cheng:
  "An efficient quantum jump method for coherent energy transfer dynamics in photosynthetic systems under the influence of laser fields"
  (includes comparison to HOM)
  New Journal of Physics 2014

- Renaud, Grozema:
  "Intermolecular Vibrational Modes Speed Up Singlet Fission in Perylenediimide Crystals"
  The Journal of Physical Chemistry Letters 2015

3.
Unifying framework:
Rate operator quantum jumps
(ROQJ)

Smirne, Caiaffa, Piilo
PRL 2020

Earlier work with QSD:
Caiaffa, Smirne, Bassi:
PRA 2017

What is the problem? Example

"Eternal" non-Markovian master equation

$$\dot{\rho} = \frac{1}{2}\sum_{k=1}^{3}\gamma_k(t)[\sigma_k\rho\sigma_k - \rho],$$

Hall et al PRA 2014

- Pauli matrices σ_k
- Decoherence rates

$$\gamma_1(t) = \gamma_2(t) = 1, \quad \gamma_3(t) = -\tanh t < 0$$ for all $t > 0$

- Map CP but breaks CP-divisibility for all $t > 0$
- "Eternal" non-Markovian according to RHP criteria
- **...however, P-divisible for all** $t > 0$

“ENM” master equation

Why Markovian MCWF does not work?

- Rate for σ_z jump $\gamma_3(t) = -\tanh t$ <0 for all t>0

→ gives negative jump probability $P_j = \delta t \; \Gamma \; p_e \; < 0$

Why non-Markovian NMQJ does not work?

- Reverse jump probility $P^{j-}_{\alpha \to \alpha'}(t) = \frac{N_{\alpha'}(t)}{N_\alpha(t)} |\Delta_{j_-}(t)| \delta t \langle \psi_{\alpha'}(t) | C^\dagger_{j_-}(t) C_{j_-}(t) | \psi_{\alpha'}(t) \rangle.$

→ singularity in the jump probability
(can not cancel something which never happened)

Note: however, fully classical Markovian description with ancillas exists

The problem and the motivation - once again…

- Processes exists which always break CP-divisibility and always preserve P-divisibility
- "In-between" Markovian and non-Markovian
- No known jump descriptions - without ancillas- exists

What is the most general stochastic jump description valid in all regimes?

Reminder about maps $\Phi_{t,0} = \Phi_{t,s}\Phi_{s,0}$:
CP-divisibility: $\Phi_{t,s}$ is CP
P-divisibility: $\Phi_{t,s}$ is P

ROQJ - Rate operator quantum jumps

- Master equation

$$\mathscr{L}[\rho(t)] = -\frac{i}{\hbar}[H_s, \rho(t)] + \sum_{\alpha=1}^{n^2-1} c_\alpha(t) \left(L_\alpha(t)\rho(t)L_\alpha(t)^\dagger - \frac{1}{2}\left\{L_\alpha^\dagger(t)L_\alpha(t), \rho(t)\right\} \right)$$

- At this stage, consider P-divisible dynamics.

- Negative rates allowed, as long as **transition rate operator** positive semi-definite (non-negative eigenvalues) for any pure state $|\psi(t)\rangle$

$$W^J_{\psi(t)} = \sum_{\alpha=1}^{n^2-1} c_\alpha(t)(L_\alpha(t) - \ell_{\psi(t),\alpha}) |\psi(t)\rangle\langle\psi(t)| (L_\alpha(t) - \ell_{\psi(t),\alpha})^\dagger$$

$$\ell_{\psi(t),\alpha} = \langle\psi(t)| L_\alpha(t) |\psi(t)\rangle$$

$$H_{\psi(t)} = H_S(t) - \frac{i\hbar}{2} \sum_{\alpha=1}^{n^2-1} c_\alpha(t) \times \left(L_\alpha^\dagger(t) L_\alpha(t) - 2\ell^*_{\psi(t),\alpha} L_\alpha(t) + |\ell_{\psi(t),\alpha}|^2 \right)$$

deterministic evolution

ROQJ

- We can diagonalize and write with eigenvalues

$$W^J_{\psi(t)} = \sum_{j=1}^{n-1} \lambda_j(t) \left|\varphi_{\psi(t),j}\right\rangle\left\langle\varphi_{\psi(t),j}\right|$$
$$= \sum_{j=1}^{n-1} V_{\psi(t),j} \left|\psi(t)\right\rangle\left\langle\psi(t)\right| V^\dagger_{\psi(t),j},$$

- Here defined

$$V_{\psi(t),j} = \sqrt{\lambda_j(t)} \left|\varphi_{\psi(t),j}\right\rangle\left\langle\psi(t)\right|$$

Transfers from current state to eigenstate of rate operator with a rate given by the eigenvalue

- Therefore deterministic evolution interrupted by jumps

$$\left|\psi(t)\right\rangle \to \frac{V_{\psi(t),j}\left|\psi(t)\right\rangle}{\|V_{\psi(t),j}\left|\psi(t)\right\rangle\|}$$

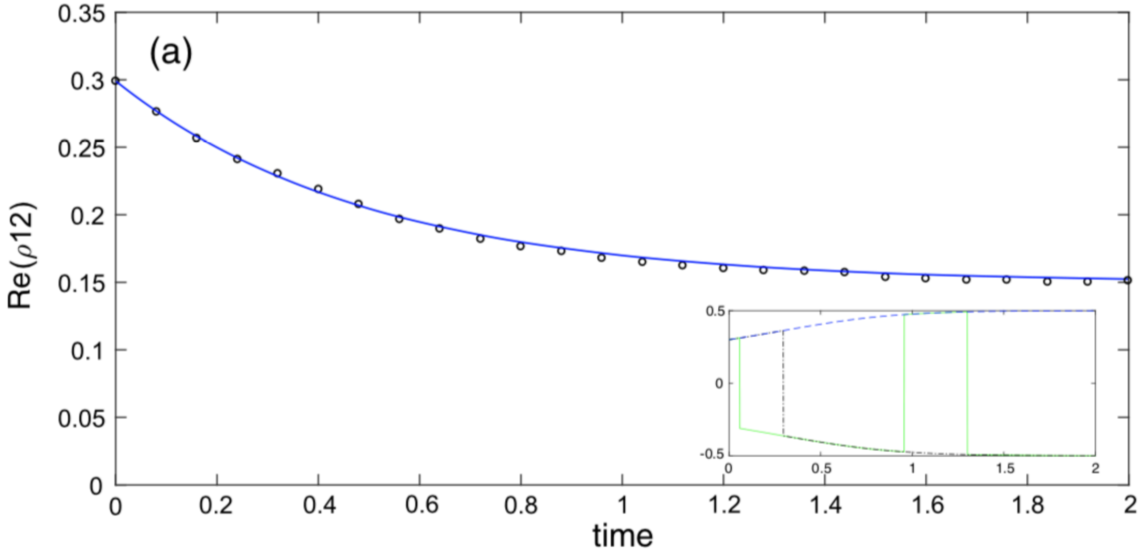
which occur with probability

$$p_j(t) = \|V_{\psi(t),j}\left|\psi(t)\right\rangle\|^2 \mathrm{d}t.$$

- Similarity to MCWF.

"ENM" master equation

$$\dot{\rho} = \frac{1}{2}\sum_{k=1}^{3} \gamma_k(t)[\sigma_k \rho \sigma_k - \rho], \qquad \gamma_1(t) = \gamma_2(t) = 1, \quad \gamma_3(t) = -\tanh t$$



Bloch vector components

1,2: $x_k(t) = \frac{1}{2}(1 + e^{-2t})x_k(0)$

3: $x_3(t) = e^{-2t}x_3(0)$

- Monotonic loss of coherence

- Simulation produces analytical results
- In general possible to prove match with master equation

Note on measurement scheme

- Markovian MCWP has measurement scheme interpretation

- No known measurement schemes in non-Markovian regime
  (Diosi PRL 2008; Gambetta, Wiseman PRL 2008)

Where is the border between the two?
How do we lose measurement scheme interpretation?

ROQJ - measurement scheme

- It is possible to show in mathematically rigorous manner that the method has continuous measurement scheme interpretation following Barchielli and Belavkin JPhysA 1991
- Therefore measurement scheme exists for master equations with negative rates as long as P-divisible
- Operations for the count (jump) defined by

$$\mathcal{I}_{\omega_t,j}\rho = V_{\omega_t,j}\rho V^{\dagger}_{\omega_t,j}t \qquad j = 1,\ldots n$$

$$V_{\psi(t),j} = \sqrt{\lambda_j(t)}\,|\varphi_{\psi(t),j}\rangle\langle\psi(t)|$$

- Trajectory upto time t

$$\omega_t = (t_1, j_1; t_2, j_2; \ldots t_m, j_m)$$

- Corresponding state transformation

$$\rho \mapsto \frac{\mathcal{I}_{\omega_t,j}\rho}{\mathrm{Tr}\left\{\mathcal{I}_{\omega_t,j}\rho\right\}}$$

ROQJ - measurement scheme

- Operations for the count (jump) defined by

$$\mathscr{I}_{\omega_t,j}\rho = V_{\omega_t,j}\rho V^{\dagger}_{\omega_t,j}\mathrm{d}t \qquad j = 1, \ldots n-1, \qquad V_{\psi(t),j} = \sqrt{\lambda_j(t)}\,|\varphi_{\psi(t),j}\rangle\langle\psi(t)|$$

Important points:

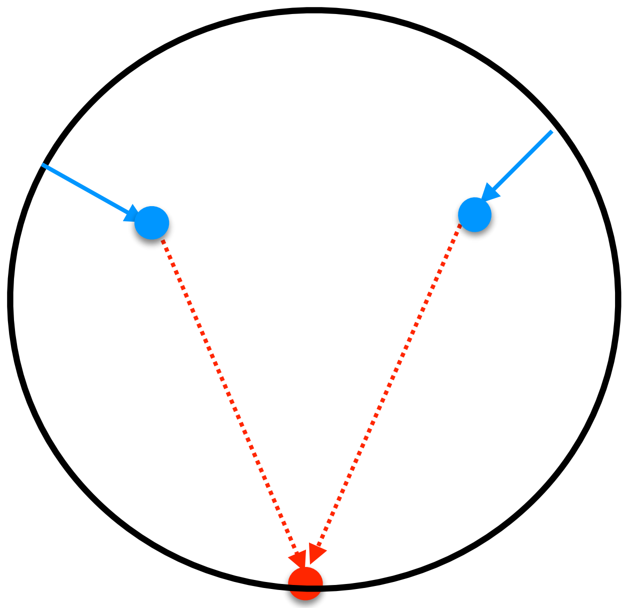
- The operations are conditioned on the whole trajectory
- For example: the past jumps influence what is the current state and may influence the diagonalization and the construction of the corresponding jump operator
- Measurement scheme: In addition of measurement record requires also computational resources and time dependent basis for the measurement depending on the trajectory

Does this describe memory effects or not?

Cartoon of different types of memory effects

Schematics with "2D" Bloch sphere
Type of stochastic realizations

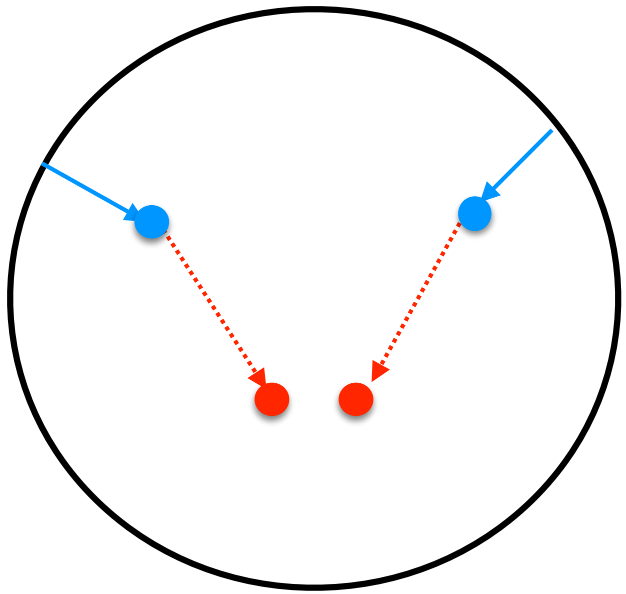
Markovian MCWF



- The jumps take the realizations to same state no matter where located prior to jump

Cartoon of different types of memory effects

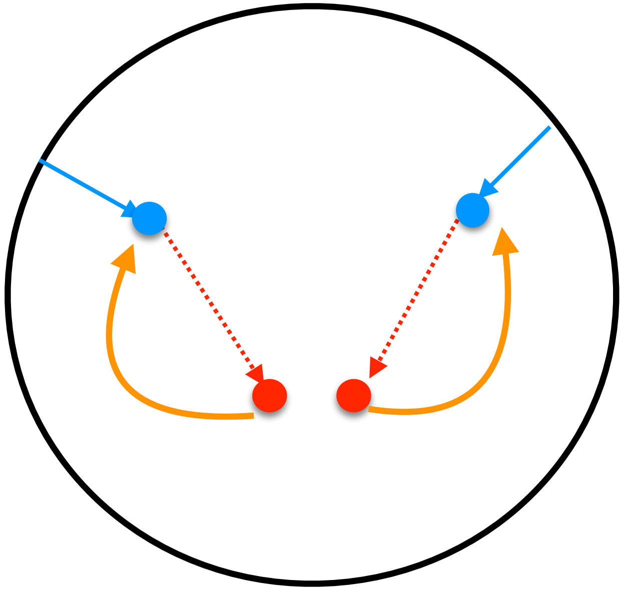
In-between region with ROQJ



- The jumps take the realizations to states which depend on the current state and therefore also on prior sequence

Cartoon of different types of memory effects

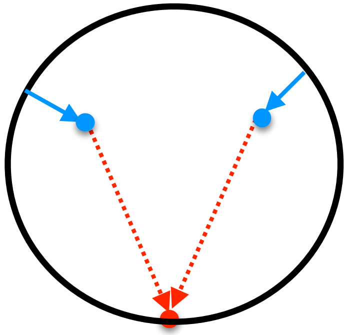
Non-Markovian quantum jumps



- The jumps take the realization to state which the realization had in the past (recovery of lost info)

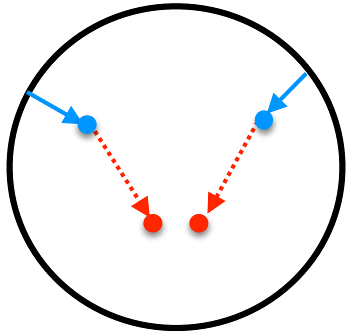
Cartoon of different types of memory effects

Markovian



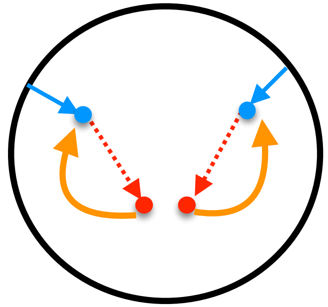
- No memory
- Always to same state

In-between



- "Dependence" from the past for single realization
- Where you go next depends where you are at the moment
- On the level of density matrix monotonic loss of coherence

non-Markovian

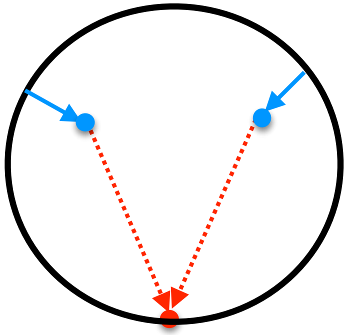


- Backflow of info for single realizations
- Backflow of info for density operator
- Going back where you were before

Does this describe memory or not?

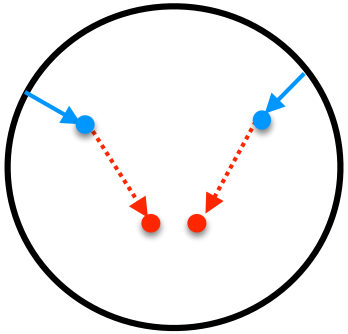
Cartoon of different types of memory effects

Markovian



- No memory
- Always to same state

In-between



- "Dependence" from the past for single realization
- Where you go next depends where you are at the moment
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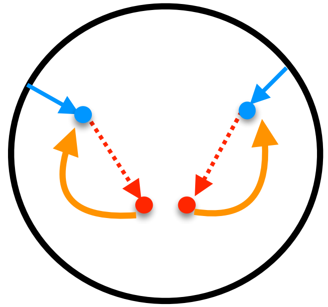
non-Markovian



- Backflow of info for single realizations
- Backflow of info for density operator
- Going back where you were before

Note that also in Markovian case dependence from the past in terms of jump probability via state populations

Does this describe memory or not?

★ If you answer "yes", what is the source of NM?

★★ Also classical Markovian rate equation solution exists (Megier et al SciRep 2017)

General scheme including non-Markovian regime

- Divide transition rate operator to positive and negative eigenvalue parts

$$W^J_{\psi_k(t)} = \sum_{j=1}^{n-1} \lambda_j(t) \, |\varphi_{\psi_k(t),j}\rangle\langle\varphi_{\psi_k(t),j}|$$
$$= \sum_{j^+} \lambda_{j^+}(t) \, |\varphi_{\psi_k(t),j^+}\rangle\langle\varphi_{\psi_k(t),j^+}| - \sum_{j^-} |\lambda_{j^-}(t)| \, |\varphi_{\psi_k(t),j^-}\rangle\langle\varphi_{\psi_k(t),j^-}|$$

- For positive part, use the earlier scheme
- For negative part, calculate the jump probabilities in similar manner as for NMQJ and use in the reverse jumps as a source the eigenstates of the transition rate operator

ROQJ - in non-P-div region

Reverse jumps, corresponding to negative eigenvalues of W, are

$$B_{\psi_k(t),\psi_{k'}(t),j^-} = \sqrt{\left|\lambda_{\psi_{k'}(t),j^-}\right|}\,|\psi_{k'}(t)\rangle\langle\psi_k(t)|$$

- Source of reverse jump is the eigenstate of W: $|\psi_k(t)\rangle = |\varphi_{\psi_{k'}(t),j^-}\rangle$
- Probability given by $p_{j^-}^{(k\to k')}(t) = \frac{N_{k'}(t)}{N_k(t)}\left|\lambda_{\psi_{k'}(t),j^-}\right| \mathrm{d}t.$

One general framework for all regimes:

- When P-div: jumps to eig. states of rate operator W
- When P-div: broken: jumps out of the eig. states of W
- ROQJ works also when neither MCWF nor NMQJ works (when P-div. with negative rates)

ROQJ - in non-Markovian (non-P-div) region

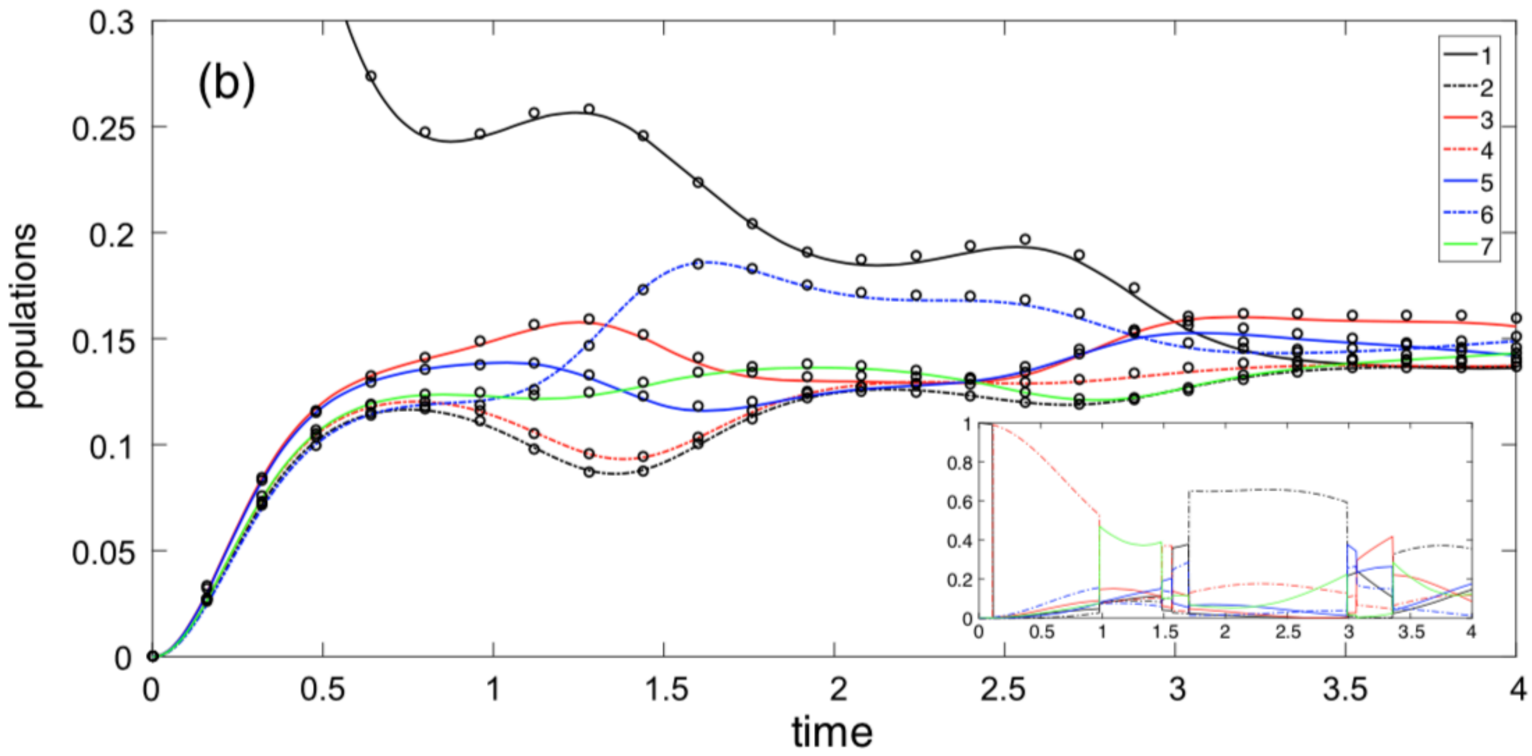
- 7-site driven system

  Unitary part: $H_S = \sum_{i \neq j} \Omega_{i,j} |i\rangle\langle j|$

  Jump (Lindblad) operators): $L_{i,j} = |i\rangle\langle j|$ (49 of them)

  Jump rates (contain negative regions):

  $c(t) = 0.5[(1 - e^{-0.5t})0.3 + e^{-0.3t} \sin(4.5t)]$



Is the rate operator unique?
Can we have a family of rate operators?

Chruscinski, Luoma, Piilo, Smirne
arXiv:2009:11312
(work still ongoing and developed…)

Different choices for rate operator

- Master equation

$$\dot{\rho} = \frac{1}{2}\sum_{k=1}^{3}\gamma_k(t)[\sigma_k\rho\sigma_k - \rho],$$

For example

- Rate operator **R1**

$$\mathbf{R1} = \sum_{k=1}^{d}\gamma_k(t)\sigma_k|\psi\rangle\langle\psi|\sigma_k + \sum_k \gamma_k(t)|\psi\rangle\langle\psi|$$

$$K1(t) = \frac{i}{2}\gamma(t)1$$ deterministic evolution

- Rate operator **R2**

$$\mathbf{R2} = \sum_{k=1}^{d}\gamma_k(t)\sigma_k|\psi\rangle\langle\psi|\sigma_k + (\gamma_1(t) + \gamma_2(t))|\psi\rangle\langle\psi|$$

$$K2(t) = \frac{i}{2}[\gamma_1(t) + \gamma_2(t)]1$$ deterministic evolution

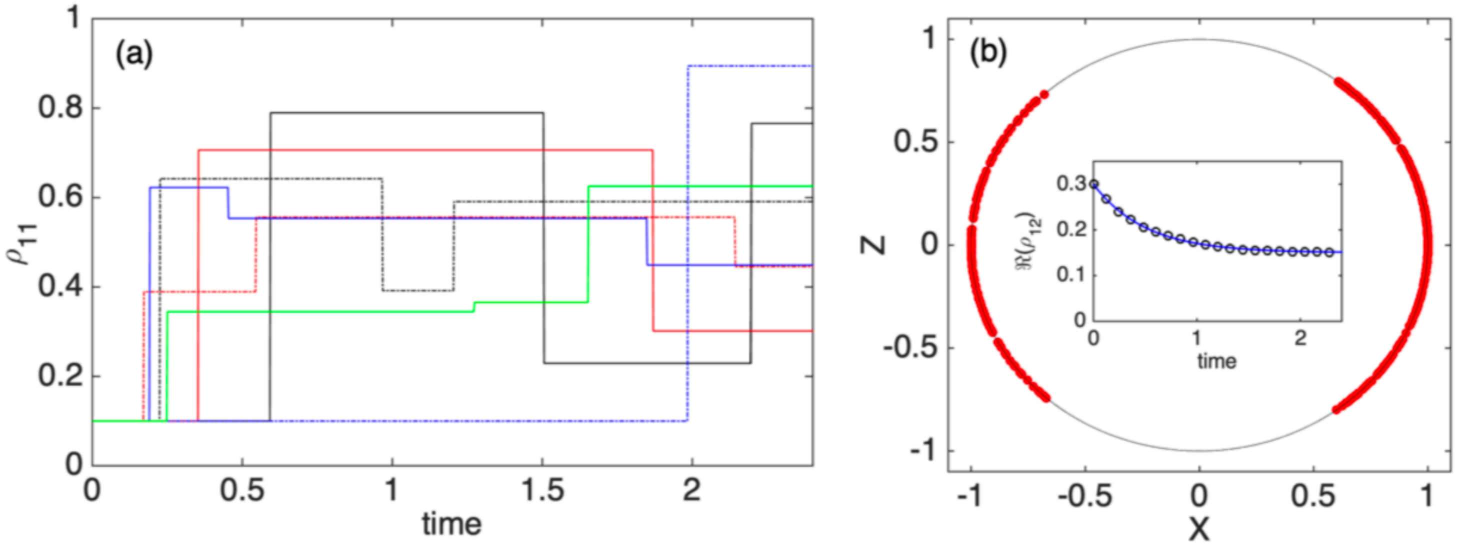
Chruscinski, Luoma, Piilo, Smirne
arXiv:2009.11312

Different choices for rate operator

With R1

Example realizations

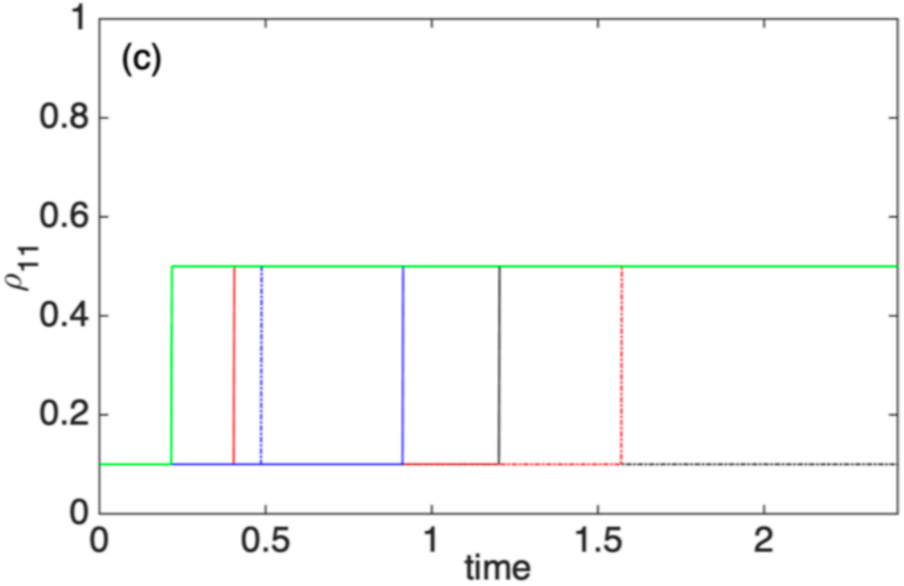
Bloch vector x and z components: final distribution



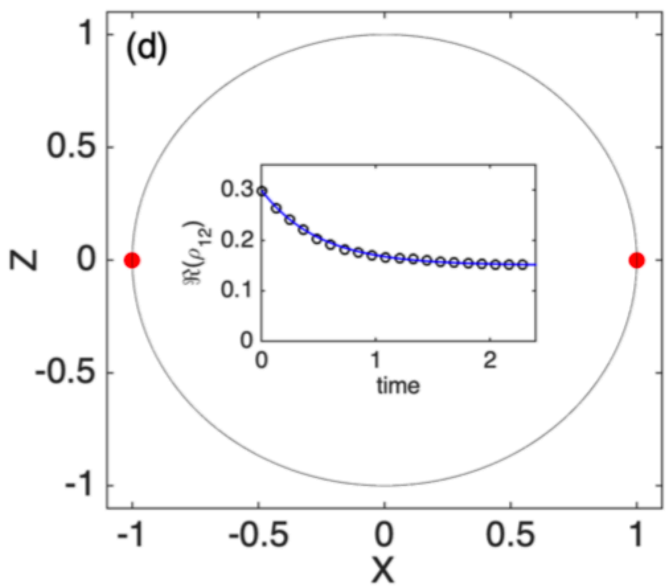
Chruscinski, Luoma, Piilo, Smirne
arXiv:2009.11312

With R2

Example realizations



Bloch vector x and z components: final distribution



Chruscinski, Luoma, Piilo, Smirne arXiv:2009:11312

CONCLUSIONS

- **MCWF** - Monte Carlo Wave Function (1992)
- **NMQJ** - Non-Markovian Quantum Jumps (2008)
- **ROQJ** - Rate Operator Quantum Jumps (2020)
  - General starting point for any regime
  - Unifies the framework for using quantum jumps to describe open system dynamics
  - Measurement scheme for master equations with negative rates (P-div, no ancillas used)
  - For the direction of having families of rate operators…

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