ROYAL INSTITUTE
OF TECHNOLOGY

On the necessity to quantize the gravitational field

Erik Aurell

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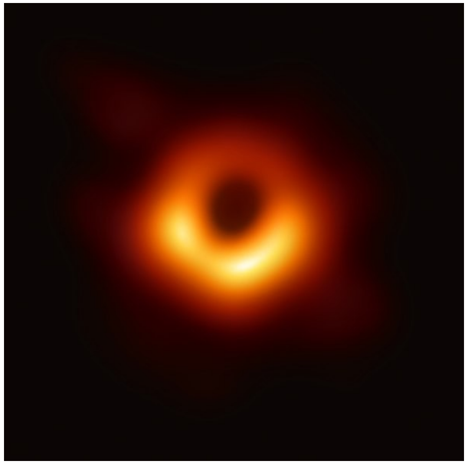
Erik Rydving, EA, Igor Pikovski, Physical Review D 104 (8), 086024 (2021)

The big picture

To combine gravity and quantum has been a/the major unsolved problem in fundamental physics. Theories of quantum gravity have been developed (string theory, loop quantum gravity, and more), but there is (1) no agreement between the different schools, and (2) no agreement what each them implies [perhaps exaggerating a bit, but not much]. There are also collapse theories by Penrose and others where gravitation is not quantized.



It can however not be doubted anymore that there are objects in the Universe where it would eventually matter if gravity is quantized.

Picture of the black hole M87* ,11 April 2017, Event Horizon Telescope (EHT), European Southern Observatory (ESO) [wikimedia commons]

Outline

1933	Bohr and Rosenfeld (BR) showed that the electromagnetic field has to be quantized to be consistent with quantum mechanics applied to ordinary bodies.
1936	Bronstein showed that the BR Gedankenexperiment does not apply to (linearized) quantum gravity.
1940ies	Quantum electrodynamics was discovered and developed.
1950-2000	Quantum field theory was further developed. Many theories of quantum gravity were invented.
2009	Baym and Ozawa published another Gedankenexperiment due to Bohr and argued that it does not apply to gravity.
2014	BR and Bronstein's arguments were revived by Dyson.
2018	Belenchia et al (Brukner & Aspelmeyer) analyzed yet another Gendankenexperiment, similar to Baym & Ozawa, arriving at the conclusion that the (linear) gravitational field has to be quantized.

Bohr & Rosenfeld (1933)

Niels Bohr & Léon Rosenfeld
“Zur Frage der Messbarkeit der elektromagnetischen Feldgrössen”
Kgl. Danske Vidensk. Selskab. Math.-Fys. Medd. **12**:3 (1933)

Reprinted [in English, translation by A Pedersen] in

Wheeler & Zurek (Eds.)
Quantum Theory and Measurement
Princeton University Press (1983)

BR aimed to show that if the electromagnetic field is not quantized it would be possible to measure a test body better than allowed by Heisenberg uncertainty relations.

One difficulty of BR is that it was written in the language of quantum field theory in the 1930ies. As one learns in courses in more modern versions of the theory, there were infinities in those theories, and renormalization had not yet been invented.

But that’s a relatively minor difficulty.

The greatest difficulty

Bitte, bitte, Landau, muss ich nur ein Wort sagen!

G Gamow (1933)
courtesy Niels Bohr Archive

BR introduce extraordinarily complicated Gedankenexperiments with many mechanic springs and other contraptions to correct for various induced fields in imagined idealized experiments.

In the most complete versions in BR there are *two* charged test bodies each with *its compensatory body*, a further *neutral test body*, *five* springs connecting these test bodies to each other and to a *rigid support*, *light signaling* between two of the test bodies and *at least two corrections* that are assumed analytically computable.

Bronstein and Dyson

A theory of the linearized quantum gravitational field was developed by M Bronstein in his 1936 PhD thesis, and later by others, e.g. by Feynman.

Bronstein observed that BR arguments do not apply to this linearized quantum gravitational field. Compensatory bodies of opposite (gravitational) charge (that is, negative mass) do not exist in Nature.

Bronstein also observed that a BR-style experiment for gravity would need to be very massive, so massive that it would be inside its gravitational radius.

Both observations were made by Dyson (2014). He found that conceivable experiments to observe a quantum of the gravitational field (a graviton) would collapse into a black hole before the experiment had finished.

Planck units

$$\text{Planck mass}: \quad m_p = \sqrt{\frac{\hbar c}{G}} \approx 2.2 \times 10^{-8}\ \text{kg},$$

$$\text{Planck length}: \quad l_p = \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \times 10^{-35}\ \text{m},$$

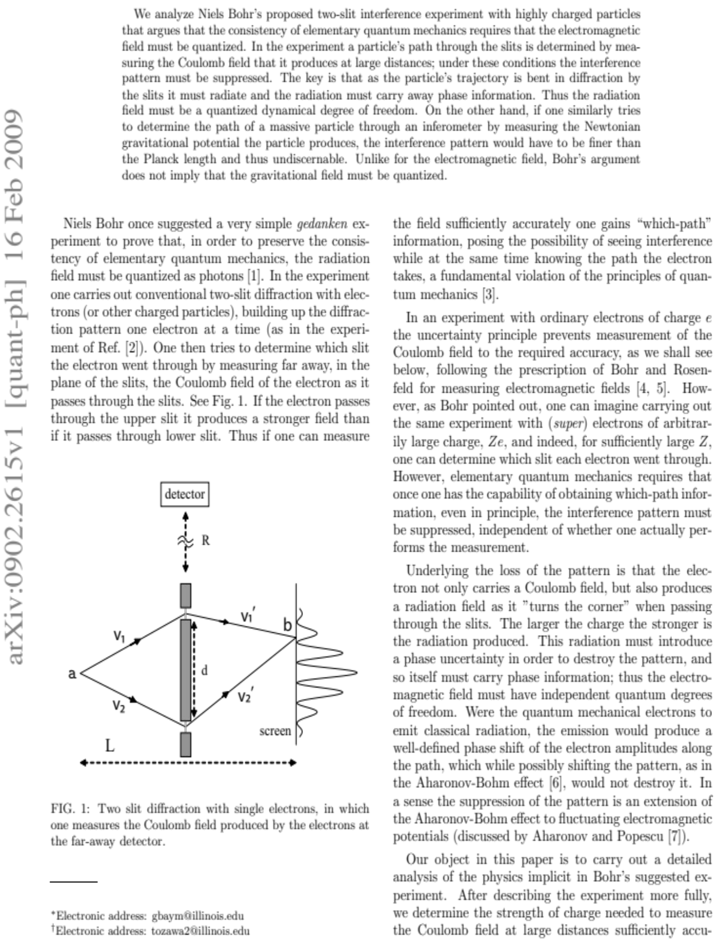
$$\text{Planck time}: \quad t_p = \sqrt{\frac{\hbar G}{c^5}} \approx 5.4 \times 10^{-44}\ \text{s}.$$

Planck units are particularly useful in the theory of black holes. The black hole radius is , the Hawking temperature is , etc.

“It is natural to suppose also that [] determines the limit of applicability of present-day notions of space and causality.”

—A.D. Sakharov, *Dokl. Akad. Nauk* **177**:70-71 (1967)

Baym & Ozawa (2009)



arXiv:0902.2615v1 [quant-ph] 16 Feb 2009

We analyze Niels Bohr's proposed two-slit interference experiment with highly charged particles that argues that the consistency of elementary quantum mechanics requires that the electromagnetic field must be quantized. In the experiment a particle's path through the slits is determined by measuring the Coulomb field that it produces at large distances; under these conditions the interference pattern must be suppressed. The key is that as the particle's trajectory is bent in diffraction by the slits it must radiate and the radiation must carry away phase information. Thus the radiation field must be a quantized dynamical degree of freedom. On the other hand, if one similarly tries to determine the path of a massive particle through an inferometer by measuring the Newtonian gravitational potential the particle produces, the interference pattern would have to be finer than the Planck length and thus undiscernable. Unlike for the electromagnetic field, Bohr's argument does not imply that the gravitational field must be quantized.

Niels Bohr once suggested a very simple *gedanken* experiment to prove that, in order to preserve the consistency of elementary quantum mechanics, the radiation field must be quantized as photons [1]. In the experiment one carries out conventional two-slit diffraction with electrons (or other charged particles), building up the diffraction pattern one electron at a time (as in the experiment of Ref. [2]). One then tries to determine which slit the electron went through by measuring far away, in the plane of the slits, the Coulomb field of the electron as it passes through the slits. See Fig. 1. If the electron passes through the upper slit it produces a stronger field than if it passes through lower slit. Thus if one can measure the field sufficiently accurately one gains "which-path" information, posing the possibility of seeing interference while at the same time knowing the path the electron takes, a fundamental violation of the principles of quantum mechanics [3].

FIG. 1: Two slit diffraction with single electrons, in which one measures the Coulomb field produced by the electrons at the far-away detector.

In an experiment with ordinary electrons of charge e the uncertainty principle prevents measurement of the Coulomb field to the required accuracy, as we shall see below, following the prescription of Bohr and Rosenfeld for measuring electromagnetic fields [4, 5]. However, as Bohr pointed out, one can imagine carrying out the same experiment with (*super*) electrons of arbitrarily large charge, Ze, and indeed, for sufficiently large Z, one can determine which slit each electron went through. However, elementary quantum mechanics requires that once one has the capability of obtaining which-path information, even in principle, the interference pattern must be suppressed, independent of whether one actually performs the measurement.

Underlying the loss of the pattern is that the electron not only carries a Coulomb field, but also produces a radiation field as it "turns the corner" when passing through the slits. The larger the charge the stronger is the radiation produced. This radiation must introduce a phase uncertainty in order to destroy the pattern, and so itself must carry phase information; thus the electromagnetic field must have independent quantum degrees of freedom. Were the quantum mechanical electrons to emit classical radiation, the emission would produce a well-defined phase shift of the electron amplitudes along the path, which while possibly shifting the pattern, as in the Aharonov-Bohm effect [6], would not destroy it. In a sense the suppression of the pattern is an extension of the Aharonov-Bohm effect to fluctuating electromagnetic potentials (discussed by Aharonov and Popescu [7]).

Our object in this paper is to carry out a detailed analysis of the physics implicit in Bohr's suggested experiment. After describing the experiment more fully, we determine the strength of charge needed to measure the Coulomb field at large distances sufficiently accu-

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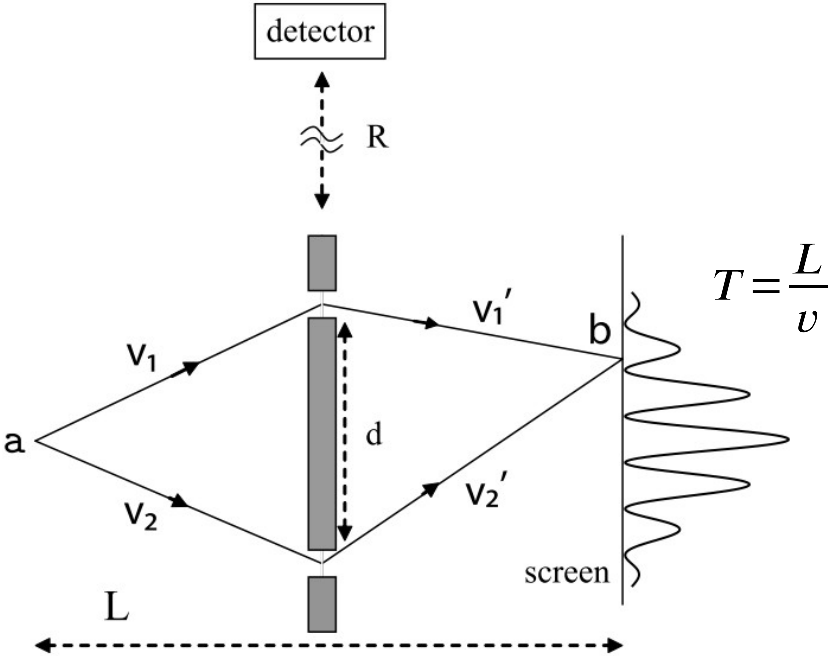
*Electronic address: gbaym@illinois.edu
†Electronic address: tozawa2@illinois.edu

Gordon Baym & Tomoki Ozawa, "Two-slit diffraction with highly charged particles: Niels Bohr's consistency argument that the electromagnetic field must be quantized", *PNAS* **106**:3035–3040 (2009).

"Niels Bohr once suggested a very simple gedanken experiment to prove that, in order to preserve the consistency of elementary quantum mechanics, the radiation field must be quantized as photons"

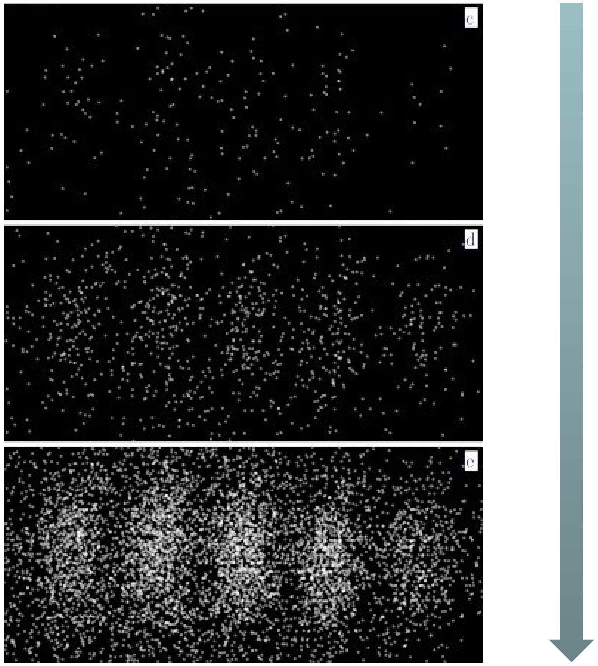
—Aage Petersen, private communication to G. Baym, Copenhagen ca. 1961.

Electromagnetic version



$$T=\frac{L}{v}$$

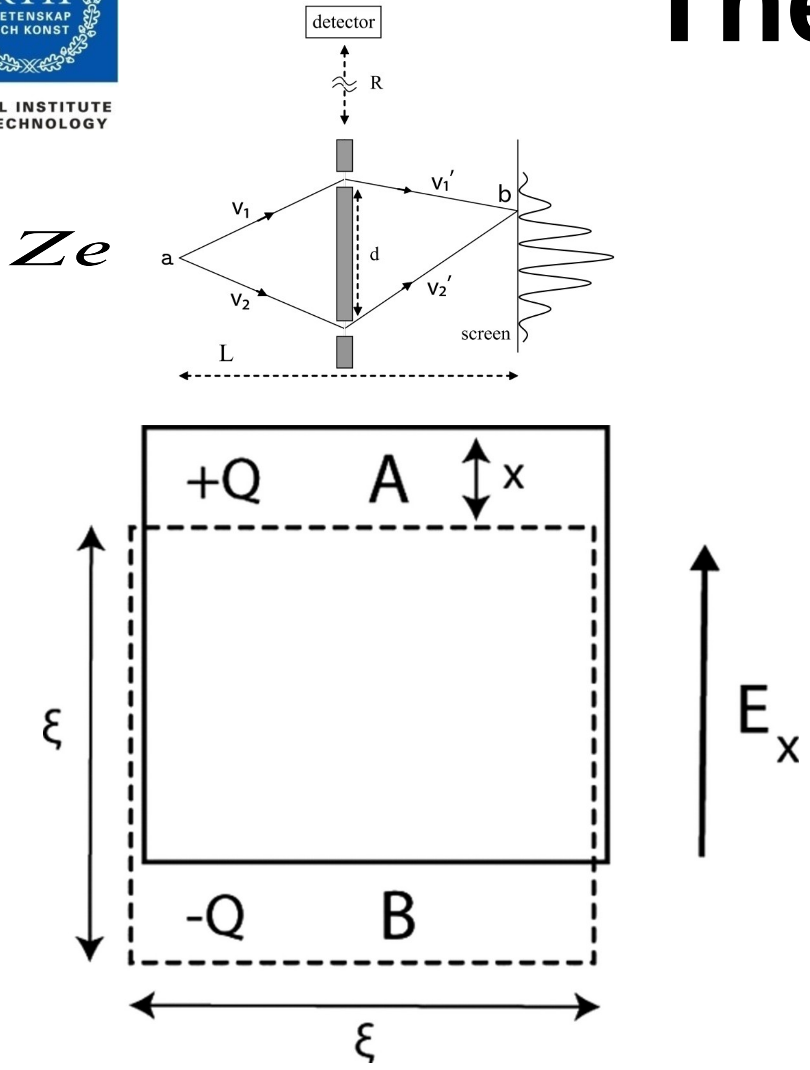
Baym & Ozawa (2009), Fig 1



Bach, R. et al. NJP (2013)

In a two-slit experiment a detector of the electric field is set up far away from the barrier. If that detector can acquire which-path information by measuring the far-field, and at the same time a diffraction pattern is observed, complementarity is broken. Assumed , so no backreaction.

The argument (1/3)



Baym & Ozawa (2009), Fig 2

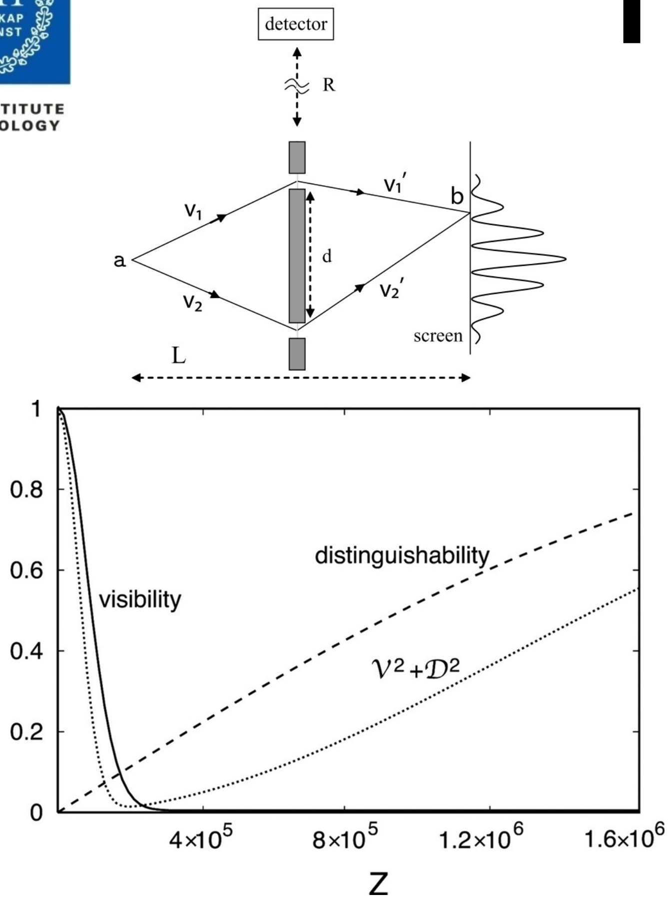
The detector, a charged body A and a compensatory body B, can determine electric field down to a minimum given by Heisenberg uncertainty applied to A (BR, in a form given by Bronstein).

The detector can get which-path information if it can distinguish the electric field from a particle going through the upper or the lower slit.

$$Z > \frac{1}{\sqrt{\alpha}}\frac{cT}{d} \qquad \alpha = \frac{e^2}{\hbar c} \approx 137$$

An electron (charge) is safe. A very charged particle seems not to be.

The argument (2/3)



Baym & Ozawa (2009), Fig 3

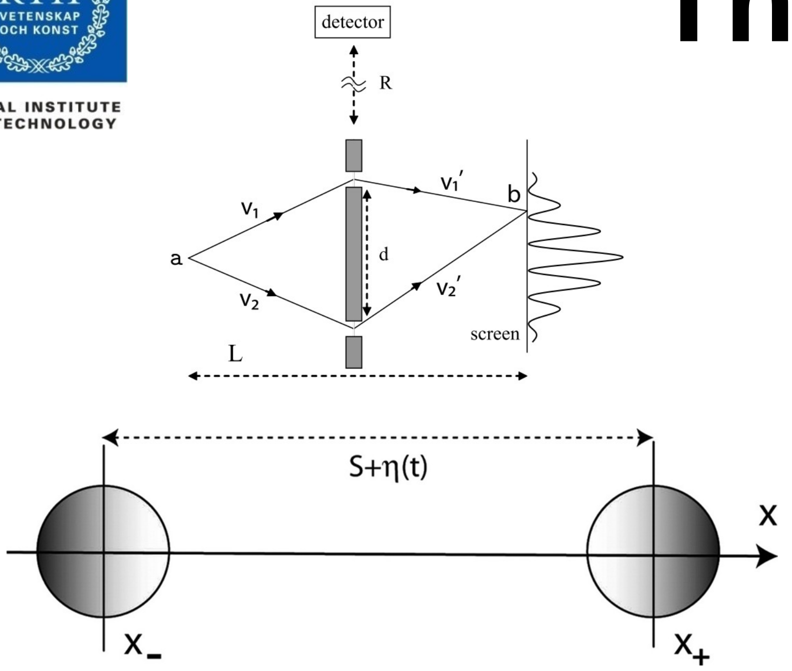
A charged particle will radiate when it changes velocity at the slits. That particle then becomes entangled with photons escaping to infinity, and is no longer fully entangled with the particle going though the other slit.

A more charged particle (larger) will radiate more photons. Visibility of interference fringes hence decreases with .

Calculation of decoherence from brehmstrahlung was done by *e.g.* Breuer & Petruccione *PRA* (2001)

A quantized electromagnetic field means that it is also not possible to acquire which-path information from a very charged particle.

The argument (3/3)



Baym & Ozawa (2009), Fig 4

The idea of a gravitational field detector for which-path information. Two mirrors can move. How far they move apart is a measure on the difference of the fields that has acted on them.

Redo the argument with a massive particle, and replace the electric field detector with a gravitational field detector. Which-path information can then be acquired if mass is

$$M > m_p \frac{R}{d}$$

The fringes are however at separation

$$\delta f \quad \frac{L}{d}\frac{\hbar}{Mv} \le l_p \frac{cT}{R} < l_p$$

A quantized gravitational field is not necessary. It suffices (Sakharov's principle) to assume that is a lower limit on position measurements.

Belenchia et al (2018)

[1] *Institute for Quantum Optics and Quantum Information (IQOQI), Boltzmanngasse 3 1090 Vienna, Austria.*
[2] *Enrico Fermi Institute and Department of Physics, The University of Chicago,*
5640 South Ellis Avenue, Chicago, Illinois 60637, USA
[3] *Vienna Center for Quantum Science and Technology (VCQ), Faculty of Physics,*
University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria

We analyse a gedankenexperiment previously considered by Mari et al. [1] that involves quantum superpositions of charged and/or massive bodies ("particles") under the control of the observers, Alice and Bob. In the electromagnetic case, we show that the quantization of electromagnetic radiation (which causes decoherence of Alice's particle) and vacuum fluctuations of the electromagnetic field (which limits Bob's ability to localize his particle to better than a charge-radius) both are essential for avoiding apparent paradoxes with causality and complementarity. We then analyze the gravitational version of this gedankenexperiment. We correct an error in the analysis of Mari et al. [1] and of Baym and Ozawa [2], who did not properly account for the conservation of center of mass of an isolated system. We show that the analysis of the gravitational case is in complete parallel with the electromagnetic case provided that gravitational radiation is quantized and that vacuum fluctuations limit the localization of a particle to no better than a Planck length. This provides support for the view that (linearized) gravity should have a quantum field description.

I. INTRODUCTION

An understanding of the fundamental nature of gravity and spacetime remains one of the most significant open issues in theoretical physics. The lack of a background spacetime structure in general relativity—the spacetime metric itself is the dynamical variable—makes it impossible to formulate a quantum theory of gravity by simply applying standard procedures that work for other fields. Although one can formulate an entirely satisfactory quantum field theory of linearized gravity—it is just a massless spin-2 field—severe difficulties arise when one attempts to go significantly beyond this description. Thus, there have been suggestions that gravity/spacetime could be fundamentally classical, or that its marriage with quantum mechanics requires a radical change of perspective on quantization [3, 4], or that quantization of gravity could be an ill-posed question in the first place [5]—although there also have been many arguments given for the necessity of a quantum description of gravity [6–11].

In order to gain more insight into the quantum properties of gravity, it is helpful to consider the gravitational field associated with a quantum source, as already discussed by Feynman [12, 13]. This is the basis of proposals for actual experiments employing macroscopic masses in superpositions [7, 14–21]. The main aim of these works is to rule out semi-classical gravity as an exact theory [22, 23], which would treat the gravitational field as classical even when the source is in a macroscopic superposition at different locations—in contrast with the expectations of standard quantum mechanics that a mass in superposition would generate a quantum superposition of gravitational fields. More recently, in [24, 25] a novel way to witness entanglement due to solely the gravitational interaction was proposed. The authors use a gravitationally induced phase shift between two previously independent masses, both in superposition of different locations, which acts fully analogous to an entangling CSIGN gate [26]. They propose to witness the entanglement through correlation measurements between additional spin degrees of freedom. The claim is that, if entanglement between the spins of the two masses is certified then gravity should be a *quantum coherent mediator* (see also [11, 27]).

However, as stressed already in [14, 28], all the previous proposals[1] can be accounted for by just considering the (non-local) gravitational potential in the Schrödinger equation describing the two particles, without any reference to dynamical degrees of freedom of the gravitational field. This has led the authors of [28] to argue that, even if successful in witnessing entanglement, experiments like [24, 25] would say nothing about the quantum nature of the gravitational field.

In this work we provide a different conclusion by revisiting a gedankenexperiment previously considered by [1], which is very similar in its essential aspects to one introduced earlier by [2]. We first analyze the electromagnetic version of this gedankenexperiment and emphasize that the quantum nature of the electromagnetic field is essential to maintain a fully consistent description. We then show that the analysis of the gravitational version of this gedankenexperiment follows in complete parallel to the electromagnetic case. In the course of our analysis of the

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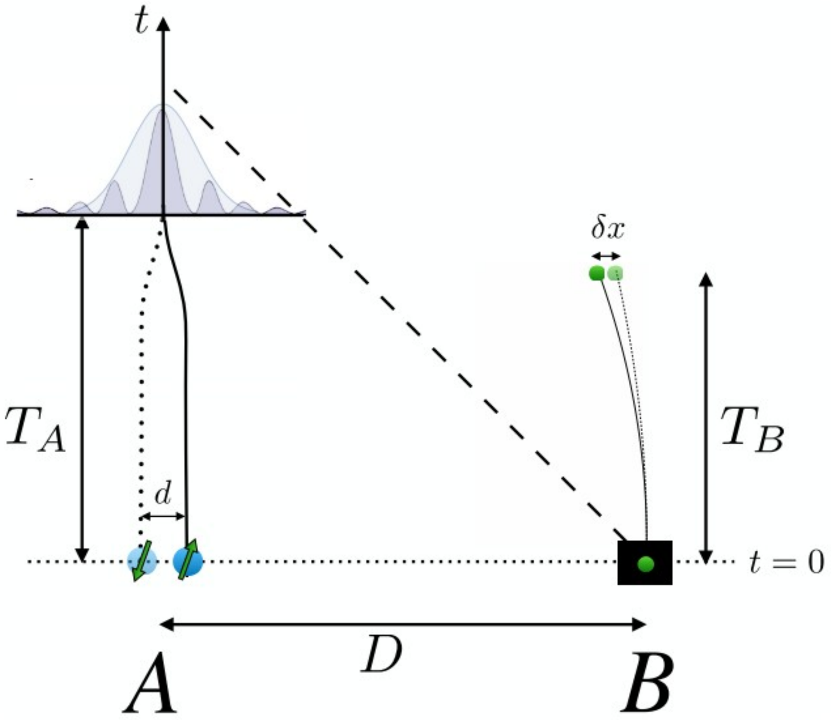
[1] With the notable exception of [18], in which a dynamical version of the Page–Geilker scenario is considered.

A. Belenchia, R. M. Wald, F. Giacomini, E. Castro-Ruiz, C. Brukner, & M. Aspelmeyer

"Quantum superposition of massive objects and the quantization of gravity" *Physical Review D* **98**:1–9 (2018)

Part of the modern trend of table-top experiments in quantum gravity *cf.* Bose et al *PRL* (2017), Marletto & Vedral *PRL* (2017), Christodoulou & Rovelli *Phys Lett B* (2019), Krisnanda et al, *npj Quantum Information* (2020)

A paradox?



Belenchia et al (2018, Fig 1

Alice has a particle in a superposition of two spatially separated states. At the end of the experiment she aims to bring the two states together to a pure state.

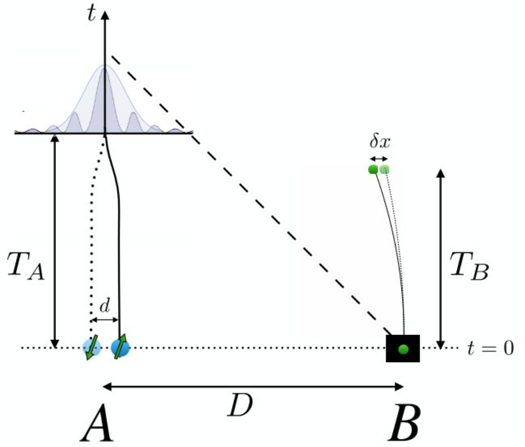
Bob has one particle in a box, which he can release, or not release.

When released Bob's particle will entangle with Alice's. If Alice could then bring back the pure state this would go against complementarity.

But if not, and if Alice and Bob are spatially separated, Bob can send one bit of information faster than light.

A paradox: either causality or complementarity is broken. *True?*

Electrodynamic case



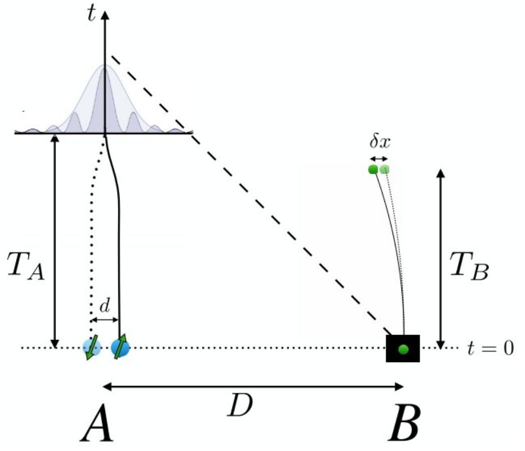
Belenchia et al first state that the uncertainty in the measuring the e-m field in a space-time region of size is .

It can be checked that this is the same BR-derived uncertainty used by Baym & Ozawa.

When Alice's particle has a too small effective dipole, Bob will not get which-path information. When Alice's particle has strong enough effective dipole, Bob could get which part information. But then Alice's particle sends out photons when the two states are brought back together. By the same argument as in Baym & Ozawa there will then not be any interference pattern.

The "effective dipole" stems from difference in the field from a source that is in a superposition at two different positions. A "real dipole" is the difference between two real sources.

Gravitational case



The effective quadrupole acceleration is the difference of Alice's particle to the left and her lab to the right, and her particle to the right and her lab to the left.

$$\Delta g \approx \left(\frac{d}{D}\right)\frac{m_A d^2}{D^4}$$

Belenchia et al point out that there will not be any effective gravitational dipole from Alice. This is because the center of mass on Alice's side is unchanged; if Alice's test particle moves to the right, then her lab (and herself) moves to the left, and vice versa.

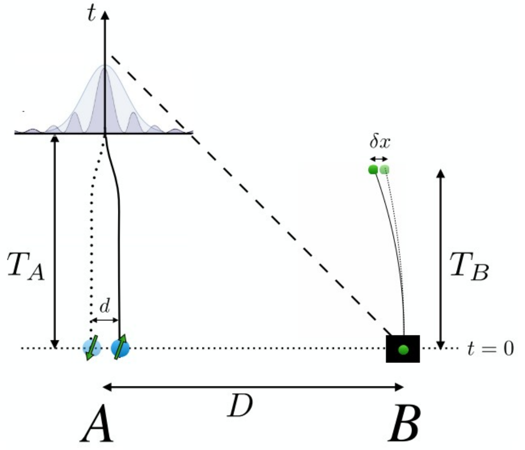
Bob can only acquire which-path information if the effective quadrupole is strong enough.

$$Q_A \equiv \left(\frac{d}{D}\right) m_A\, d^2 > m_p D^2$$

This quadrupole changes as is decreased to zero, which is a source of gravitational waves.

A quantized gravitational field means that it is also not possible to acquire which-path information from a very massive particle.

But is it necessary?



The strength of the effective quadrupole influences the interference fringes for Alice.

But then we should check the width of the interference fringes for Alice. They are, in analogy with Baym & Ozawa,

is the correction to Belenchia et al derived in Rydving, Aurell & Pikovski [in preparation]. Retaining this term or not doing so makes no difference.

If Bob acquires which-path information the interference fringes are too close to be observed by Alice.

It is not necessary to have quantized gravitational field to resolve the paradox.

What does it mean?

(***Negative***) The absence of an argument for a thesis is not an argument for the antithesis. That Belenchia et al argument does not seem to work is not an argument against quantization of gravity.

(***Negative***) According to Wikipedia the first Gedankenexperiment in physics was Gallileo dropping weights from the Tower of Pisa (it was likely never done in reality, but discussed in *Discorsi e dimostrazioni matematiche*, 1638). The term was coined by Ørsted in 1820. The most famous modern users were Einstein and Bohr. Maybe Gedankenexperiments are inherently difficult. Quantum gravity is also difficult. Their combination may be difficult squared.

(***Speculative***) Perhaps the difficulty of constructing a BR-like argument for the quantization of the gravitational field points to that if gravity is quantum, it does not have to be a quantum field theory.

Thanks

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