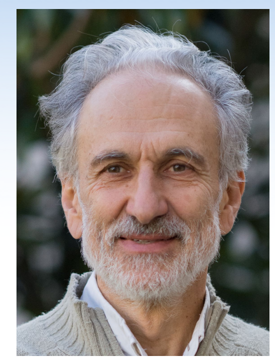
Fast is hot: energetics of information erasure and the overhead to Landauer's bound at low dissipation



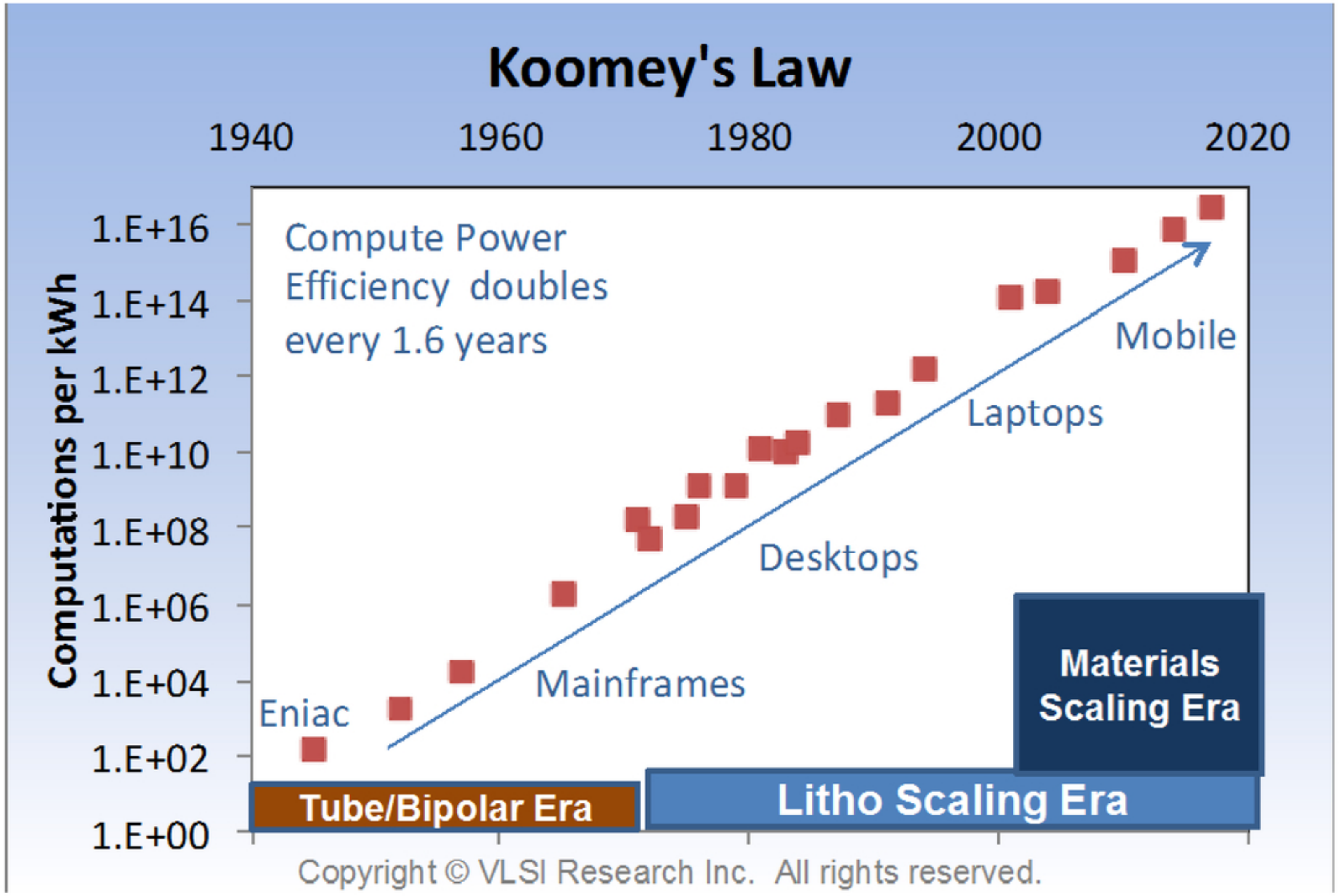



Salambô Dago, Jorge Pereda, Sergio Ciliberto, Ludovic Bellon

Laboratoire de Physique, Université de Lyon, ENS de Lyon, CNRS

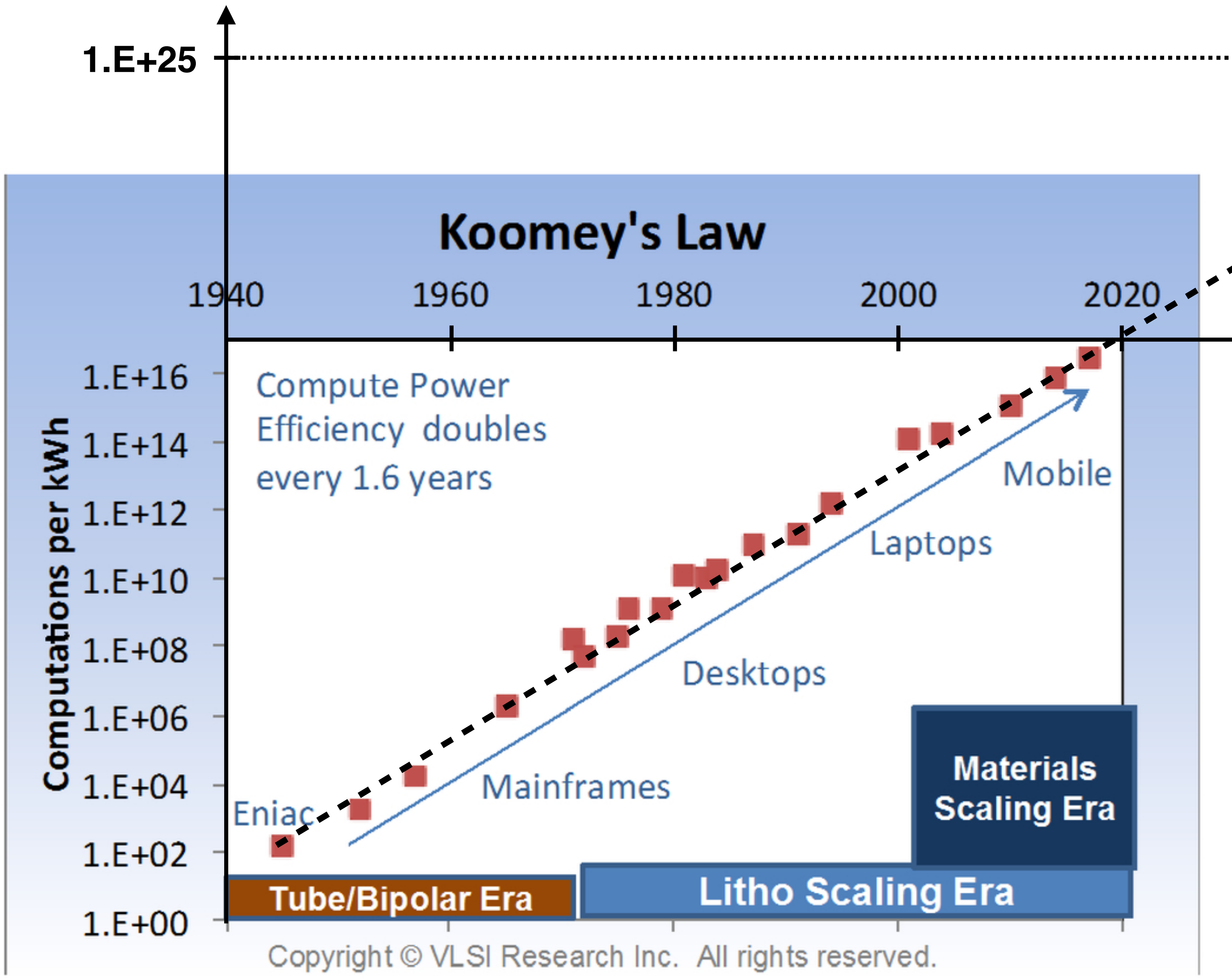
Energetic cost of information processing

- Our brain: ~20 W (vs ~100 W for our body)
- Internet: 10% of electricity worldwide
- Supercomputers: costs are driven by Flops/W



Physical Limit ? **Rolf Landauer** (IBM, 1961):

1-bit erasure costs at least $k_B T \ln 2 = 3 \times 10^{-21}$ J



R. Landauer, *IBM Journal of Research and Development* **5**, 183 (1961)

Energetic cost of information processing

Physical Limit ? **Rolf Landauer** (IBM, 1961):

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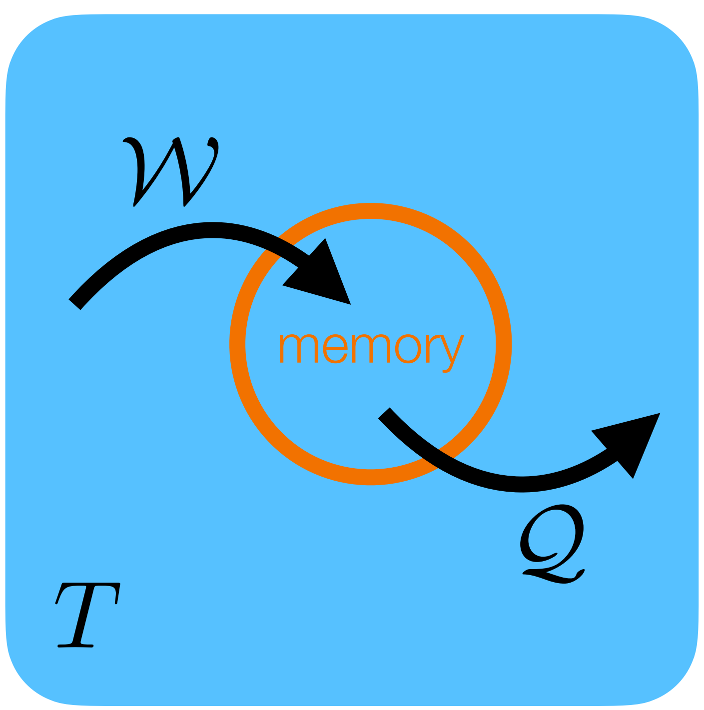
$$S = k_B \ln N$$

[Boltzmann]

$$\{0,1\} \xrightarrow{\text{erasure}} \{0\}$$

$$N = 2 \qquad N = 1$$

$$\Delta S = -k_B \ln 2 \geq -\frac{\langle \mathcal{Q} \rangle}{T}$$



$$\boxed{\langle \mathcal{Q} \rangle = \langle \mathcal{W} \rangle \geq k_B T \ln 2}$$

R. Landauer, *IBM Journal of Research and Development* **5**, 183 (1961)

Physical Limit ? **Rolf Landauer** (IBM, 1961):

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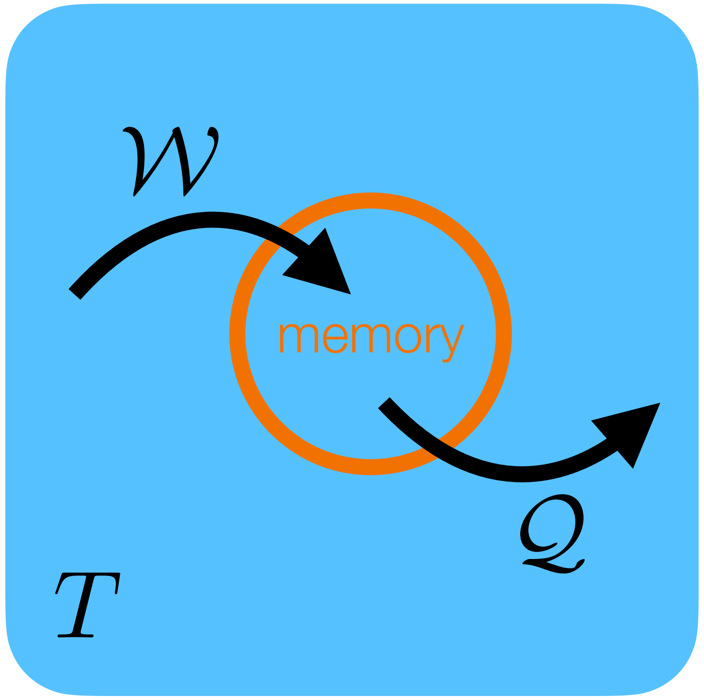
Experimental demonstration ?

Distance to the bound ?

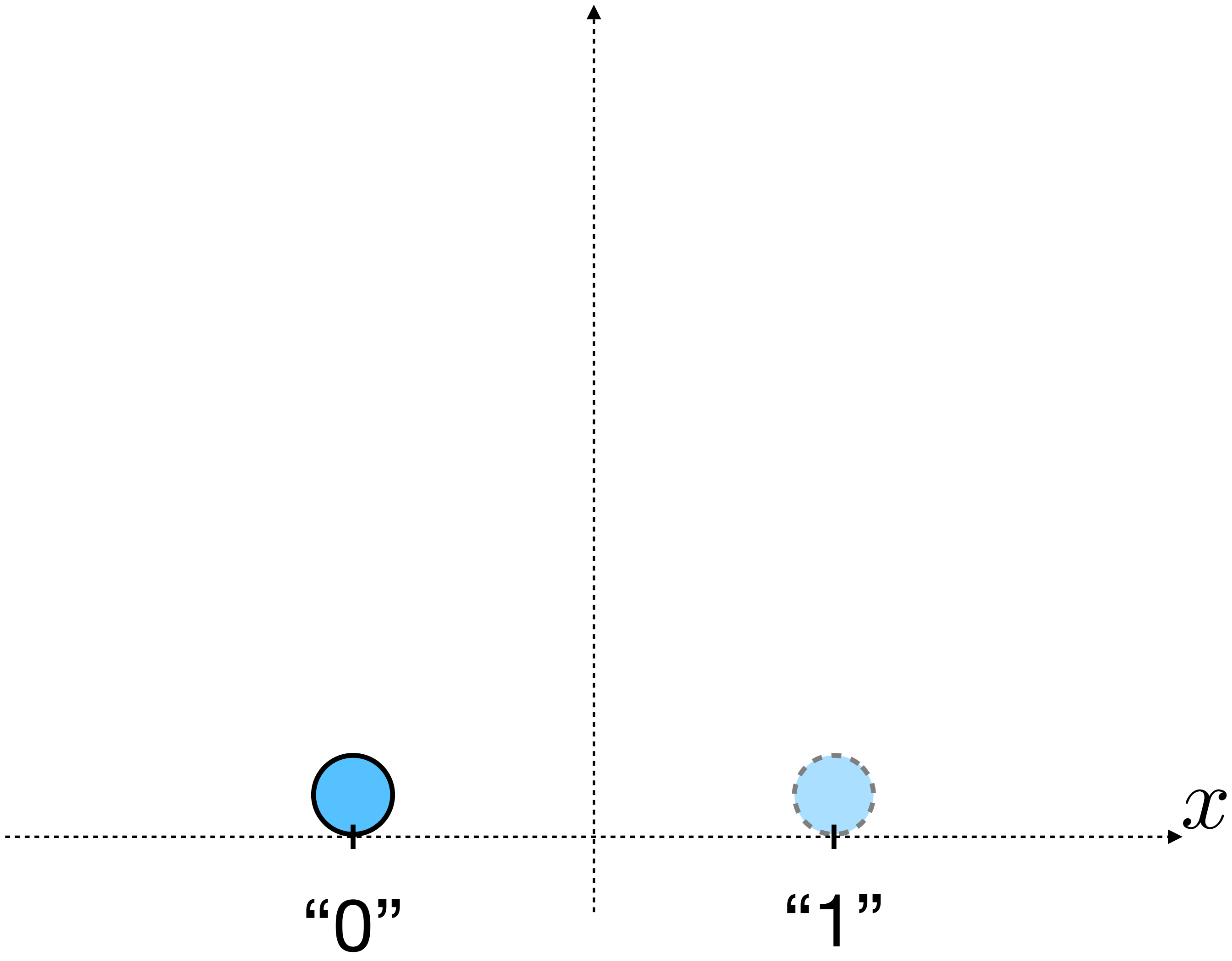
Fast operation ?

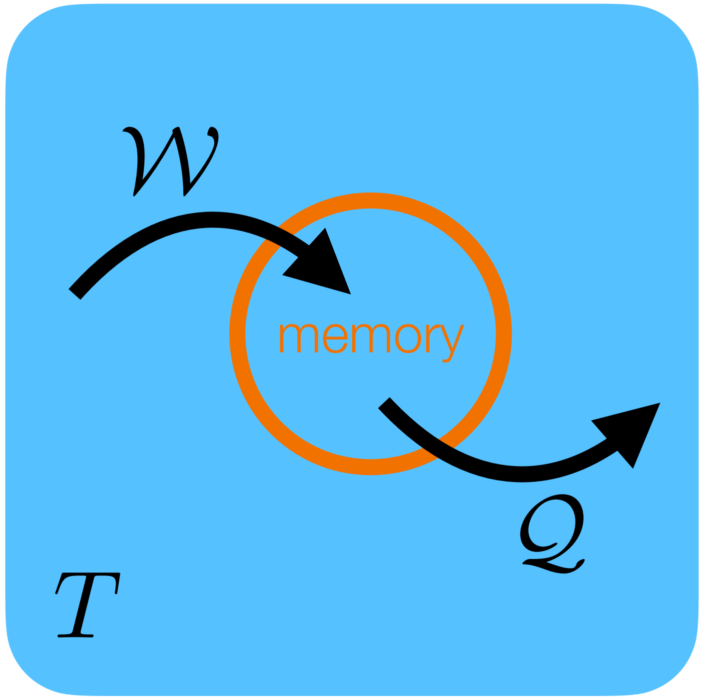
Energetics ?



$$\boxed{\langle \mathcal{Q} \rangle = \langle \mathcal{W} \rangle \gtrsim k_B T \ln 2}$$

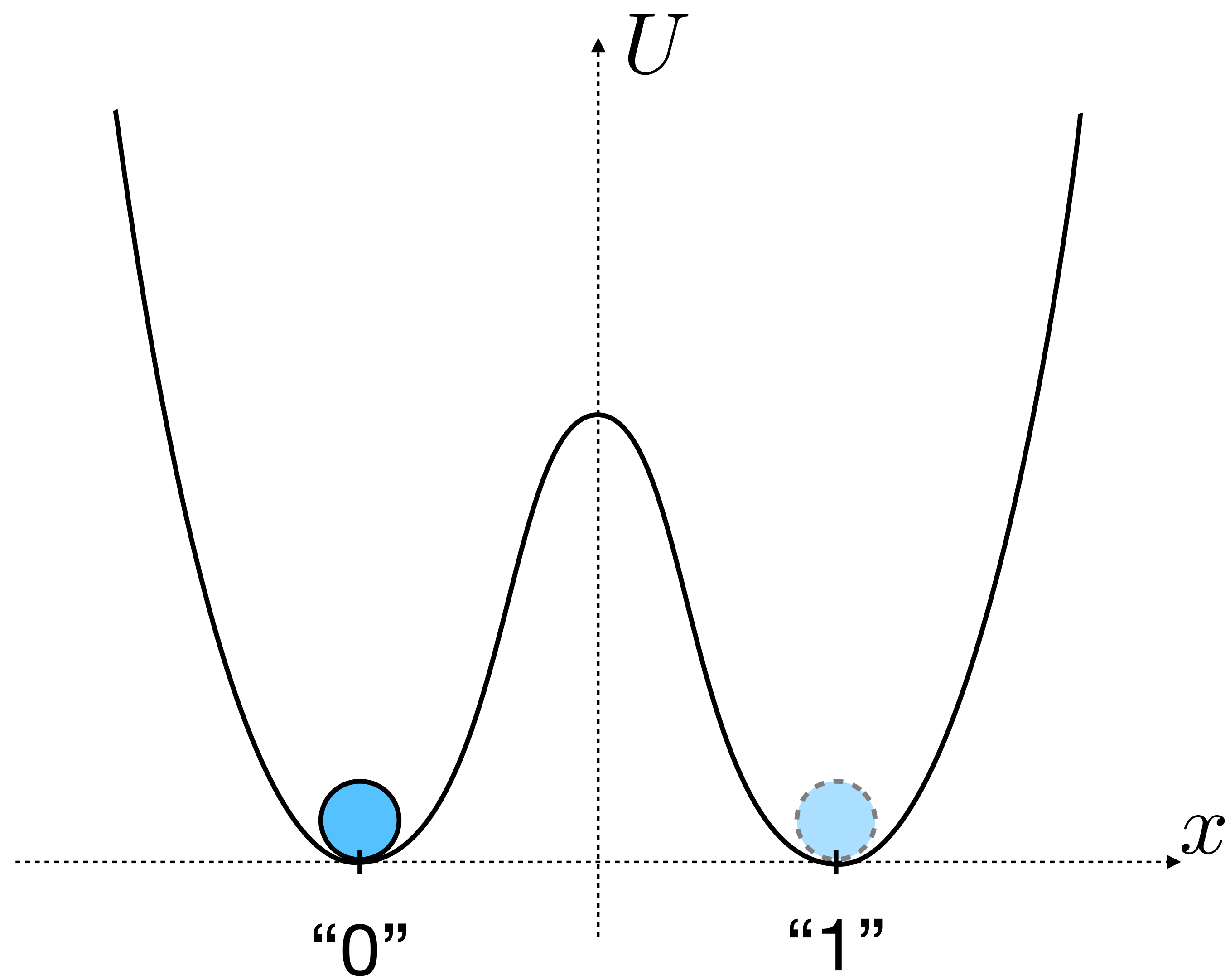
- 1 DOF

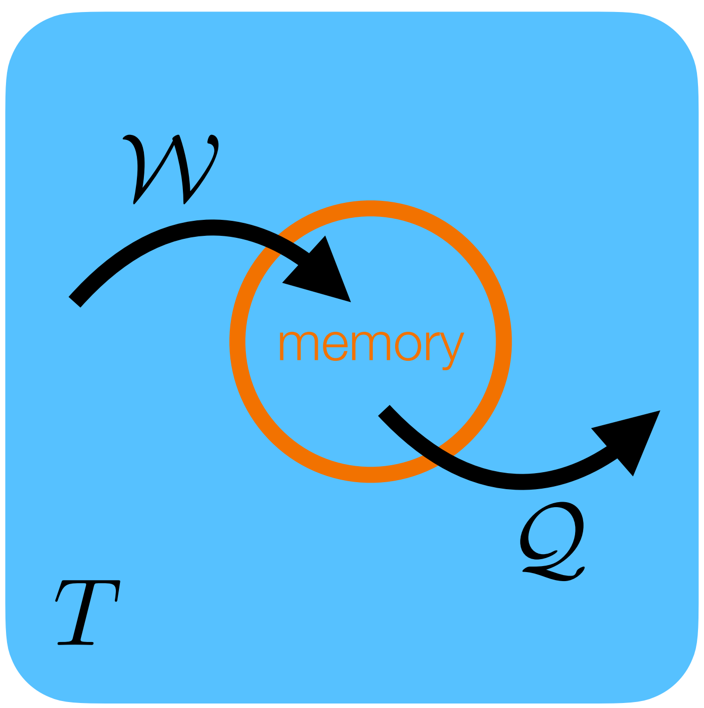




$$\boxed{\langle \mathcal{Q} \rangle = \langle \mathcal{W} \rangle \sim k_B T \ln 2 = 3 \times 10^{-21}\ \mathrm{J}}$$

- 1 DOF
- Bistable potential

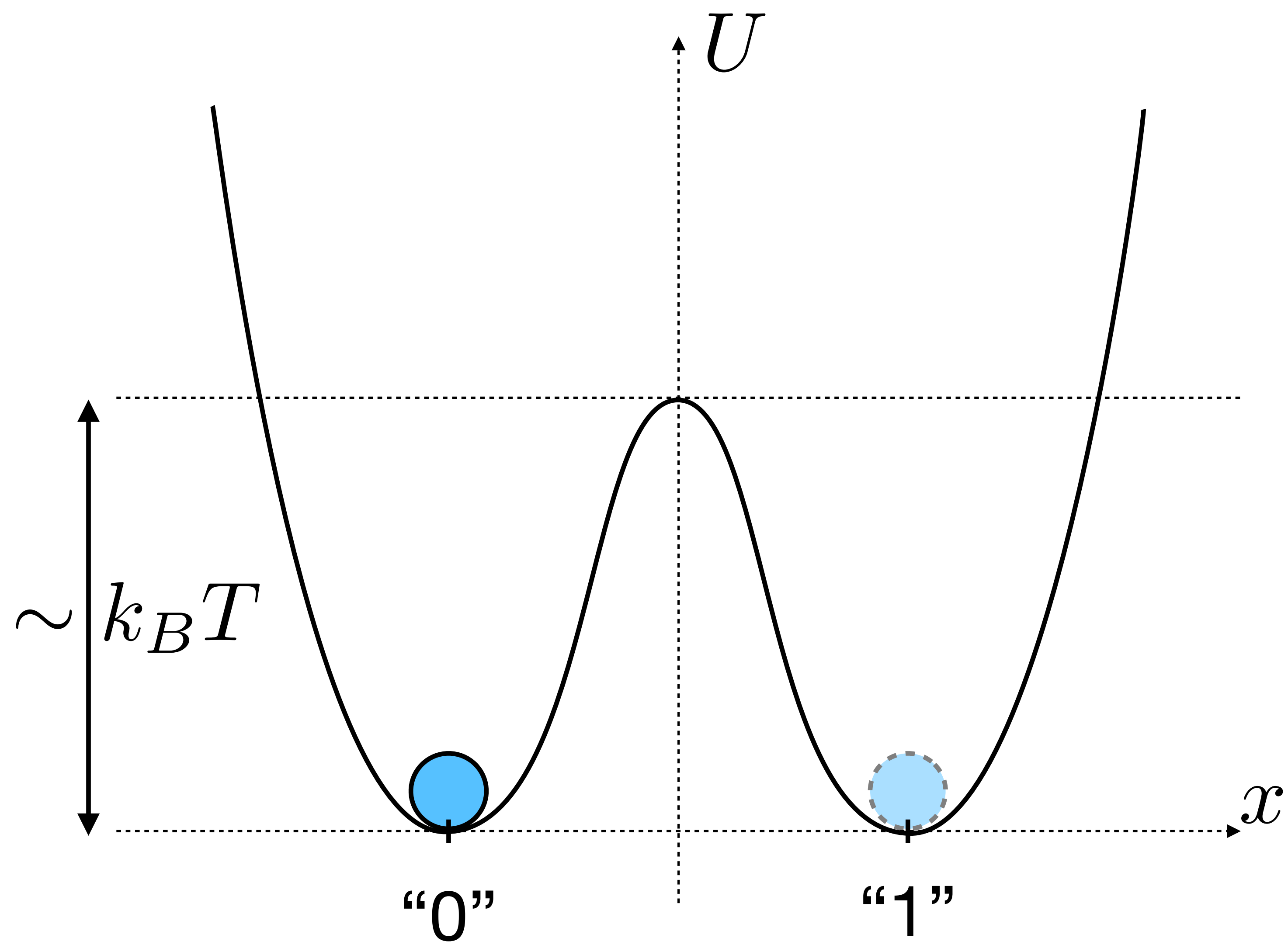


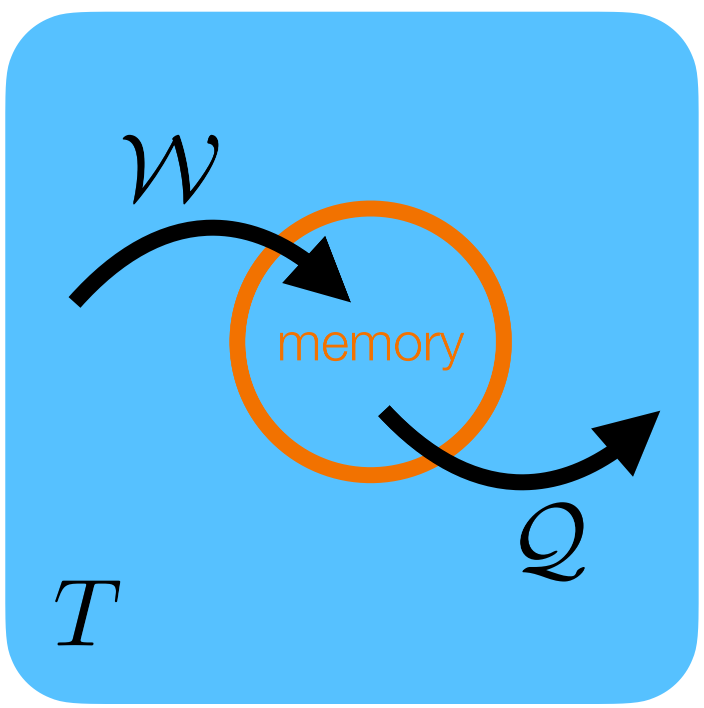


$$\boxed{\langle \mathcal{Q} \rangle = \langle \mathcal{W} \rangle \sim k_B T \ln 2 = 3 \times 10^{-21}\ \mathrm{J}}$$

Experiment: requirements

- 1 DOF
- Bistable potential
- $k_B T$ scale





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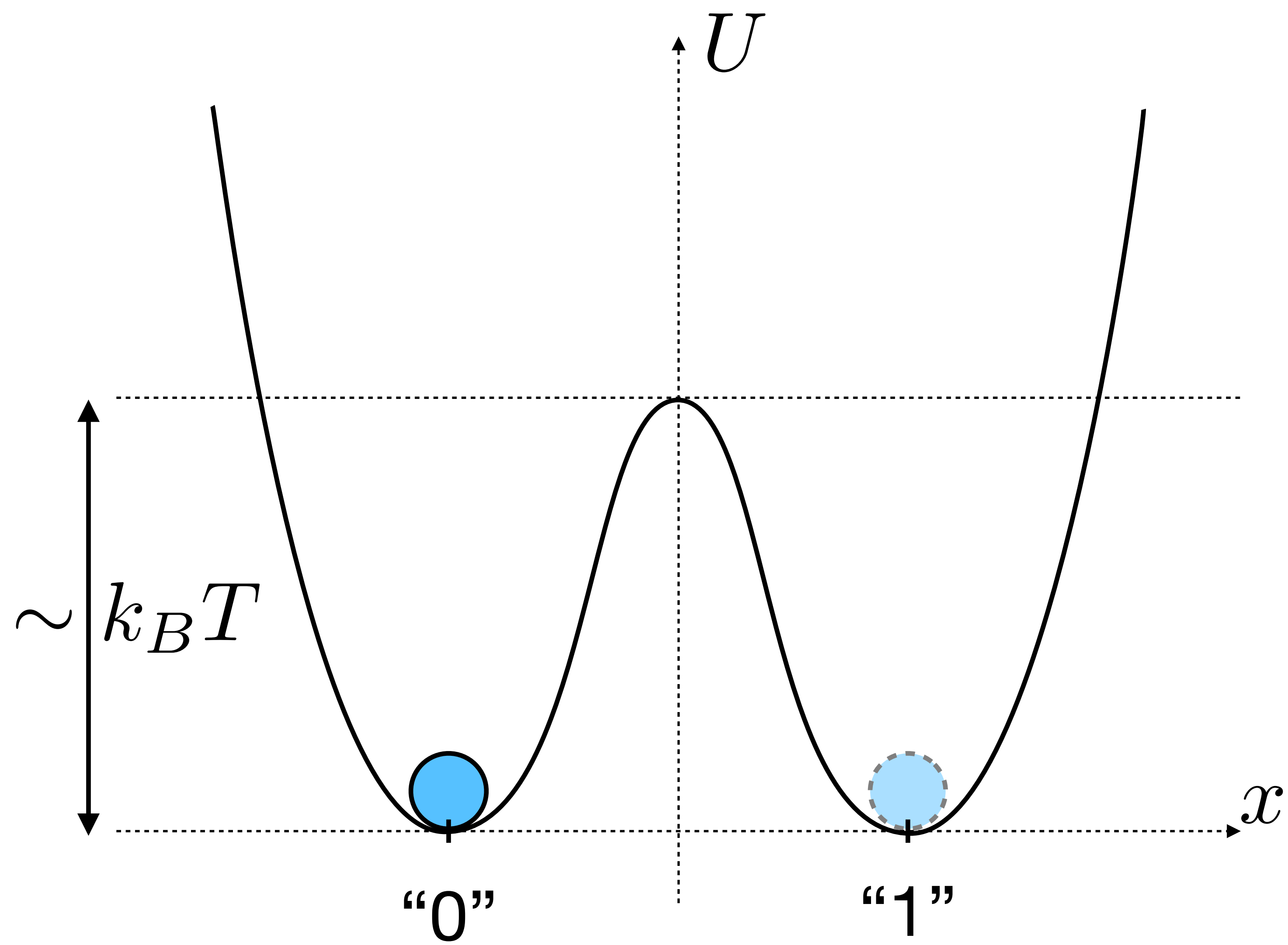
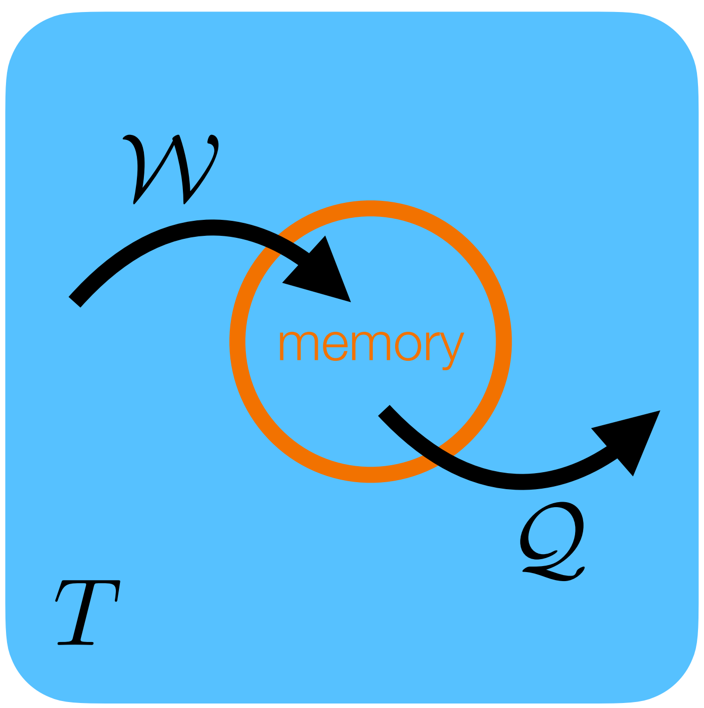
- Tunable potential





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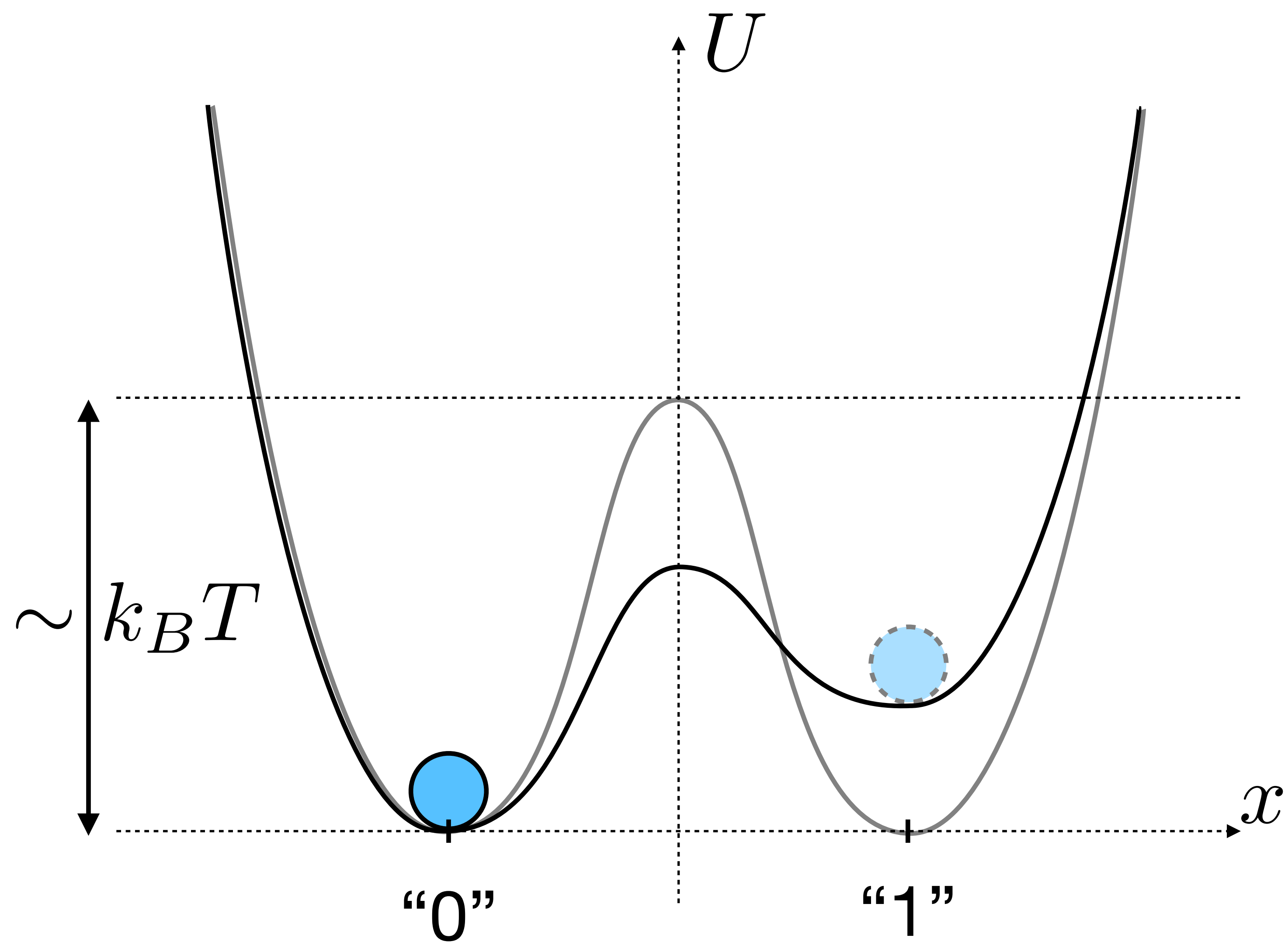

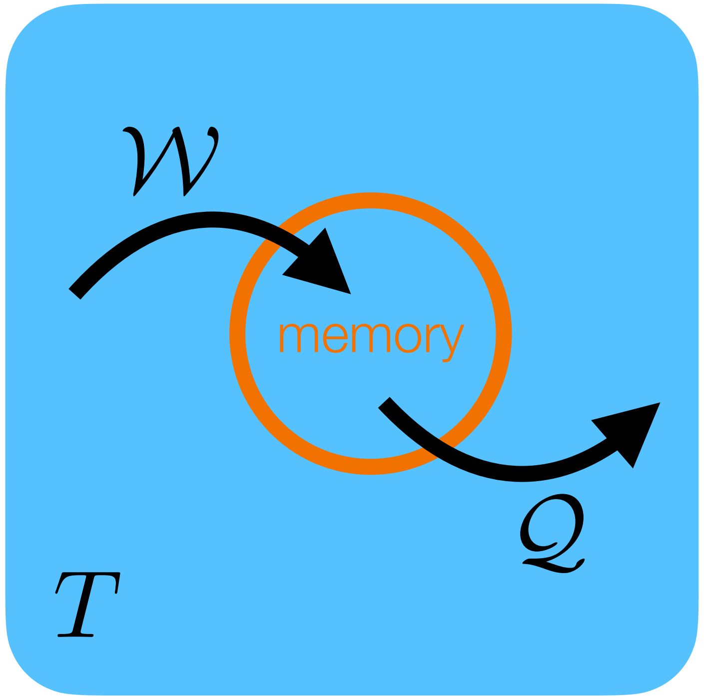

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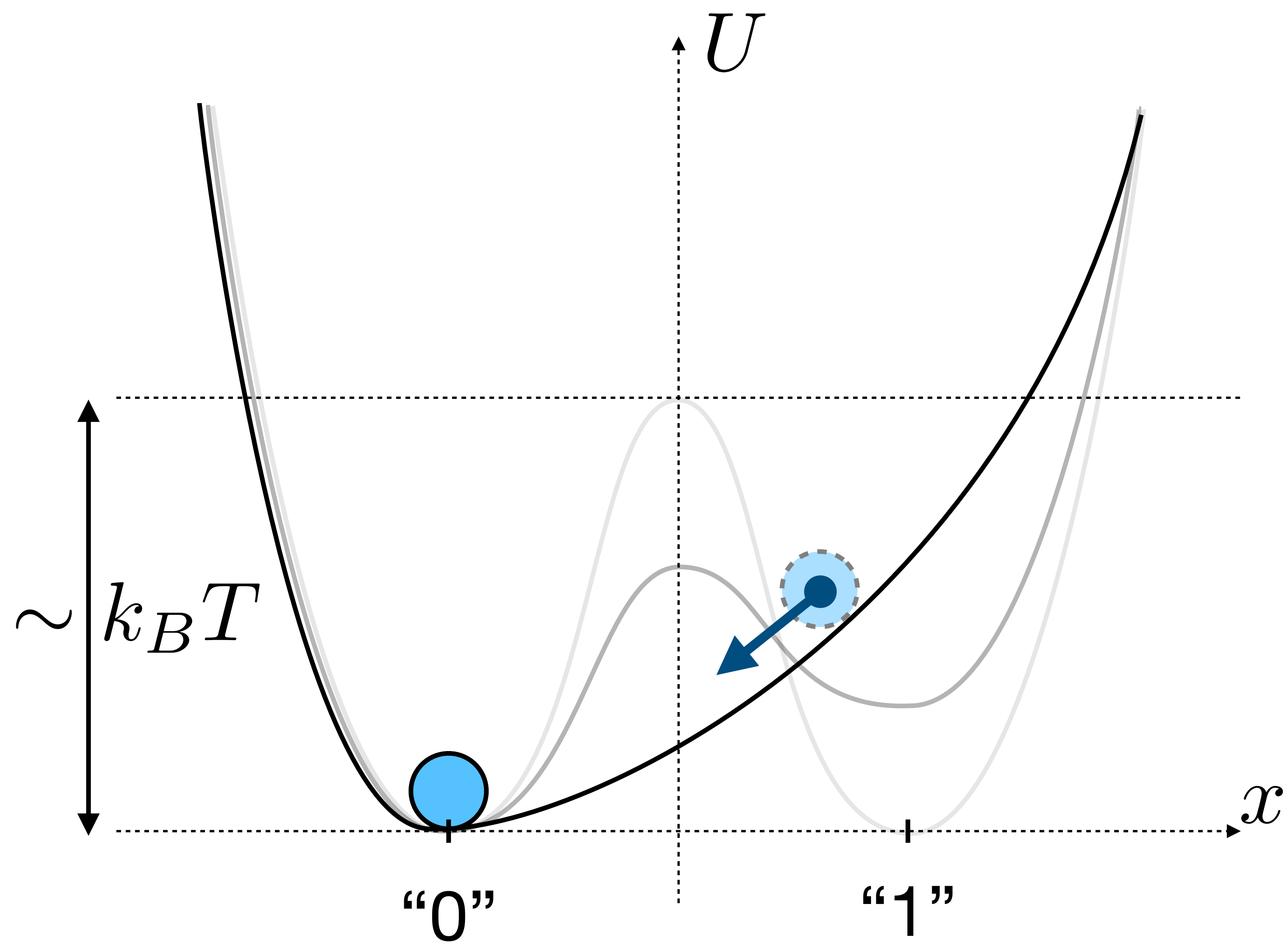

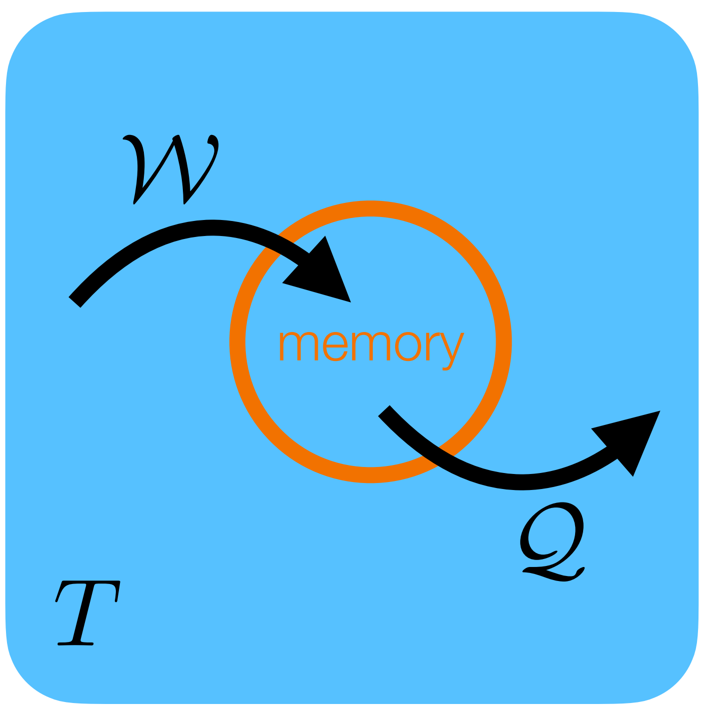

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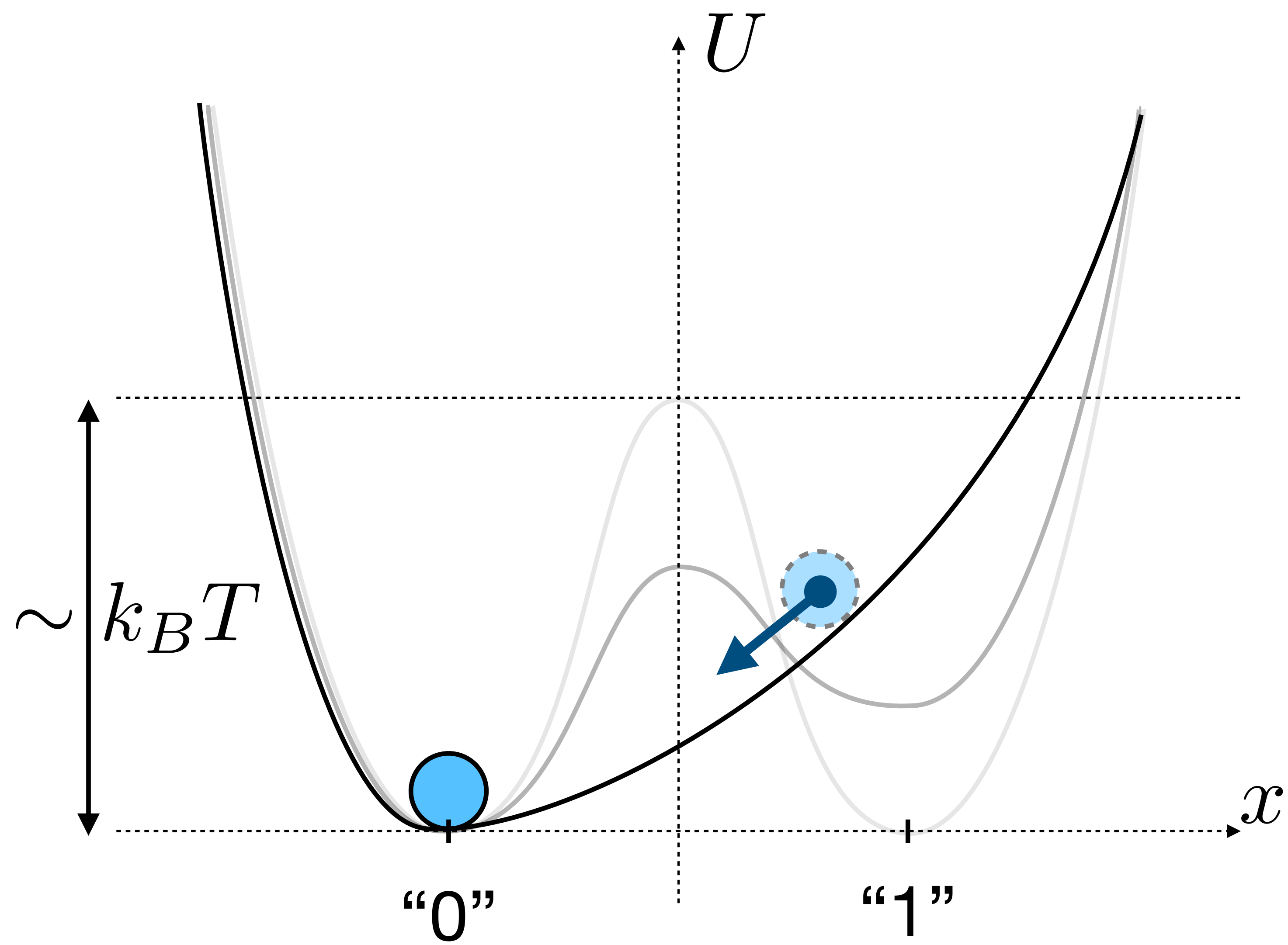

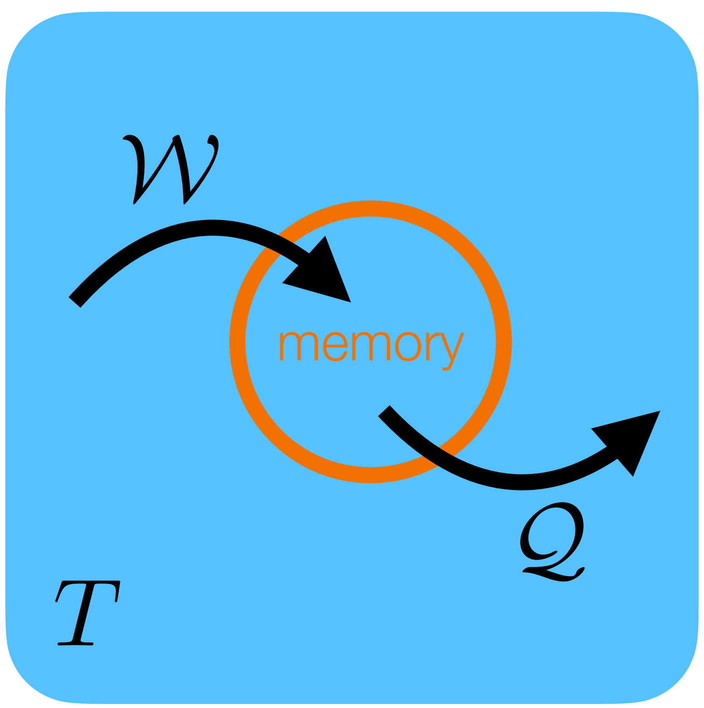

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Stochastic thermodynamics

- 1 DOF
- Bistable potential
- $k_B T$ scale
- Tunable potential $\longrightarrow U(x, \lambda)$
- Measure $\mathcal{Q}$ & $\mathcal{W}$

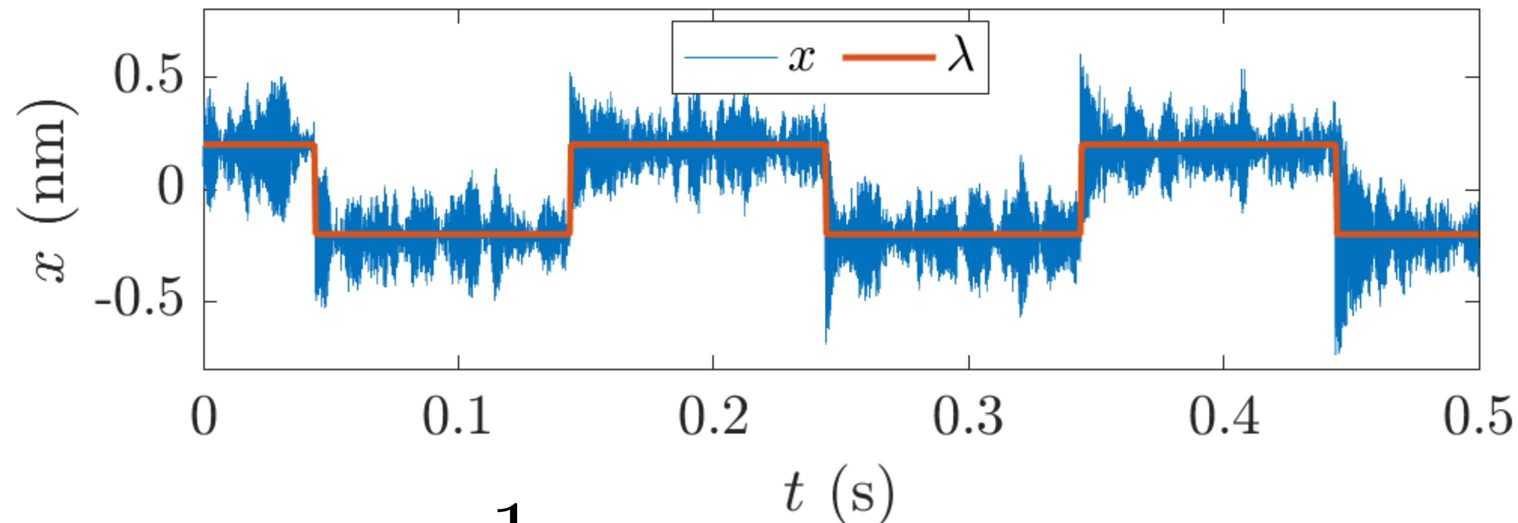
white noise, variance 1

Overdamped Langevin eq. $$\gamma \dot{x} + \frac{\partial U}{\partial x} = \sqrt{2 k_B T \gamma} \eta$$

$$\mathcal{W} = \int_0^\tau \frac{\partial U}{\partial \lambda} \dot{\lambda} dt \qquad \mathcal{Q} = -\int_0^\tau \frac{\partial U}{\partial x} \dot{x} dt$$

$$\Delta U = \mathcal{W} - \mathcal{Q}$$

K. Sekimoto, Stochastic Energetics, *Lecture Notes in Physics* **799** (Springer, 2010)



- 1 DOF
- Bistable potential
- $k_B T$ scale
- Tunable potential $\longrightarrow U(x,\lambda) = \frac{1}{2}k(x-\lambda)^2$
- Measure $\mathcal{Q}$ & $\mathcal{W}$

Underdamped Langevin eq. $$m\ddot{x} + \gamma\dot{x} + \frac{\partial U}{\partial x} = \sqrt{2k_B T\gamma}\eta$$ white noise, variance 1

$$\mathcal{W} = \int_0^\tau \frac{\partial U}{\partial \lambda}\dot{\lambda}dt \qquad \mathcal{Q} = -\int_0^\tau \frac{\partial U}{\partial x}\dot{x}dt - \Delta K$$

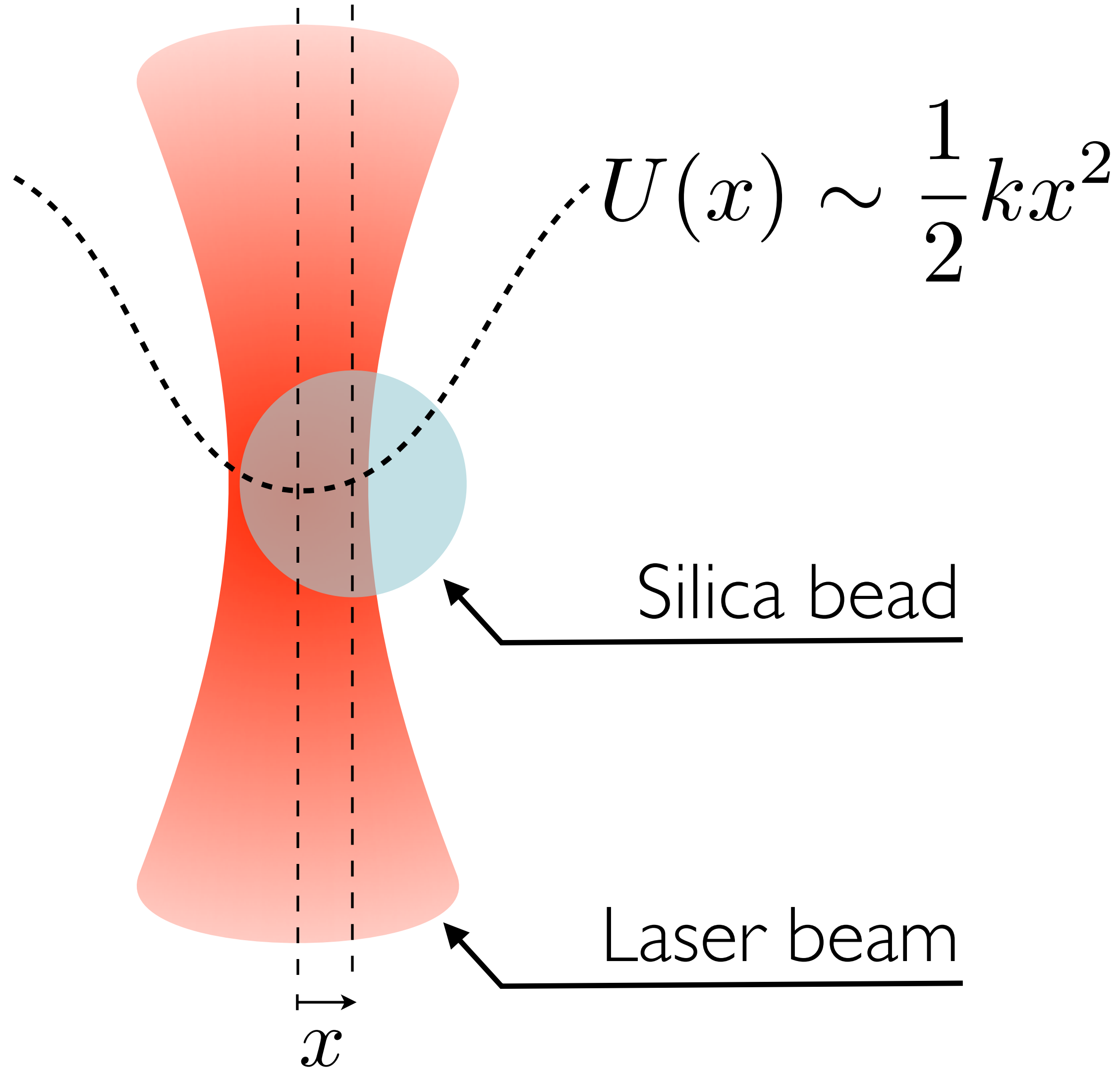
$$\Delta K + \Delta U = \mathcal{W} - \mathcal{Q}$$

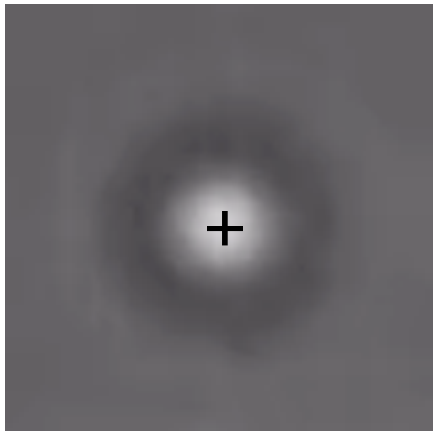
S. Albert, A. Archambault, A. Petrosyan, C. Crauste-Thibierge, L. Bellon, S. Ciliberto, *EPL* **131**, 10008 (2020)

Optical tweezers

- 1 DOF
- Bistable potential
- $k_B T$ scale
- Tunable potential

✓ Measure $\mathcal{Q}$ & $\mathcal{W}$





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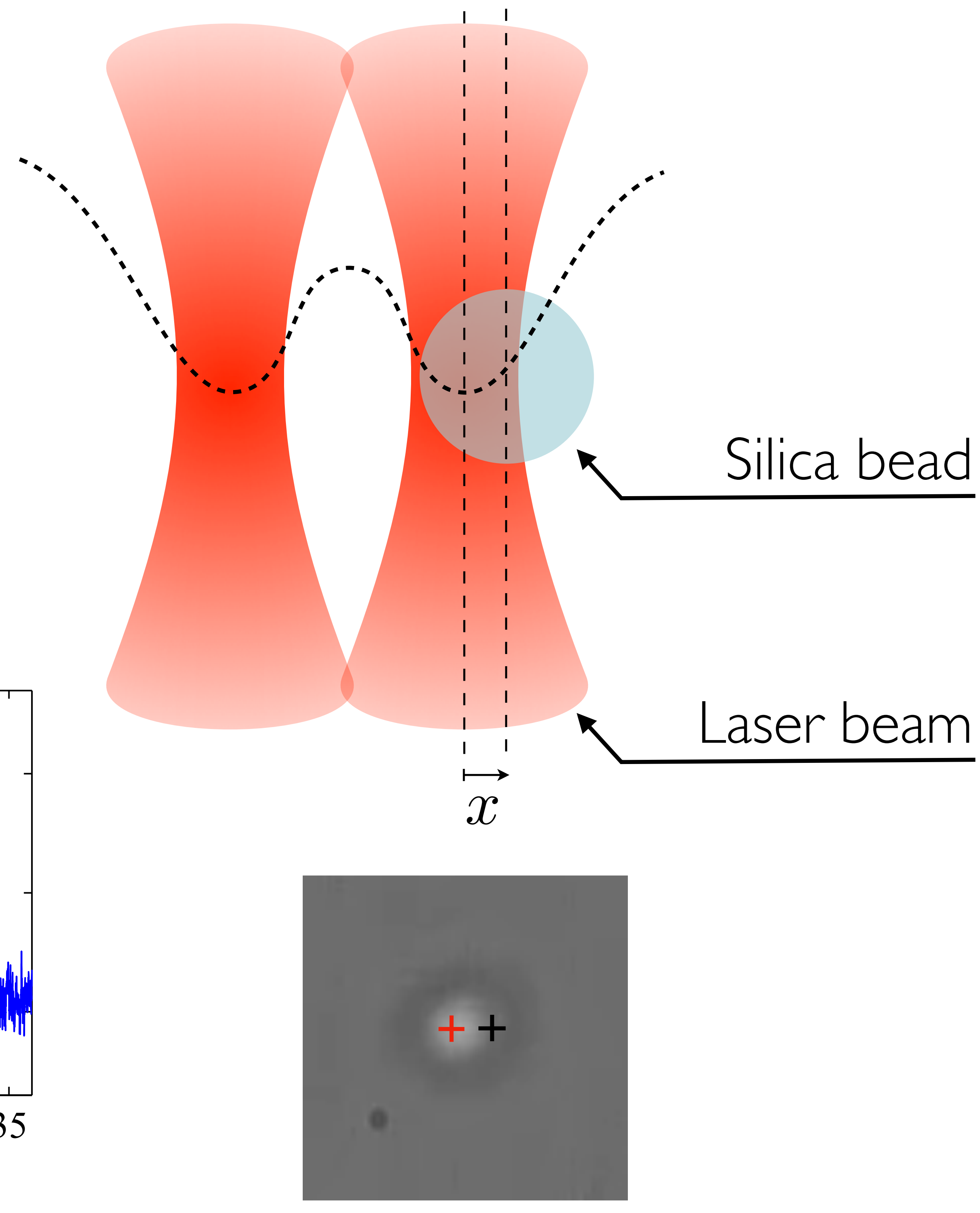

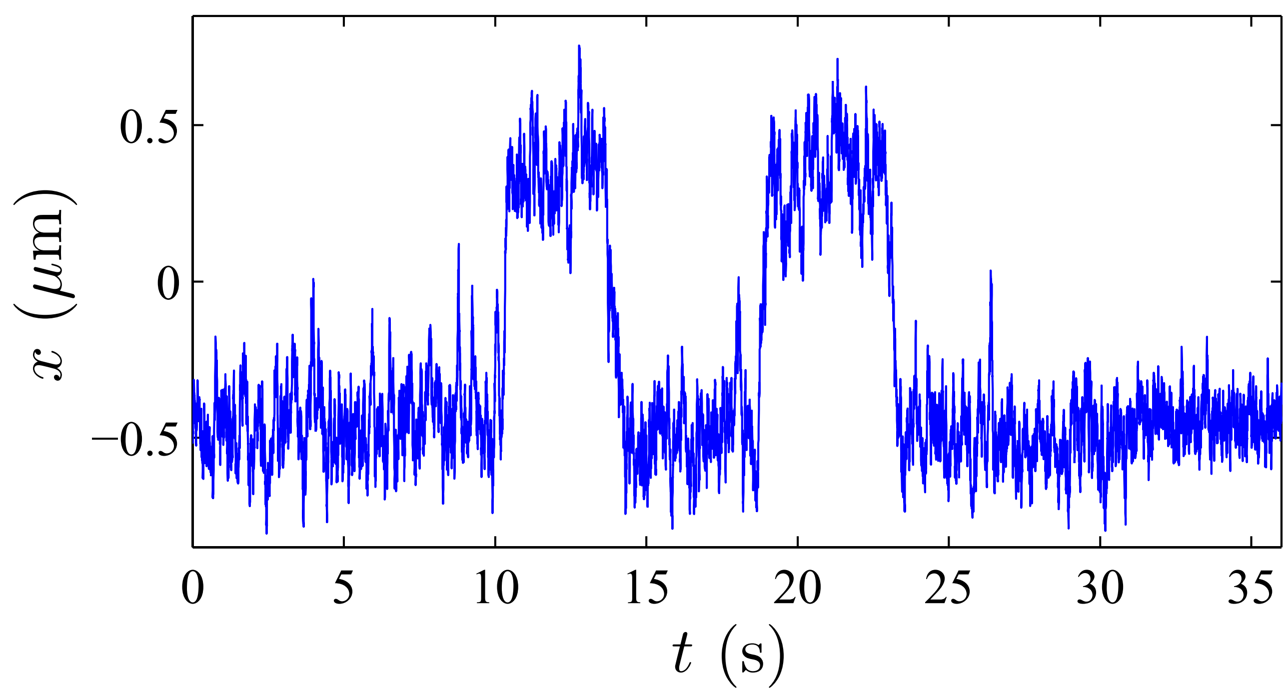

A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, E. Dillenschneider, E. Lutz, *Nature* **483**, 187 (2012)

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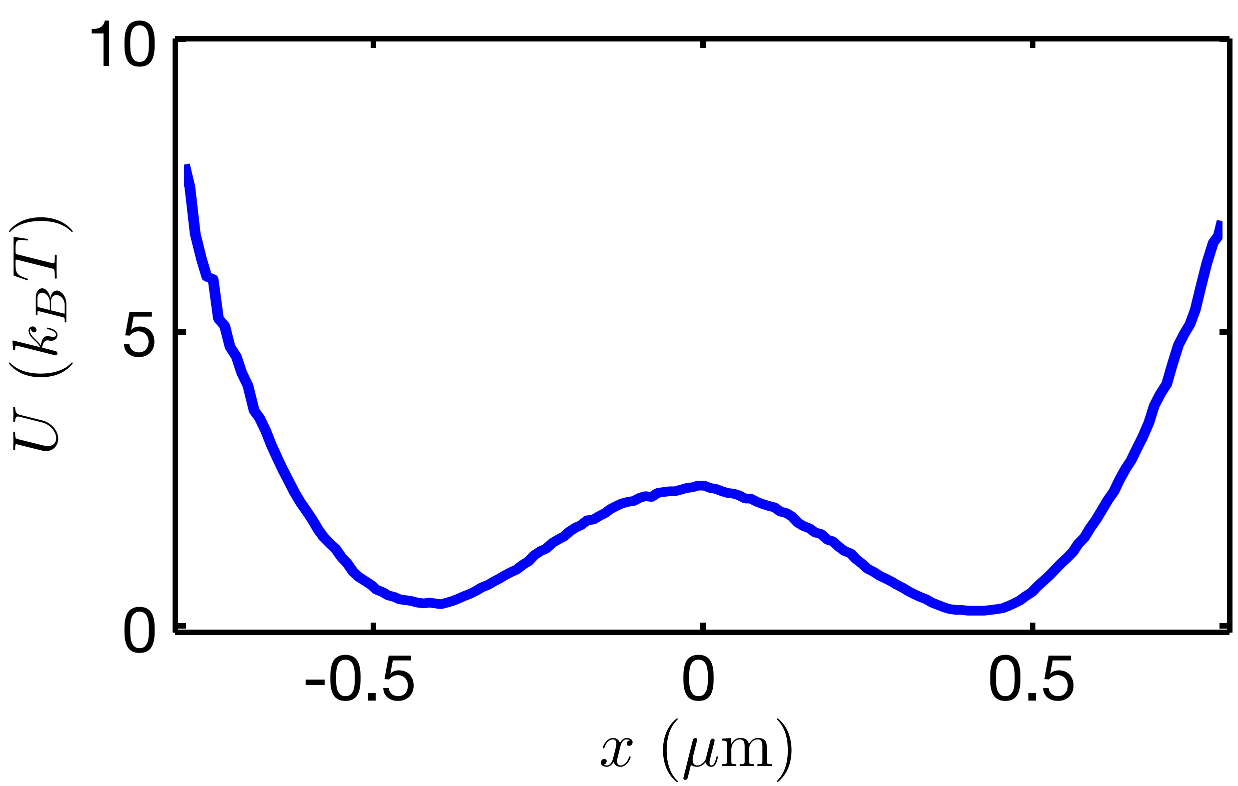

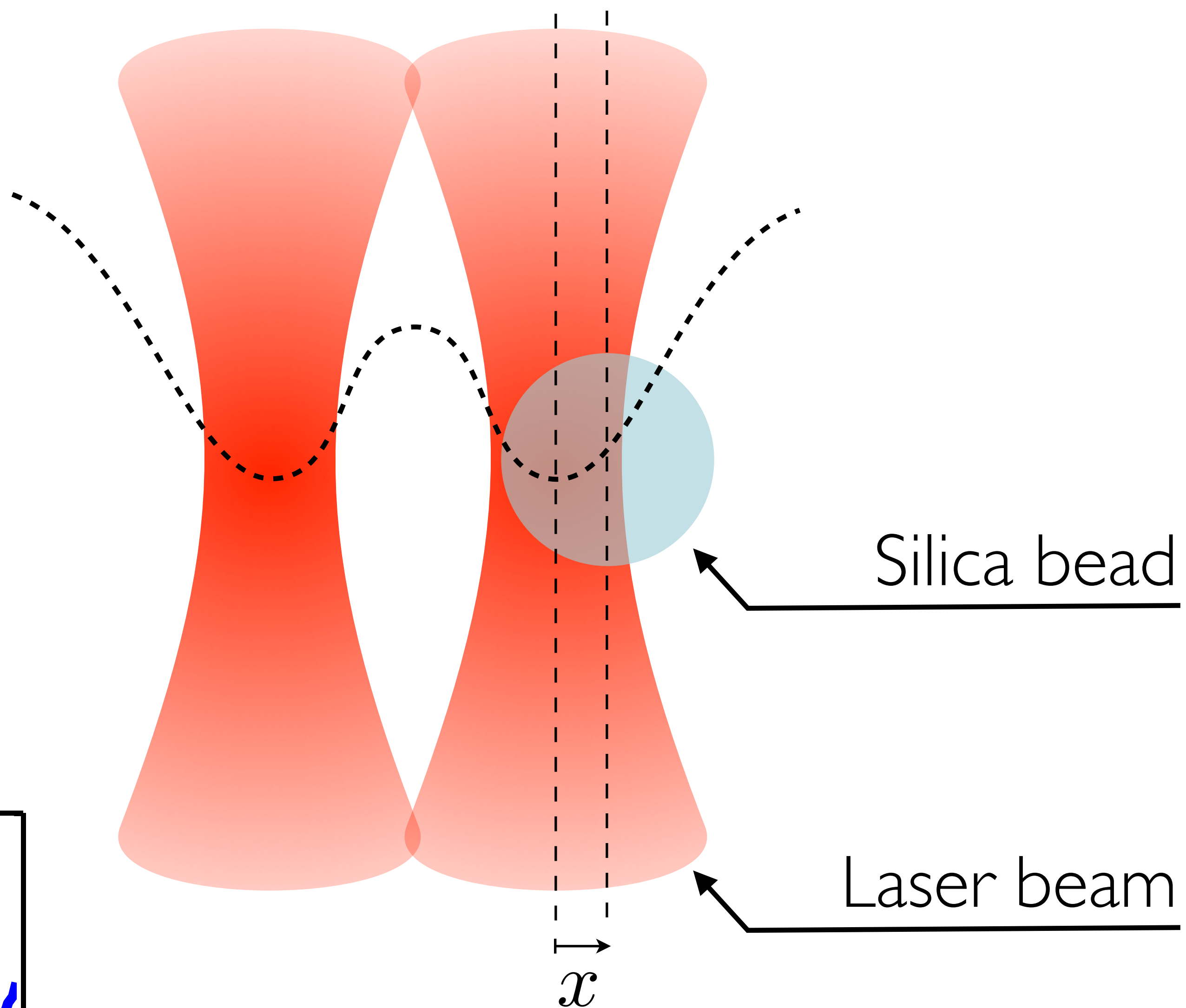

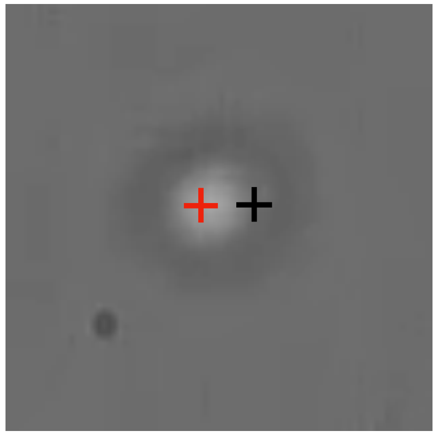

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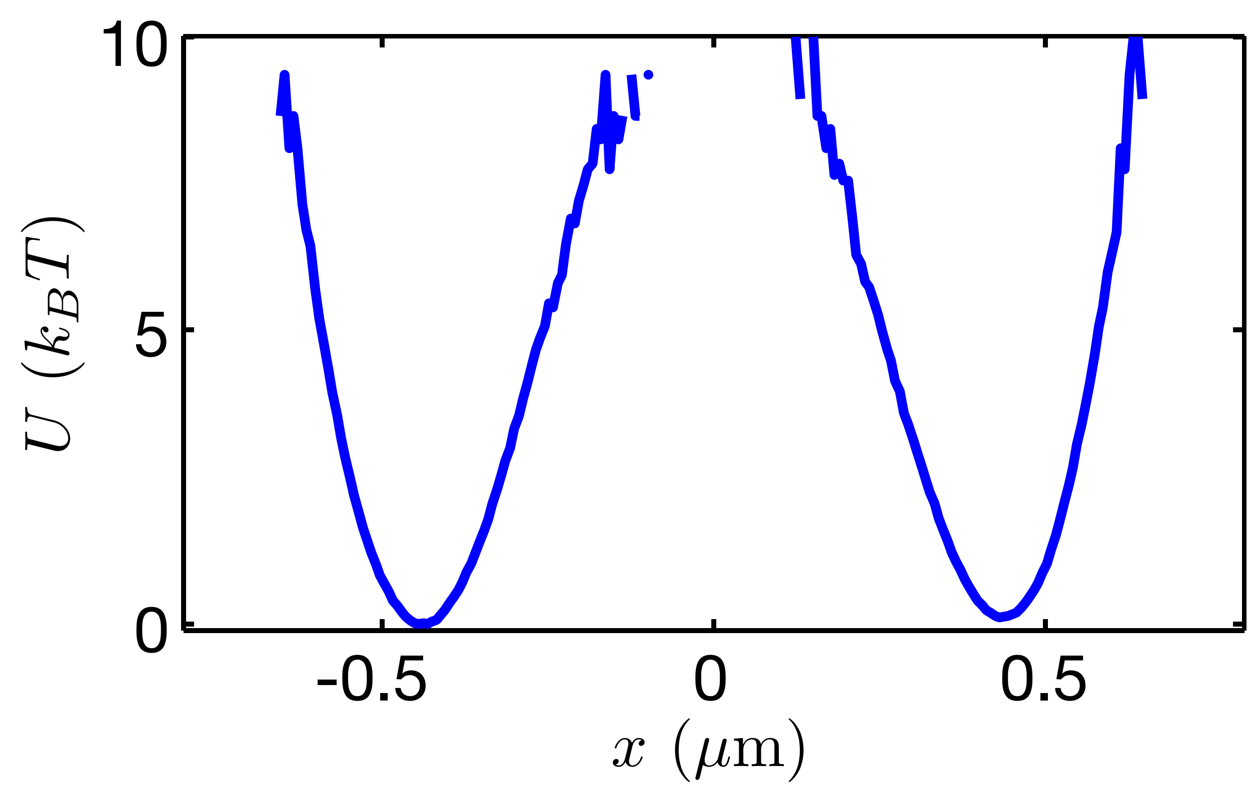

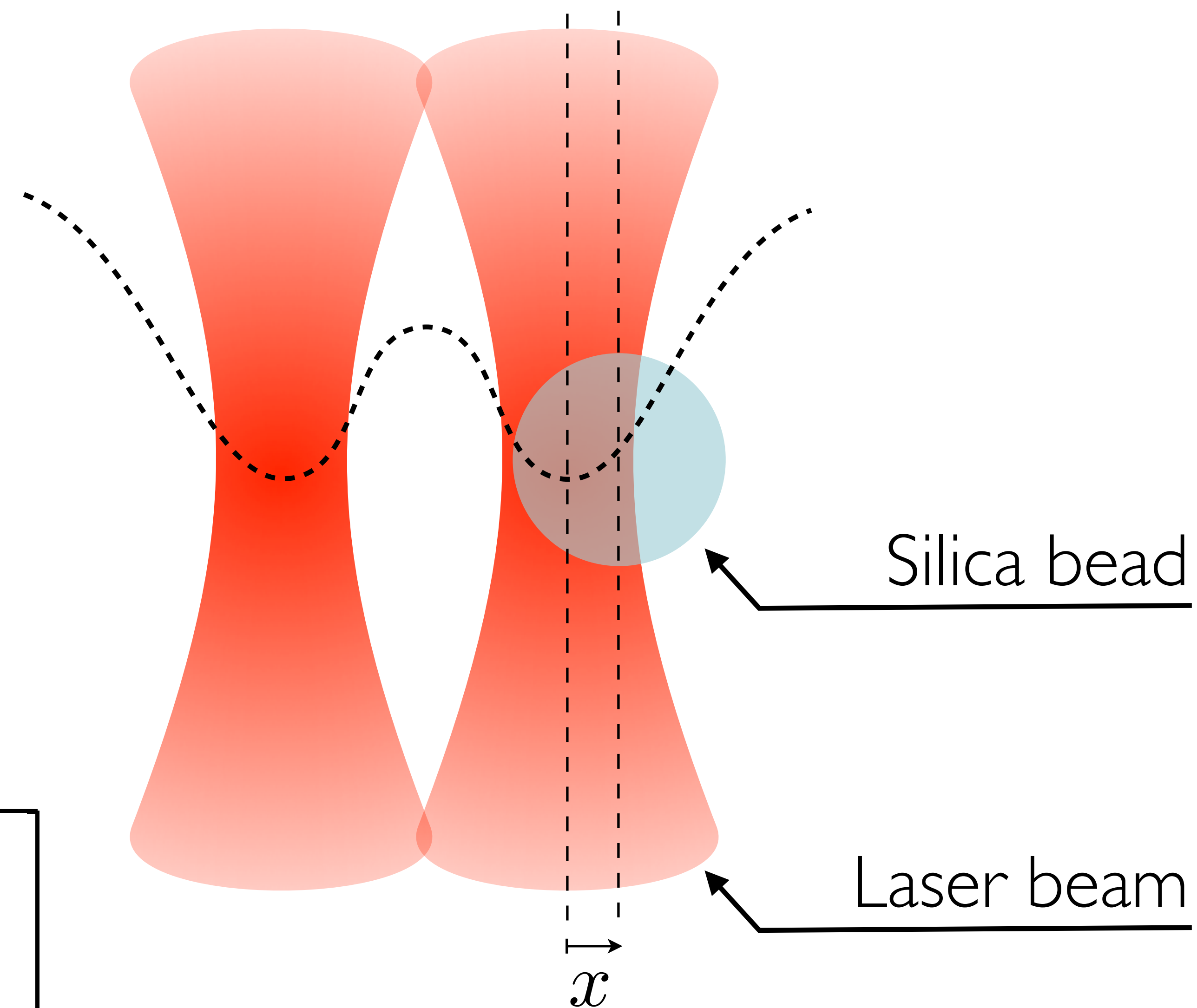

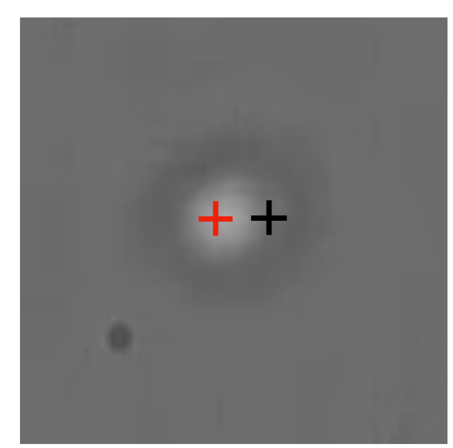

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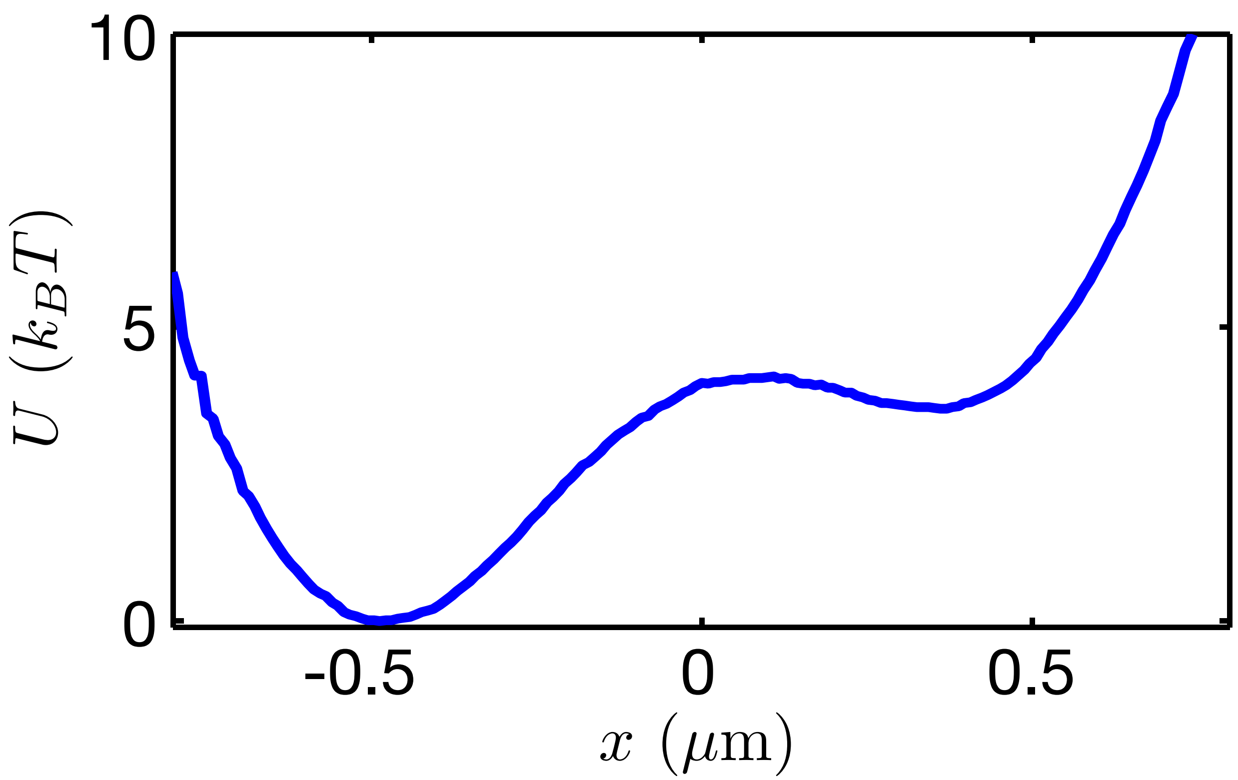

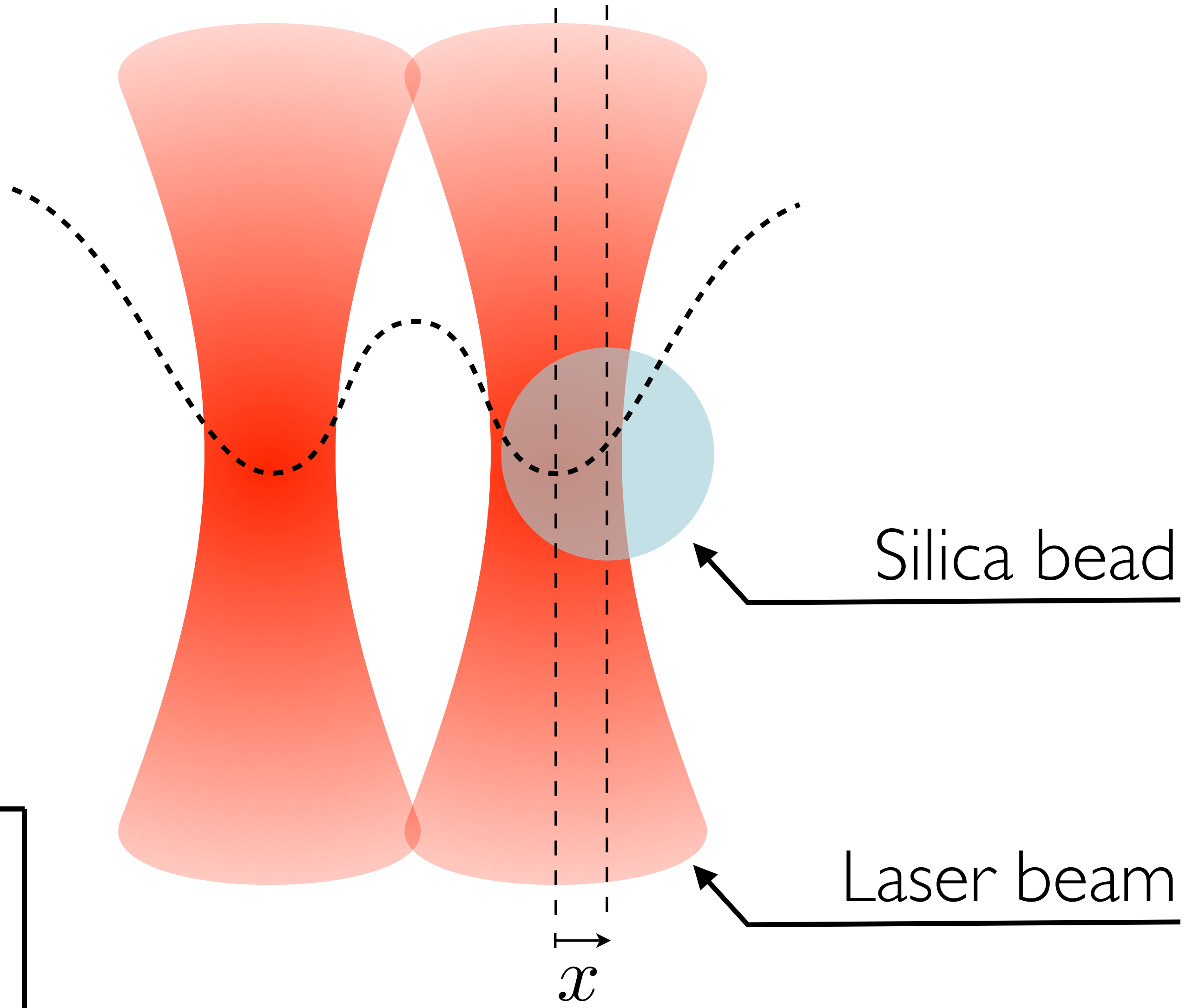

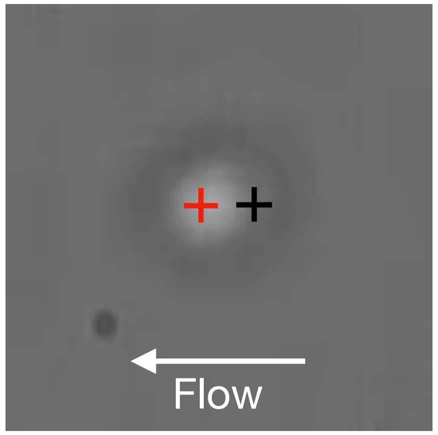

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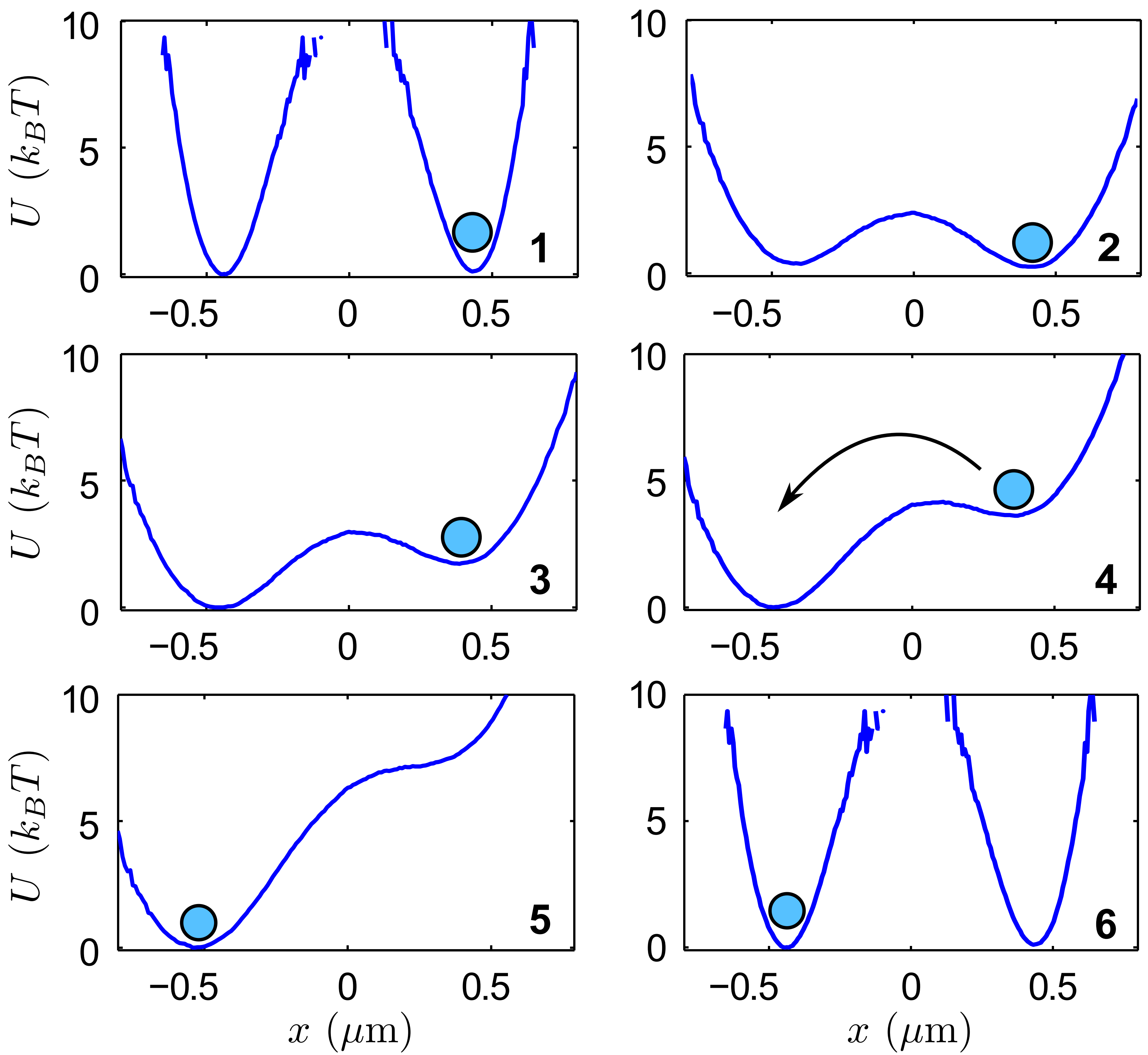
Erasure protocol



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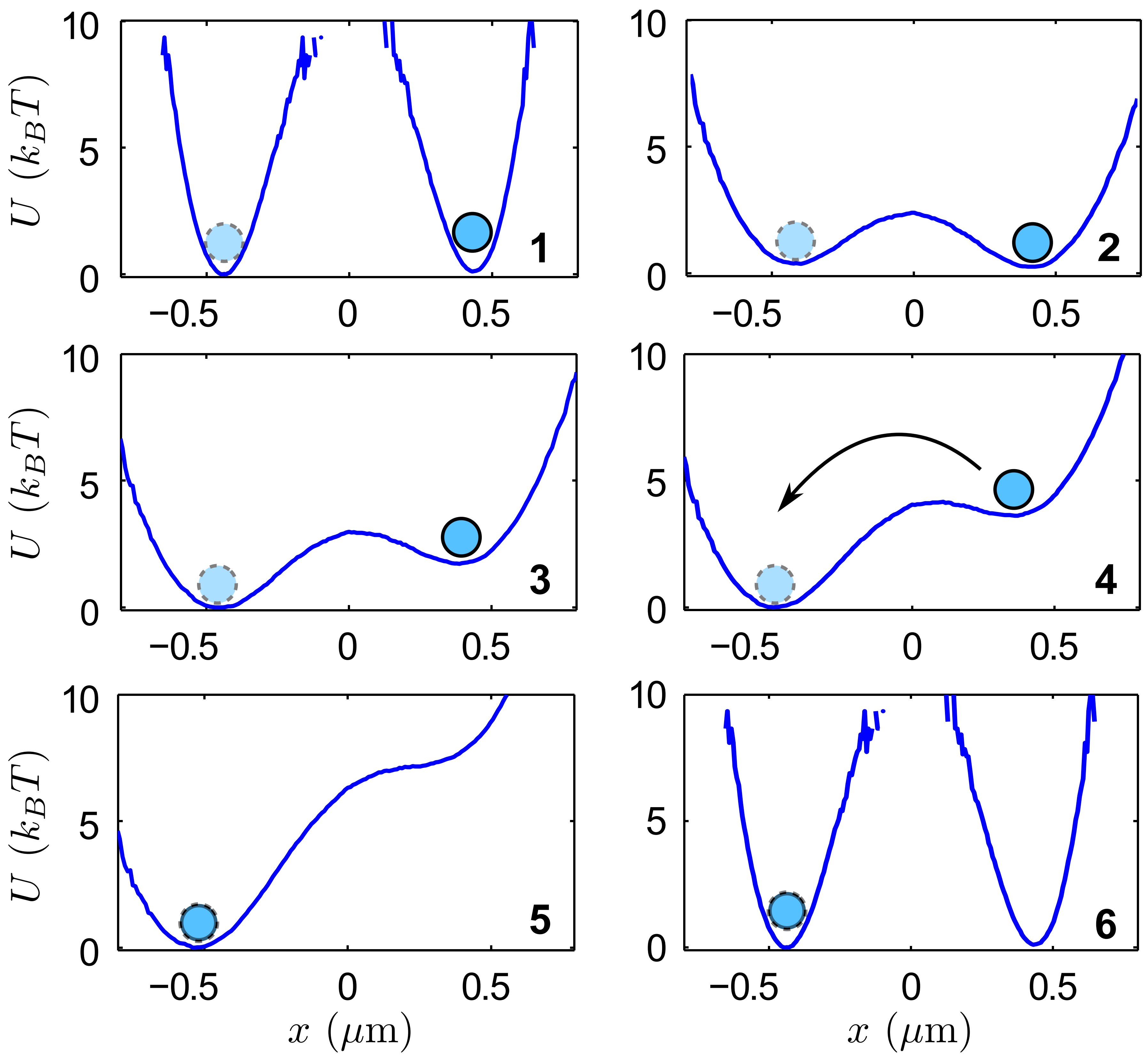

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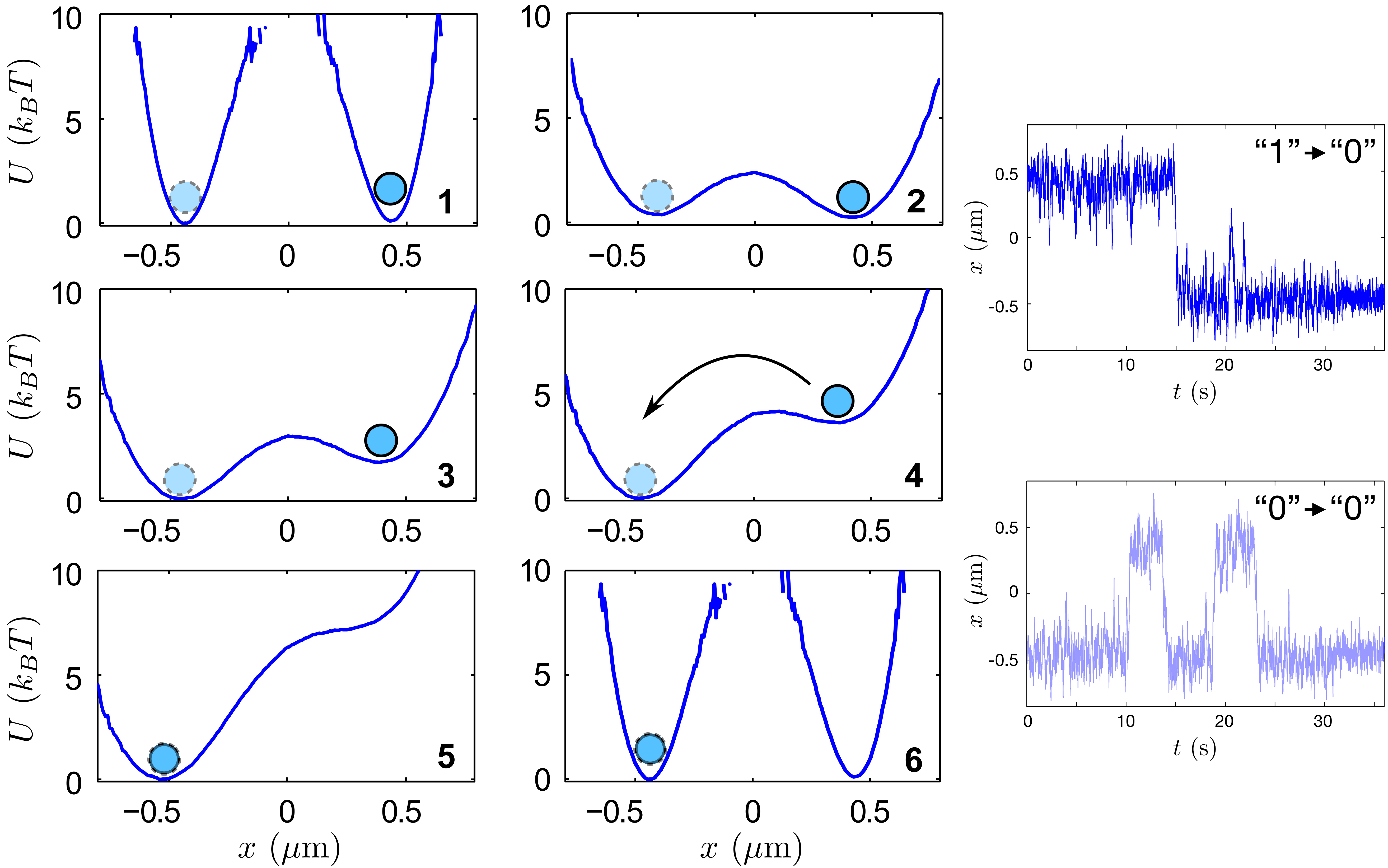

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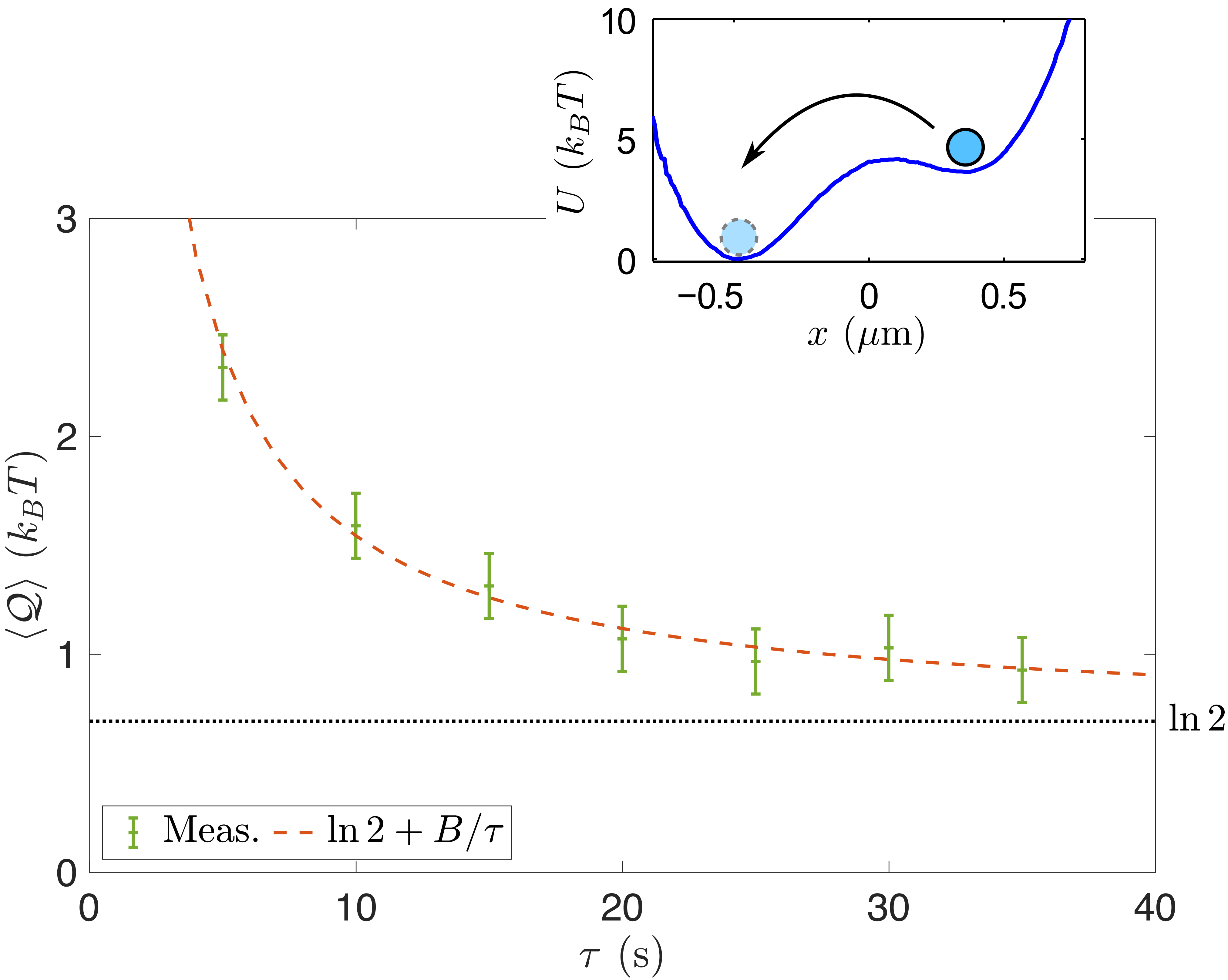
Landauer's limit



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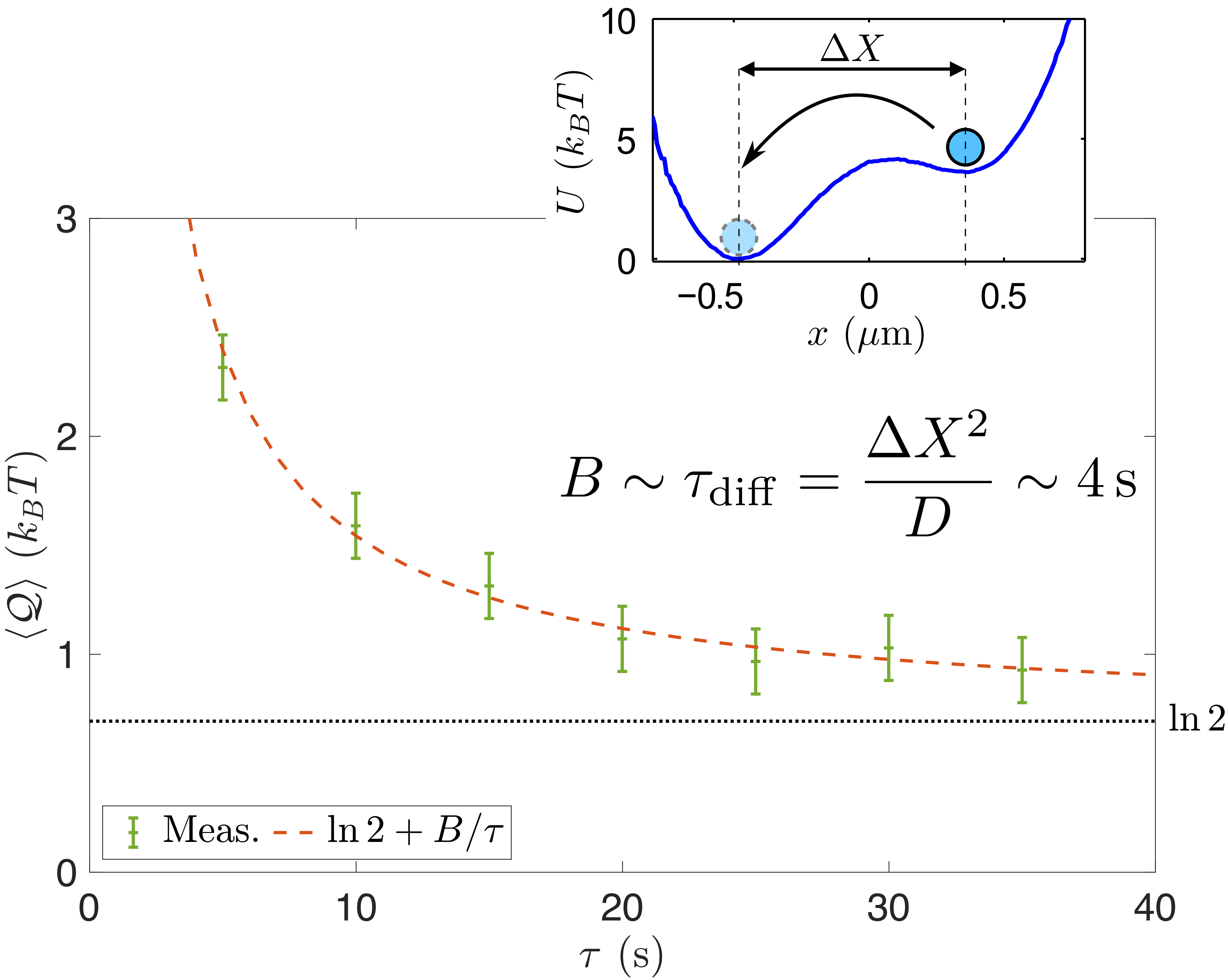
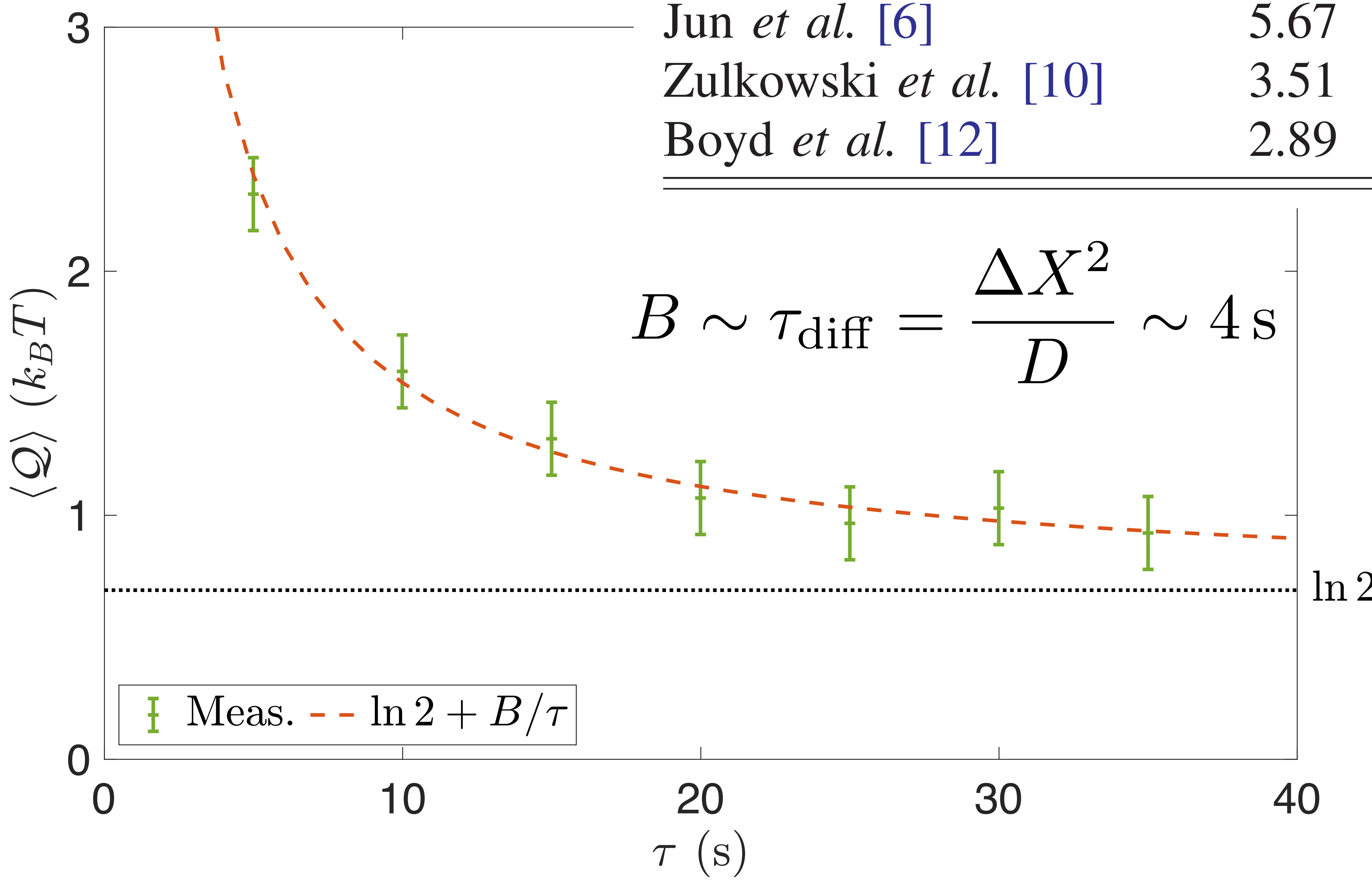


$$B \sim \tau_{\mathrm{diff}} = \frac{\Delta X^2}{D} \sim 4\,\mathrm{s}$$

A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, E. Dillenschneider, E. Lutz, *Nature* **483**, 187 (2012)

Experiment/numerics	$W/k_BT - \ln 2$
Bérut *et al.* [5,36,37]	10.2
Gavrilov *et al.* [8]	7.20
Jun *et al.* [6]	5.67
Zulkowski *et al.* [10]	3.51
Boyd *et al.* [12]	2.89

$$\times \frac{\Delta X^2}{4D\tau}$$



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Karel Proesmans, Jannik Ehrich, John Bechhoefer, *Phys. Rev. Lett.* **125**, 100602 (2020)

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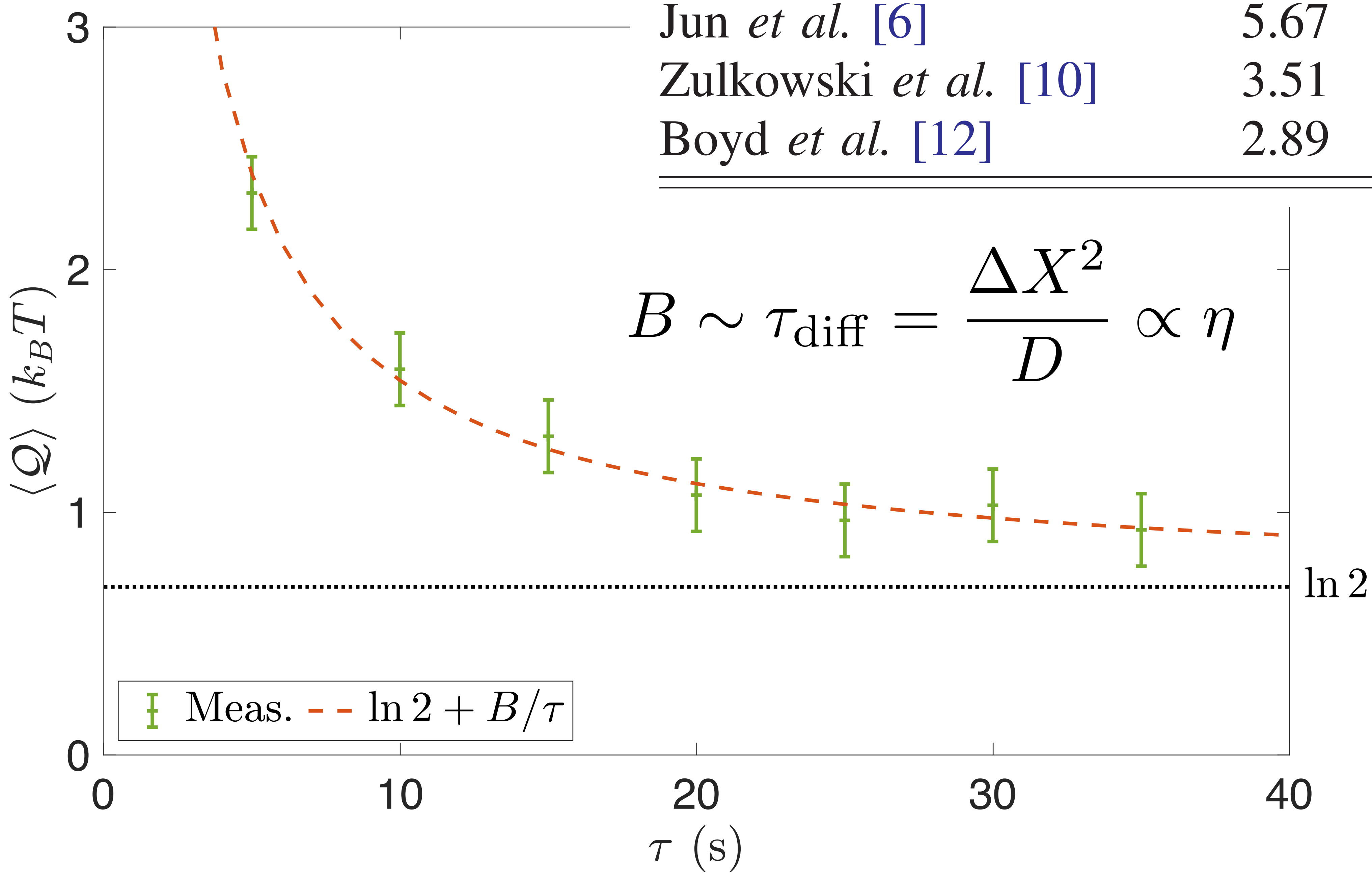
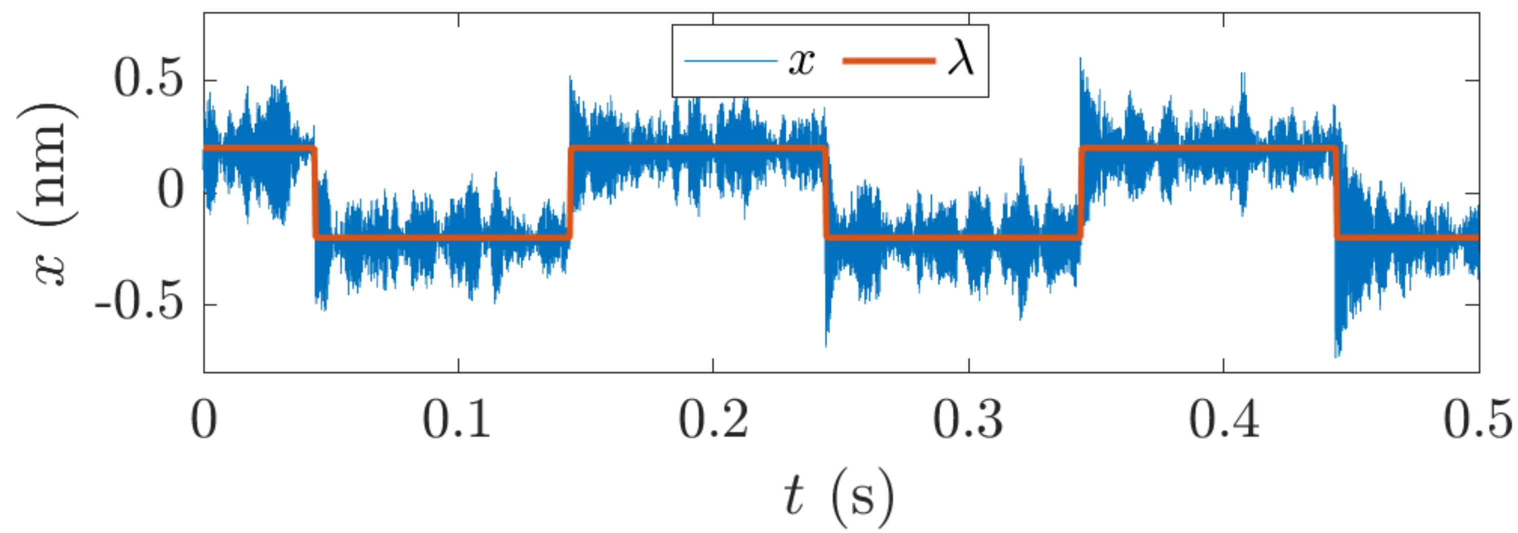


$$B \sim \tau_{\mathrm{diff}} = \frac{\Delta X^2}{D} \propto \eta$$

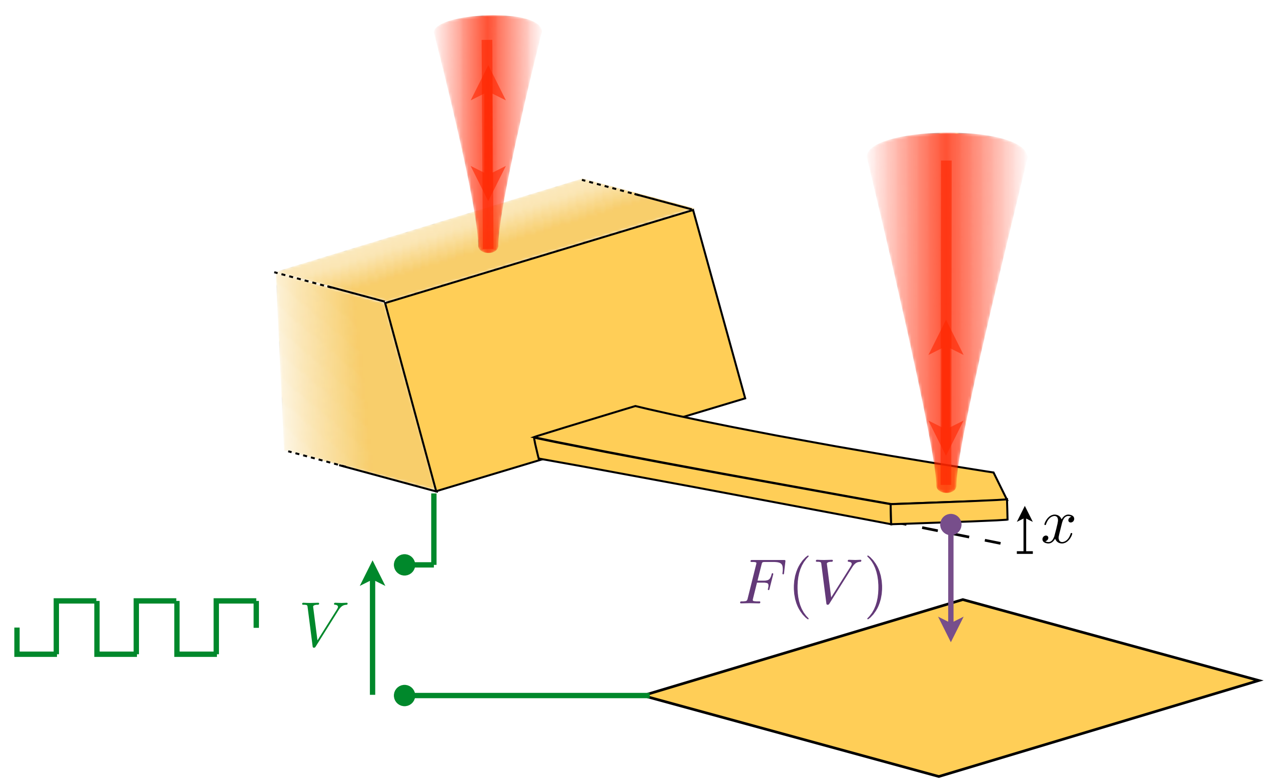
Karel Proesmans, Jannik Ehrich, John Bechhoefer, *Phys. Rev. Lett.* **125**, 100602 (2020)

Underdamped oscillator

- 1 DOF
- Bistable potential
- $k_B T$ scale
- Tunable potential
- Measure $\mathcal{Q}$ & $\mathcal{W}$



$$U(x,\lambda) = \frac{1}{2}k(x-\lambda)^2$$



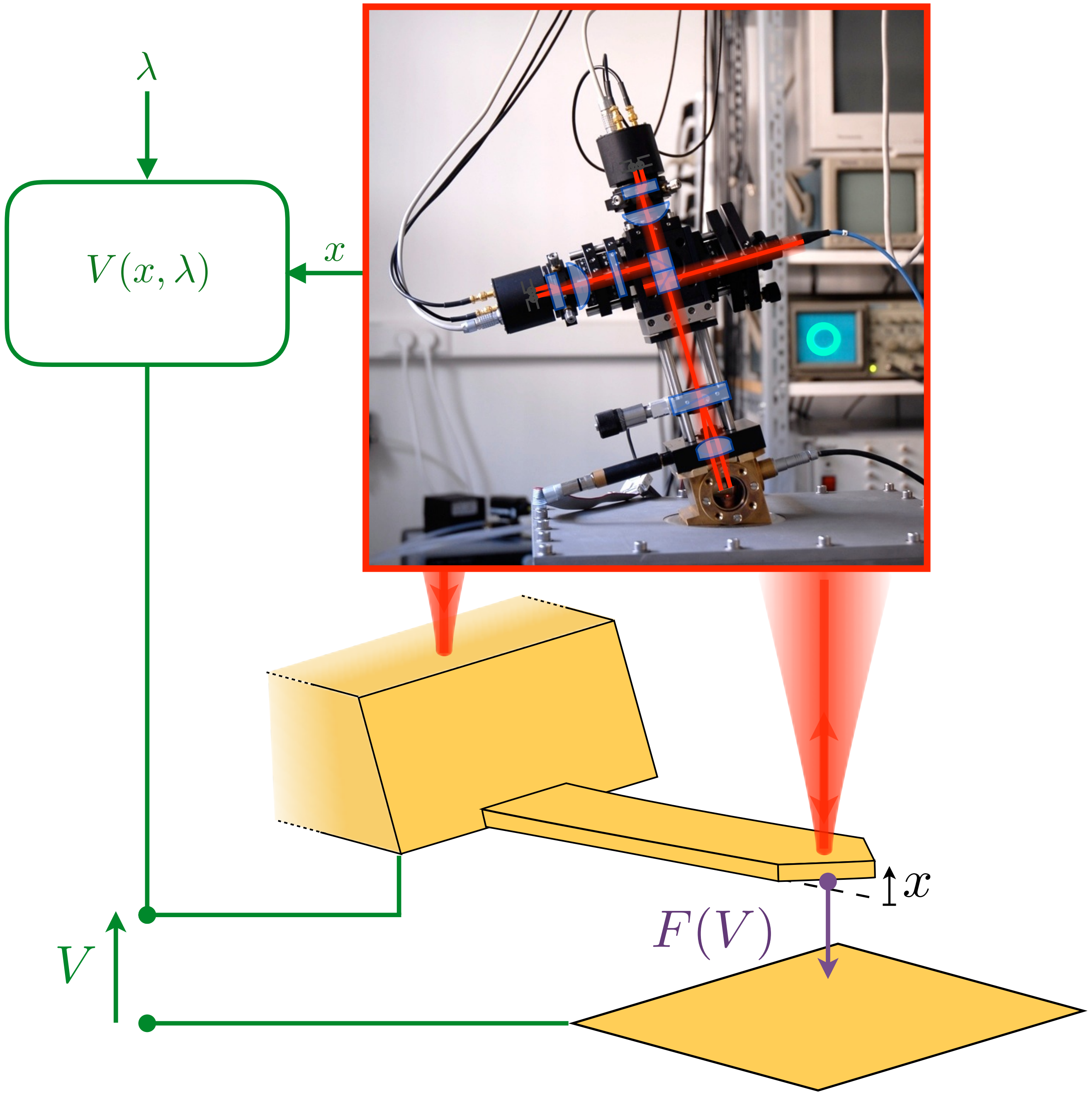
$$\sigma = \sqrt{\frac{k_B T}{k}} \sim 1\,\mathrm{nm}$$

$$f_0 = \sqrt{\frac{k}{m}} = 1270\,\mathrm{Hz}, \quad Q \sim 10 \longrightarrow \tau_{\mathrm{relax}} = \frac{Q}{\pi f_0} \sim 2.5ms$$

S. Albert, A. Archambault, A. Petrosyan, C. Crauste-Thibierge, L. Bellon, S. Ciliberto, *EPL* **131**, 10008 (2020)

- 1 DOF
- Bistable potential
- $k_B T$ scale
- Tunable potential
- Measure $\mathcal{Q}$ & $\mathcal{W}$

$U(x,\lambda)$ arbitrary !



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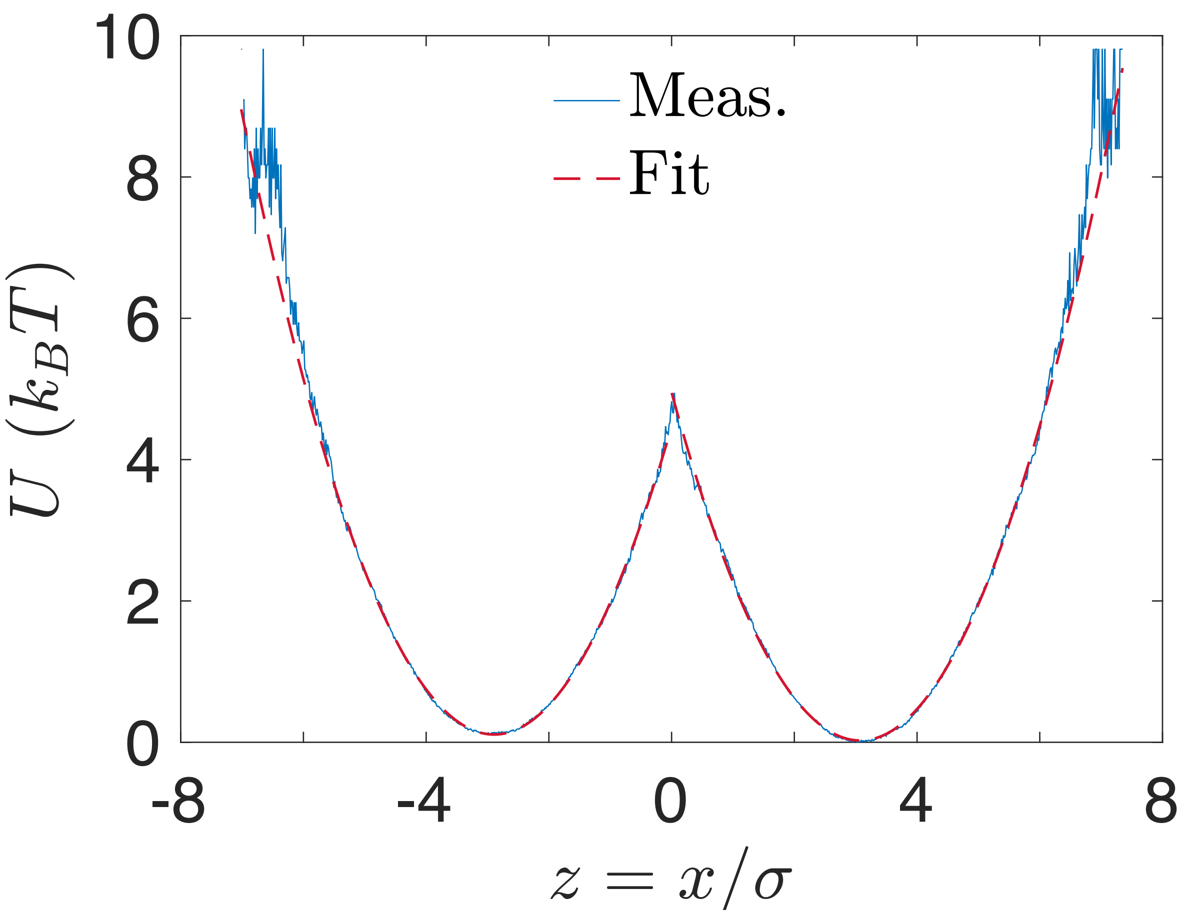
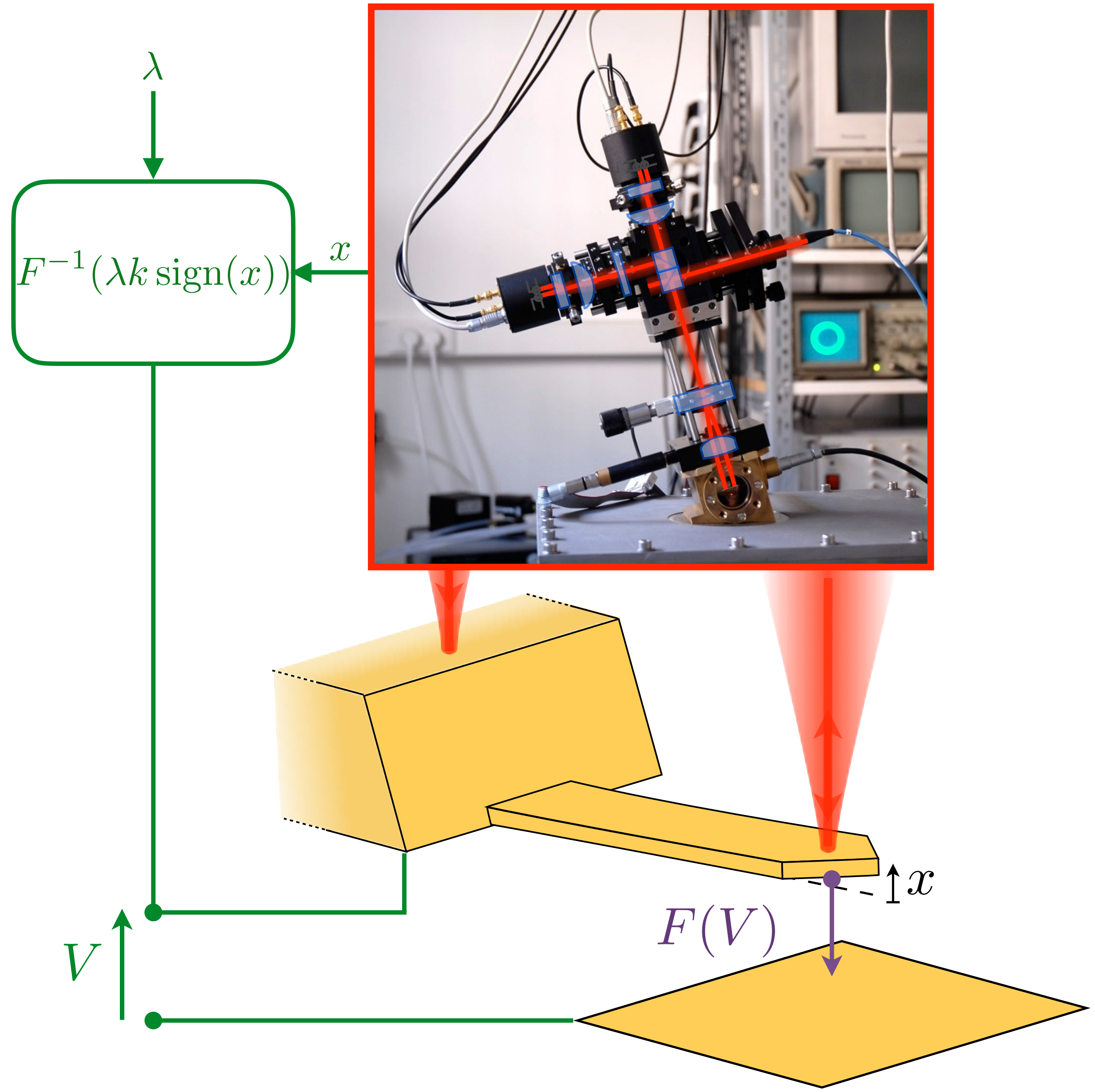
S. Dago, J. Pereda, N. Barros, S. Ciliberto, L. Bellon, *Phys. Rev. Lett.* **126**, 170601 (2021)

Bistable underdamped oscillator



$$U(x,\lambda) = \frac{1}{2}k(|x| - \lambda)^2$$



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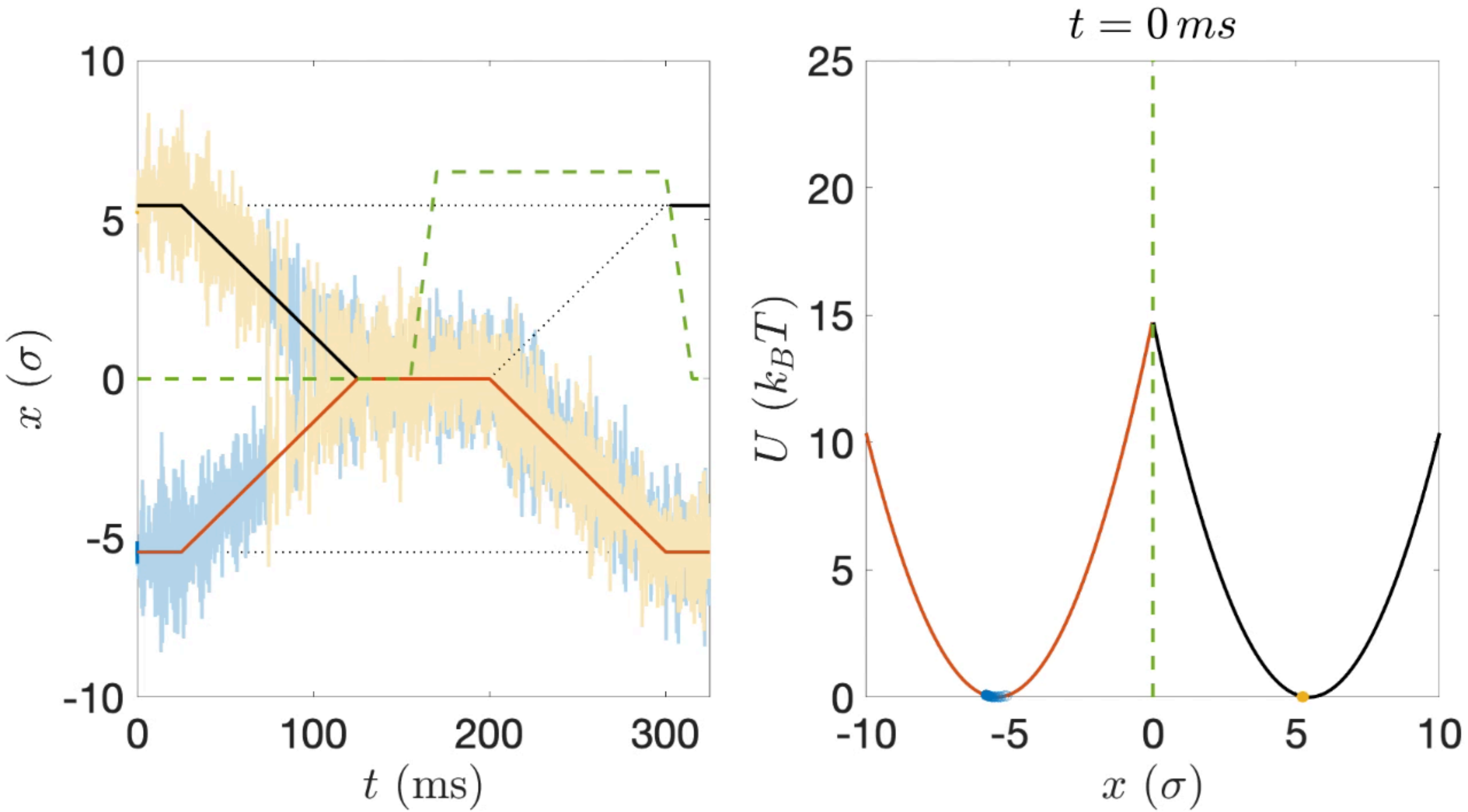
S. Dago, J. Pereda, N. Barros, S. Ciliberto, L. Bellon, *Phys. Rev. Lett.* **126**, 170601 (2021)

Erasure protocol

Click here to play video online



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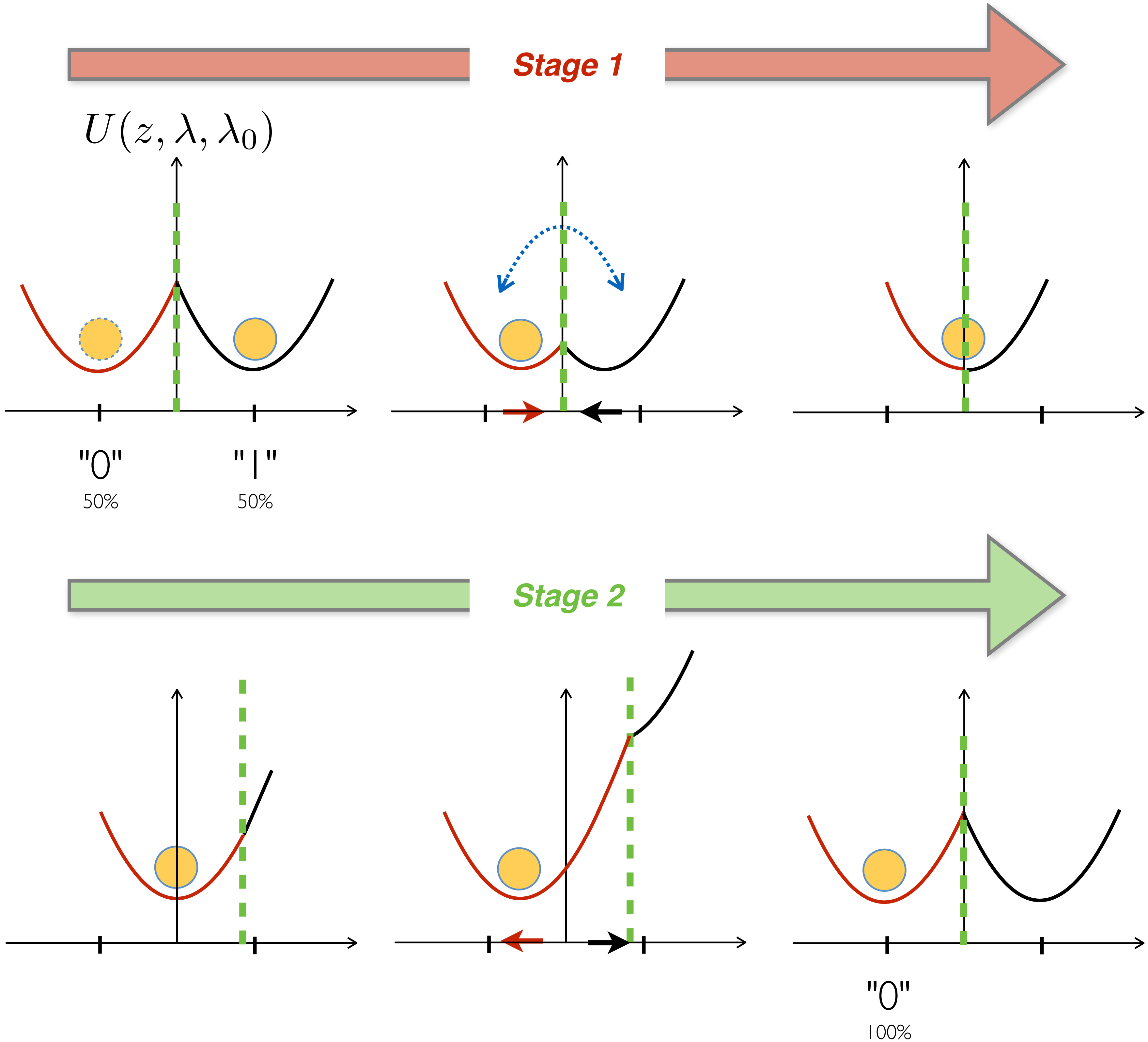
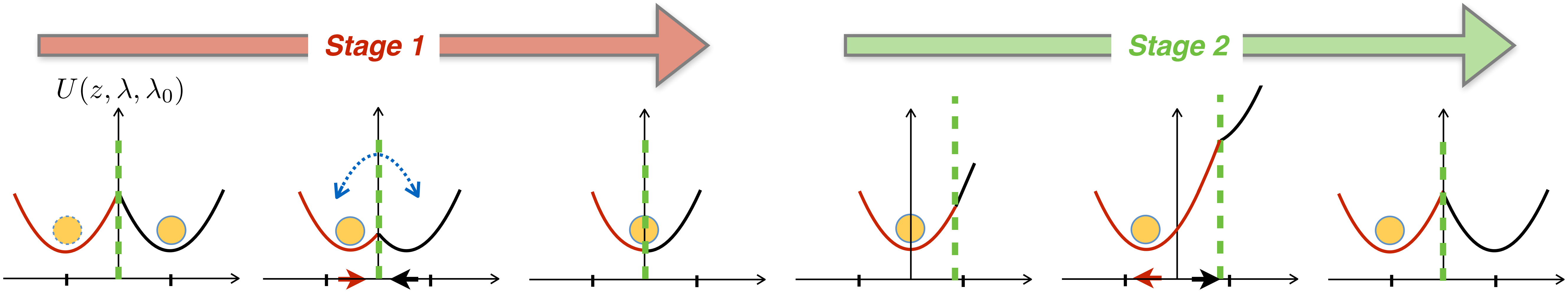
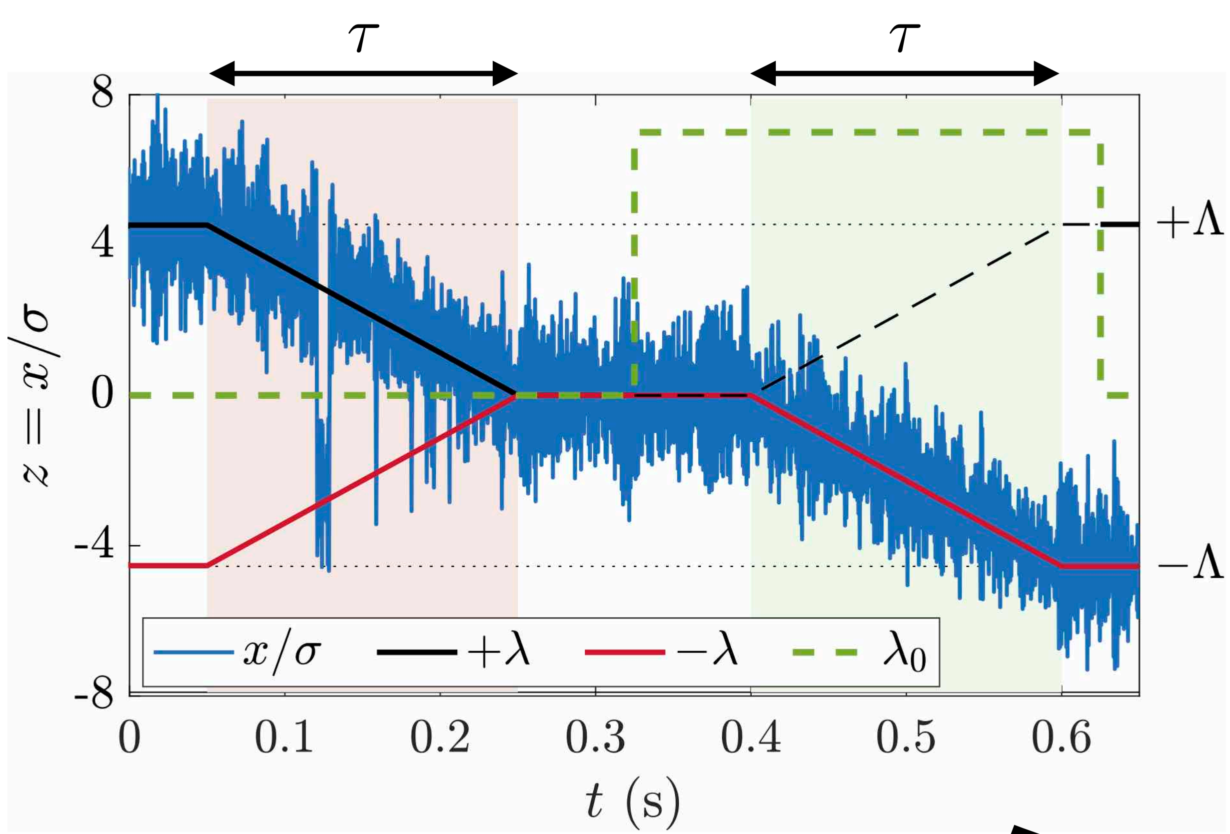
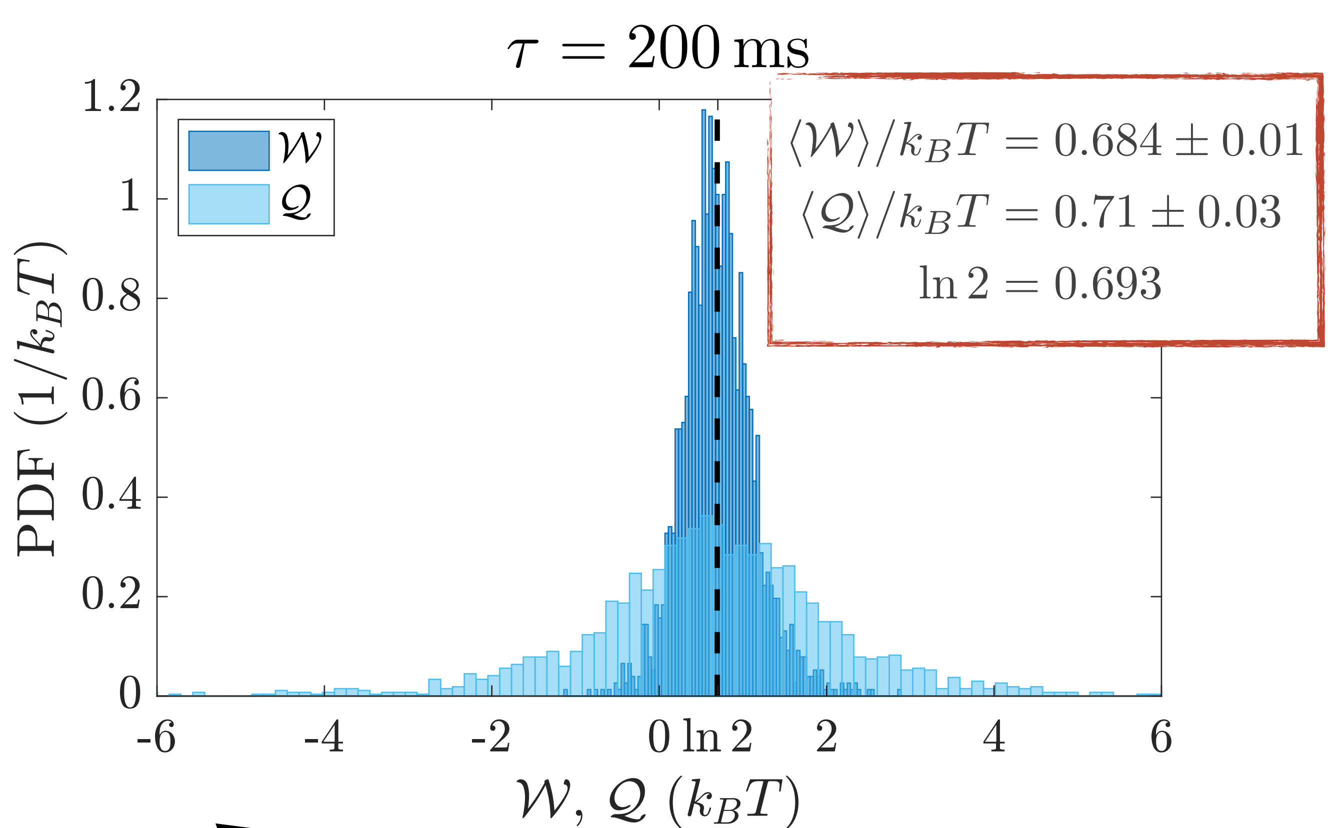


S. Dago, J. Pereda, N. Barros, S. Ciliberto, L. Bellon, *Phys. Rev. Lett.* **126**, 170601 (2021)

Landauer's Limit





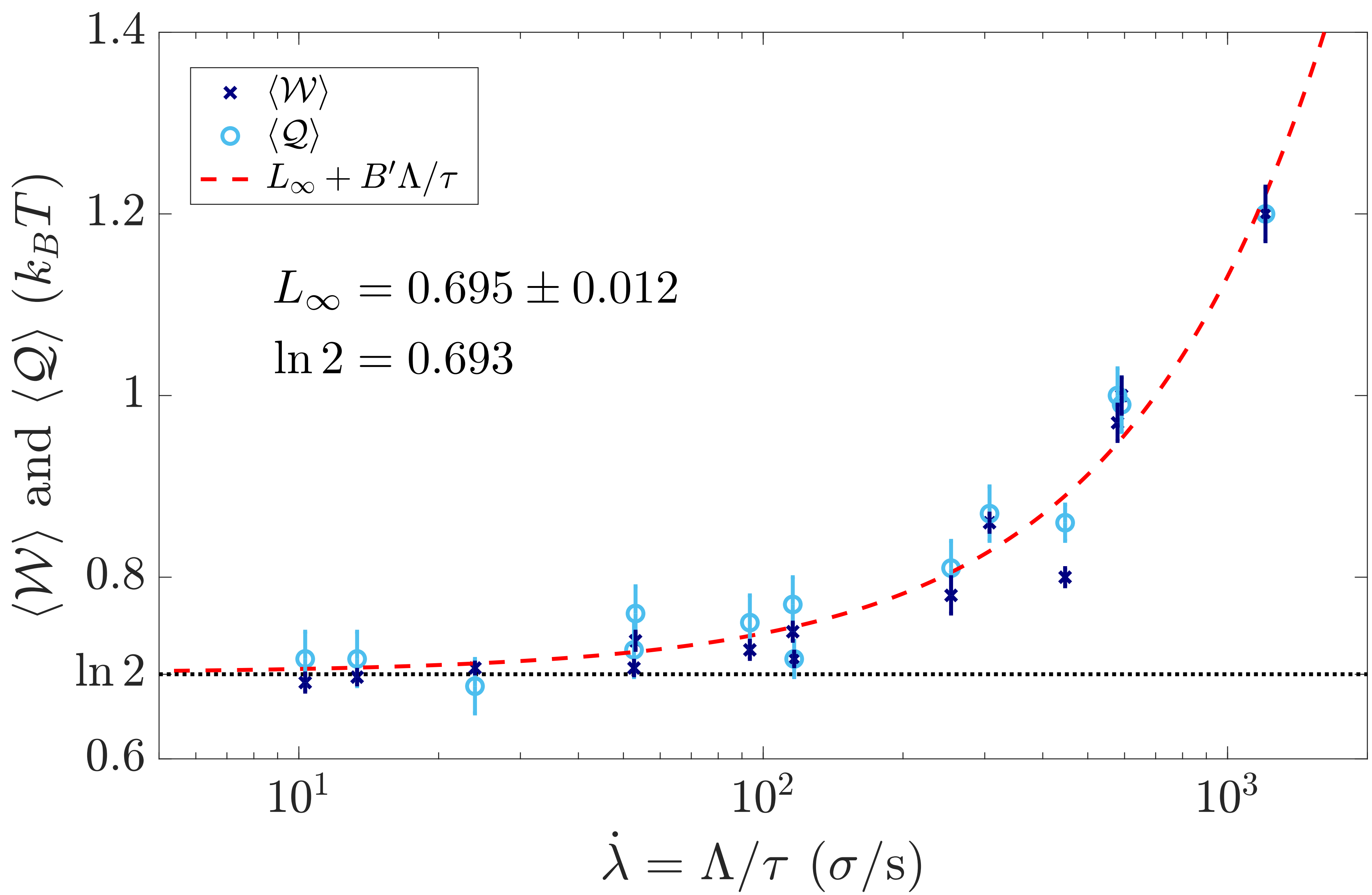


2000 trajectories

S. Dago, J. Pereda, N. Barros, S. Ciliberto, L. Bellon, *Phys. Rev. Lett.* **126**, 170601 (2021)

Landauer's Limit

$$\langle \mathcal{Q} \rangle = \langle \mathcal{W} \rangle = k_B T \left(\ln 2 + \frac{B}{\tau} \right)$$



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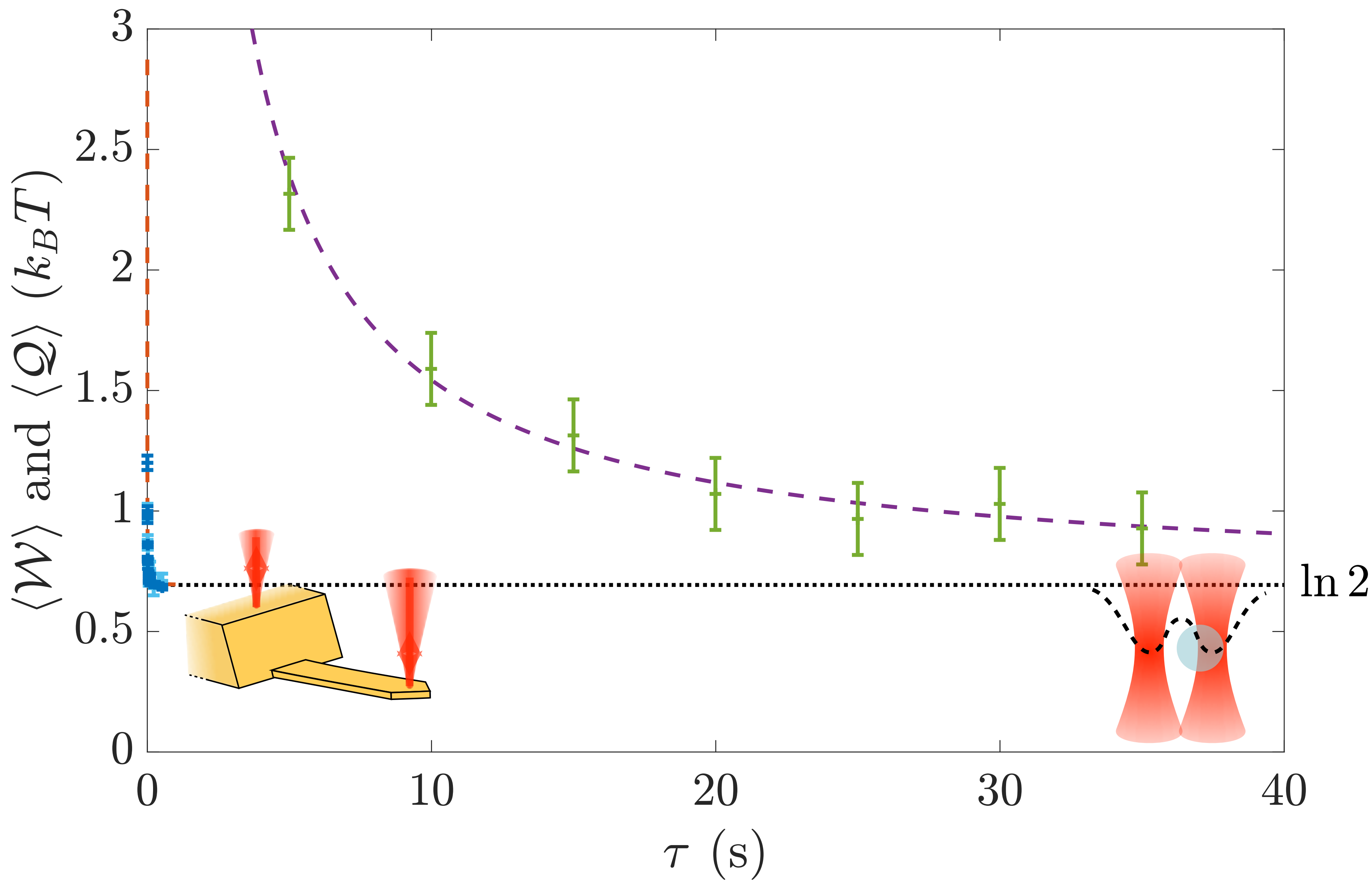

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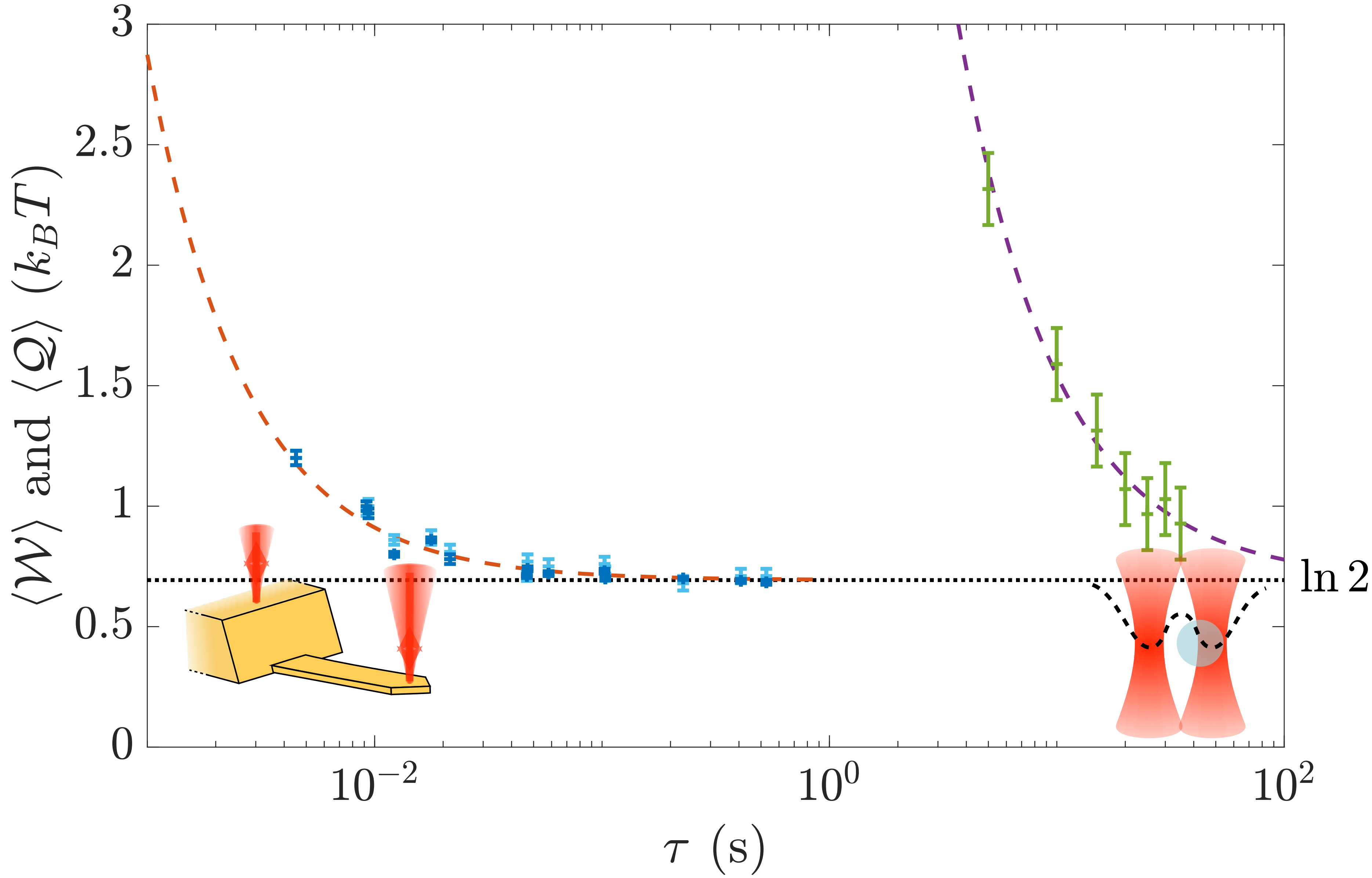
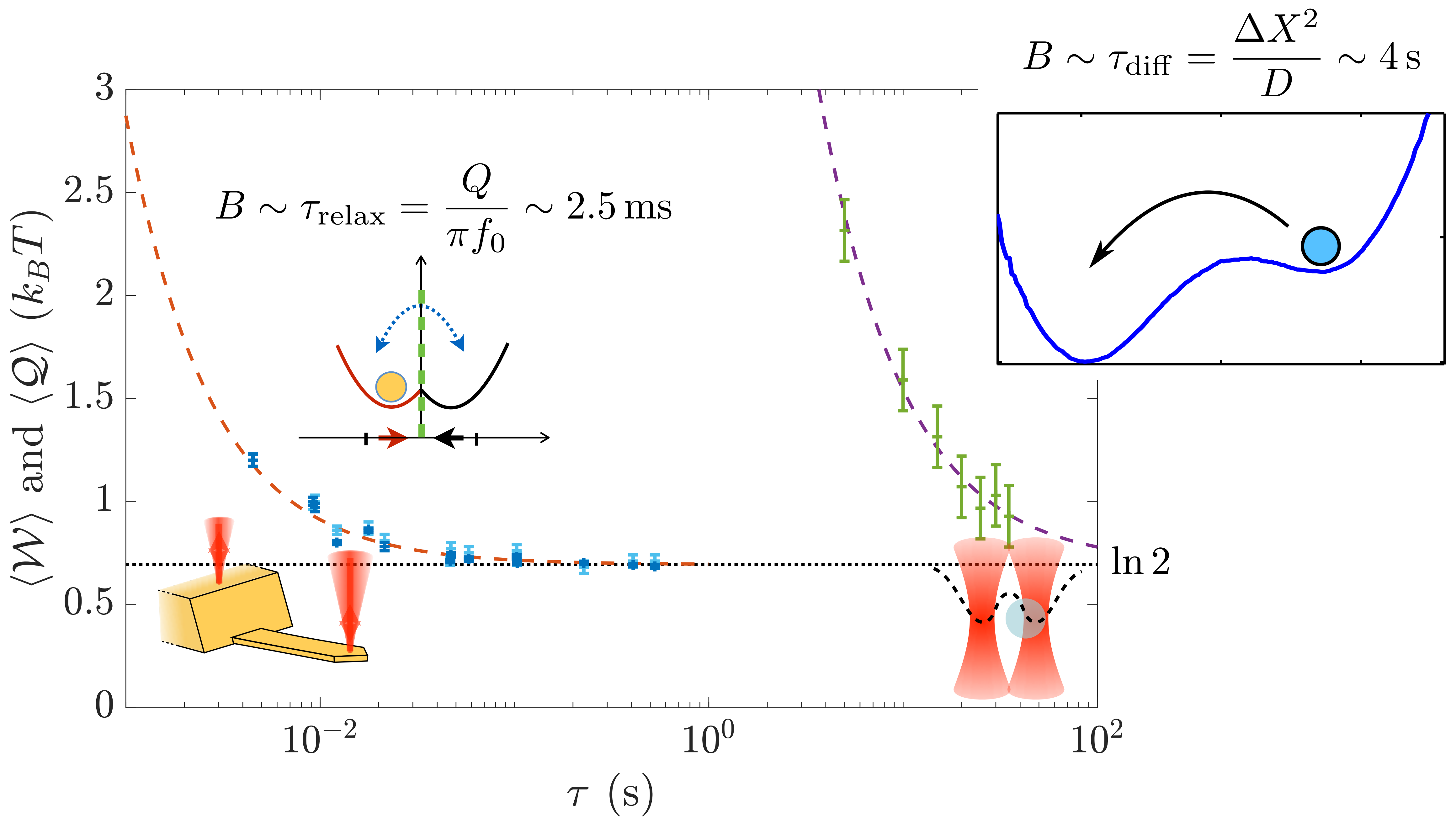


S. Dago, J. Pereda, N. Barros, S. Ciliberto, L. Bellon, *Phys. Rev. Lett.* **126**, 170601 (2021)

Energetics of the erasure process

$$\langle \mathcal{Q} \rangle = \langle \mathcal{W} \rangle = k_B T \left(\ln 2 + \frac{B}{\tau} \right)$$



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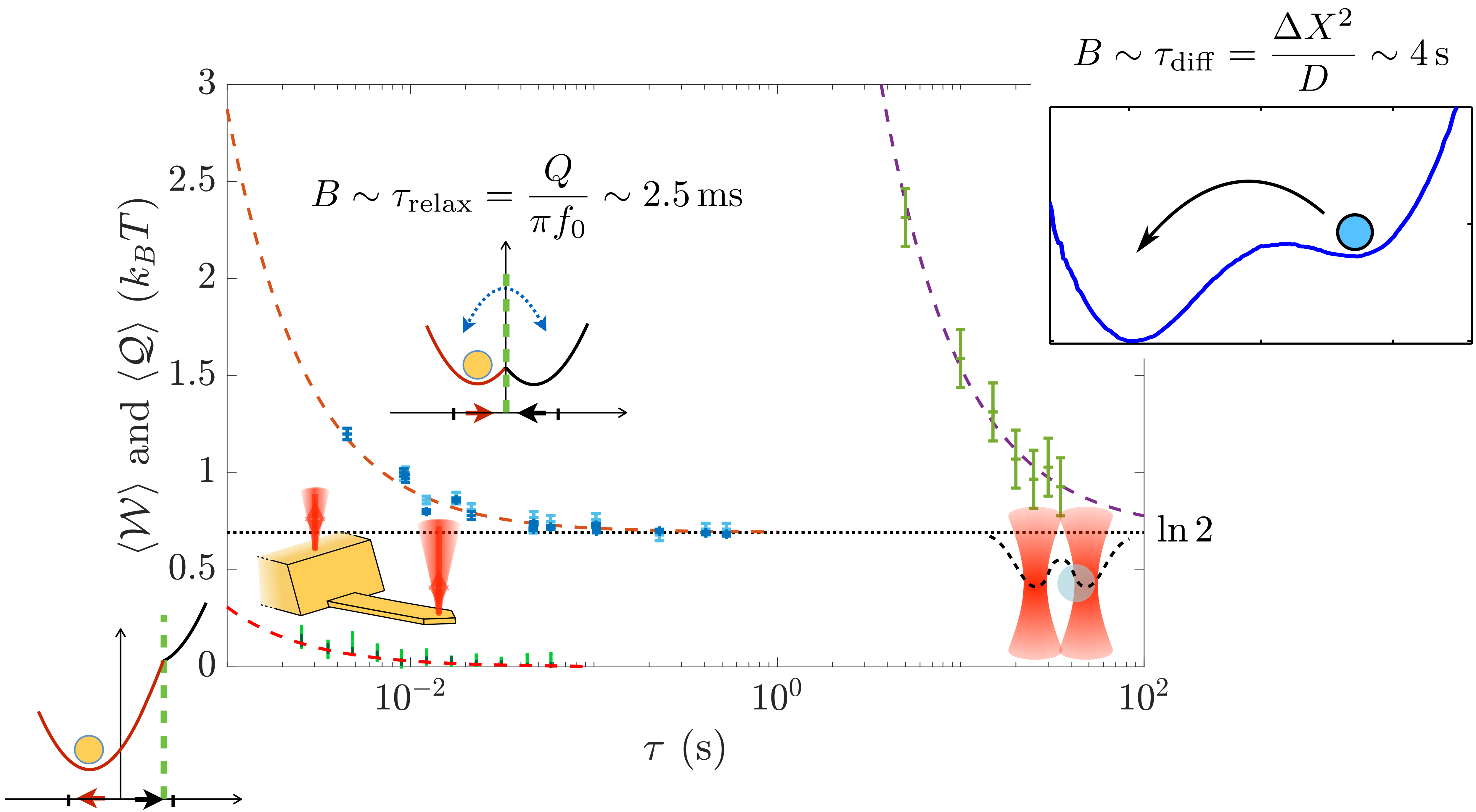

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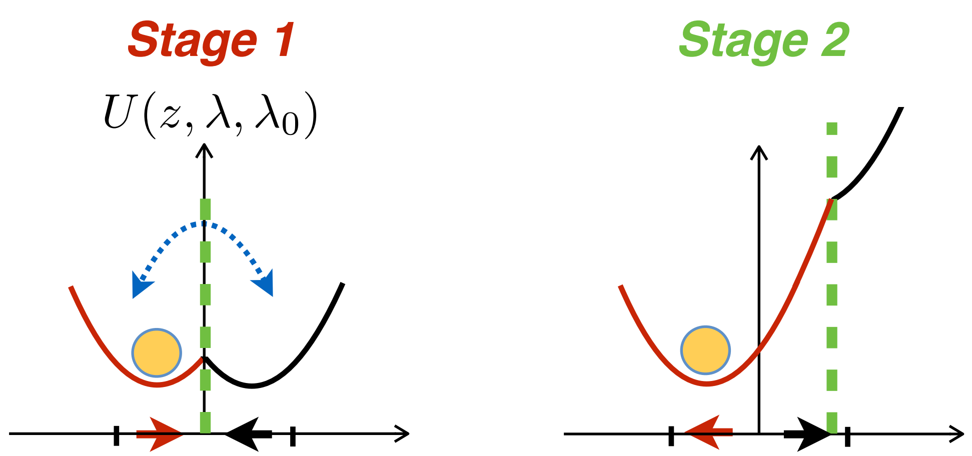
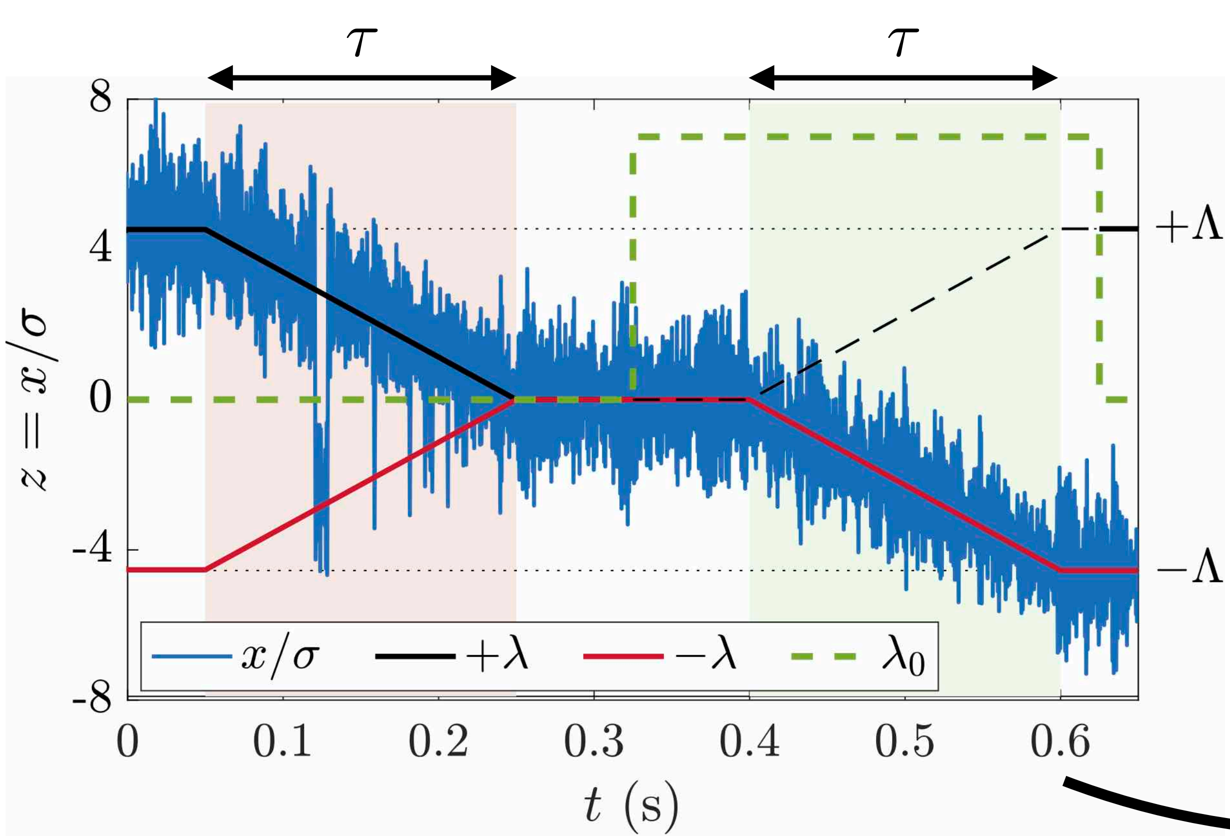
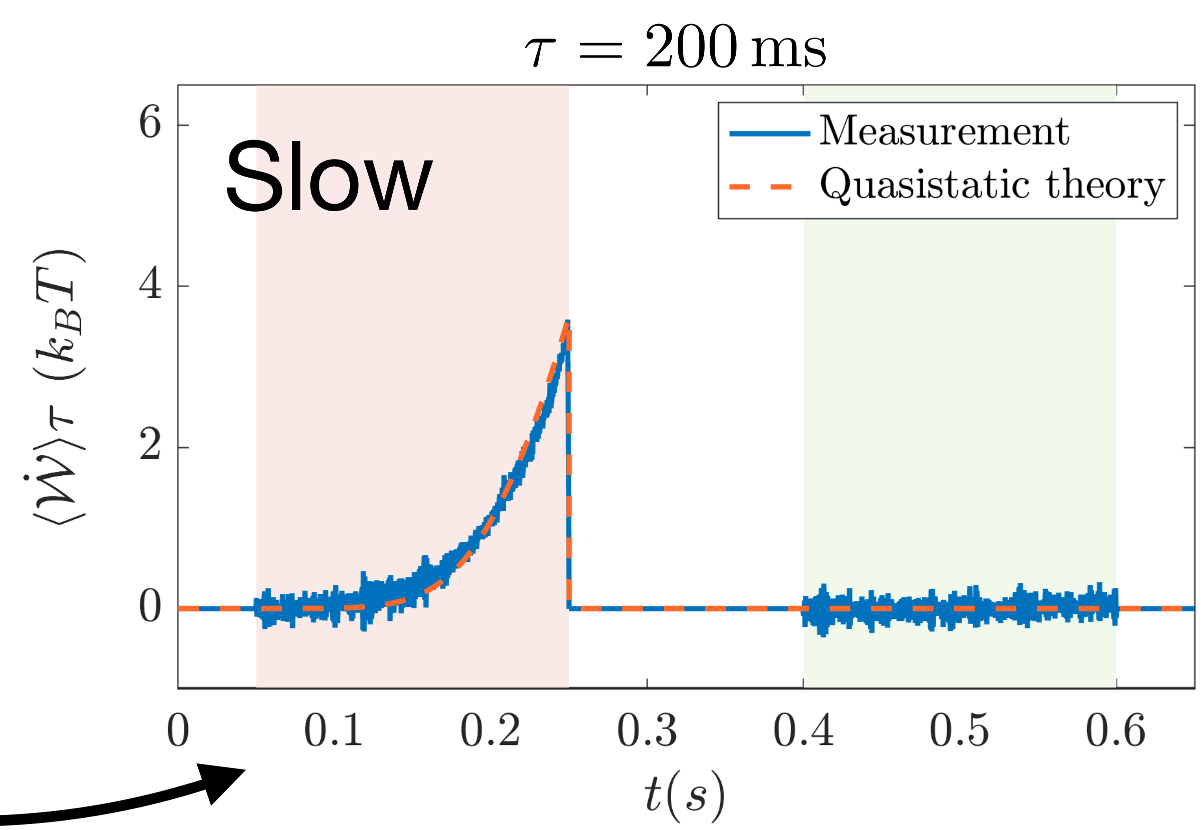


$$\langle \dot{\mathcal{W}} \rangle = \langle \frac{\partial U}{\partial \lambda} \dot{\lambda} \rangle = \langle k(\lambda - |x|)\dot{\lambda} \rangle$$

$$P(x) = \frac{1}{Z} e^{-\frac{1}{k_B T} U(x,\lambda)}$$ *Quasistatic Equilibrium*

$$\langle \dot{\mathcal{W}} \rangle = -k_B T \frac{d}{dt} \ln \left[1 + \mathrm{erf}\left(\sqrt{\frac{k}{2 k_B T}} \lambda \right) \right]$$





2000 trajectories

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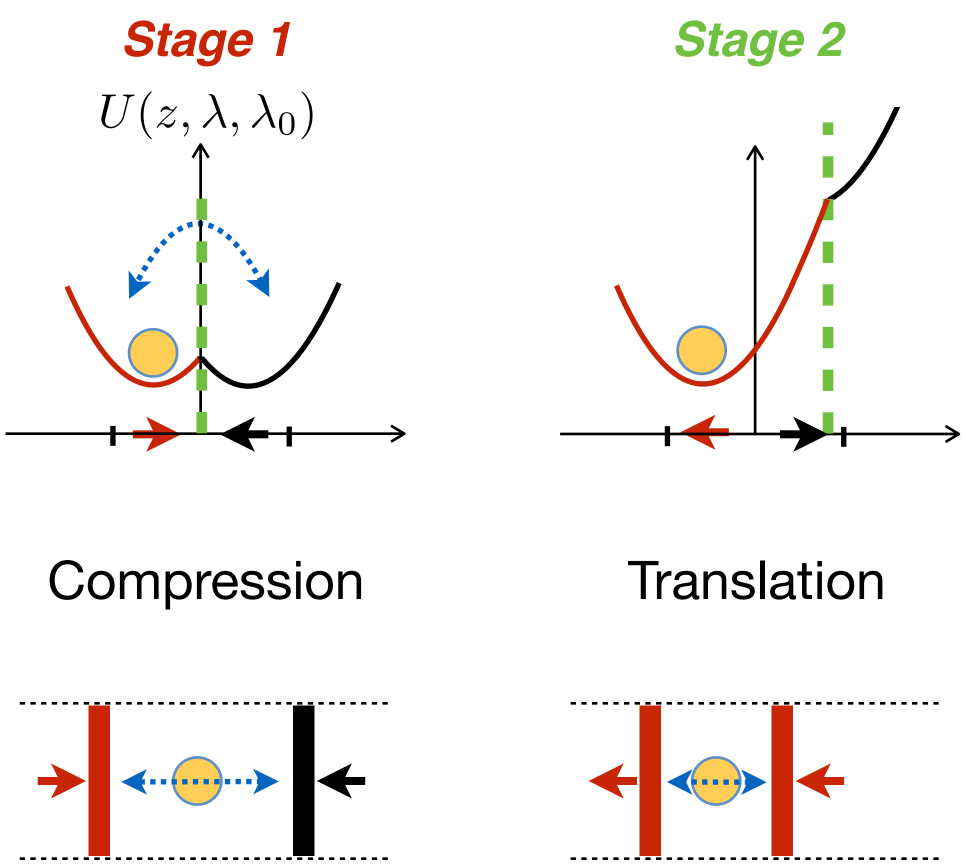

$$p = k_B T / V$$

$$\begin{aligned} \delta\mathcal{W} &= -p\,\mathrm{d}V \\ &= -k_B T \mathrm{d}\ln V \\ V &= V_0 \to V_0/2 \end{aligned}$$

$$\delta\mathcal{W} = 0$$

$$\langle \dot{\mathcal{W}} \rangle = \langle \frac{\partial U}{\partial \lambda} \dot{\lambda} \rangle = \langle k(\lambda - |x|)\dot{\lambda} \rangle$$

$$P(x) = \frac{1}{Z} e^{-\frac{1}{k_B T} U(x,\lambda)}$$

Quasistatic Equilibrium

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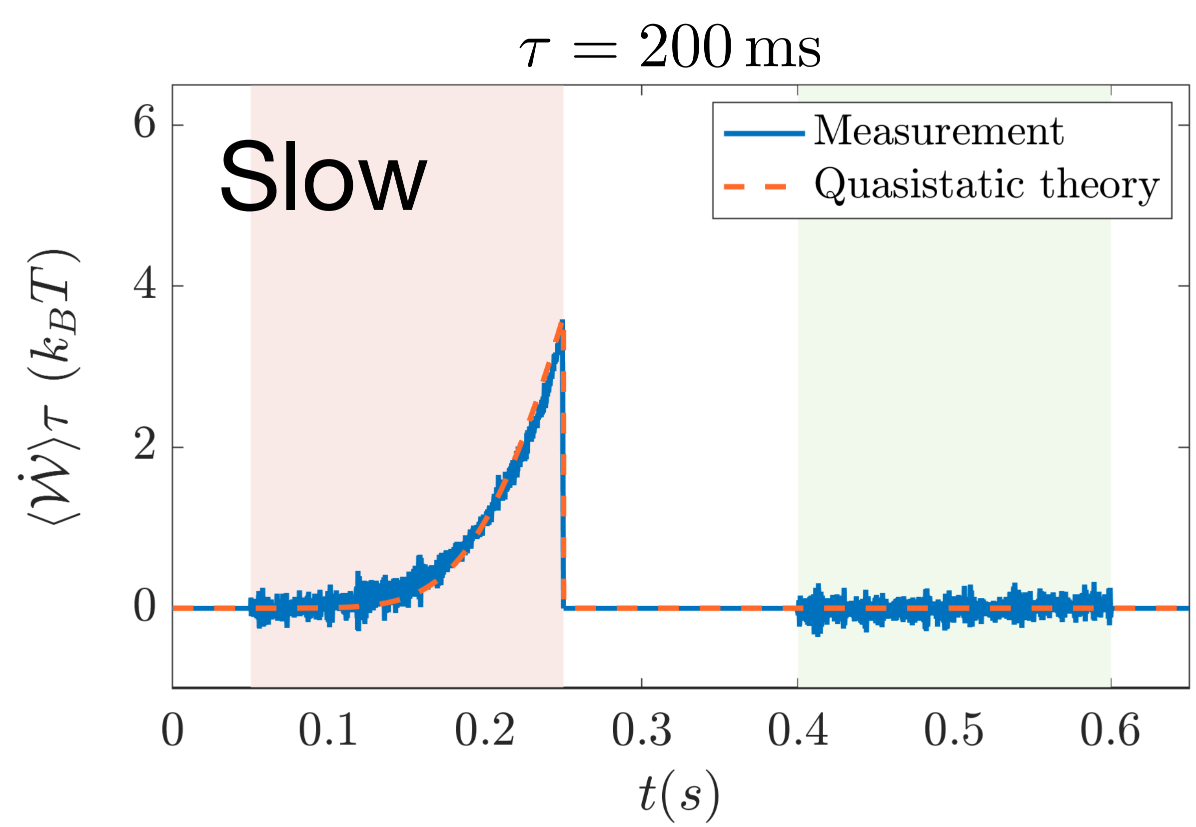

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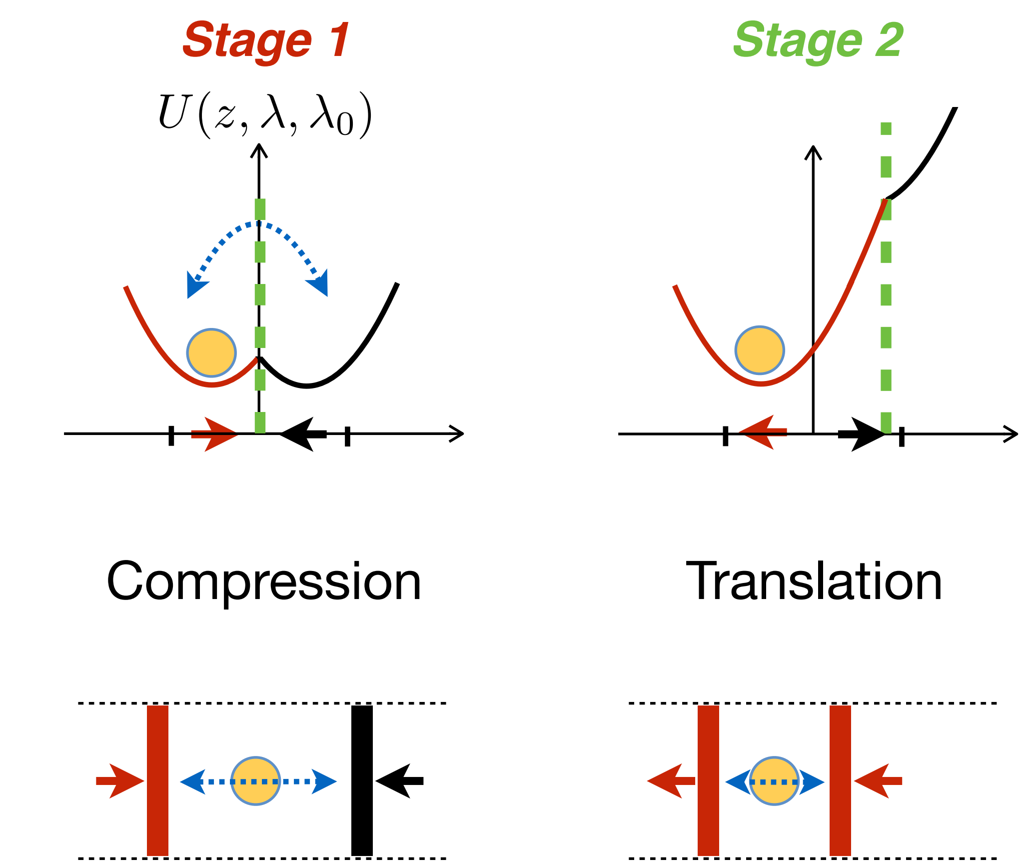

$$p = k_B T / V$$

$$\delta\mathcal{W} = -p\,\mathrm{d}V$$

$$= -k_B T \mathrm{d}\ln V$$

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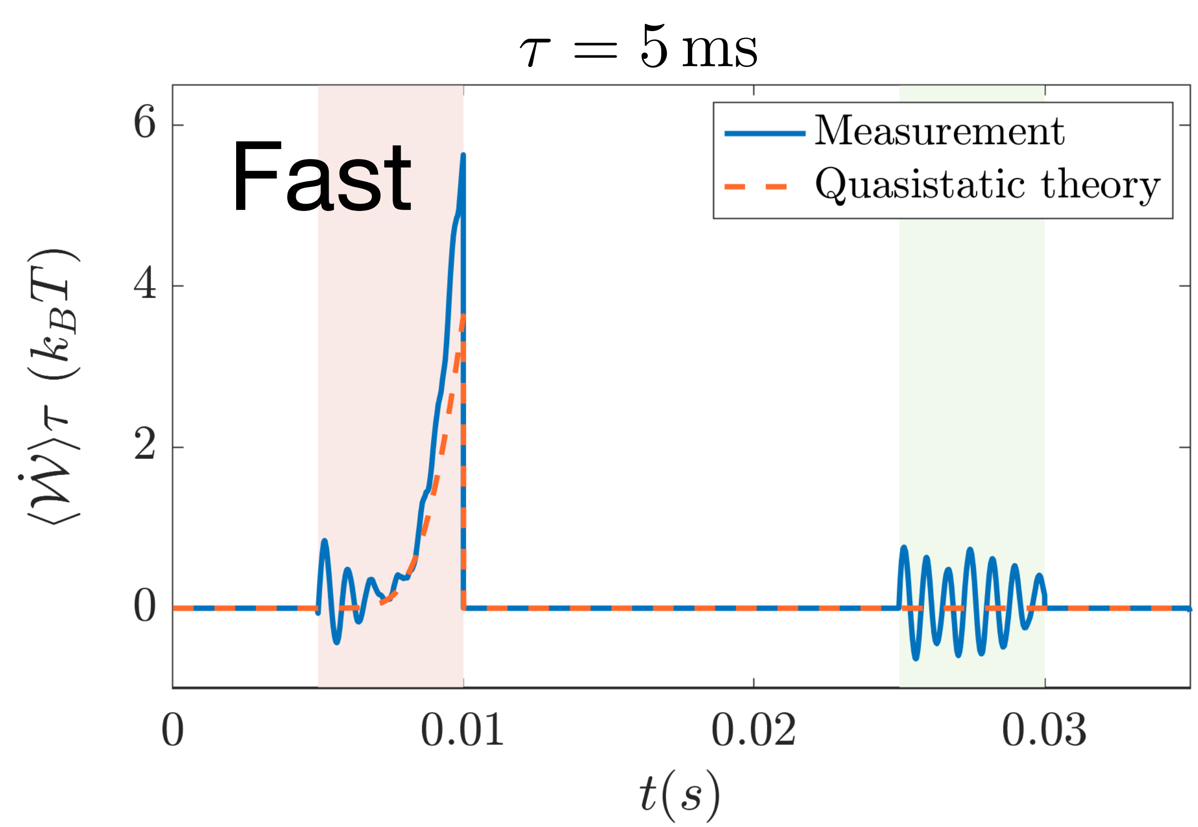
$$\langle \dot{\mathcal{W}} \rangle = \langle \frac{\partial U}{\partial \lambda} \dot{\lambda} \rangle = \langle k(\lambda - |x|)\dot{\lambda} \rangle$$

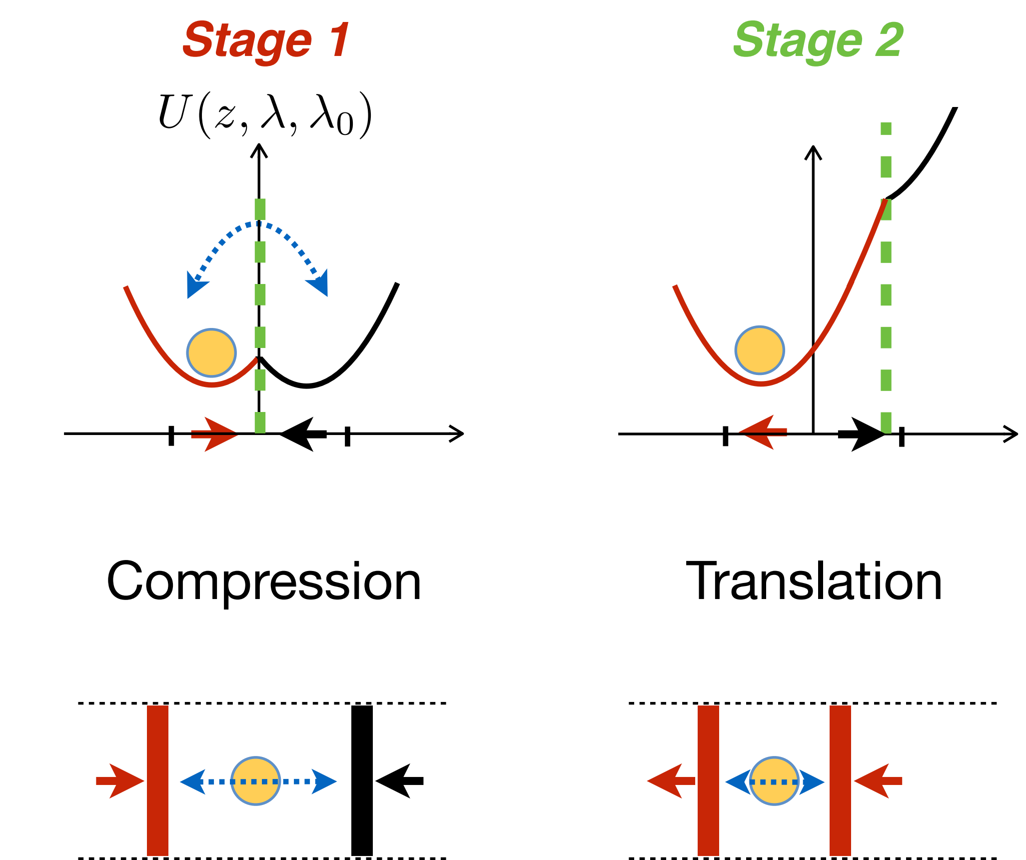
$$\cancel{P(x) = \frac{1}{Z} e^{-\frac{1}{k_B T} U(x,\lambda)}}$$

Quasistatic Equilibrium

$$\cancel{\langle \dot{\mathcal{W}} \rangle = -k_B T \frac{d}{dt} \ln \left[1 + \mathrm{erf}\left(\sqrt{\frac{k}{2k_B T}} \lambda \right) \right]}$$



S. Dago, L. Bellon, *arXiv: 2105.12023* (2021)



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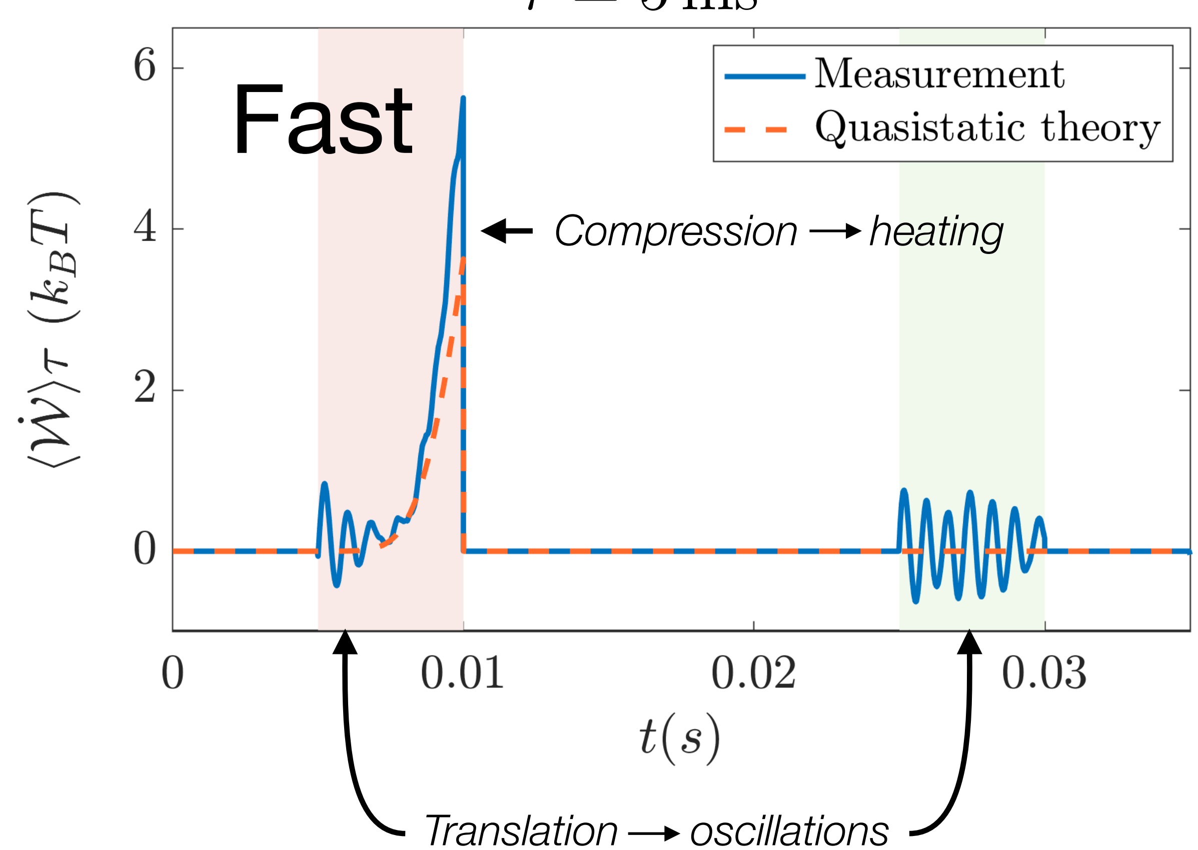
Quasistatic Equilibrium

$$\langle\dot{\mathcal{W}}\rangle = -k_B T \frac{d}{dt}\ln\left[1 + \mathrm{erf}\left(\sqrt{\frac{k}{2k_B T}}\lambda\right)\right]$$

$$\tau = 5\,\mathrm{ms}$$



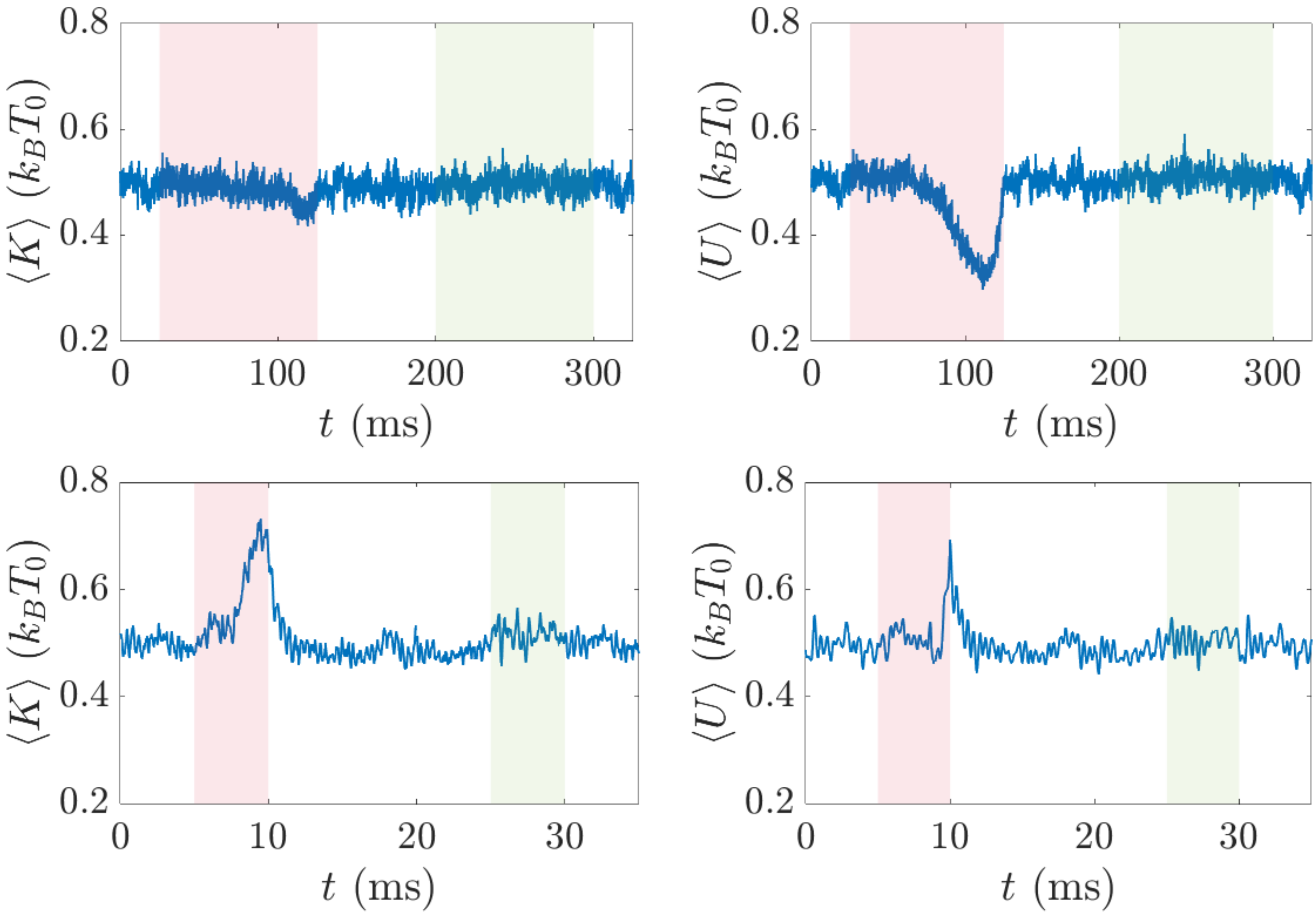
Fast is hot !

Slow

$\tau = 100\,\mathrm{ms}$
$\sim 125\, f_0^{-1}$
$\sim 40\, \tau_{\mathrm{relax}}$

Fast

$\tau = 5\,\mathrm{ms}$
$\sim 6\, f_0^{-1}$
$\sim 2\, \tau_{\mathrm{relax}}$



S. Dago, L. Bellon, *arXiv: 2105.12023* (2021)

Energetics of the erasure process

Slow

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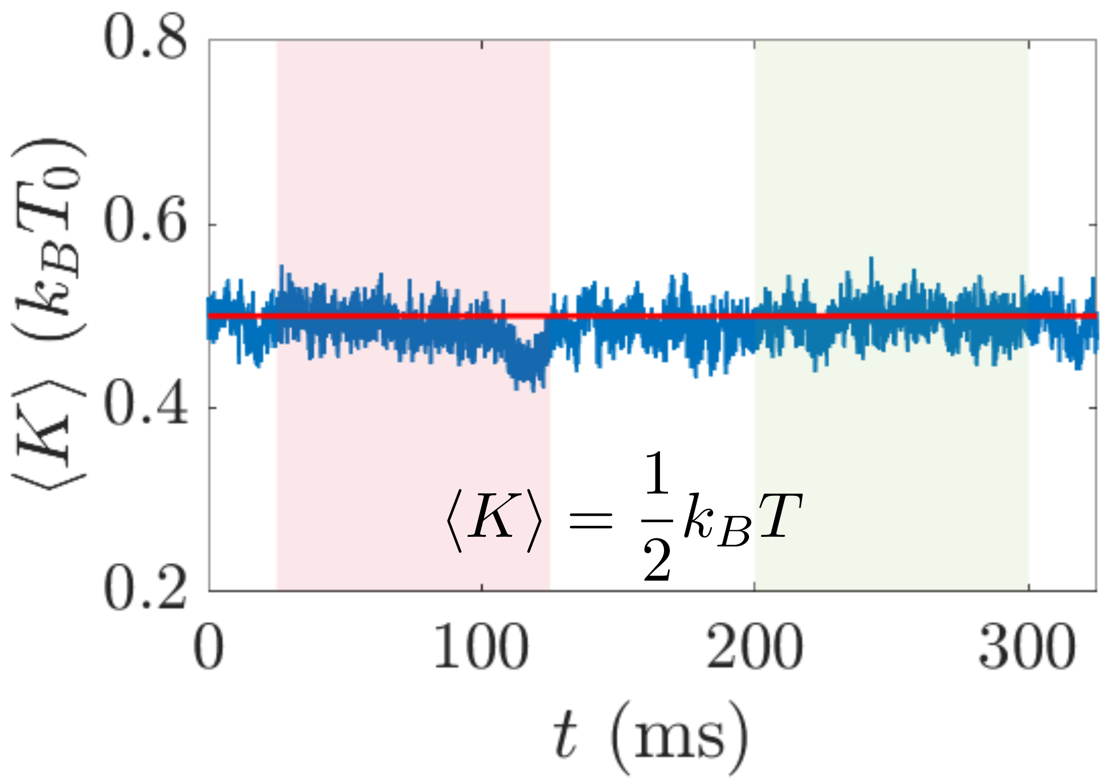

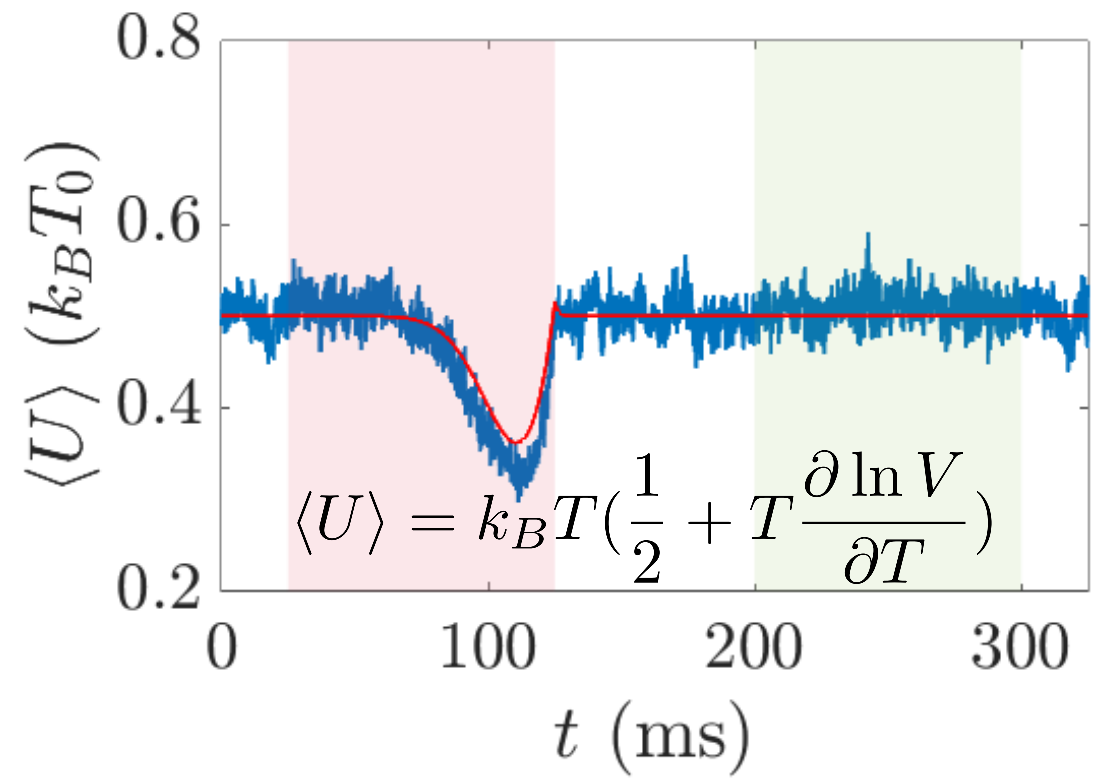

$$P(x,v) = \frac{1}{Z} e^{-\frac{1}{2}\beta m v^2} e^{-\frac{1}{2}\beta k(|x|-\lambda)^2}, \quad \beta = \frac{1}{k_B T}, \quad Z = \frac{2\pi}{\sqrt{km}\beta} V, \quad V = 1 + \mathrm{erf}\left(\sqrt{\frac{k\beta}{2}}\lambda\right)$$

$$\langle E \rangle = \langle U + K \rangle = -\frac{\partial \ln Z}{\partial \beta} = k_B T + k_B T^2 \frac{\partial \ln V}{\partial T}$$

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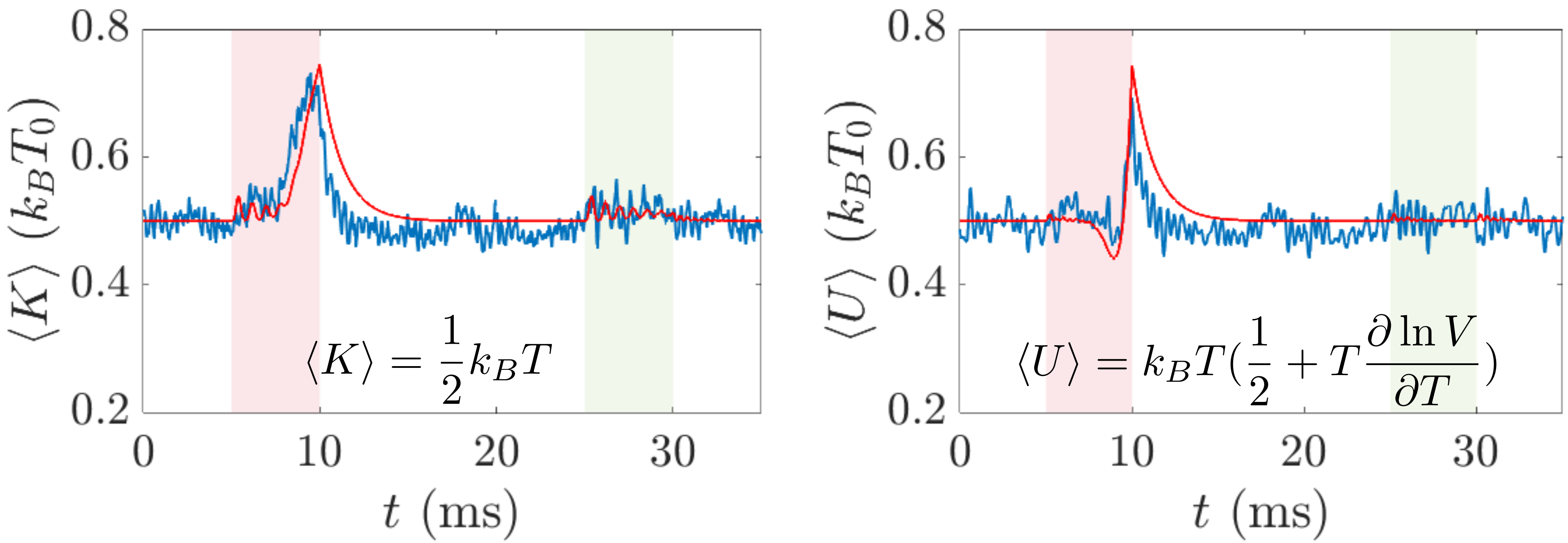

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$$\frac{\partial \langle E \rangle}{\partial T}\dot{T} + \frac{\partial \langle E \rangle}{\partial \lambda}\dot{\lambda} = \langle \dot{E} \rangle = \langle \dot{\mathcal{W}} \rangle - \langle \dot{\mathcal{Q}} \rangle = -k_B T \frac{\partial \ln V}{\partial \lambda}\dot{\lambda} + \frac{\omega_0}{Q} k_B (T - T_0)$$

S. Dago, L. Bellon, *arXiv: 2105.12023* (2021)

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$\sim 6\, f_0^{-1}$

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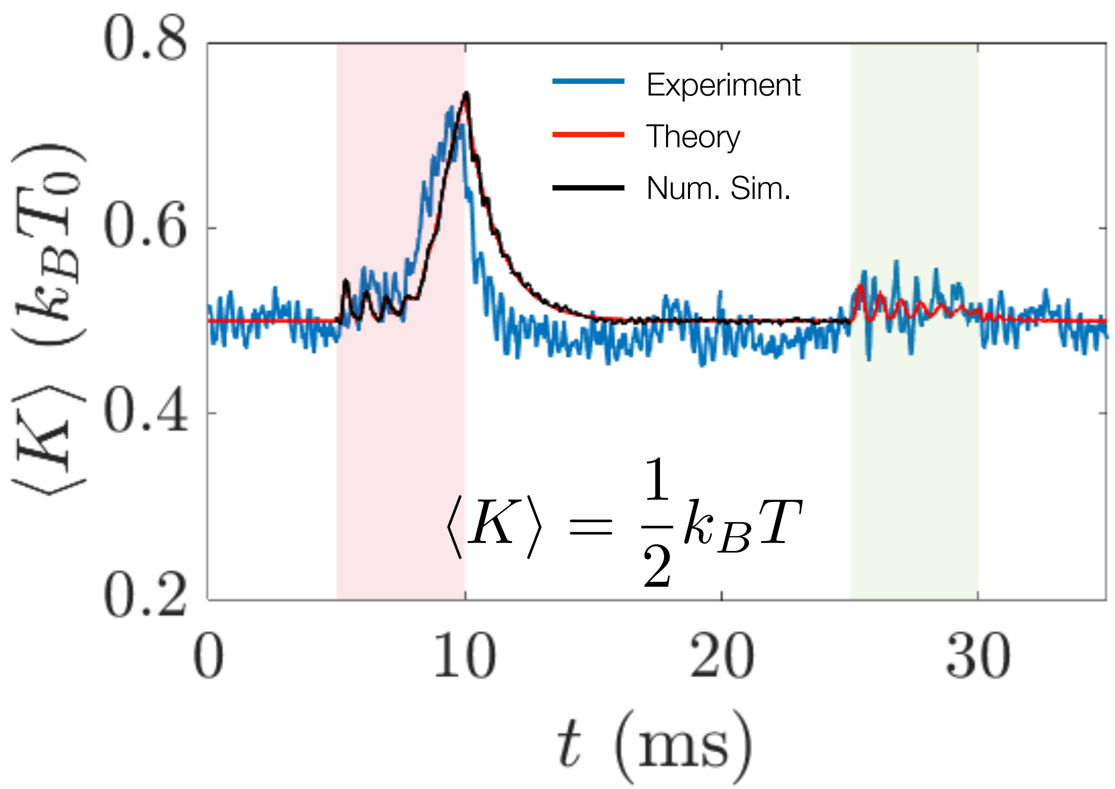

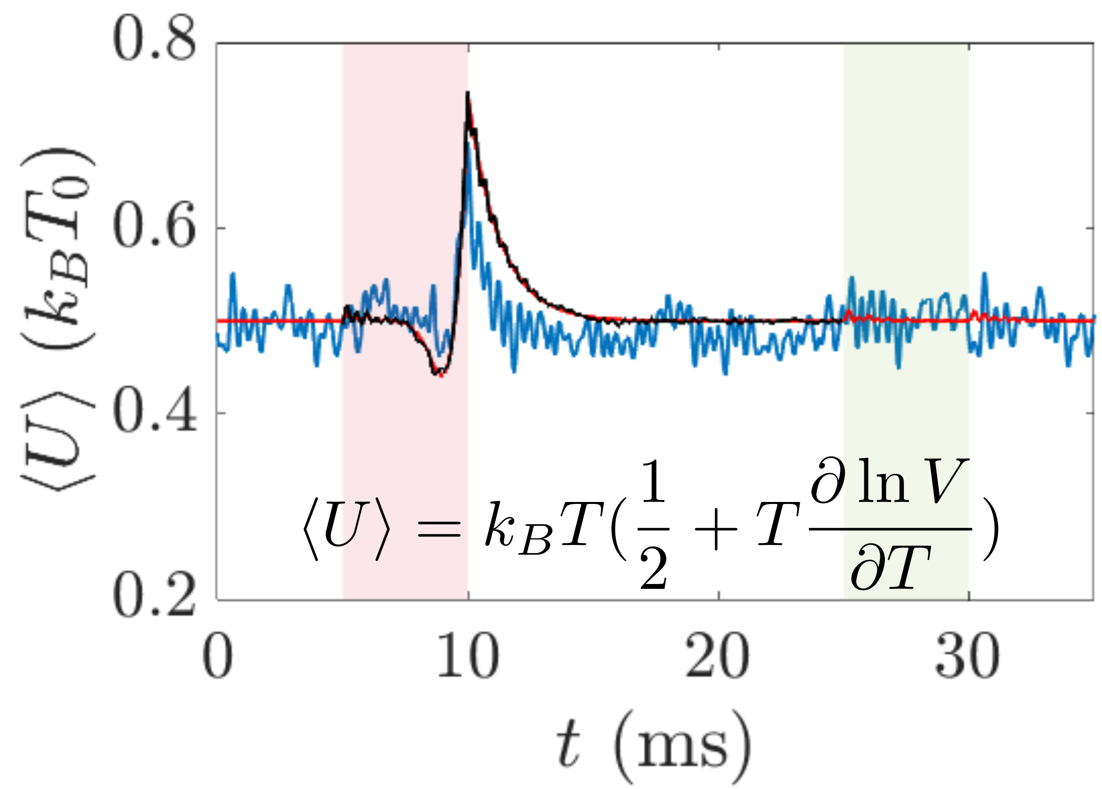

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S. Dago, L. Bellon, *arXiv: 2105.12023* (2021)

Fast is hot !

[Click here to play video online]

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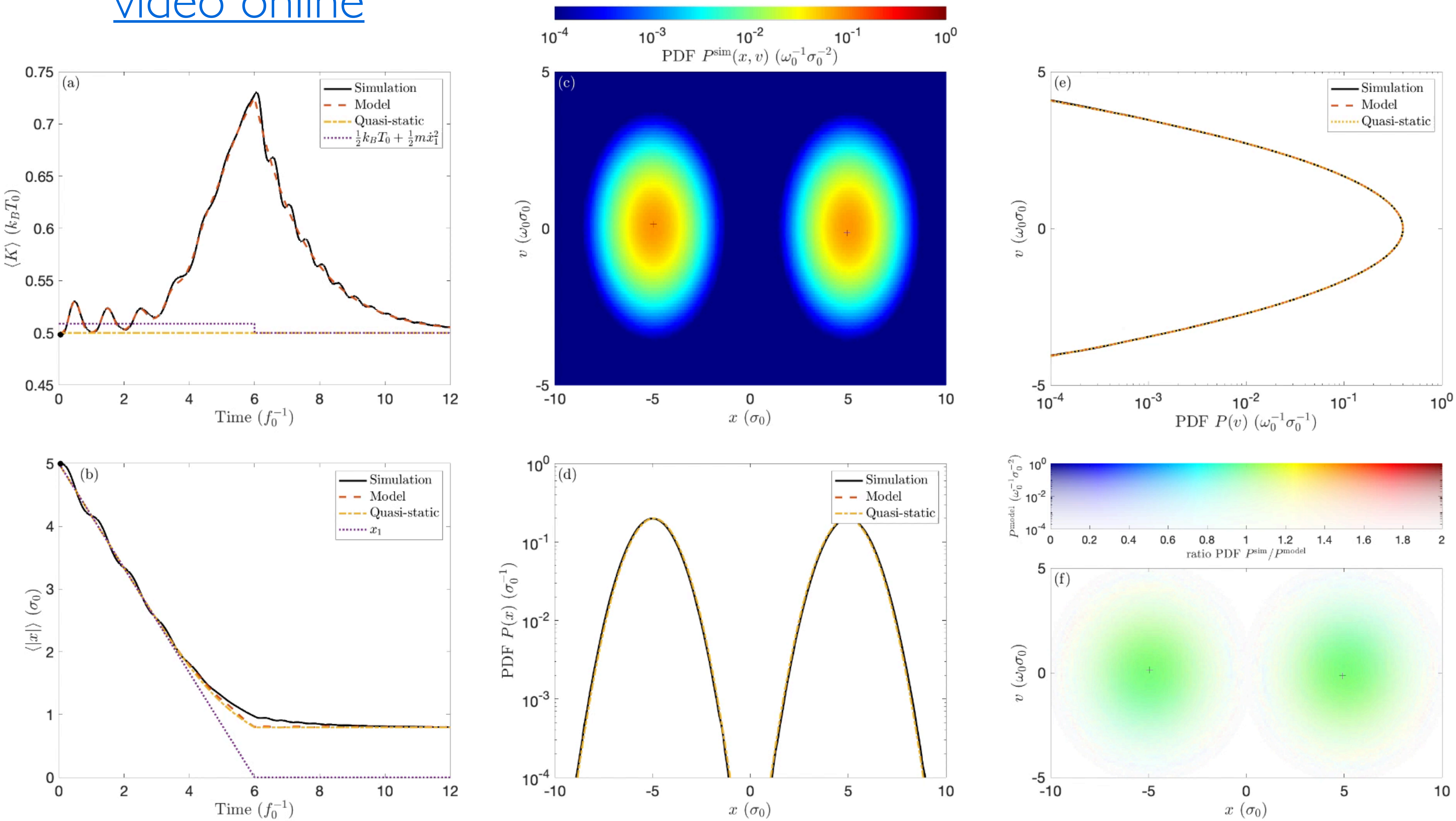
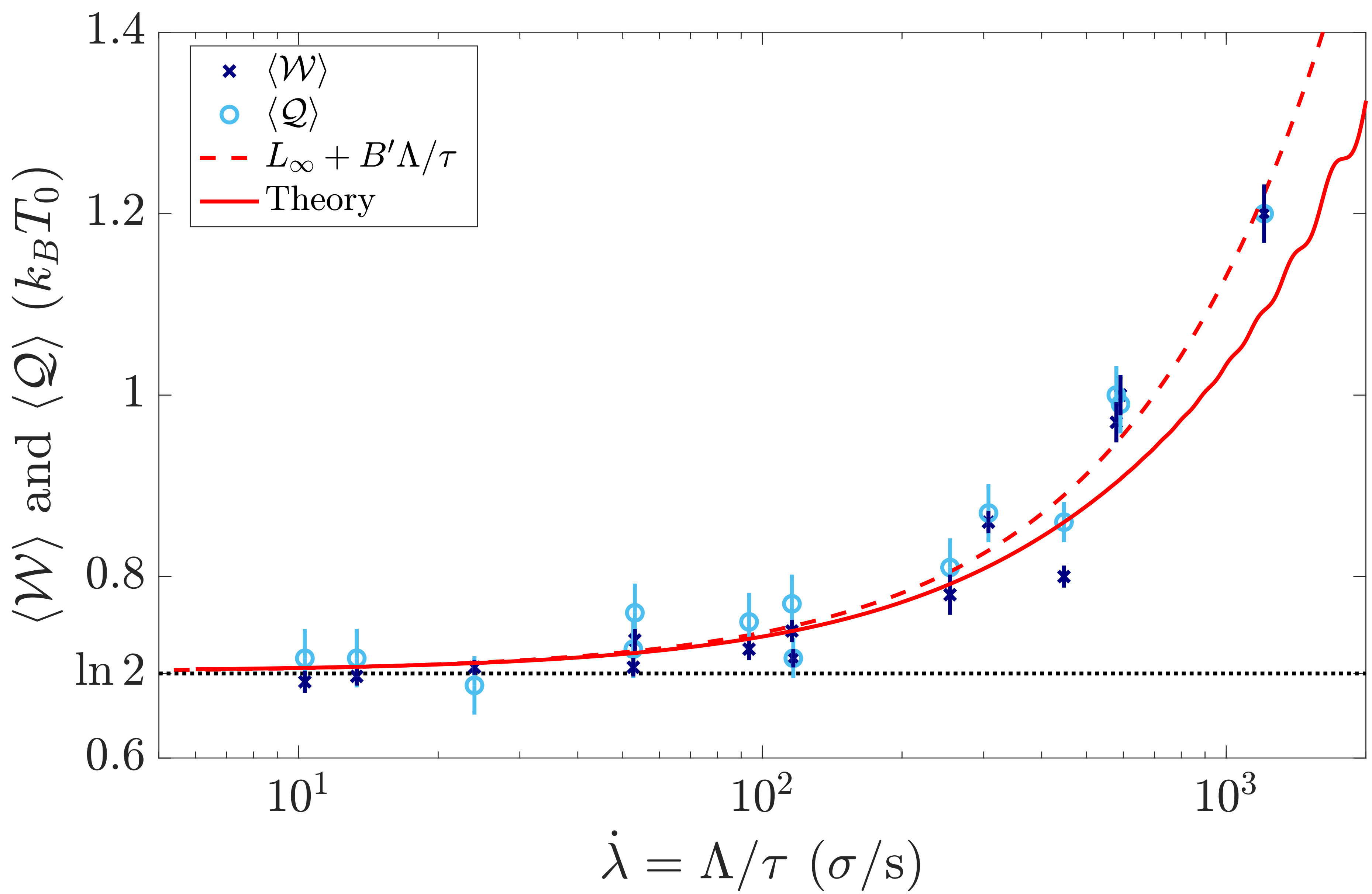


S. Dago, L. Bellon, *arXiv: 2105.12023* (2021)

Fast is hot !

$$\langle \mathcal{Q} \rangle = \langle \mathcal{W} \rangle \sim \int k_B T d \ln V$$



S. Dago, L. Bellon, *arXiv: 2105.12023* (2021)

Adiabatic limit

$$Q \gg 1 \longrightarrow \langle \dot{\mathcal{Q}} \rangle = -\frac{\omega_0}{Q} k_B (T - T_0) \sim 0$$

$$\left[k_B \ln Z + \frac{\langle E \rangle}{T} \right]_0^\tau = \Delta S = \int_0^\tau \frac{\langle \dot{\mathcal{Q}} \rangle}{T} dt = 0$$

$$\langle E \rangle = k_B T + k_B T^2 \frac{\partial \ln V}{\partial T}$$

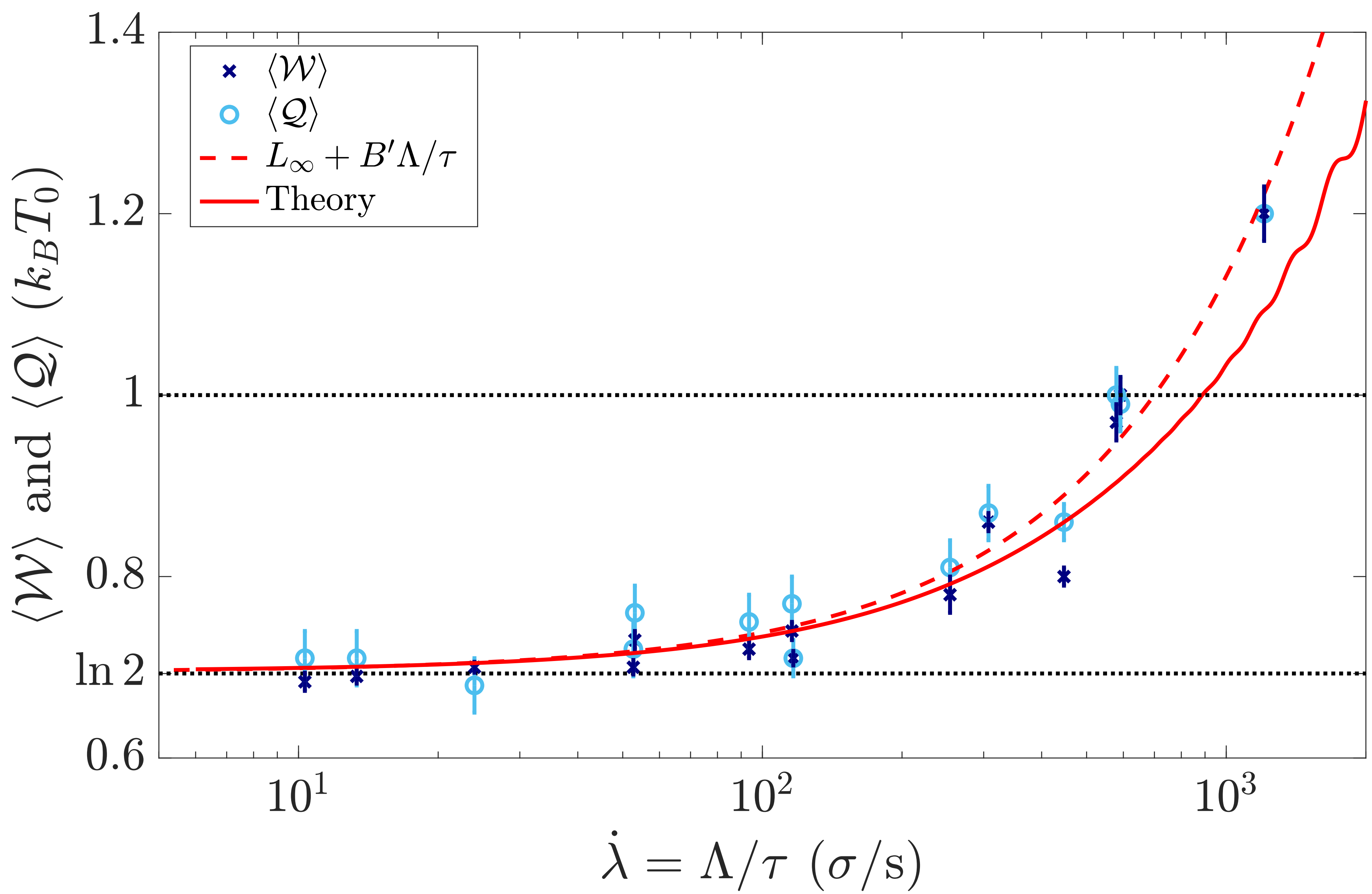
$$Z = \frac{2\pi}{\sqrt{km}} k_B T V$$

$$V = 1 + \mathrm{erf}\left(\sqrt{\frac{k\beta}{2}} \lambda \right)$$

$$\longrightarrow \begin{aligned} TV &= T_0 V_0 \\ T(\tau) &= 2T_0 \\ \langle \mathcal{W} \rangle &= \Delta \langle E \rangle = k_B T_0 \end{aligned}$$

Fast is hot, but not too hot !

$$Q \gg 1 \longrightarrow \underbrace{\langle \mathcal{W} \rangle = \langle \mathcal{Q} \rangle}_{\text{(delayed)}} \sim k_B T_0$$

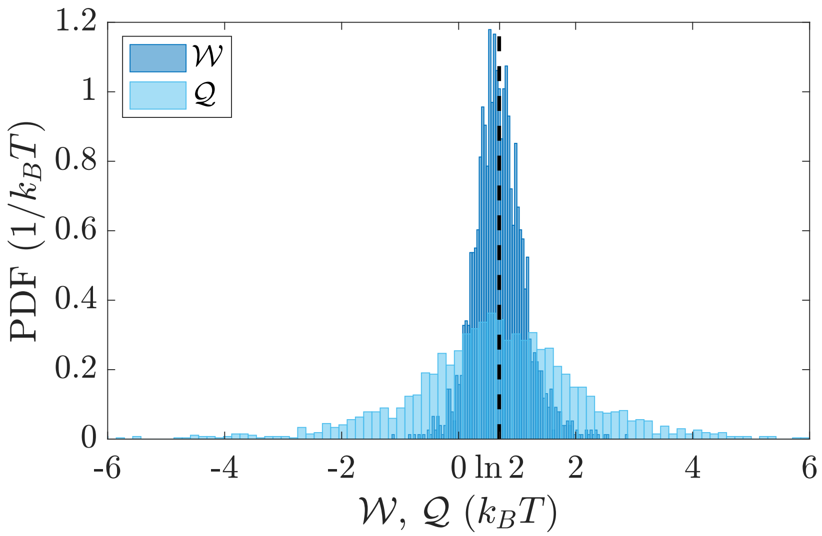


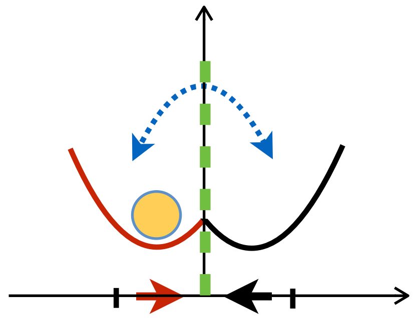
Adiabatic Limit

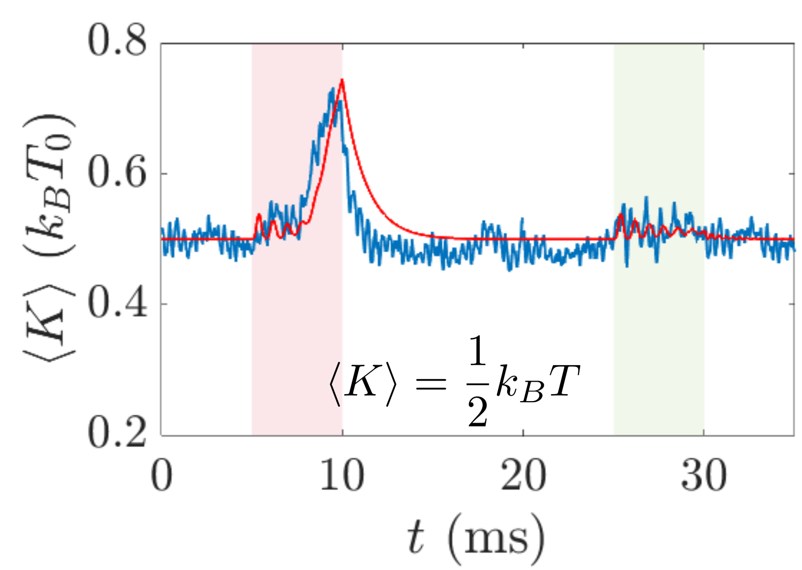
Landauer's Bound

Underdamped stochastic thermodynamics

- Fast operation, high statistics
- Illustration of Landauer's bound
- $k_B T_0 \ln 2$ comes from compression
- Fast is hot
- Full energetics description
- Adiabatic limit: $\langle \mathcal{W} \rangle = \langle \mathcal{Q} \rangle = k_B T_0$







Thank you for your attention

Kiitos huomiostasi :-)

[according to Google translate]

To go further…

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IBM Journal of Research and Development **5**, 183 (1961)
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