PME 49
Helsinki, Finland
2026

ORAL COMMUNICATIONS

CONTEXTUALIZING THE TRANSITION TO MIDDLE-SCHOOL USING TIMSS DATA

Emilia Aaltonen, Anne-Maria Ernvall-Hytönen, Peter Hästö, Dimitri Tuomela

University of Helsinki

The transition between elementary and middle school mathematics has been linked to declines in motivation, achievement and engagement, which may relate to curricular and pedagogical discontinuities (Gueudet et al., 2016; Kaur et al., 2022).We elucidate these discontinuities by comparing Finnish elementary- and middle-school teachers' views about instruction practices, social learning environment, and teacher profession using TIMSS 2023 data (Fishbein et al., 2025).

We combined data from teacher questionnaires of grade 4 (n=423) and grade 8 (n=400) teachers, identified 99 items with identical questions and compared the means of the two groups by independent samples t-tests. Due to the large number of tests, we only consider differences at significance level $p<0.001$ of which there were 46. In the second phase of the analysis, we analyzed these 46 items qualitatively to identify common themes.

Based on the surveyed teachers' views, the following differences between elementary and middle-school emerged. In instructional practices, use of digital devices and homework differed the most, organizing mathematics instruction somewhat, whereas teachers' assessment practices content contextualization were similar. Regarding the social learning environment, parents', peers' and teachers' achievement expectations varied, factors limiting teaching differed somewhat, and safety and behavior were similar. Related to the teacher profession, work-load and pressure experience differed whereas work satisfaction and professional development activity were similar. Details about the categories will be given in the presentation.

The identified differences between elementary- and middle school teachers are related to at least some of the curricular and pedagogical discontinuities across the transition. We will utilize the results in future design research on supporting students' transition in mathematics.

References

Fishbein, B., Taneva, M., & Kowolik, K. (2025). *TIMSS 2023 User Guide for International Database*. Boston College, TIMSS & PIRLS International Study Center.

Gueudet, G., Bosch, M., DiSessa, A. A., Kwon, O. N., & Verschaffel, L. (2016). *Transitions in mathematics education.* Springer Nature.

Kaur, T., McLoughlin, E., & Grimes, P. (2022). Mathematics and science across the transition from primary to secondary school: A systematic literature review. *International journal of STEM education, 9*(1), 13.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 414. PME.

PILOTING THE IMPLEMENTATION OF MENTAL STARTER INTERVENTION IN THE NIGERIAN CONTEXT

Lawan Abdulhamid[1], Balarabe Yushau[2]

[1]University of the Witwatersrand, [2]Abubakar Tafawa Balewa University

Performance in Nigerian public primary schools is critically below curricular expectations, with many pupils graduating without the necessary foundational numeracy skills. Studies have linked this poor pupils' performance to inadequate preparation of primary school teachers. It is this situation that motivates this study to focus on an intervention to support pre-service teachers' competence in the teaching of mental mathematics. Mental mathematics involve helping learners to move from unit counting to more fluent and strategic working with number. For instance, to calculate 278 + 309, one can strategically rewrite the sum as 287 + 300. This can fluently be answered as 587. This cognitive skill is fundamental to developing mathematical proficiency and fostering problem-solving abilities in young learners. We draw on the Mental Starter Assessment Project (MSAP) developed in the South African context. Each unit of MSAP include eight structured starters lessons, with guided tasks and individual practice. Thus, the study is guided by the following research question: How does the implementation of structured mental starter lessons enhance pre-service teachers' professional learning within the Nigerian context?

The study adopts Strom and Viesca's (2023) teacher learning-practice framework, emphasising a shift from "being" to "becoming," where teacher learning is viewed as an ongoing, emergent, and non-linear process of transformation rather than a fixed state. Twelve pre-service teachers (PSTs) participated in the study. Each PST implemented two instructional units—jump strategy and rounding and adjusting—over a six-week period. Lesson Starters (LS) 3 in each unit was observed and followed by structured feedback, with a subsequent observation conducted during LS 7. The study reports initial findings on LS 3, analysed across implementation of strategies, responsiveness to learners' thinking, adherence to the teacher guide, and pacing. Preliminary findings from the observation of LS 3 revealed that the PSTs have weak alignment with the teacher guide, limited strategic questioning, and extended pacing. In response, structured feedback was provided to support improved pedagogical practice. The upcoming observation of LS 7 will assess whether PSTs' enactments reflect changes in response to feedback and whether these shifts translate into improved learner outcomes. Their teaching develops non-linearly through enactment, reflection, and feedback, which acts as a space for refining practice.

References

Strom, K. J., & Viesca, K. M. (2023). Towards a complex framework of teacher learning-practice. In Non-linear perspectives on teacher development (pp. 13-28). Routledge.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 415. PME.

RELATIONAL THINKING IN SOLVING NUMBER SENTENCES: A TEXTBOOK ANALYSIS

Reisid May Sumbilon[1], Maria Alva Aberin[2]

[1]Villanueva National High School, [2]Ateneo de Manila University

Relational thinking is a type of thinking that deals with finding relations between quantities in a number sentence. According to Kızıltoprak & Köse (2017), relational thinking is mostly concerned with finding or examining the given quantities rather than knowing the result of operations. Studies on students' conceptions of equivalence show that exposure to non-standard number sentences is critical for fostering relational understanding (Singh & Kosko, 2017). Relational thinking therefore depends not only on cognitive readiness but also on structured opportunities to engage with equations that require structural comparison.

This study examines relational thinking as a curricular affordance embedded in elementary mathematics textbooks. A content analysis was conducted on two Grade 1–6 textbook series. Number sentences were classified according to their relational potential: basic operational, flexible operational, basic relational, and full relational (structures requiring compensatory reasoning or involving two unknowns), drawing from the continuum proposed by Singh and Kosko (2017).

Findings indicate that operational structures overwhelmingly dominate across grade levels. Most textbook exercises follow the traditional format $a + b = c$, reinforcing a computational view of equivalence. Although relational-type equations occasionally appear, they are not systematically emphasized. The predominance of operational forms may limit opportunities for students to develop a structural conception of equivalence. Furthermore, no clear progression in relational complexity was observed from Grade 1 to Grade 6, suggesting limited curricular scaffolding toward more advanced relational reasoning. The findings therefore raise a central issue for mathematics education: relational thinking may not fail to develop because students are incapable, but because curricular materials insufficiently demand structural reasoning.

References

Kızıltoprak, A., & Köse, N. (2017). Relational thinking: The bridge between arithmetic and algebra. International Electronic Journal of Elementary Education, 10.

Singh,R. & Kosko, K. (2017). Exploring the structure of equivalence items in an assessment of elementary grades. In Galindo, E., & Newton, J., (Eds.). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Indianapolis, IN* (pp. 211-218)

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 416. PME.

PRE-SERVICE MATHEMATICS TEACHERS' MOVES FOR SUPPORTING STUDENT REASONING

Fatma Acar[1], Melek Pesen[2], Mervenur Belin[2], Habibe Toker Kekik[3]

[1]MoNE, [2]Boğaziçi University, [3]Alanya Alaaddin Keykubat University

Teacher actions that support students' reasoning are critical for the conceptual understanding of mathematics. Different frameworks have been developed to evaluate how teachers and teacher candidates promote mathematical reasoning. For instance, Ellis et al. (2019) developed the "Teacher Moves for Supporting Student Reasoning (TMSSR)" framework consisting of four categories (Eliciting, Responding, Facilitating, Extending) to analyze teacher's instructional moves. Each category includes low and high potential moves in terms of supporting students' reasoning. In this study, we aimed to reveal the moves of two teacher candidates in terms of supporting student reasoning via examining their two teaching practices on different subjects within the scope of the Mathematics Teaching Practice course. The data sources were teacher candidates' lesson plans, audio recordings of the lessons, self-evaluation reports, and the evaluations of the mentors. Four researchers coded the transcribed lesson recordings individually and collectively according to the TMSSR framework (Ellis et al., 2019).

Preliminary findings showed that the teacher candidates mostly have response- and guidance-oriented moves during their teaching practices, while giving less attention to deepen students' mathematical reasoning. Further, teacher candidates exhibited low-potential moves such as eliciting answers, validating correct answers, cueing, and providing information. The teacher candidates' moves were mostly in the "Responding to students' reasoning" category which might be due to the introductory nature of the lesson topics, time constraints, and the tendency to stick to the lesson plans. The findings inform implications for mathematics teacher education. Analyzing teacher moves demonstrated that teachers' tone of voice and facial expressions are important and it requires the joint use of multimodal data. Furthermore, the cultural background of the teachers should not be neglected. In some cases, it was difficult to decide on the coding, and an intermediate coding category was needed. For instance, when responding to students' answers, some teacher candidates' moves included both the correction of the students' wrong answers and the emphasis of the underlying reason, which does not exactly match with any code in the framework. Further results will be discussed during the presentation.

References

Ellis, A., Özgür, Z., & Reiten, L. (2019). Teacher moves for supporting student reasoning. *Mathematics Education Research Journal, 31*(2), 107-132.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 417. PME.

AFFORDANCES IN DIFFERENT FORMATS: WHAT DO YOUNG STUDENTS USE AND WHY?

Erica Aldenius, Lovisa Sumpter
Department of Teaching and Learning, Stockholm University

Today, students are increasingly encountering knowledge assessments in digital formats, and the digital format has in many places replaced the paper-and-pencil format. However, the affordances and constraints of the format alter students' opportunities to demonstrate mathematical knowledge and skills, and equivalence in results does not necessarily imply equivalence in the processes used by students (Grapin & Sayac, 2022). The affordance theory aims to capture such affordances and constraints (e.g., Gibson, 1979). The present pilot study aims to explore what Swedish students, aged 9 to 10, experience as an affordance (or constraint) in the paper and digital formats, and how they make use of those affordances when solving mathematical tasks. Data generation consists of interviews with and video recordings of seven students in a lab setting. The students worked alone on four mathematical tasks, choosing the format for each. The tasks were selected from the Swedish national tests for grade 3, and an interview guide was developed based on them. The data then consists of students' solutions in their chosen format and the reason they chose that format. The deductive analysis was conducted by examining what affordances the students considered the different formats offered and, based on this, drawing conclusions about whether the formats gave the students different opportunities to solve the tasks (Komatsu & Jones, 2020). The preliminary results show that students identify different affordances in the two formats, such as "easier to erase on a computer", or "easier to calculate using an algorithm on paper". Many students find digital formats suitable for tasks like selecting options or writing short answers. However, for problem-solving tasks requiring presenting solutions, paper offers better opportunities to work and communicate. Hence, the students express that their ability to demonstrate knowledge varies across formats. One implication is that when assessing students, the chosen format impacts the validity (e.g., Grapin & Sayac, 2022) as well as the possibility for students to demonstrate their knowledge fairly.

References

Gibson, J. J. (1979). *The ecological approach to visual perception*. Houghton Mifflin.

Grapin, N., & Sayac, N. (2022). From paper-pencil to tablet-based assessment: a comparative study at the end of primary school. *Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)*, Bozen-Bolzano, Italy, February 2022.

Komatsu, K., & Jones, K. (2020) Interplay between paper-and-pencil activity and dynamic-geometry-environment use during generalisation and proving. *Digital Experiences in Mathematics Education 6*, 123–143. https://doi.org/10.1007/s40751-020-00067-3

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 418. PME.

MATHEMATICS ACHIEVEMENT ACROSS SCHOOL SYSTEMS IN ONE CHILEAN REGION: A MOTIVATOR FOR CHANGE

Felipe Almuna, Lynda Ball

Universidad Austral de Chile and Faculty of Education, University of Melbourne

System-level difficulties in mathematics learning exist in Chile, particularly for the most economically and culturally disadvantaged (Gónzalez-Pizarro et al. 2024), with the COVID-19 pandemic resulting in many students having little face-to-face teaching for almost two years (Hofflinger et al., 2024). This study examines the performance of each school system—i.e., public (PU), private-subsidised (PS) and private-paid (PP) — in one Chilean region, overall and by topic, for the 2022 (post COVID-19) 1.5-hour Years 4 and 10 national mathematics tests. It compares average school performance (by topic) and high/low performance by system. Research questions: (i) What was the performance on the 2022 Year 4 and 10 national mathematics tests, overall and by topic, for each system? (ii) How did high/low school performance compare for each system? Publicly available summary data for each school (Year 4, n=500; Year 10, n=163), reporting a score of 1-10 for each topic, was analysed to give (a) *average school score* (i.e., broad overview of performance), (b) *highest average school score* (and percentage of schools with score) to compare results for highest performing schools by system, (c) *lowest average school score* (and percentage of schools with score), to explore whether being a 'low performing school' is similar for each system and (d) *percentage of schools with average scores* greater than 5, (/10) overall, and by topic. The PP system performed better in every topic, suggesting a systemic issue. However, highest performing Year 4 schools performed similarly, regardless of system, but there are few schools in this group. Lowest performing schools in the PP system scored higher, than *lowest average school score* in PU and PS systems. However, the highest performing school in PU system at Year 4 performed similarly (i.e., average school score) to PP system, so some schools do perform differently to what might be expected based on school system. But having the *lowest average school score* (5) in PP system is different to being the lowest performing PU (2) or PS (2.5) school. Further results will be discussed in the presentation.

References

González-Pizarro, F., López, C., Vásquez, A. & Castro, C. (2023) Inequalities in Computational Thinking among Incoming Students in a STEM Chilean University. *IEEE Transactions on Education*, *67*(2), 180-189

Hofflinger, Á., Rodríguez-Ramírez, R., & Vegas, E. (2024). Chile: From Closure to Recovery: Tracing the Educational Impact of COVID-19. In *Improving Education Systems After COVID-19* (pp. 17–36). https://doi.org/10.1007/978-3-031-69284-0_2

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 419. PME.

EXPLORING ADAPTIVE SUPPORT THROUGH AN INSTRUCTED AI ASSISTANT FOR MATHEMATICAL MODELLING

Alina Alwast

University of Hamburg

Mathematical modelling is cognitively demanding, with barriers arising at various stages of the modelling process (Niss & Blum, 2020). To overcome these barriers while preserving the learner's autonomy, adaptive support is essential (e.g. Stender & Kaiser, 2015). Generative artificial intelligence (genAI), with its interactive nature and capacity for immediate feedback, has the potential to assist learners in solving mathematical problems (Pepin et al., 2025). This raises a central question: Can genAI be applied to mathematical modelling in a way that provides effective, adaptive support?

Against this background, the study's specific aim is to examine how a modelling-specific AI assistant supports pre-service teachers during mathematical modelling processes. To this end, an AI assistant was developed, which is designed to act as a mentor following theoretical principles for supporting modelling processes, such as strategic assistance. Five groups of pre-service teachers (consisting of two to three members each) worked on modelling problems with the modelling-specific AI assistant. The chat histories and video recordings of their work processes were then analysed using qualitative content analysis.

The preliminary findings show that the AI assistant provided multiple forms of support. On the one hand, the AI assistant reliably initiated the discourse by enquiring about the group's particular challenges and the current status of their work. It provided general strategic support related to mathematical modelling (e.g. encouraging the formulation of assumptions) and explicitly highlighted its proneness to errors to enhance the user's awareness. Conversely, an inherent tendency to offer detailed information at times overrode the intended scaffolding approach (despite system-level instructions), thereby shifting the interaction to content-related support.

References

Niss, M., & Blum, W. (2020). *The Learning and Teaching of Mathematical Modeling*. Routledge.

Pepin, B., Buchholtz, N. & Salinas-Hernández, U. (2025). A Scoping Survey of ChatGPT in Mathematics Education. *Digital Experiences in Mathematics Education*, *11*, 9-41.

Stender, P., & Kaiser, G. (2015). Scaffolding in complex modelling situations. ZDM – Mathematics Education, 47(7), 1255–1267. https://doi.org/10.1007/s11858-015-0741-0

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 420. PME.

REVERSING THE DIRECTION OF LOGICAL IMPLICATION IN A GEOMETRIC EXPLORATION TASK

Takeshi Ando
Graduate School of Comprehensive Human Sciences, University of Tsukuba

Exploration tasks, which require students to identify all mathematical objects satisfying a given property, can bring the structure of logical implication to the forefront. Such tasks require students to prove p⇒q while also considering q⇒p to ensure completeness. This approach is expected to clarify how students understand assumptions and conclusions in proofs and can contribute to improving students' reasoning and proof activities in classroom instruction. Building on the one-directional reasoning framework of Dawkins & Norton (2022), this study develops a framework that incorporates both directions of necessity and sufficiency and analyzes how a pair of high school students manipulated and reversed the direction of implication while working on a geometric exploration task. For this exploration, designed to make both assumptions and conclusions in proofs salient, students constructed equilateral triangles externally on two sides AB and AC of a triangle △ABC (△ABD and △ACE), ensuring they did not overlap with △ABC. The given property q was that the four points D, B, C, E are concyclic. Students were asked to determine all possible shapes of △ABC (property p) such that q holds. Initially, the students reasoned in the direction of p⇒q, generating candidate triangles and attempting to verify concyclic deductively. However, during trial-and-error exploration, one student suggested drawing a circle and placing the four points on it. This marked a turning point: instead of deriving q from candidate p, the students instantiated q diagrammatically and explored which triangles satisfied this assumption. Their reasoning thus reversed from p⇒q to q⇒p. The reversal appears to have been motivated by a desire for exhaustiveness. Simply listing familiar triangle types did not guarantee all solutions were identified, whereas assuming concyclic constrained the search space and enabled a more systematic exploration. The diagram acted as a mediating representation, facilitating the shift in reasoning direction. While both directions of implication emerged during the activity, they were not explicitly integrated into a coherent necessary-and-sufficient structure. Arguments based on the assumption of q were not fully re-examined to confirm that each identified p indeed implies q. This case suggests that, although the reversal of implication can arise spontaneously in exploration, bidirectional reasoning does not automatically lead to logical integration of both directions.

References

Dawkins, P., & Norton, A. (2022). Identifying mental actions for abstracting the logic of conditional statements. *The Journal of Mathematical Behavior, 66*.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 421. PME.

EFFECT OF METACOGNITIVE STRATEGY INSTRUCTION ON LOW-ACHIEVING GRADE 8 STUDENTS' PERFORMANCE IN COORDINATE GEOMETRY: A QUASI-EXPERIMENTAL STUDY

Mohamad Farouk Hamuth[1], Khemduth Singh Angateeah[2]

[1]Open University of Mauritius, [2]Mauritius Institute of Education

Geometry is a foundational strand of lower-secondary mathematics. However, many low-achieving grade 8 learners experience geometry as a collection of rules rather than a connected system of relationships. They often rely on memorised procedures without conceptual monitoring of what coordinates and diagrams imply. Metacognitive strategy instruction offers a practical route to support low-achieving learners by making thinking visible and teachable during problem solving (Montague, 2008). Using a quantitative approach, this quasi-experimental study examined whether instruction based on metacognitive strategy (Kuzle, 2013) leads to greater gains in coordinate geometry performance than regular instruction for low-achieving grade 8 students. All grade 8 students (N=47) from two schools were considered. However, after data cleaning, only 12 girls from School A (treatment n=5, control n=7) and 11 boys from School B (treatment n=5, control n=6) were retained and randomly allocated to the control and treatment groups. The same intervention was implemented in both schools. Treatment groups received instruction structured by Kuzle seven recursive phases (Reading, Understanding, Analysing, Exploring, Planning, Implementing and Verifying), supported by bilingual (English/Mauritian Creole) metacognitive prompt sheets to scaffold planning, monitoring and verification. The control groups were taught using traditional methods. Within each school, groups were comparable at pre-test ($p > .05$); both groups improved ($p < .05$), but treatment gain scores were higher ($p < .05$), with a more pronounced advantage in the School B cohort. The findings suggest that structured metacognitive scaffolding can support low-achieving learners in developing more coherent conceptual understanding of coordinate relationships. However, given the small sample size, the findings should be interpreted cautiously and warrant further large-scale investigation before informing policy decisions.

References

Kuzle, A. (2013). Patterns of Metacognitive Behavior During Mathematics Problem-Solving in a Dynamic Geometry Environment. *International Journal of Science and Mathematics Education*, 17(5), 665–685.

Montague, M. (2008). Self-regulation strategies to improve mathematical problem solving for students with learning disabilities. *Learning Disability Quarterly*, 31(1), 37-44.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 422. PME.

PROPORTIONAL REASONING AS COGNITIVE REGULATION: A MULTIPLE-CASE STUDY WITH SECONDARY STUDENTS WITH ATTENTION DIFFICULTIES

Christiyanti Aprinastuti[1], Anastasia Ajeng Wulan Tantri[2], Linda Devi Fitriana[3]
[1]University of Debrecen, [2]The Excellent Study, [3]Jakarta State University

Proportional reasoning (PR) is frequently derailed by the over-use of additive strategies and the overgeneralization of proportional models to non-proportional situations. This literature distinguishes the transition from additive to multiplicative thinking and warns about the 'illusion of proportionality' (Tourniaire & Pulos, 1985). Inhibition and working-memory demands are implicated in PR, where learners must suppress additive heuristics and stabilise multiplicative structure (Cragg & Gilmore, 2014). We adopt the claim that PR operates as cognitive regulation, coordinating executive function (EF) and metacognitive monitoring during problem-solving. The research questions are: (1) how do attention and EF relate to students' strategy use? (2) how does graduated scaffolding influence coherence, transfer, and correction of overgeneralized proportionality? This multiple-case study followed three secondary students (13–15) with attention difficulties as they solved proportional-reasoning tasks under a five-level scaffolding sequence (unsupported → representational scaffolds → invariance prompts → strategic cues → worked examples). Strategy use (additive/mixed/multiplicative/overgeneralized) and coherence (0–3) were coded. Initially, the findings showed that all relied on additive/mixed strategies; two shifted to multiplicative after Level-1 scaffolds, and the third after Level-3 prompts, with coherence rising from 0–1 to 2–3; two students showed transfer. Attention lapses aligned with additive regression, while externalising invariants appeared to lower working-memory demands; on a non-proportional lure, all three first overgeneralized, then two corrected after invariance prompts. We conclude that proportional reasoning functioned as a regulatory mechanism: representational scaffolds reduced EF load and, together with brief metacognitive cues, supported inhibition of impulsive additive responses; instruction should cue invariance early, fade scaffolds strategically, and use short reappraisal prompts, with future work scaling sample size, adding controlled EF measures, and contrasting dynamic vs. static representations.

References

Agnoli, S., Mastria, S., Kirsch, C., & Corazza, G. E. (2022). Executive functions and creative thinking in young students. *Thinking Skills and Creativity*, 45, 101052.

Cragg, L., & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education*, 3(2), 63–68.

Tourniaire, F., & Pulos, S. (1985). *Proportional reasoning: A review of the literature. Educational Studies in Mathematics*, 16(2), 181–204.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 423. PME.

AN INTERDISCIPLINARY APPROACH TO SUPPORT STUDENTS' CONCEPTUALIZATIONS ON LINEAR FUNCTIONAL RELATIONSHIPS

Nil Arabacı, Gülseren Karagöz Akar, Sevda Yerdelen Damar

Boğaziçi University

Mathematical and scientific thinking are inherently interconnected ways of understanding and explaining the world around us. However, students struggle to transfer and apply mathematical ideas, concepts, and formulas to unfamiliar situations in physics (Michelsen, 2006). Meaningful connections between mathematics and physics can be strengthened through a quantitative literacy framework in relation to quantitative reasoning (Brahmia et al., 2023). We further argue that an interdisciplinary approach can be beneficial for connecting ideas across two disciplines by bringing kinematics quantities and functions together.

In this study, we focus on the following research question: How do students reason on linear functional relationships within an instruction involving interdisciplinary mathematics and physics tasks focusing on quantitative literacy? To focus on the research question, we developed design-based research in which the first author conducted four classroom teaching experiment sessions. The participants of the study were 10 science high school students. In all the teaching sessions, the students discussed examples of the linear functional relationship in the context of position-time and velocity-time relationships.

Results indicated that during the initial session of the classroom teaching experiment, none of the students were able to represent the position–time and velocity–time relationships as functions. On the contrary, by the fourth session, students could explain why the relationship between kinematics quantities indicates a function while writing the function rule and explaining where the constant term and the coefficient in that rule come from. Based on the results, we claim that an interdisciplinary approach grounded in quantitative literacy helped elaborate on students' conceptualization of linear functional relationships. Potential implications and directions for subsequent research will be discussed in the oral communication.

References

Brahmia, S. W., & Olsho, A. (2023). Operationalizing and assessing quantitative reasoning in introductory physics. In G. Karagöz Akar et al. (Eds.), *Quantitative reasoning in mathematics and science education*. Springer International Publishing.

Michelsen, C. (2006). Functions: A modelling tool in mathematics and science. *ZDM – The International Journal on Mathematics Education, 38*(3), 269–280.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 424. PME.

HIGH-LEVERAGE NOTICING: FROM STUDENT CONTRIBUTIONS TO PEDAGOGICAL OPPORTUNITIES

Zeynep Arslan, Derya Çelik

Trabzon University

Teaching mathematics requires teachers to navigate a continuous flow of student contributions. Yet, not all contributions hold the same pedagogical potential. Professional noticing (PN) conceptualises teachers' work as attending to, interpreting, and deciding how to respond to students' mathematical thinking (Jacobs, Lamb, & Philipp, 2010). However, distinguishing which student thinking are pedagogically worth pursuing remains a practical and analytic challenge in classroom settings.

The High-Leverage Noticing (HLN) Model addresses this challenge by framing noticing as identifying moments in which student contributions hold pedagogical potential. Drawing on work that characterises certain instances of student thinking as Mathematically Significant Pedagogical Opportunities (MOST) (Leatham et al., 2015), the model specifies how such criteria can inform each component of the noticing triad. In the HLN Model, attending is oriented toward identifying the mathematical point embedded in student contributions. Interpreting involves examining the mathematical centrality and developmental accessibility. Deciding conciders whether such moments create intellectual need within the flow of instruction and advance understanding.

The model was implemented in a practicum module with eight preservice middle school mathematics teachers. Reflective journals, lesson plans, and classroom episodes were analysed to examine how structuring noticing influenced engagement with student mathematical thinking. Findings indicate a qualitative shift in preservice teachers' noticing skills. Although attending statements decreased overall, this reflected greater selectivity rather than reduced engagement. Participants increasingly focused on contributions holding pedagogical potential and sustained their analysis across attending, interpreting, and deciding. By structuring PN through MOST, the HLN Model reorganises noticing by foregrounding an intentional stance toward student contributions that meet these criteria.

Acknowledgement

This study was supported by TÜBİTAK-BİDEB (2214-A International Research Fellowship Programme).

References

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, *41*(2), 169–202.

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education*, *46*(1), 88–124.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 425. PME.

FEELING AND MAKING SENSE OF QUADRATIC FUNCTION THROUGH MODELLING FREE FALL

Guillermina Ávila García[1], Iván Saúl Ortega Castañeda[2]

Instituto Politécnico Nacional, [1]CECyT 11, [2]ESIQIE

Introduction: This study analyses how high school students construct meaning from free fall, examining how the sensorimotor experience fosters the perception of nonlinearity, generating the need for mathematical modelling using Tracker software.

Theoretical Framework: We articulated Objectification Theory (Radford, 2023) for the phenomenological dimension; Anthropological Didactics Theory (Chevellard, 1999) for the reconstruction of mathematical organizations; and the Theory of Registers of Semiotics Representation (Duval, 2006) for cognitive conversion.

Methodology: A qualitative and interpretive approach was used with eight students working in small teams. Data included video recordings, written productions, and Tracker files. The analysis focused on identifying gestures and verbal expressions that demonstrate objectification processes and register conversions.

Results and Discussion: Sensorimotor experience focused on "feeling" rapid acceleration. When linear fitting failed, to map the curvature, students consciously switched to a quadratic fit in Tracker, recognizing the functional necessity (Chevallard, 1999) of a t^2term to match their physical observation. Video analysis showed this conversion facilitated attributing physical meaning to the acceleration parameter (g) within the symbolic model.

Conclusions: Meaning-making emerges from articulating sensory experience with symbolic representations. Bodily interaction fosters awareness of nonlinearity, while digital modelling allows for progressive objectification in formal mathematical terms. Physically, free fall provides a robust context where position depends quadratically on time ($y \propto t^2$); therefore, digital modelling allows students to move from "feeling" motion to formalizing the quadratic nature of acceleration.

References

Chevellard, Y. (1999). L'analyse des practiques enseignantes en théorie anthropologique tu didctique. *Recherches en didactique des mathématiques, 19*(2), 221-266. https://www.univ-irem.fr/IMG/pdf/analyse_des_pratiques_univ_d_ete_la_rochelle.pdf

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational studies in mathematics, 61*(1), 103-131.

Radford, L. (2023). *La teoría de la objetivación : Una perspectiva vygotskiana sobre saber y devenir en la enseñanza y el aprendizaje de las matemáticas*. Ediciones Uniandes. https://doi.org/10.51570/Educ202312

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 426. PME.

A MATHEMATICS TEACHER'S MORAL NEGOTIATION BETWEEN INSTRUCTIONAL VISION AND PRACTICE

Emine Fazlı[1], Emin Aydın[1], Ömer Avcı[2]

[1]Marmara University, [2]Istanbul Medeniyet University

In this exploratory research, we examine how a novice mathematics teacher negotiates the gap between her instructional vision and her classroom practice, focusing on the moral reasoning embedded in her instructional decisions. We adopted a qualitative single case study design. The participant was a novice mathematics teacher at a socioeconomically disadvantaged public high school in Istanbul. Data from classroom observations and interviews were analysed using Moustakas's (1994) Transcendental Phenomenology approach to identify patterns in her moral negotiation. The findings suggest that this negotiation can be interpreted as a form of selective suspension of morality (Bandura, 2016). Rather than abandoning her moral standards, she justifies pedagogically limited practices through three main mechanisms. She externalizes responsibility by framing students' low achievement and lack of interest as constraints that prevent her from preparing lessons or using varied materials. She diffuses responsibility by attributing her teacher-centred practices to broader systemic and institutional expectations. She engages in moral rationalization by redefining limited, procedural teaching as ethically acceptable and reconstructing her responsibility merely around completing tasks rather than ensuring high-quality mathematics instruction. This selective suspension allows the novice teacher to maintain her moral self-worth and preserve her original instructional vision while working within a challenging school environment. In the presentation, we further examine how this process reframes her instructional vision as a future-oriented ideal rather than a present responsibility. While this study focuses on a single novice mathematics teacher, it has broader implications for mathematics education. It demonstrates that high-quality instruction depends not only on content knowledge and pedagogical skill but also on moral reasoning. These findings signal the need to further investigate teachers' moral reasoning to support teachers in ensuring high-quality mathematics instruction.

Additional information

This study uses data from the first author's master's thesis, supervised by the second and third authors, funded by Marmara University BAP (Project No. 11226).

References

Bandura, A. (2016). Moral disengagement: *How people do harm and live with themselves*. Worth Publishers.

Moustakas, C. (1994). *Phenomenological research methods*. Sage publications.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 427. PME.

CO-DESIGNING A GEOGEBRA-BASED ADAPTIVE LEARNING ENVIRONMENT FOR LINEAR FUNCTIONS

Selina Baldinger, Jonas Mayrhofer

Johannes Kepler University Linz, Center for Open Digital Education

Despite their increasing use, digital mathematics learning environments (DMLEs) often rely on a one-size-fits-all approach that overlooks inherent learner heterogeneity (Awang et al., 2024). Adaptive learning addresses this challenge by tailoring, for instance, task difficulty and scaffolding strategies to evolving student needs, making it a promising approach for future mathematics learning (Engelbrecht & Borba, 2024). However, to ensure that DMLEs actually reflect classroom realities requires involving pedagogical agents, such as teachers, in their development. Co-design offers a promising solution by directly embedding pedagogical expertise into adaptive DMLE design (Voogt et al., 2015). In our case study, we utilize the five iterative phases of the ADDIE (*analysis*, *design*, *development*, *implementation*, *evaluation*) model to co-design a GeoGebra-based adaptive DMLE on linear functions with one Austrian in-service secondary mathematics teacher experienced in DMLE development. Audio recordings, field notes, and material prototypes from four co-design sessions were analysed using reflexive thematic analysis to link ADDIE phases with practical design rationales. Results demonstrate how the teacher's pedagogical expertise grounded the adaptive DMLE in classroom realities. Going beyond theoretical knowledge, the teacher shaped the design by setting time constraints, focusing the mathematical scope on slope and intercept, and defining adaptive difficulty and scaffolding strategies to target common student misconceptions. A subsequent teacher-led pilot refined the prototype's linguistic precision and usability and ultimately yielded a practice-informed strategy for future classroom evaluations. Practically, this study contributes a prototype and concrete adaptive specifications for a classroom-ready adaptive DMLE on linear functions. For the research community, it offers a transparent, replicable account of structured collaboration, illustrating how bridging theoretical design with everyday classroom realities yields more effective educational technology.

References

Awang, L. A., Yusop, F. D., & Danaee, M. (2024). Insights on usability testing: The effectiveness of an adaptive e-learning system for secondary school mathematics. *International Electronic Journal of Mathematics Education, 19*(3), em0782.

Engelbrecht, J., & Borba, M. C. (2024). Recent developments in using digital technology in mathematics education. *ZDM–Mathematics Education, 56*(2) 281–292.

Voogt, J., Laferrière, T., Breuleux, A., Itow, R. C., Hickey, D. T., & McKenney, S. (2015). Collaborative design as a form of professional development. *Instr. Sc., 43*(2), 259–282.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 428. PME.

HOW MATHEMATICS TEACHER EDUCATORS RELATE PRIOR EXPERIENCES TO THEIR EXPERTISE

Jonei Cerqueira Barbosa

Federal University of Bahia

Research on mathematics teacher educators (MTEs) has expanded significantly in recent years. Recent scholarship conceptualises MTE work through the lens of expertise, understood as what MTEs do and why when educating teachers, with particular attention to their pedagogical communication (Barbosa, 2025). If MTE expertise is distinctive, understanding how it is constituted matters. Prior experiences may be consequential, yet how MTEs connect these experiences to the expertise they enact remains underexplored (Goos, 2025). This study asks: How do MTEs relate their prior experiences to the expertise they enact in mathematics teacher education?

Using an interpretivist qualitative design, I conducted semi-structured interviews with four MTEs working across different course types in an undergraduate mathematics teacher education programme. Thematic analysis of the interview transcripts identified five themes: experiences in school settings, academic trajectory and research, professional experience as an MTE, professional discussions, and literature and material selection. These themes depict a network linking past and present practices: school experiences serve as contrastive or inspirational lenses; academic training structures sequencing and criteria; accumulated professional experience refines sensitivity; peer discussions provide resources and intersubjective support; and literature guide task design and pedagogical reorganisation.

I propose two constructs to interpret the findings. Biographical-disciplinary capital synthesises school and academic experiences into a repertoire that anchors the initial constitution of expertise. Sociomaterial mediation captures how expertise is updated through professional practice, peer discussions, and engagement with artifacts and literature. These dimensions interact dynamically and recursively, suggesting that MTE expertise is continuously constituted and reorganised. The findings indicate that supporting MTE development requires making prior experiences explicit. Future research should compare different forms of biographical-disciplinary capital and sociomaterial mediation across settings to refine these constructs.

References

Barbosa, J. C. (2025). Bridge-building as a dimension of mathematics teacher educators' expertise. In C. Cornejo et al. (Eds.), *Proceedings of the 48th Conference of the Int. Group for the Psychology of Mathematics Education* (Vol. 1, pp. 83–90). PME.

Goos, M. (2025). The complexity of mathematics teacher educator expertise. *Journal of Mathematics Teacher Education*, *28*(2), 445–457.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 429. PME.

HOW DO NORWEGIAN PRE-SERVICE MATHEMATICS TEACHERS ENACT HUMAN RIGHTS RESPONSIBILITIES?

Yasmine Abtahi[1], Richard Barwell[2], Emma Carlenius[3], Lisa Steffensen[3]

[1]University of South-Eastern Norway; [2]University of Ottawa, Canada; [3]Western Norway University of Applied Sciences

In the context of global challenges, injustice and threats to democracy, it is important that human rights values to be included in mathematics teaching and learning. Norway provides an interesting case, since the curriculum explicitly requires such values – democratic participation, human dignity and ethical awareness – to be incorporated into all subjects, including mathematics. When these values are built into teaching, mathematics can be more meaningful for learners and youth can contribute to a socially just democratic society. The question then arises of how pre-service teachers (PSTs) understand these values and incorporate them into their teaching. We used a participatory design to first understand the perspectives of youth and then those of PSTs. We applied Robinson et al.'s (2020) framework in which educators' *responsibilities* are categorised in terms of: knowledge and values (learning about human rights); attitude and environment (to create an environment for teaching through human rights values); and agency and action (to educate for human rights). Our research question for this OC is: how do PSTs express their responsibilities about, through and for human rights in response to the perspectives of youth.

16 lower-secondary students in Oslo discussed mathematics and human rights over one year. They produced artefacts (e.g., artwork, a rap song, film) expressing their ideas with the explicit purpose of sharing these artefacts with PSTs. Next, PSTs at a Norwegian university participated in focus groups and activities in their mathematics teaching courses to: (1) discuss their ideas about mathematics and human rights; (2) view and discuss the artefacts; (3) design mathematics teaching activities in response to the artefacts. Our analysis of data from one group with four PSTs shows the artefacts prompt shifts in their thinking, with comments such as: "people are basically just portrayed as numbers, that they don't have any significance", and the real world "becomes so overwhelming that it spills over and […] blinds the person". The artefacts also influence PSTs' design of mathematics teaching activities, e.g. by integrating poverty, wealth tax, or media statistics, to include human rights values in mathematics.

Acknowledgements: ViMTE project funded by Research Council of Norway, additional team members: T. M. Foyn, K. H. Hauge, B. Herbel-Eisenmann, S. Kacerja, M. Loeb.

Reference

Robinson, C., Phillips, L. & Quennerstedt, A. (2020). Human rights education: developing a theoretical understanding of teachers' responsibilities. *Educational Review*, *72*(2), 220–241.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 430. PME.

STUDENTS' VIEW OF MATHEMATICAL PROBLEMS IN THE CONTEXT OF DESIGNING A MATHEMATICAL ESCAPE ROOM

Caterina Bassi, Domenico Brunetto

Department of Mathematics, Politecnico di Milano

Research highlights a gap between experts' and students' views of mathematical problems. Experts characterize problem solving as involving the absence of a ready-made procedure and the activation of productive thinking (Perkins, 2000), whereas students often see a problem as a story with numbers to combine to obtain an answer, paying little attention to context (Di Martino, 2018; Lesh & Doerr, 2003). This view tends to become increasingly stereotyped over schooling (Di Martino, 2018).

Drawing on the "view of mathematics" component of the attitude construct (Di Martino & Zan, 2010), we investigate upper secondary students' views of mathematical problems within a project in which they collaboratively designed a mathematical escape room for lower secondary students. We address two questions: (RQ1) What are students' views at the beginning of the project? (RQ2) Does designing a mathematical escape room foster greater attention to context? If so, how? To answer RQ1, we conducted an inductive qualitative content analysis of students' responses to an open-ended prompt; many proposed stereotyped problems. Regarding RQ2, analysis of the designed escape rooms showed that such stereotypes largely persisted—unsurprising, given the limited scope of the intervention. However, preliminary findings from a similar project including an explicit reflection on narrative–problem coherence suggest increased attention to context. Designing a mathematical escape room thus appears promising, provided that attention to context is explicitly discussed with students.

References

Di Martino, P., & Zan, R. (2010). 'Me and maths': towards a definition of attitude grounded on students' narratives. *Journal of Mathematics Teacher Education*, *13*(1), 27-48. https://doi.org/10.1007/s10857-009-9134-z

Di Martino, P. (2018). Pupils' view of problems: the evolution from kindergarten to the end of primary school. *Educational Studies in Mathematics, 100*, 291-307. https://doi.org/10.1007/s10649-018-9850-3

Lesh, R., & Doerr, H. (2003). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh, & H. M. Doerr (Eds.), *Beyond constructivism: models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 3-33). Mahwah, NJ: Lawrence Erlbaum.

Perkins, D. (2000). *Archimedes' bathtub: the art of breakthrough thinking*. New York: W.W. Norton and Company.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 431. PME.

INTERACTIONS OF MATHEMATICAL MODELLING AND COMPUTATIONAL THINKING IN DATA-BASED PROJECTS

Lukas Bayer

University of Koblenz

Mathematical modelling is the process of translating problems from a given context into mathematics. Similarly, computational thinking (CT) refers to thinking processes needed for translating problems (and solutions) from a given context into the domain of computer science. Therefore, CT and mathematical modelling share practices, such as generalising from special cases (cf. Kallia et al., 2021). However, their interaction in mathematics learning remains underexplored. This study addresses the following research question: Which CT practices appear in data-based modelling projects of upper secondary school students, and how are they deployed in relation to mathematical modelling? Data were collected during a mathematical modelling week with seven data-based projects involving 40 upper secondary school students. Groups documented their modelling processes through computational essays (Odden et al., 2023), combining programming code and explanatory text. These essays were deductively coded using qualitative content analysis based on the CT model given by Kallia et al. (2021) to reconstruct deployed CT practices. Preliminary results highlight two CT practices mainly deployed by students: *(1) algorithmic thinking* to construct computational models as implementations of mathematical models, and *(2) generalisation* to construct adaptable and transferable solutions. Additionally, mutual influence of modelling and CT was observed in learners' transitions between domains: they often transitioned from computational results to mathematical models by visualising the results of the computational model with a mathematical representation. These visualisations were used to evaluate intermediate results in the mathematical domain rather than the real context. Interpretation of these results is limited, as they rely on modelling artefacts rather than observed processes. Overall, the preliminary results suggest that CT and mathematical modelling are supportive of each other, due to the productive back-and-forth between algorithmic implementation and mathematical reasoning.

References

Kallia, M., van Borkulo, S. P., Drijvers, P., Barendsen, E., & Tolboom, J. (2021). Characterising computational thinking in mathematics education: A literature-informed Delphi study. *Research in Mathematics Education*, *23*(2), 159–187.

Odden, T. O. B., Silvia, D. W., & Malthe-Sørenssen, A. (2023). Using computational essays to foster disciplinary epistemic agency in undergraduate science. *Journal of Research in Science Teaching*, *60*(5), 937–977. https://doi.org/10.1002/tea.21821

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 432. PME.

DO STUDENTS' ESTIMATES AND USE OF THE SAMPLE PROPORTION IMPROVE AS GRADE LEVEL PROGRESSES THROUGH SECONDARY SCHOOL?

Nuria Begué[1], Hortensia Ramos-Pérez[2], María Magdalena Gea[2], Alondra Stuardo-Aguayo[2]

[1]University of Zaragoza, [2]University of Granada

Understanding sampling is key to statistical reasoning; in particular, the sample proportion relies on the sampling process to infer information about a population (Ben-Zvi et al., 2015). However, the literature reflects difficulties associated with its understanding (Batanero et al., 2020). In order to complement previous research, the aim of this study is to analyse the responses of 104 Spanish students (aged 12 to 15) in a task adapted from the literature that requires the estimation of the sample proportion and its application in a resampling context. We used the content analysis (Neuendorf, 2018) to identify and classify the students' answers. Regarding the population proportion estimate, 35.6% of students apply proportional reasoning. Of those students, 34.3% provide the correct estimate. One quarter of students (25%) provide an estimate without a strategy, and of these, only 7.1% provide the correct value. The remainder apply additive reasoning (12.5%), respond imprecisely (12.5%), state they are unable to provide an estimate (1%), or do not respond (13.5%). In the total population estimate, only 6.7% apply proportional reasoning, with 42.9% of their responses being correct; meanwhile, the use of strategies involving additive reasoning increases (46.2%), not using a strategy (27.9%), imprecise responses (8.7%), or non-response (10.6%). Correct answers were also scarce when calculating an expected value using the sample proportion. Our results highlight the need to reinforce the understanding of sampling from an early age, considering proposals that promote different levels of algebraic reasoning in a stochastic sense (i.e., Burgos et al., 2022).

References

Batanero, C., Begué, N., Borovcnik, M. & Gea, M.M. (2020). Ways in which high-school students understand the sampling distributions for proportions. *Statistics Education Research Journal*, *19*, 32-52 https://doi.org/10.52041/serj.v19i3.5

Ben-Zvi, Bakker, A., & Makar, K. (2015). Learning to reason from samples. *Educational Studies in Mathematics*, *88*, 291–303. https://doi.org/10.1007/s10649-015-9593-3

Burgos, M., Batanero, C. & Godino, J. D. (2022). Algebraization levels in the study of probability. *Mathematics*, *10*(1), 91. https://doi.org/10.3390/math10010091.

Neuendorf, K. A. (2018). Content analysis and thematic analysis. In P. Brough (Ed.), *Advanced research methods for applied psychology: Design, analysis and reporting* (1.ª ed., pp. 211–223). Routledge. https://doi.org/10.4324/978131551797

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 433. PME.

NUANCED TEACHING AND TEACHING PERSPECTIVES: REFINING OUR LENSES FOR OBSERVING CHANGE

Mary Beisiegel, Kelsey Quaisley

Oregon State University

In our work offering professional development (PD) intended to broaden and improve mathematics graduate teaching assistants' (MGTA) teaching practices (Pilgrim et al., 2025), we aimed to understand whether and how MGTAs' teaching practices changed over time. We videorecorded various lessons that the MGTAs facilitated six times over the course of two years while they participated in a PD program that is focused on student engagement in active learning and group problem solving. Initially, we solely used observation tools to track change (e.g., Toolkit for Assessing Mathematics Instruction (TAMI), Hayward et al., 2017). However, these tools did not offer enough insight for us to fully understand the nuanced changes the MGTAs were making to their teaching practices. We then turned our attention to additional data sources (interviews, artifacts and reflections from PD seminars, post-observation debriefs) and a different lens – the Theory of Planned Behaviour (TPB, Azjen, 1991), which explores attitudes, intentions, norms, perceived behavioural control, and resulting behaviours. This combination of theoretical framework and additional data allowed us to notice the oftentimes nuanced changes in how the MGTAs thought about and aimed to teach. It also allowed us to understand that the MGTAs were persistent in and dedicated to their growth in teaching. One participant, Ayla, is an example of the deeper understanding we were able to achieve. Close to the end of the PD program, we observed Ayla in her classroom and saw that students were not engaging with each other, despite Ayla's prompts. In exploring her journals, interviews, and debriefs through TPB, we came to understand that she had persevered in her beliefs and attitudes toward supporting students engaging with each other and that the changes she was making were subtle yet indicative of profound change. In this O.C., we share further examples of nuanced changes that we observed in MGTAs' classrooms through deeper analysis with TPB.

References

Ajzen, I. (1991). The theory of planned behavior. *Organizational Behavior and Human Decision Processes*, *50*(2), 179-211.

Hayward, C. N., Laursen, S. L., & Weston, T. J. (2017). TAMI-OP: Toolkit for Assessing Mathematics Instruction – Observation Protocol. Boulder, CO: Ethnography & Evaluation Research, University of Colorado Boulder.

Pilgrim, M. E., Quaisley, K., Yu, F., Beisiegel, M., Oliva, P. D., Moschetti, M., & Segal, R. (2025). Guiding Principles for Equity-Oriented Professional Development for Mathematics Graduate Teaching Assistants. *Notices of the Am. Math. Society, 72*(2).

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 434. PME.

A DISCIPLINARY DIVIDE: AN ANALYSIS OF K-12 EDUCATOR ATTITUDES TOWARD INTEGRATED STEM

Cory A. Bennett, Wendy Ructhi & Elizabeth Metts

Idaho State University

Shifting STEM learning experiences from siloed content explorations to interdisciplinary integration requires understanding the affordances and challenges in schools. English (2016) indicates that meaningful STEM integration requires "boundary-crossing" where disciplines are mutually reinforcing and Stohlmann et al. (2012) argue that successful integration hinges on teachers' perceptions of feasibly integrating content. As such, this study examined teachers' perceptions towards the feasibility of integrating STEM content by examining attitudes toward integration between elementary and secondary STEM teachers and how STEM backgrounds influence perceived feasibility. A survey was completed by 50 K-12 educators; 20 were elementary teachers and 30 were secondary STEM teachers. Of the 30 secondary, 14 taught science and 17 taught mathematics and one taught both. The statistical analysis included Mann-Whitney U tests to compare the independent groups.

Findings suggest there may be an integration gap between elementary and secondary mathematics teacher that does not exist between elementary and secondary science teachers. A statistically significant difference was found between elementary and secondary mathematics teachers' perspectives on the feasibility of integration whereas there were no statistically significant differences ($p=0.01$) between elementary and secondary science teachers in the same metrics. This suggests the barrier to integration may not necessarily be between elementary and secondary contexts but may be specifically related to mathematics classrooms.

The results may challenge the assumption that content specialization leads to better STEM integration. While secondary science teachers seem to successfully incorporate other STEM disciplines, including mathematics and data sciences, secondary mathematics teachers believe they face unique structural and pedagogical hurdles that lower their perception on the feasibility of integration. Essentially, mathematics teachers may feel structurally and pedagogically isolated.

References

English, L. D. (2016). STEM education K-12: Perspectives on integration. *International Journal of STEM Education*, 3(3). https://doi.org/10.1186/s40594-016-0036-1

Stohlmann, M., Moore, T. J., & Roehrig, G. H. (2012). Considerations for teaching integrated STEM education. *Journal of Pre-College Engineering Education Research (J-PEER)*, 2(1), 28–34. https://doi.org/10.5703/1288284314653

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 435. PME.

HOW IS MATHEMATICAL REASONING RECONTEXTUALISED IN SWEDISH AND ENGLISH CURRICULUM DOCUMENTS?

Ewa Bergqvist[1], Candia Morgan[2], Magnus Österholm[1], Lotta Vingsle[1], Jill Adler[3]

[1]Umeå University, [2]UCL Institute of Education, [3]University of Witwatersrand

Mathematical reasoning is an aim for student learning in policy documents worldwide, but the concept is often ascribed different meanings (e.g., Reid, 2022). Our overarching project aims to understand how mathematical reasoning is construed and recontextualised within and between discourses in the curriculum chain in Sweden and England. By comparing the curriculum chains in Sweden and in England, we hope to contribute to a general understanding of processes of recontextualization. Here, we compare how curriculum documents in Sweden and England construe reasoning, in relation to three reform frameworks based on mathematics education research. Previous research on conceptualisations of reasoning within research has resulted in categorisation of a range of meanings, for example, generalising, conjecturing, identifying patterns, comparing, justifying, and proving (e.g., Jeannotte & Kieran, 2017). Similarly, we found 39 named types of reasoning across three frameworks: Adding it Up, NCTM Standards, and the KOM-project (Morgan et al., 2024). Five types of reasoning were frequent and common to all frameworks: argument, (logical) chain, explanation, justification, and proof. The data in this study are the current mathematics curriculum documents and official commentary and guidance materials relevant for lower secondary school. Our analyses identify parts of the texts describing processes and objects construed as types of reasoning and compare the results between the two countries. We focus particularly on the *objects* that were given as goals of reasoning processes or named as types of reasoning, for example, argument or explanation. The preliminary results show that while *argument* and *justification* are common in both frameworks and curriculum documents, the curriculum documents also emphasise other aspects. For example, *response/refutation* (Sw. "bemötande") is common in the Swedish curriculum and *generalisation* in the English curriculum. We will continue with more detailed analyses and based on these, discuss possible reasons for differences between the recontextualizations of reasoning in the two countries.

References

Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. *Educational Studies in Mathematics*, *96*(1), 1-16.

Morgan, C., Bergqvist, E., Adler, J., & Österholm, M. (2024). Discourses of mathematical reasoning: analysis of three reform frameworks. In T. Fujita (Ed.) *Proceedings of the British Society for Research into Learning Mathematics 44*(1). BSRLM.

Reid, D. A. (2022). 'Reasoning' in national curricula and standards. Presented at the 12th *Congress of the European Society for Research in Math Education*, Bozen-Bolzano, Italy.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 436. PME.

CONCEPTUALISATION OF A VIGNETTE TEST TO ASSESS MATHEMATICS TEACHERS' LESSON PLANNING SKILLS

Katharina Berking[1], Benjamin Rott[1], Gabriele Kaiser[2], Johannes König[1]

[1]University of Cologne, [2]University of Hamburg

Lesson planning is a complex cognitive process that must align learning objectives, curricular requirements, and learners' prior knowledge. Despite its importance for the teaching profession, (Mathematics) teachers' lesson planning skills (LPS) remain under-researched. To address this research gap, a text vignette test was developed to assess the LPS of lower secondary Mathematics' teachers in Germany. Drawing on Cevikbas et al. (2024), we conceptualise those skills within professional teacher competence as situation-specific, mediating between cognitive dispositions and performance (Blömeke et al., 2022). The test consists of two text vignettes (in randomised order) representing typical planning situations in lower secondary mathematics, and 23 items which are linked to one of the two vignettes and refer to five cognitive demands teachers need to meet during lesson planning: *pedagogical adaptivity*, *clarity of learning objectives*, *structuring the lesson*, *content transformation* and *ICT (= information and communication technology) integration*, adapted from prior research (e.g., König et al., 2021). To assess test quality, the instrument was administered to a subsample of N = 198 pre-service lower secondary Mathematics teachers at the start of the German induction phase as part of an ongoing, large-scale, longitudinal study. Findings indicate a reliable measurement (EAP-Reliability = 0.589, $\alpha = 0.670$), and substantial agreement in double-coded open response items (20% of the data, $\kappa = 0.67$, 95% *CI* [0.54, 0.80]), although some poorly fitting items were identified. A revised version of the test is currently being implemented in the longitudinal study and additional validation is required. Overall, initial findings suggest that the test may be promising to assess LPS and provide insights into the development of these skills, with potential implications for teaching quality.

References

Blömeke, S., Jentsch, A., Ross, N., Kaiser, G., & König, J. (2022). Opening up the black box: Teacher competence, instructional quality, and students' learning progress. *Learning and Instruction*, *79*, 101600. https://doi.org/10.1016/j.learninstruc.2022.101600

Cevikbas, M., König, J., & Rothland, M. (2024). Empirical research on teacher competence in mathematics lesson planning: Recent developments. *ZDM – Math ed*, *56*(1), 101–113.

König, J., Krepf, M., Bremerich-Vos, A., & Buchholtz, C. (2021). Meeting Cognitive Demands of Lesson Planning: Introducing the CODE-PLAN Model to Describe and Analyze Teachers' Planning Competence. *The Teacher Educator*, *56*(4), 466–487. https://doi.org/10.1080/08878730.2021.1938324

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 437. PME.

TEACHERS' KNOWLEDGE IN GAI-SUPPORTED MATHEMATICS EDUCATION

Maria Lucia Bernardi[1], Helena Rocha[2]

[1]University of Bari Aldo Moro

[2]Faculdade de Ciências e Tecnologia, Universidade NOVA de Lisboa

The growing use of Generative AI (GAI) tools such as ChatGPT in mathematics education raises new questions about teachers' knowledge. Existing models, such as the Knowledge for Teaching Mathematics with Technology (KTMT; Rocha, 2020), provide robust accounts of how teachers integrate technology into pedagogy and content. However, these models were developed before the advent of GAI and do not fully account for its distinctive demands, such as fluent error or non-determinism.

In this study, we reconsider KTMT in GAI contexts by extending three interrelated knowledge domains: AI Knowledge (AIK), AI Mathematical Knowledge (AIMK), and AI Pedagogical Knowledge (AIPK), capturing respectively teachers' understanding of how chatbots operate, their ability to evaluate AI-generated mathematical content critically, and their capacity to adapt AI output to their teaching and students' learning.

With the aim of examining the mobilization of these domains, a qualitative pilot study was conducted in Portugal, involving four pre-service mathematics teachers (PMTs) in a workshop in which they used ChatGPT to solve a mathematics problem and to design a lesson. Data from discussions, chatbot interactions, and surveys were analysed through episode coding based on the extended model. All three domains emerged and interacted dynamically: PMTs drew on AIK when reasoning about how prompting shapes AI output, on AIMK when evaluating mathematical correctness and identifying conceptual omissions, and on AIPK when revising AI-generated lesson plans according to their own pedagogical principles. These findings offer preliminary evidence for the need to reconsider existing teachers' knowledge models in light of emerging technology and provide initial empirical support for the effectiveness of the proposed extended version of the KTMT model in the GAI context.

Acknowledgement

This study was supported by Portuguese funds through FCT, I.P., within the scope of the project AI-TecTeachers (2024.15124.PEX). The research of ML Bernardi is funded by a PhD fellowship within the framework of the Italian "D.M. n. 118/23"- PNRR, M4, C1, Inv. 4.1 – PhD Project "Tech4Math-Math4STEM" (CUP H91I23000500007).

References

Rocha, H. (2020). Using tasks to develop pre-service teachers' knowledge for teaching mathematics with digital technology. *ZDM, 52*, 1381–1396.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 438. PME.

MATHEMATICS EDUCATION IN A RURAL SCHOOL IN SOUTHERN BRAZIL

Andrei Luís Berres Hartmann, Marcus Vinicius Maltempi

São Paulo State University, Unesp

In Southern Brazil, many students in rural schools come from family farming communities. This study examines how mathematics education (ME) is enacted when schooling explicitly connects curricular content with students' cultural and generational knowledge. Grounded in ethnomathematics (D'Ambrosio, 2018), research on culture and popular knowledge in ME (Knijnik, 2006), and critical/Freirean perspectives, we ask: What is revealed in the pedagogies and practices of a rural school when analysed through a lens attentive to culture, power, and participation? The study is qualitative and ethnographic, conducted in a rural school organized through the pedagogy of alternation (periods at school alternating with periods in family contexts). Participants were 66 students and 8 teachers across disciplines. Data include participant observation (Feb–Jul 2023), field notes, transcribed interviews, school documents, and photographs. Content analysis examined how mathematical knowledge was selected, legitimized, and connected to rural life. Four findings emerged. (1) The school mobilized contextualized mathematics linked to family farming, especially financial decision-making and everyday quantitative reasoning. (2) Mathematics-related activities supported dialogue, intersubjective relations, and critical participation, positioning students as agents who question and justify decisions affecting their communities. (3) Alternation enabled school–family–community integration, creating recurring pathways for community knowledge to enter the classroom and for curricular mathematics to circulate back into practice. (4) Family farming was treated not only economically but also as a locus of sustainability, shaping which problems, data, and criteria counted as "good" solutions. These results indicate that this school format supports ethnomathematical and critical approaches, with implications for task design, teacher education, and institutional structures that connect community knowledge and school mathematics. Internationally, it offers an empirically grounded account of sustainability as a curricular organizer, informing debates on epistemic justice, rurality, and the socio-ecological turn in mathematics education.

Acknowledgment: Grant 2021/11937-0, FAPESP

References

D'Ambrosio, U. (2018). Etnomatemática, justiça social e sustentabilidade. *Estudos avançados, 32*, 189-204.

Knijnik, G. (2006). *Educação matemática, culturas e conhecimento na luta pela terra*. Santa Cruz do Sul: EDUNISC.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 439. PME.

NEGOTIATING IDENTITY AND SENSE-MAKING IN AI-SUPPORTED MATHEMATICS LEARNING: EXPERIENCES OF GIFTED STUDENTS

Emrah Bilgiç[1] Şahin Danişman[2]

[1]Sakarya University, [2]Düzce University

The rapid integration of artificial intelligence [AI] into mathematics learning environments is transforming students' engagement with mathematical problem solving (Opesemowo & Ndlovu, 2024), yet little is known about how mathematically gifted learners psychologically experience these tools and how AI reshapes their mathematical sense-making. This study explores how mathematically gifted high school students make sense of mathematical problem solving in AI-supported learning environments, focusing on how these tools shape students' academic identity within mathematical engagement. The study was conducted as a qualitative phenomenological inquiry aimed at understanding students' lived experiences. Participants were 24 mathematically gifted high school students (14 female, 10 male) enrolled in Science and Art Centers in Türkiye. Data were collected through individual face-to-face structured interviews with open-ended questions on AI use, decision-making, perceived competence, ownership of learning, and mathematical self-understanding, and analyzed using reflexive thematic analysis.

Students described AI-supported mathematics learning as an actively regulated process in which they sought to preserve intellectual ownership by first attempting problems independently and limiting AI assistance to hints or partial support. While strategic AI use enhanced understanding and confidence, encountering more efficient AI-generated solutions sometimes triggered temporary self-doubt and highlighted a tension between independence and reliance. Students distinguished between ownership of process and product, perceiving achievement as legitimate only when grounded in personal effort and often reworking solutions to restore authorship. AI use also prompted reflection on mathematical identity: seeking help challenged internalized competence norms, leading many to redefine being "good at mathematics" as the ability to think strategically, critically evaluate solutions, and effectively guide AI. The findings suggest that effective integration of AI in mathematics education should support reflective and agentic use of technology while preserving students' ownership and developing mathematical identity.

References

Opesemowo, O. A. G., & Ndlovu, M. (2024). Artificial intelligence in mathematics education: The good, the bad, and the ugly. *Journal of Pedagogical Research, 8*(3), 333-346. https://doi.org/10.33902/JPR.202426428

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 440. PME.

ENHANCING NUMBER SENSE THROUGH SUBITIZING AND GROUPITIZING

Nathalie Bisaillon, Roxanne Bilodeau, Raphaëlle Dufour, Françoise Lachaîne

Université de Montréal

Many students struggle with grouping and the multiplicative aspect of place value, key aspects of number sense. This study examines how number sense develops through a teaching sequence based on subitizing and groupitizing. It addresses the following research question: how do students' responses to number sense tasks change, particularly in the ways they structure and coordinate quantities in additive, multiplicative, and equivalence thinking? Number sense is defined as a flexible understanding of numbers and operations beyond procedures (Reys, 1994). This study builds on a developmental hypothesis in which number sense progresses from subitizing and groupitizing to more structured forms. Subitizing and groupitizing are considered as tools for structuring quantities as wholes, enabling the construction of flexible mental representations essential for understanding numeration and supporting number sense development. The participants were eight students aged 8–9 who were struggling with mathematics. A pre- and post-test assessed number sense through additive, multiplicative, and equivalence. Tasks were presented on a screen within a 30-minute session, with limited exposure time to encourage reliance on mental structuring rather than counting one by one. Qualitative analyses of the students' responses focused on correctness, organization of quantities, and coordination of representations. The analysis shows consistent changes in students' number sense across participants. In additive tasks, responses shifted from sequential counting to increasingly structured groupings, reflecting a growing ability to treat quantities as wholes. In multiplicative tasks, students produced structured arrays more often, reflecting emerging coordination of quantities. In equivalence tasks, responses improved but remained fragile. The time-limited nature of the intervention makes it difficult to determine whether these remaining difficulties reflect persistent obstacles or could have been overcome. However, these changes suggest that subitizing and groupitizing appear to underpin students' structuring of quantities across levels of thinking, supporting the development of mental representations essential to number sense and emphasizing the role of visuospatial abilities in numerical understanding (Gunderson et al., 2012).

References

Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: The role of the linear number line. *Developmental Psychology, 48*(5), 1229–1241.

Reys, B. J. (1994). Promoting number sense in the middle grades. *Mathematics Teaching in the Middle School, 1*(2), 114–120. https://doi.org/10.5951/MTMS.1.2.0114

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 441. PME.

THE CONTRIBUTION OF GENERATIVE AI (CHATGPT) TO PRE-SERVICE MATHEMATICS TEACHERS' PROBLEM-POSING SKILLS AND TPACK

Yaniv Biton[1,2], Ruti Segal[3]

[1]Shaanan Academic College of Education, Haifa

[2]Center for Educational Technology, Tel-Aviv

[3]Oranim Academic College of Education, Tivon

The rapid emergence of generative artificial intelligence (GenAI) has opened new possibilities for supporting teaching practices, particularly in areas where teachers experience difficulty, such as mathematical problem posing (PP). Despite its central role in mathematics education, PP remains a challenging task for many teachers (Cai & Jiang, 2017). This study examines how interaction with ChatGPT supports pre-service mathematics teachers' PP practices and contributes to their technological pedagogical content knowledge.

Fifteen pre-service high school mathematics teachers participated in a preparatory course focused on teaching advanced mathematics. After solving several mathematical problems, participants were asked to generate and refine new problems by interacting with ChatGPT using structured prompts. Data included 30 prompts, corresponding AI responses, and teachers' reflections. A qualitative analysis using constant comparative methods identified key themes in both ChatGPT's suggestions and teachers' responses.

Findings indicate that ChatGPT functioned as a pedagogical partner that supported multiple dimensions of problem posing. Its contributions included improving clarity and mathematical precision, increased cognitive demand, the incorporation of real-world contexts, encouraging inquiry-based thinking, and suggesting differentiation strategies for diverse learners. From a TPACK perspective, the use of ChatGPT supported the integration of technological, pedagogical, and content knowledge by enabling teachers to design richer mathematical tasks and reconsider instructional decisions.

The results suggest that integrating AI tools into teacher education programs can provide meaningful opportunities for developing both pedagogical skills and adaptive expertise in technology-rich environments.

References

Cai, J., & Jiang, C. (2017). An analysis of problem-posing tasks in Chinese and U.S. elementary mathematics textbooks. International Journal of Science and Mathematics Education, 15, 1521–1540. http://dx.doi.org/10.1007/s10763-016-9758-2

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 442. PME.

STUDENTS' FUNCTIONAL THINKING IN GRADE 7

Anneli Blomqvist

Department of Teaching and Learning, Stockholm University

Previous research has identified functional thinking as a key component relevant to students' mathematical development involving functions, generalising relationships, and expressing them (e.g., Blanton et al., 2019). Functional thinking encompasses progression levels, with the main ones being recursive patterning, corresponding relationships, and covariational relationships, with the latter one as the most sophisticated (e.g., Deng et al., 2025). At the level of covariational relationships, students tend to be more limited in their functional thinking, suggesting a need for tasks beyond pattern-related ones to support the transition to complex levels of functional thinking, where covariational relationships are essential. One way to investigate this progression is to analyse students' arguments with respect to levels of functional thinking, where students' arguments are anchored in different mathematical properties (e.g., Lithner, 2008). Against this background, this study explores students' arguments regarding the progression of functional thinking, with particular emphasis on the sophisticated levels.

The present study aims to present results from semi-structured observations conducted in a lab setting with Swedish grade 7 students (N=17) while working on tasks involving functions, emphasising the covariational relationship. Drawing on transcript data, students' arguments were analysed to identify how mathematical properties related to functional thinking were manifested across different progression levels. The results indicate that the mathematical properties associated with functional thinking emerge at various progression levels. Students' limited generalisations, particularly in pattern-related tasks, indicate challenges in transition beyond specific cases. One implication is that pattern-related tasks alone may be insufficient to support the generalisation of relationships between quantities, meaning that a variety of tasks is needed when assessing students' functional thinking.

References

Blanton, M. L., Isler-Baykal, I., Stroud, R., Stephens, A., Knuth, E., & Gardiner, A. M. (2019). Growth in children's understanding of generalizing and representing mathematical structure and relationships. *Educational Studies in Mathematics*, *102*(2), 193–219. https://doi.org/10.1007/s10649-019-09894-7

Deng, X., Ding, R., & Huang, R. (2025). Investigating a learning progression of functional thinking for elementary students. *Educational Studies in Mathematics*, 1–27.

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, *67*(3), 255–276. https://doi.org/10.1007/s10649-007-9104-2

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 443. PME.

EXPLORING PLACE VALUE REPRESENTATIONS IN MATHEMATICS WORKBOOKS

Tammy Booysen

Rhodes University

Place value is a concept that learners find difficult to understand. The use of visual representations (VR) could assist in developing grade 3 learners' understanding of place value when using workbooks. Visual representations in workbooks are central to developing place-value understanding in mathematics. Workbooks are key curriculum tools used in many countries. The research sought to explore: What are the similarities and differences between the use of visual representations in place value activities in Singaporean and South African workbooks? As a South African primary mathematics teacher educator I wanted to analyse use of representations in our government provided workbooks and chose the Singaporean workbooks for comparative purposes as their learner performance contrasts significantly with South African learners. In international rankings they rank towards the top while South Africa at the bottom.

I used an adapted version of the Visual Representation Framework by Fotakoupoulou and Spiliotopoulou (2008) to analyse the VRs. The categories used to analyse the two workbooks were: a) type of VR, b) relation of VR to content, c) relation of VR to reality, d) function of the VR, e) dimensionality of VR. The study builds on constructivism which suggests that tools and signs are central to facilitating learning (Daniels, 2005). The workbooks are considered the tools, and the place value VRs serve as signs that assist learners in the learning process.

The findings suggest that the South African workbook provides more place-value VRs, particularly, Diene's blocks and flard cards. While the Singaporean workbooks provide fewer VRs particularly, photos, flard cards, and place value charts. Similarly, the VR in both textbooks had an exemplifying function. The adapted VRF by Booysen et al., (2024) has exemplifying (type a), this is a worked example and can be found in South African workbook. While exemplifying (type b) function is a practice example and is the most dominant function in Singaporean workbook. I argue that analysis of the similarities and differences of VR across countries is useful for teacher education because brings the attention to the function of the VR in mediating learning.

References

Daniels, H. (Ed.). (2005). *An introduction to Vygotsky* (2nd ed.). Routledge.

Booysen,. T. I. & Westaway, L. (2024). The nature of place value visual representations in the national workbook series. In *The 15th International Conference of Mathematics Education (ICME) conference proceedings,* Sydney.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 444. PME.

STUDENTS-WITH-GENAI CO-CREATING MATHEMATICAL MODELS: TOWARDS A NEW PEDAGOGICAL PARTNERSHIP?

Marcelo de Carvalho Borba, Aldo Peres Campos e Lopes

São Paulo State University UNESP, Federal University of Itajubá

The research investigates how student agency mediated by Generative Artificial Intelligence (GenAI) manifests in sociocritical mathematical modeling activities in higher education. The central problem lies in the theoretical gap regarding the active role of GenAI as a non-human agent in the production of mathematical knowledge, particularly in contexts that value critical thinking and social responsibility. The study is grounded in the theoretical framework of humans-with-media (Borba, 2021), which conceptualizes the human-technological collective as the fundamental unit of knowledge production. From this perspective, GenAI is not treated as a mere instrument, but as an agent that participates in, influences, and is influenced by human actions. Additionally, the concept of machine agency, distinguishing between good agency and bad agency, is mobilized to categorize the observed interactions analytically. Methodologically, the research is qualitative in nature and was conducted with engineering students enrolled in a Differential Equations course at a Brazilian federal university during the first semester of 2025. The students developed mathematical models in groups, with explicit authorization and documentation of GenAI use. The results indicate that GenAI exercised good agency by functioning as a cognitive scaffold, simplifying complex phenomena and overcoming operational obstacles, such as code generation and graph construction, thereby expanding the space for students' critical analysis. Conversely, it exhibited bad agency by inducing epistemic dependency and producing mathematically incorrect responses at more advanced levels, requiring students to act as critical "filters," a competence they did not always possess. It is concluded that agency within the humans-with-GenAI collective is necessarily negotiated and conditioned by students' prior knowledge, which transforms the tool from a potential source of misinformation into a formative resource. The study contributes to refining the analytical categories of AI agency in mathematics education. It highlights the need for further investigations into pedagogical strategies that promote critical and formative uses of artificial intelligence.

Acknowledgment to CNPq, funding agency of the Brazilian Government 409575/20224-0.

References

Borba, M. C. (2021). The future of mathematics education since COVID-19: humans-with-media or humans-with-non-living-things. *Educational Studies in Mathematics*, 108(1-2), 385–400. https://doi.org/10.1007/s10649-021-10043-2

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 445. PME.

RESEARCH CYCLES TO UNDERSTAND HOW STUDENTS LEARN SOLIDS OF REVOLUTION

Vahid Borji[1], Deborah Moore-Russo[2] and Rafael Martínez-Planell[3]

[1]Charles University, [2]University of Oklahoma, [3]Universidad de Puerto Rico

This study builds on prior research on students' understanding of integrals and solids of revolution (SoR), which has documented students' reliance on procedural knowledge and difficulties with geometric and conceptual interpretations (e.g., Mofolo-Mbokane et al., 2013). More recent studies have emphasized challenges in coordinating graphical, geometric, and symbolic representations, as well as limitations in students' visuospatial reasoning (e.g., Blanco et al., 2019). The present study proposes a theoretical model of how students may construct SoR and discusses subsequent classroom testing and subsequent improvements of the theoretical model. Grounded in APOS theory, this study investigates students' understanding of SoR using the theoretical model (called a genetic decomposition; GD) as a design tool for instructional activities and interview tasks. The research questions are: 1) how well can the GD model students' constructions of SoR, and 2) how the GD can be refined or improved based on data? The study was conducted in a first-year university calculus course. GD-based activities were designed and implemented. They emphasized geometric reasoning, partitioning, and Riemann sums, as described in the GD. Data were collected through semi-structured interviews with 11 students using think-aloud protocols and then analysed using the GD. Preliminary results indicate that most students were able to construct key components of the GD, particularly in visualizing rotations and correctly representing solids. Compared to earlier research, students demonstrated improved coordination between geometric and symbolic reasoning, suggesting the effectiveness of GD-informed instruction. However, significant difficulties remain. Some students relied on memorized procedures without connecting them to the underlying geometric meaning, showing the need for further research to study rotations about non-standard axes. These findings provide empirical support for the GD as a viable model of student learning of SoR. They also highlight the need for the GD to clearly consider infinitesimal elements and for instructional tasks to explicitly require students to construct and interpret these elements to connect Riemann sums to integrals.

References

Blanco, T. F., Diego-Mantecón, J. M., & González Sequeiros, P. (2019). Visualization Processes in a Task that Requires Generating and Representing Solids of Revolution. *Bolema: Boletim de Educação*, *33*(64), 768–789.

Mofolo-Mbokane, B., Engelbrecht, J., & Harding, A. (2013). Learning difficulties with solids of revolution: classroom observations. *International Journal of Mathematical Education in Science and Technology*, *44*(7), 1065–1080.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 446. PME.

USING CHATGPT FOR MATHEMATICAL MODELLING: PRE-SERVICE PRIMARY TEACHERS' REFLECTIONS

Rita Borromeo Ferri, Neslihan Bulut

University of Kassel, Institute of Mathematics

The rapid emergence of generative artificial intelligence (AI), particularly ChatGPT, has increased discussions about its role in mathematics education and teacher preparation. While recent research highlights the pedagogical potential and mathematical risks of AI-generated content (Noster et al., 2024; Schorcht et al., 2024), little is known about how pre-service primary mathematics teachers (PSMTs) incorporate ChatGPT into their developing professional roles. This study examines how PSMTs conceptualize ChatGPT as a tool for mathematical modelling (MM) and which competencies they consider necessary for its responsible use. The study involved 19 primary pre-service mathematics teachers in their second or third semester at a German university. During a 13-week Education for Sustainable Development (ESD)-MM seminar, one 90-minute session was dedicated to ChatGPT. During this session, the PSMTs worked in small groups to develop and critically evaluate an ESD-MM task by using ChatGPT. Data include a collaborative in-class presentation and individual written reflections. A qualitative thematic analysis was conducted.

The findings show common acceptance of ChatGPT as support tool, accompanied by explicit concerns. While all participants described ChatGPT as a supportive tool for idea generation, none considered it a substitute for the teacher. While 79% thought it was generally suitable for modelling task development, this approval was dependent on certain conditions. Specifically, 21% questioned its ability to generate authentic MM problems due to its limited openness and modelling depth. Risks frequently mentioned included mathematical discrepancies (84%) and insufficient consideration of learners' context (79%). Participants stressed the importance of strong mathematical knowledge and critical evaluation skills. Additionally, 95% of participants highlighted the importance of didactic competence. ChatGPT is perceived as valuable for inspiration, yet its use is understood to increase—rather than reduce—professional responsibility. Further results will be discussed in detail in the presentation.

References

Noster, N., Gerber, S., & Siller, H. S. (2024). Pre-service teachers' approaches in solving mathematics tasks with ChatGPT. *Digital Experiences in Mathematics Education, 10*(3), 543–567. https://doi.org/10.1007/s40751-024-00155-8

Schorcht, S., Buchholtz, N., & Baumanns, L. (2024). Prompt the problem: Investigating the mathematics educational quality of AI-supported problem solving by comparing prompt techniques. *Frontiers in Education, 9,* 1386075. Frontiers Media SA.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 447. PME.

MEDIATING REFLECTION: CRITICAL-FRIEND DIALOGUE AS A METHODOLOGICAL TOOL IN MATHEMATICS TEACHER EDUCATOR SELF-STUDY

Gülay Bozkurt[1], Edith Lindenbauer[2], Melike Yiğit Koyunkaya[3]

[1]Johannes Kepler University, [2]University College of Education Upper Austria
[3]Dokuz Eylül University

Reflection is widely recognized as central to mathematics teacher educators' (MTEs) professional practice (Beswick & Goos, 2018). In self-study research, it serves as both object and method, yet it is often treated as an individual cognitive activity, leaving under-theorized the dialogic structures that mediate it. This study examines critical-friend dialogue as a methodological tool in MTE self-study, asking: How does critical-friend dialogue function as a mediational structure that reorganizes reflective practice? Drawing on Schuck and Russell (2005), we conceptualize critical friendship not as collegial exchange but as a structured methodological practice. Through questioning and re-voicing, the critical friend surfaces implicit assumptions and opens reflective accounts to challenge. We therefore frame dialogue as mediation, a process through which reflections are articulated, interrogated, and reorganized. Rather than foregrounding learning outcomes, we analyze how the dialogic structure itself operates and which interactional moves enable or constrain criticality. The study adopts a collaborative self-study design involving two MTEs from different countries who each taught a 14-week technology-integrated mathematics teacher education course, one emphasizing digital resource design, the other digital task design. Across the semester, both engaged in systematic written reflections and weekly recorded critical-friend dialogues. The dialogues followed a recurring reflective cycle: one MTE articulated intentions, described enactment, and identified tensions; the critical friend posed probing questions and requested clarification to deepen the reflection. Analysis focused on how dialogic moves mediated reflection, particularly moments of re-voicing intentions and surfacing implicit rationales. Preliminary findings indicate that critical-friend dialogue functioned as a methodological mediator by externalizing tacit reasoning, structuring reflective depth, and revealing tensions in the dual role of the critical friend as both supporter and challenger. The study suggests that the methodological value of critical-friend dialogue emerges from the interactional quality of dialogue.

References

Beswick, K., & Goos, M. (2018). Mathematics teacher educator knowledge: What do we know and where to from here? *J. of Mathematics Teacher Education*, *21*(5), 417–427.

Schuck, S. & Russell, T. (2005). Self-study, critical friendship, and the complexities of teacher education. *Studying Teacher Education, 1*(2), 107-121.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 448. PME.

WHAT'S LEFT FOR US TO DO? HUMANS' ROLE IN MATHEMATICAL MODELLING IN THE AGE OF GENERATIVE ARTIFICIAL INTELLIGENCE

Andreas Brandsæter[1,2], Rebecca Ellis[3], Jennifer Lawson[4], Morten F. Ingvaldsen[1,2], Ingeborg L. Berget[1]

[1]Volda University College, [2]NTNU – Norwegian University of Science and Technology, [3]Concord Consortium, Concord, [4]University of Calgary

This study investigates how Large Language Models (LLMs) engage with mathematical modelling tasks and what their growing capabilities imply for mathematics education. Through an analysis of eight diverse modelling problems, ranging from classical estimation tasks to project-based modelling and regression, we examined whether LLMs can solve such problems independently and which aspects of the modelling process still require human involvement.

Across most problems, LLMs produced solutions that appeared mathematically sound and convincing at first glance. Closer inspection, however, revealed important limitations, including a lack of transparency, unrealistic assumptions, and insufficient attention to contextual factors. While LLMs handled computational components effectively, they often neglected realism and human considerations, resulting in plausible yet flawed solutions. In some cases, particularly when a definitive answer could be found online, LLMs bypassed modelling altogether, as in estimating the container transshipment area of the Port of Hamburg, where the model retrieved the actual area rather than constructing an estimate based on assumptions. Tasks with unclear or multiple objectives also proved challenging. For example, in problems such as optimizing bus stop placement, LLMs handled generic formulations well but struggled when local context was required, at times proposing bus stops in very remote areas with no nearby housing. Extrapolation posed a further difficulty: in a regression task predicting patients' temperature normalization from medication dosage, the LLM produced a reasonable model but failed to comment on uncertainty or the validity of predictions beyond the observed data range, even suggesting that temperatures would normalize within seconds at sufficiently high doses.

These findings have important implications for mathematics education. The availability of powerful AI tools does not diminish the value of teaching modelling but calls for a reconsideration of how modelling is taught and what we expect students to accomplish in real-world contexts. To preserve meaningful engagement, modelling tasks should be designed so that essential steps such as making assumptions, justifying choices, and interpreting results cannot be bypassed. Overall, the results highlight the continued necessity of human judgement throughout the modelling cycle, with LLMs best understood as assistants to human modelers rather than replacements.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 449. PME.

ANALYZING A STEP FUNCTION GRAPHING TASK

Daniela Buksárová, Ingrid Semanišinová
Pavol Jozef Šafárik University in Košice

Many studies point out that students lack a complete understanding of the concept of a function, with covariational reasoning being the least developed aspect. This contrasts with the claim that "ideas of continuous covariation are epistemologically necessary for students and teachers to develop useful and robust conceptions of functions" (Thompson & Carlson, 2017, p. 423). Therefore, we have selected a task through which we aim to examine the current state of covariational reasoning among Slovak secondary school students. Inspired by The Parking Fees scenario and task (Ronau et al., 2014), it presents a real-life situation in which students are asked to sketch a step function graph. Our aim was to determine their level of covariational reasoning and identify any other difficulties relating to the concept of a function.

Twenty-four 18-year-old high school students solved nine tasks as part of a review of functions. This task was the first task in the series, and it took them the longest to solve (10–15 minutes). We anticipated the following difficulties: a) choosing the proper scale for the graph axes ($\sum$=2); b) sketching a linear function instead of a step function ($\sum$=6); c) sketching isolated points ($\sum$=10); d) sketching a graph that resembles a staircase (with segments parallel to the y-axis) ($\sum$=2); e) problems with the endpoints of individual segments in a step graph ($\sum$=5). After analysing the students' solutions, we found that our expectations had been met (see the number of students above). Additionally, 19 students had difficulty sketching the interval from 0 to 20 minutes, when parking is free. One student explicitly stated: "I don't know how to sketch the first 20 minutes in a graph.". Four students identified price as the independent variable instead of time, and two had difficulty determining function values at the interval endpoints where the value changes or a jump occurs. These findings support the claim that students have limited ability to work with different function representations when expressing invariant relationships between two co-varying quantities in applied contexts (Thompson & Carlson, 2017). The results highlight the need for targeted pedagogical interventions to improve students' understanding of functions.

References

Ronau, R. N., Meyer, D., Crites, T., & Dougherty, B. J. (2014). *Putting essential understanding of functions into practice in grades 9-12*. National Council of Teachers of Mathematics.

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421–456). National Council of Teachers of Mathematics.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 450. PME.

A COMPARISON OF GROUP AND INDIVIDUAL PROBLEM-POSING PERFORMANCES OF PRE-SERVICE ELEMENTARY MATHEMATICS TEACHERS: THE CONTEXT OF PAPER FOLDING

Ayşe Tuğba Çakan[1], Berna Cantürk Günhan[1] and Burçak Boz Yaman[2]

[1]Dokuz Eylül University, [2]Muğla Sıtkı Koçman University

Problem posing is the process of generating a new problem based on a given context or reformulating an existing one (Stoyanova & Ellerton, 1996). This process functions as an assessment tool by revealing individuals' mathematical understanding and eliciting critical skills such as creativity (English, 1997). Paper-folding activities, which embody geometric principles and have an experiential nature, provide a rich context for problem-posing processes. This study aims to compare the group and individual problem-posing performances of pre-service elementary mathematics teachers within the context of paper folding activities and to collect their views on these problem-posing activities. Conducted within a holistic single-case design, the study involved nine pre-service teachers whose spatial visualization skills were at or above the average level. The participants engaged in paper folding based problem-posing activities both individually and in two groups of four to five members. The problem-posing situations were designed based on the free, semi-structured, and structured classification proposed by Stoyanova and Ellerton (1996), and the generated problems were evaluated using a rubric developed in the context of this research. In addition, individual interviews conducted after the implementation were analyzed to examine the participants' understanding and performances on the types. The results did not indicate a clear overall superiority between group and individual performances. However, according to the problem type higher scores were obtained in the free problem-posing for both individual and group working, whereas in the structured problem posing, group and individual scores were close to each other. The interviews revealed that group working supported the emergence of diverse perspectives and creative ideas, whereas working individually was associated with decision-making and responsibility-taking. Overall, the findings suggest that, in addition to the types of working, factors such as problem type influence problem-posing performance.

References

English, L. D. (1997). The development of fifth grade children's problem-posing abilities. *Educational Studies in Mathematics*, 34, 183–217.

Stoyanova, E. & Ellerton, N. F. (1996). A framework for research into students' problem posing in school mathematics. P. C. Clarkson (Ed.), *Technology in mathematics education* (pp. 518–525). Mathematics Education Research Group of Australasia.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 451. PME.

EXPLORING CHILDREN'S USE OF GESTURE TO DESCRIBE GEOMETRIC FEATURES OF A HIDDEN OBJECT

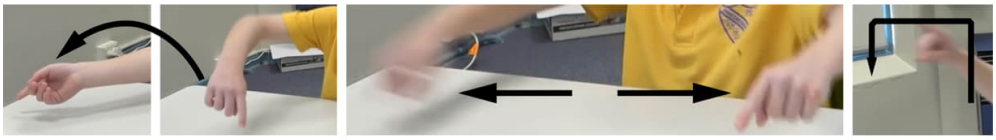
Katherin Cartwright, Jennifer Way

The University of Sydney

Embodied modes play an important role in children's mathematical learning, assisting in making representational connections along the pathway to abstract thinking (Nahdi et al., 2024). Ehrlich et al. (2006) consider the interplay between gesture and verbal representations in relation to geometric features–noticing children's verbal *and* gestural responses. Theoretical perspectives including Ehrlich et al.'s (2006) geometric feature categories and the van Hiele (1999) model were taken to explored young children's descriptions of an unseen object to answer the research question: *What geometric features are revealed through children's verbal and gestural responses?*

Deductive micro-analysis was employed to explore qualitative data from video-recorded task-based interviews with 106 4-to-6-year-olds. The 'feely bag' task was designed to elicit verbal and gestural responses. The most prevalent mode of communication was verbal, 98 of the 106 children, 39 of these were speech-only. There were 57 children who used gesture, most, 93%, used co-speech gesture, e.g., *'I could feel the block was like [G], and that [G] then I think it was like that [G]'* (Figure 1).

Figure 1: Sequence of co-speech gestures in place of words



1. Start and end position of arch | 2. Draws horizontal sides of block | 3. Draws sides

Speech-only children referred to the *perceptual whole* most often (90%), compared with co-speech gesture children who referred to *perceptual features* (62%) most often. Children's verbal and gesture representations generally aligned to Level 1 or 2 of the Van Hiele model. A distinct difference was noticed at Level 2, with 32 gesturers and only 4 speech-only. Co-speech gesturers seem to communicate 'more' mathematics. Van Hiele's model does not specifically include the interpretation of embodied modes in relation to geometric thinking, therefore further embodied research is needed.

References

Ehrlich, S. B., Levine, S. C., & Goldin-Meadow, S. (2006). The importance of gesture in children's spatial reasoning. *Developmental Psychology, 42*(6), 1259.

Nahdi, D. S., Jatisunda, M. G., Cahyaningsih, U., Rasyid, A., & Hidayah, R. (2024). Mapping geometric minds: Exploring 3D thinking skills of elementary school students using the Van Hiele model. *Journal of Education For Sustainable Innovation*, *2*(1), 94-106.

van Hiele, P. M. (1999). Developing geometric thinking through activities that begin with play. *Teaching Children Mathematics*, *5*(6), 310-316.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 452. PME.

DIMENSIONS OF TEACHER AGENCY IN THE USE OF MATHEMATICS TEXTBOOKS

Ayla Carvalho[1,2], Rúbia Amaral[1]

[1]São Paulo State University (Unesp), [2]University of Michigan

Classrooms are dynamic spaces, with teachers exercising agency as curriculum designers who mediate between official mandates and enacted practice through resources like textbooks. Thus, this study is guided by the following research question: *how does teacher agency manifest in the ways mathematics teachers use textbooks in their instructional practice?* Building on Priestley et al. (2015), teacher agency is the capacity to act, shaped by many factors and sustained from past experiences, projective orientations toward future goals, and a practical-evaluative dimension in which teachers make situated judgments based on available possibilities. This qualitative study conducted online semi-structured interviews with two Brazilian public secondary-school mathematics teachers, selected because their daily textbook access occurs within a tightly policy-regulated context that creates productive tensions for observing teacher agency. By analyzing teachers' discourses, it was possible to observe how the official curriculum is enacted through tightly prescribed routines aligned with standardized testing, thereby shaping teachers' actions. Past experience informs textbook use, as it relates to teachers' prior knowledge and professional trajectories. From both projective and practical-evaluative dimensions, time pressure steers planning and in-the-moment decisions toward test-oriented coverage, contributing to a diminished sense of individual autonomy in textbook use (Carvalho & Amaral, 2025). The analysis showed that standardized assessments position the official curriculum as a structuring force shaping how teachers enact their agency in textbook use. Thus, agency emerges as a negotiated construction rather than an individual trait, as accountability pressures narrow the boundaries of what teachers can imagine and enact beyond performance metrics.

Acknowledgements

Thanks to São Paulo Research Foundation (FAPESP) for the grant #2025/09749-1.

References

Carvalho, A. M., & Amaral, R. B. (2025). The influence of the professional space on mathematics teachers' agency in the use of textbooks. *Educação Matemática Pesquisa: Revista Do Programa De Estudos Pós-Graduados Em Educação Matemática*, *27*(5), 133–159. https://doi.org/10.23925/1983-3156.2025v27i5p133-159

Priestley, M. R., Biesta, G., & Robinson, S. (2015). *Teacher agency: An ecological approach.* Bloomsbury Publishing.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 453. PME.

SEMIOTIC REGISTER USE AND COORDINATION IN MIDDLE SCHOOL STUDENTS' FRACTION PROBLEM SOLVING

Gözde Kübra Celkanlı[1], Orhan Çanakçı[2], Mahmut Kertil[2]

[1]Ucsehitler Lower Secondary School, [2]Marmara University

Mathematical understanding relies on the coordination of multiple semiotic registers rather than on the manipulation of symbols alone (Duval, 2006, 2017). According to Duval, mathematical thinking requires transformations within and between representational systems such as symbolic, graphical, numerical, and verbal forms. Fraction concepts, in particular, demand flexible movement across registers, as students must interpret part–whole relations, proportional reasoning, and operational procedures through various representations. Despite extensive evidence that students experience difficulties in transitioning between representations, limited research has examined how register coordination emerges during problem-solving processes. This study therefore investigates semiotic register coordination in middle school students' fraction problem solving across paper-and-pencil and digital contexts. This qualitative action research study was conducted with sixth-grade students attending a public middle school. Following an initial readiness test, a heterogeneous group of ten students was selected, and the implementation was carried out over four weeks in both paper-and-pencil and digital learning environments. In the digital setting, students worked in pairs using tablets equipped with tools such as fraction bars and number lines, while data were collected through think-aloud protocols, screen recordings, and written documents. Content analysis was employed to examine students' use of semiotic registers and the transformations between them. Findings indicate that in paper-and-pencil contexts, students predominantly relied on symbolic procedures and tended to remain within a single register. Register coordination was limited, and transitions between representations were rarely observed unless explicitly prompted. In contrast, the digital environment encouraged more frequent use of visual and graphical registers, as available tools facilitated engagement with multiple representations. However, students often used these representations operationally rather than reflectively, and difficulties emerged when coordinating transformations across registers. Overall, the findings suggest that while digital environments can promote exposure to diverse representations, meaningful register coordination depends on deliberate instructional support that foregrounds the reasons for transitioning between representations, rather than merely providing representational tools.

References

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics, 61*(1), 103–131.

Duval, R. (2017). *Understanding the mathematical way of thinking-The registers of semiotic representations* (pp. 28-43). Springer International Publishing

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 453. PME.

MATHEMATICS TEACHER EDUCATORS' EXPERIENCES WITH AND VISIONS FOR AI IN TEACHER EDUCATION

Mustafa Cevikbas

Vrije Universiteit Brussel (VUB)

The rapid diffusion of artificial intelligence (AI) is reshaping how mathematical ideas can be explained, represented, and assessed. Yet research has largely focused on technical affordances and students' AI use, leaving mathematics teacher educators' (MTEs) visions and experiences underexplored. This gap matters because MTEs shape norms of pedagogical reasoning and professional judgment in teacher preparation and influence how AI will be taken up in future classrooms. Accordingly, this research adopts a human-centered AI literacy perspective emphasizing pedagogical purposes, ethical responsibility, and professional agency (UNESCO, 2024). The study examines how MTEs (a) describe their experiences with AI, (b) conceptualize its role in teacher education, (c) identify opportunities, and (d) articulate concerns and conditions for responsible integration. An international qualitative exploratory design used an online open-ended survey with 31 MTEs from 17 countries. Data were analyzed via thematic analysis using a hybrid inductive–deductive approach; a checked subset indicated substantial agreement ($\kappa = .78$). Most participants reported professional AI use (87%), yet most had no structured training (71%), relying on self-learning/peer exchange. AI use focused on preparation and design rather than replacing teaching. MTEs framed AI as (1) a pedagogical assistant, (2) a catalyst for innovation, (3) an object of critical study (AI literacy/critique as content), and (4) a threat/ethical dilemma (bias, profit-driven logics, erosion of human-centered teaching). Opportunities included time efficiency, richer resources, rapid feedback, differentiation, and support for problem posing/solving; some used AI outputs as artifacts for critique to strengthen PSTs' evaluative judgment. Key risks involved overreliance, reduced autonomy and reflective/cognitive skills, ethical issues (privacy, bias, transparency, integrity), and inaccurate outputs, alongside equity/access concerns. In line with UNESCO (2024), AI literacy is treated as a multidimensional competence (conceptual understanding, pedagogical design, ethical awareness, reflective judgment), and participants stressed verifying and contextualizing AI outputs given risks to epistemic trust. Rather than "whether," the key issue is how to integrate AI to augment professional judgment: (a) embed AI literacy systematically; (b) design tasks requiring PSTs to validate and contextualize AI outputs; and (c) develop institutional guidance and professional learning addressing ethics, transparency, and equity. Overall, the study highlights both the promise and the peril of AI, calling for ethically grounded integration that augments rather than replaces human professional judgment.

References

UNESCO (2024). *AI literacy competency framework for teachers*. Paris: UNESCO.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 455. PME.

MAKING SENSE OF TIME: LEARNING EXPERENCES TO HELP YOUNG LEARNERS CONSTRUCT MATHEMATICAL UNDERSTANDING OF TIME

Min Lie Chan, Guat Bee Loh
Academy of Singapore Teachers

Understanding time is challenging for children aged 7-8 as it is abstract and not directly perceptible (Burny et al., 2009). As a result, many struggle to comprehend units of time and how these units relate to a given duration. Children often focus on reading individual clock times rather than understanding the interval between them, and many rely on informal strategies such as "counting numbers" on the clock face. Crossing hour boundaries (e.g., from 9:50 to 10:20) is particularly problematic, as children may subtract smaller numbers from larger ones without considering the hour change. Van de Walle (2020) advocates the use of an empty timeline to support children in solving elapsed-time problems, however, Primary Two students were observed to have difficulty sketching the timeline accurately to find duration. Timelines are often treated as embellishments rather than scaled representations in where equal spacing reflects equal units, hindering children's coherent understanding of duration. This exploratory case study investigates whether a carefully sequenced instructional design can strengthen Primary Two students' understanding of telling time and calculating duration. The intervention comprised three components: (1) students constructed one-hour (60-minute) rectangular paper strips marked at 5-minute intervals to reinforce the equivalence of 1 hour and 60 minutes while connecting the timeline to analogue clock reading; (2) students worked in groups to manipulate analogue clocks to show given start and end times, and use the one-hour (60-minute) paper strips to scaffold timeline drawings to find duration between the two time points (3) students completed individual practice tasks to assess their mastery of finding duration. Qualitative insights are gathered through classroom observations, teachers' reflection and student work samples. Preliminary findings indicate that students shifted from counting labels or performing unstructured numerical subtraction to coordinating clock representations with unit-based reasoning along a timeline. They were able to identify start and end times more reliably, justify their answers using the hour strips and explain durations in terms of "hours and minutes" rather than "what numbers I subtract." Spatial proportionality in their timelines also showed marked improvement. Our findings offer educators ways to enhance student understanding of time and duration.

References

Burny, E., Valcke, M., & Desoete, A. (2009). Towards an agenda for future research on children's time assessment. *Learning and Instruction*, 19(6), 481-493.

Van de Walle, J.A. 2020. Elementary and middle school mathematics teaching developmentally. Boston: Pearson education. 10th ed.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 456. PME.

GEOMETRIC AND MAP-BASED SPATIAL ABILITIES ACROSS DOMAINS

Li-Jung Chang, Tsu-Jen Ding, Hui-Yu Hsu, Chen-Yu Yao

National Tsing Hua University, Taiwan R.O.C.

Spatial ability is crucial for learning both geometry and cartography. In psychology, object-based mental rotation and self-based spatial perspective taking have been identified as distinct yet related constructs rather than as a single unitary ability (e.g., Hegarty & Waller, 2004). However, prior psychological research has rarely examined how these two spatial abilities manifest across different academic disciplines. The study aimed to examine the extent to which geometry-based mental rotation and map-based spatial perspective taking are mutually explanatory, or whether they represent two distinguishable spatial competencies that may influence one another, which are cores to understanding cross-domain transfer and for informing instructional design.

To address this issue, forty-three university students completed two within-subject tasks: a geometric mental rotation task and a map-based spatial perspective-taking task, with angular disparity varied across multiple rotation angles. Reaction time and accuracy were analyzed using repeated-measures ANOVA. Reaction time results revealed a task effect, with geometric mental rotation eliciting longer response times than the map-based task, as well as a significant interaction between task and angular disparity, indicating a linear increase in angular cost for geometry but a reduced angular sensitivity for maps. Accuracy results showed no overall task difference, indicating comparable correctness across domains, yet revealed a significant interaction with angular disparity. Accuracy in the geometric task decreased as angular disparity increased, reflecting the difficulty of constructing mental images, while orthogonal orientations did not lead to additional declines in accuracy, whereas the map-based task showed an accuracy decrease at 90°, with no clear differences between larger angular disparities, suggesting reorientation processes. Together, these dissociable patterns in speed and accuracy indicate that geometric mental rotation and map-based spatial perspective taking draw on spatial resources but are not mutually explanatory abilities. Extending previous studies, the present findings show that the distinction between geometric and map-based spatial abilities is not attributable solely to angular disparity, but to the structural constraints governing spatial transformation—coordinate-axis alignment in geometry versus the directionality of alignment processes in map-based tasks. From a geometric perspective, instruction may benefit from leveraging coordinate-axis alignment to support reasoning about rotation and transformation.

References

Hegarty, M., & Waller, D. (2004). A dissociation between mental rotation and perspective-taking spatial abilities. *Intelligence*, *32*(2), 175-191.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 457. PME.

CULTIVATING PRE-SERVICE TEACHERS' PERCEIVED BELIEFS REGARDING MEASUREMENT AND GEOMETRY IN SCHOOL MATHEMATICS

Narumon Changsri, Maitree Inprasitha, Chanika Senawongsa, Jitlada Jaikla,
Nisakorn Boonsena, Khon Kaen University

Geometry and measurement are essential strands in school mathematics, fostering logical reasoning and practical application in daily life (NCTM, 2000). Despite their importance, there is relatively little research on pre-service teachers' beliefs about these domains, even though beliefs developed during teacher preparation strongly influence teaching practices. This research investigates the second-year pre-service mathematics teachers' perceived beliefs regarding measurement and geometry in school mathematics. Over a four-month period, the study employed reflective practice through the transformative lesson study incorporating the Open Approach (TLSOA Model), integrated into a 15-week. This process included Co-Plan, in which the Lesson Study (LS) team collaboratively designed problem situations that focus to challenge cognitive conflict and affective interruption; Co-Do, in which participants engaged in problem-solving and shared their ideas; and Co-See, in which participants engaged in reflection. Data were gathered from 27 participants' written reflections on five problem-solving scenarios, critical analysis of problem situations in the textbook, and micro-teaching, with sequential reflections collected at the two-month mark and upon the conclusion of the four-month study. The qualitative data were examined using thematic analysis, guided by a framework of perceived beliefs with a specific focus on Emotional Experiences model (Inprasitha, 2001).

The findings reveal that initial belief systems (AS1) reflected an instrumentalist view emphasizing procedural certainty and teacher-centered transmission. Cognitive and affective interruptions (I), including astonishment, empathetic disruption, and realistic discomfort, subsequently destabilized these foundations. Cognitive evaluative schema (CES) facilitated reflective meaning-making through ambiguity tolerance, intellectual satisfaction, and moral-professional commitment. Ultimately, revised belief systems (AS2) demonstrated three interconnected transformations: repositioning measurement and geometry toward a problem-solving view of mathematics; reframing the teacher's role as a problem-situation designer; and endorsing problem situations that enable students to generate, communicate, and refine their own mathematical ideas.

References

Inprasitha, M. (2001). Emotional Experiences of Students in Mathematical Problem Solving. *Dissertation*, Doctoral Program in Mathematics Education, University of Tsukuba.

NCTM. (2000). Principles and Standards for School Mathematics. Reston, VA: NCTM.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 458. PME.

PROSPECTIVE MATHEMATICS TEACHERS' LEARNING THROUGH NARRATIVE AND IMAGINATION

Olive Chapman
University of Calgary

Inquiry-based (IB) teaching is a challenge for teachers to implement particularly if their experience with learning mathematics did not align with it. This paper reports on a pilot study to explore an approach to support prospective secondary mathematics teachers' (SPTs') learning of IB teaching. The aim was to gain insights on how they made sense of IB teaching through creating a narrative based on their imagination teaching a mathematics lesson. Theoretical framing of the study includes: (1) Narrative as a way of representing experience. It helps to make knowledge more engaging, helps us think and do things more effectively, and aids in the process of meaning making. (2) Imagination as the ability to think about what might be possible. It enables students to pick up new ideas, try them out, and find a place for them amidst the things they have already learned (Egan, 2005). (3) Inquiry-based teaching as approaches that support students' individual and collaborative construction of knowledge through inquiry/exploration of mathematics concepts and finding their own path to solutions. Participants were 16 SPTs enrolled in a mathematics education course. They had a 3- or 4-year undergraduate degree in mathematics but no school experience with IB learning. In the course, they were exposed to theories of IB leaning, tasks, and discourse. They explored examples of tasks but not examples of complete IB lessons. The assignment used for this study required them to imagine themselves conducting an IB lesson of a mathematics concept of their choice from their G7-9 curriculum. The guideline included: The lesson must include an introduction, a task they created, their intervention in at least three groups of their students with different experiences while working on the task, and whole-class discussion. The narrative must have a temporal flow as lived and include dialogues. Data sources were the four narratives of the four SPT groups. For analysis, each narrative was categorized based on the structure of the lesson, each category was coded to identify factors of inquiry-based teaching which were compared across the groups. Findings indicated that two groups had fully IB lessons, one was mostly IB, and one was partly teacher-centered and partly IB. The fully and mostly IB groups had appropriate IB lesson introductions**,** tasks, teacher-student interactions and lesson conclusions**.** The partial IB group used teacher-centered intervention/interactions and lesson conclusion directed to what they wanted students to know. The findings suggest that the narrative-imagination approach has potential to help the SPTs to make sense of and develop a holistic image of IB teaching in a way that they could apply in their teaching. But the SPTs whose tasks focused on exploring patterns/relationships of concepts were more successful in imagining IB lessons.

Reference

Egan, K. (2005). *An imaginative approach to teaching*. San Francisco: Jossey-Bass.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 459. PME.

NONLINEAR DYNAMICS IN EARLY ARITHMETIC DEVELOPMENT: PROBLEM-SOLVING AS A BIFURCATION MECHANISM IN CHILDREN WITH INTELLECTUAL DISABILITIES

Garyfalia Charitaki[1], Anastasia Alevriadou[2], Spyridon-Georgios Soulis[3]
[1]University of Thessaly, [2]Aristotle University of Thessaloniki, [3]University of Crete

Research on early arithmetic development in children with Intellectual Disabilities (IDs) has primarily relied on linear prediction models, despite evidence that learning may involve qualitative shifts (Silver, 2013). This study examines whether the relationship between problem-solving and arithmetic attainment exhibits nonlinear characteristics. Participants were 64 children with IDs aged 6-9 years attending special and inclusive settings in Greece. Linear and cubic models were compared, followed by cusp catastrophe modelling to test whether problem-solving functions as a bifurcation variable in predicting arithmetic attainment. Age, fine motor skills, developmental benchmarks, and parental attitudes toward mathematics and literacy were specified as asymmetry parameters, given their documented role in early numeracy development (Susperreguy et al., 2020). Cubic models outperformed linear alternatives. Asymmetry parameters significantly shaped the developmental landscape, influencing the likelihood of nonlinear transitions. Qualitative analysis of children's responses across early numeracy tasks (counting, measurement, classification, probability) identified four problem-solving profiles: naïve, transitional, emerging strategic, and advanced. The transitional profile was characterized by instability and strategy switching and aligned with the bifurcation region identified in the quantitative model. This suggests that apparent inconsistency may reflect developmental reorganization rather than deficit. Findings support a nonlinear account of early arithmetic development in children with ID and position problem-solving as a mechanism capable of reorganizing mathematical performance.

ACKNOWLEDGMENT

This research is conducted in the operating framework of the University of Thessaly Innovation, Technology Transfer Unit and Entrepreneurship Center "One Planet Thessaly", under the "University of Thessaly Grants for Scientific Publication Support" action and is funded by the Special Account of Research Grants of the University of Thessaly.

References

Silver, E. A. (2013). Foundations of cognitive theory for mathematics problem solving. In *Cognitive science and mathematics education* (pp. 33–60). Routledge.

Susperreguy, M. I., et al. (2020). Expanding the home numeracy model. *Early Childhood Research Quarterly*, 50, 16–28.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 460. PME.

CULTIVATING SPATIAL ABILITY THROUGH DIMENSIONAL PUZZLES FOR EARLY CHILDHOOD STUDENTS

Sudthiporn Chatathicoon, Sampan Thinwiangthong, Duangmanee Leelalawan, Thawee Sanamkam, Chuanchuen Malila, Supasit Khonyai, Naphaporn Woranetsudathip, Niyom Wongphongkam

Khon Kaen University

Spatial ability represents a critical skill for the cultivation of spatial sense and the facilitation of meaningful mathematics learning. Spatial ability refers to the capacity to comprehend visual-spatial relations among objects (Zhang, 2016). Early childhood learners require systematic cultivation of spatial ability to establish a foundational essential for learning at the primary education level.

This research was designed to analyse spatial ability in early childhood learners through their engagement in solving two- and three-dimensional puzzles. The study employed qualitative research methodology, incorporating video analysis and analytical narrative interpretation. The target group comprised forty kindergarten-level early childhood learners from Khon Kaen University Demonstration School, selected through purposive selection. The learners received instructional intervention based on the I-WISDOM model through a ten-lesson learning unit focused on rice and farmer. Data analysis utilized the spatial ability analytical framework established by Linn and Petersen (1985).

The research findings revealed that early childhood learners demonstrated spatial ability comprising three dimensions. First, spatial perception: learners accurately identified visual-spatial relationships depicted on various faces of cubes and coordinate grids. Second, mental rotation: learners effectively manipulated and arranged images in both two-dimensional representations (coordinate grids) and three-dimensional forms (cubes) with accuracy and efficiency. Third, spatial visualization: learners analysed and executed complex visual-spatial operations involving detailed and multistep procedures. Additionally, some learners experienced difficulties and systematic errors in performance execution, indicating areas requiring further developmental attention and instructional support.

References

Linn, M. C., & Petersen, A. C. (1985). Emergence and characterization of sex differences in spatial ability: A meta-analysis. *Child Development, 56*(6), 1479–1498.

Zhang, X. (2016). Linking language, visual-spatial, and executive function skills to number competence in very young Chinese children. *Early Childhood Research Quarterly, 36,* 178–189.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 461. PME.

SCAFFOLDING THROUGH PROBLEMS

Akshi Chauhan, Shweta Naik, Aniket Sule

Homi Bhabha Centre For Science Education, TIFR Mumbai

While research on teachers' problem posing predominantly engages teachers in researcher-designed tasks or professional development settings, this study focuses on teachers' practice of problem posing - reformulation or generation of problems - within their own instructional contexts. We adopt an integrated lens of Schoenfeld's (2011) goals, resources, and orientations and Ball et al.'s (2008) Mathematical Knowledge for Teaching (MKT) to analyse how pedagogical intent and content knowledge shape their problem posing. We argue that orientations and goals inform the pedagogical intent behind tasks, drawing on MKT domains to notice affordances and adapt problems. In this context, this study examines: how teachers' problem posing practices are shaped by their knowledge resources, goals, and orientations? Drawing on a broader study, we focus on narrative accounts of two purposefully selected secondary teachers, Medha and Anay, each with over a decade of experience. Using in-depth semi-structured interviews, we conducted a case-based narrative analysis guided by an integrated lens of Schoenfeld (2011) and Ball et al. (2008). Both cases point to an implicit 'inventory of problems' that teachers draw upon and iteratively transform. Medha's narrative, centred on combinatorics, shows how systematically she modifies or adds constraints to the problems, enabling a progression from intuitive counting to formal applications and strategic problem solving, while foregrounding anticipated student errors. In contrast, Anay's narrative, focusing on deriving the sine rule, illustrates how problems are reformulated to preserve the 'joy of discovery' and student engagement, sequencing problems from numeric to algebraic, while supporting higher-order reasoning. Both cases mobilise integrated use of MKT domains, particularly Specialised Content Knowledge (SCK), Knowledge of Content and Students (KCS), and Knowledge of Content and Teaching (KCT). What distinguishes these cases is that Medha's reformulations emphasise conceptual structuring, whereas Anay's are strongly oriented by fostering student agency and valuing curiosity. Notably, the problems remain close to textbook forms - it is the teachers' deliberate sequencing and reformulation that expands their pedagogical potential. These narratives thus offer resources to elicit deeper reflections in professional development contexts.

References

Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education*, *59*(5), 389–407.

Schoenfeld, A. H. (2011). Toward professional development for teachers grounded in a theory of decision making. *ZDM*, *43*(4), 457–469.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 462. PME.

UNDERSTANDING TIME IN MULTIGRADE CLASSROOMS: A DESIGN-BASED STUDY IN MATHEMATICS EDUCATION

Yolanda Chávez-Ruiz[1], Alejandro Maravilla-Cruz[2]

[1]Centro Regional de Formación Profesional Docente de Sonora
[2]Escuela Normal para Maestras de Jardines de Niños

Teaching time is a challenge in mathematics education due to its abstract nature, which is invisible and cannot be directly manipulated. Classic and contemporary studies show that students face difficulties in constructing notions of duration, sequencing, periodicity, and measurement, especially when teaching focuses on reading the clock. Recent curriculum research also shows fragmented attention to school time, with little coordination between grades and limited integration of physical, linguistic, and sociocultural dimensions (van Bommel & Walla, 2025). These difficulties are intensified in multigrade classrooms, where age and cognitive diversity require flexible and inclusive teaching approaches.

This study aims to design, implement, and document a didactic proposal for teaching time in multigrade classrooms, aimed at broadening the understanding of time as a mathematical quantity and as a social and cultural phenomenon. The theoretical framework is based on the psychology of mathematics education, which analyzes the transition from subjective conceptions of time to conventional measurement systems, as well as on recent studies that question the centrality of the clock as the only teaching tool for addressing this quantity.

The research was conducted using a qualitative, design-based approach. It was implemented in multigrade schools in Mexico and Colombia, with the participation of two teachers and 50 students from different grades.

The documentation of the process included video recordings of sessions, notes, photographic records, student productions, and interviews with teachers.

Preliminary results show that students were able to distinguish between estimation and measurement, recognize the need for more accurate instruments, and establish relationships between natural phenomena, social practices, and time measurement systems. Interaction between students from different grades encouraged peer mediation processes and progressively more elaborate explanations.

References

van Bommel, J., & Walla, M. (2025). Teaching and learning of time in early mathematics education: A systematic literature review. *Education Sciences*, *15*(8), 1003. https://doi.org/10.3390/educsci15081003

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 463. PME.

A STUDY ON INSTRUCTIONAL INTERVENTIONS FOR PRE-SERVICE MATHEMATICS TEACHERS' PERFORMANCE IN PROBLEM POSING

Suanrong Chen[1], Yehui Zhang[1], Jia Pang[1], Jinfa Cai[2]

[1]The School of Mathematics, Yangzhou University, Jiangsu

[2]The Mathematics Department, University of Delaware

This study builds on prior problem-posing research, particularly using posing tasks to understand student thinking (Cai, 2022) and the link between teacher questioning competence and professional knowledge/beliefs (Chapin et al., 2013). Professional development in problem posing helps teachers apply questioning strategies (Li et al., 2019). Yet, effective methods for cultivating PMTs' problem-posing skills and integrating problem evaluation into instruction remain lacking. This study addresses this gap through a problem-posing intervention in a Mathematical Proposition Techniques course.

In a 5-week P-PBL intervention, 38 senior PMTs improved problem-posing performance—greater diversity, richer content, and deeper integration. They designed advanced problems (e.g., probability, limits, integrals, vectors) and adopted multiple geometric perspectives (e.g., projection theorem, Monge's circle theorem). However, problem reasonableness declined; many novel, challenging problems lacked appropriateness, revealing a creativity-feasibility imbalance. The instructor focused on posing strategies but not on evaluating question quality.

Teaching problem-posing strategies alone is insufficient; fostering evaluation awareness is equally important. Future instruction should integrate problem-evaluation interventions to improve reasonableness and educational value. These findings inform teacher preparation programs, especially for developing PMTs' ability to pose meaningful, sound problems.

References

Cai, J., Chen, T., & Sun, Q. (2022). "Problem posing" themed training promotes the professional development of primary school mathematics teachers: An interview with Professor Jinfa Cai of the University of Delaware. Journal of Teacher Education, 9(5), 1–8. https://doi.org/10.13718/j.cnki.jsjy.2022.05.001.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2013). Classroom discussions: Using math talk to help students learn (3rd ed.). Math Solutions.

Li, X., Song, N., Chen, T., et al. (2019). A study on the performance of "problem posing" among primary school mathematics teachers. Journal of Mathematics Education, 28(2), 1-6.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 464. PME.

DIFFERENCES AND COMMONALITIES IN STUDENTS' FOUR THINKING SKILLS: RATIO AND PROPORTION

Wei-Ting Chen[1], Kai-Lin Yang[1], Ting-Ying Wang[1], Ying-Hao Cheng[2]
Jian-Cheng Chen[3]

[1]National Taiwan Normal University, [2]University of Taipei
[3]National Taipei University of Education

According to the PISA framework for 21st-century competencies, critical, reflective, systems, and creative thinking are essential capacities for students in contemporary society (OECD, 2018). However, limited research has examined the distinctive characteristics of students' performance across these four domains. To address this gap, this study explored the differences and commonalities among these thinking skills within ratio and proportion. Three thinking-skills assessment questionnaires were developed and randomly administered to 150 students from six classes. Students' responses were coded and analyzed to identify patterns in their reasoning.

In critical thinking, students more readily evaluated statements containing explicit numerical relationships than abstract claims requiring definition-based reasoning. In reflective thinking, they demonstrated greater strength in comparing procedural efficiency than in judging contextual effectiveness. In systems thinking, students typically relied on a given variable (n) to construct algebraic expressions, favoring symbolic abstraction over multiple representations. In creative thinking, responses were often shaped by familiar formulas and task-provided elements. Across all four domains, students' reasoning was grounded primarily in visible numerical relationships or explicitly stated information, with comparatively limited engagement in abstract principles or integrated structural reasoning.

This pattern suggests that thinking processes become more visible when explicitly embedded into task structures, implying that structured and scaffolded instruction more effectively promotes higher-order thinking. Further teaching experiments are needed to compare infused (explicit) and immersed (implicit) approaches (see Tiruneh et al., 2018) in fostering the development of the four thinking skills in mathematical literacy.

References

Organisation for Economic Co-operation and Development (OECD). (2018). *PISA 2021 mathematics framework draft*. https://pisa2021-maths.oecd.org/files/PISA%202021%20Mathematics%20Framework%20Draft.pdf

Tiruneh, D. T., De Cock, M., & Elen, J. (2018). Designing learning environments for critical thinking: examining effective instructional approaches. *International journal of science and mathematics education*, *16*(6), 1065-1089.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 465. PME.

AGENCY-DRIVEN INDICATORS OF MATHEMATICAL RESILIENCE: MACHINE LEARNING AND THE HOLISTIC RESILIENCE INDEX (HRI)

Yoxiu Chen, Cheng-Yu Hsieh, Su-Wei Lin
National University of Tainan, Taiwan

Mathematical resilience among socio-economically disadvantaged students is a key concern in equity-oriented mathematics education. This study examines (1) which PISA 2022 variables predict mathematically resilient students and (2) how these predictors can be interpreted through the Holistic Resilience Index (HRI). Mathematically resilient students are defined as those in the lowest quartile of the PISA Economic, Social and Cultural Status (ESCS) index and the highest quartile of mathematics achievement, following OECD's classification of disadvantaged high achievers (OECD, 2023). The HRI is a four-dimensional interpretive framework: perseverance (effort, self-efficacy), stress resistance (buffering adversity), emotional control (mathematics anxiety), and sense of belonging (feeling safe, cooperation, student–teacher relationships). It is used as a post hoc lens rather than as a model component, building on Cheung et al. (2024), who used a similar Random Forest–SHAP approach without an overarching framework. Approximately 60 PISA 2022 variables, including student attitudes, school climate, and learning environment indicators, were entered into a Random Forest classifier ($n \approx 690{,}000$; 79 economies, with PISA student weights). An optimal subset of 29 variables was identified via cross-validated feature selection based on F1 performance, and SHAP values were used to interpret predictor importance and direction. The model achieved an F1 of 0.591 on a held-out test set. Results reveal a clear pattern across HRI dimensions. Sense of belonging has the strongest impact: feeling safe is the top positive predictor, followed by cooperation and student–teacher relationships. Mathematics anxiety is the strongest negative predictor, and frequent bullying victimization is also strongly negative. Effort, self-efficacy, and curiosity positively support perseverance and stress resistance. Student gender is salient but non-malleable and is interpreted as reflecting differential exposure to contextual conditions. Overall, agency-driven, school-based factors outweigh fixed structural conditions, suggesting that relational safety and supportive learning environments are central to fostering mathematical resilience.

References

Cheung, K.-c., Sit, P.-s., Zheng, J.-q., Lam, C.-c., Mak, S.-k., & Ieong, M.-k. (2024). A machine-learning model of academic resilience in the COVID-19 pandemic era. *British Journal of Educational Psychology, 94*, 1224–1244.

OECD. (2023). *PISA 2022 Results (Volume II): Learning During – and From – Disruption*. OECD Publishing.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 466. PME.

LEARNING TO NOTICE MATHEMATICAL SENSE-MAKING: A FIRST-CYCLE DBR STUDY WITH LESSON STUDY

Brantina Chirinda

University of the Witwatersrand

This paper reports on the first cycle of a design-based research (DBR) project investigating how teachers in under-resourced South African schools support mathematical problem solving as sense-making. The study addresses the research question: How does participation in lesson study support teachers' attention to and facilitation of students' sense-making processes during problem solving?

The study is situated in a resource-constrained context in Cape Town, South Africa, where structural inequalities shape classroom practice. Mathematical problem solving is conceptualized as sense-making—students' active construction of meaning through talk, gesture, representation, and interaction, rather than answer-focused procedures (Schoenfeld, 1985). This framing positions teacher noticing—including attention to embodied and social forms of meaning-making—as central to instructional practice.

The intervention integrates lesson study as the pedagogical structure for teacher learning within a DBR framework involving iterative design, enactment, and refinement. Four Grade 7 teachers from four nearby primary schools participated in collaborative cycles of planning, teaching, observation, and reflection. Data analysis followed grounded theory techniques, involving iterative coding of teacher discourse and practices to trace shifts in pedagogical attention during Cycle 1.

Findings show three key shifts in teacher learning. First, teachers moved from prioritizing correct answers to attending to students' reasoning processes and the emergence of multiple strategies. Second, teachers developed increased sensitivity to students' embodied and social sense-making practices, including gestures, diagrams, and peer interactions, making these features more explicit in their instructional decisions (Choy, 2025). Third, teachers increasingly framed struggle as productive, delaying intervention to allow students to explore and articulate their thinking. However, challenges persisted in selecting cognitively demanding tasks and supporting student reflection within limited instructional time. As a first-cycle DBR study, these findings are interpreted as initial design conjectures, suggesting that lesson study can support the development of teacher noticing of multimodal sense-making in resource-constrained contexts, while highlighting areas for further design refinement.

References

Choy, B. H. (2025). Noticing to learn from lesson study: the role of critical events. *Journal of Mathematics Teacher Education, 28*, 1059–1076.

Schoenfeld, A. H. (1985). *Mathematical problem-solving*. Academic Press.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 467. PME.

MAKING SENSE OF THE WORLD: INTERWEAVING MATHEMATICS AND POLITICAL APPORTIONMENT

Sean Chorney

Simon Fraser University

This oral communication describes a classroom design in which Grade 8 students (13-year olds) engage mathematically with the political practice of apportionment. Rather than teaching proportional reasoning through textbook procedures, the study immerses students in real-world electoral data, asking: What does students' interweaving of apportionment with their own perspectives look like, and what themes emerge? The study draws on métissage (Campbell-Chudoba, 2023): a theory of identity formation through the interweaving of personal experience, values, and the world. Mathematics is not context-free; learners engage with it through their own histories, reshaping both themselves and their understanding. Student responses are also understood through story making (Jørgensen, 2024): a relational mode in which personal narratives intertwine with mathematical and political reasoning. Data was collected from 59 Grade 8 students across two Western Canadian classrooms. Students worked on (1) the 2024 British Columbia provincial election (allocating 93 legislative seats) and (2) a student council scenario (distributing 12 seats across three grade levels), with a final question asking whether fairness and equality are the same. Five themes emerged. First, some students distributed extra seats equally across parties, prioritizing uniform treatment even at the cost of proportionality. Second, others argued the party with the most votes deserved more seats: that popularity should translate into power. Third, another group advocated for smaller parties, reasoning that extra seats would amplify underrepresented voices. Fourth, in the council scenario, some students argued older grades deserved more seats due to experience, reframing representation as a matter of voice and maturity. Fifth, students distinguished fairness from equality, drawing on personal analogies such as food portions, height restrictions, and accessibility seating to articulate that fairness depends on context, not uniformity. Students did not simply compute; they weighed considerations and drew on lived experience. The focus was less on calculation and more on developing a general sense of what proportional reasoning is about. The métissage framework illuminates how students interweave justice with mathematical tools through story making, indicating that proportional reasoning can be experienced as relevant and meaningful in socially situated ways.

References

Campbell-Chudoba, R. (2023). Métissage. In *Varieties of qualitative research methods* (J. M. Okoko et al., Eds., pp. 309–315). Springer.

Jørgensen, K. M. (2024). *The ethics of sustainability in management—Storymaking in organizations*. Routledge.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 468. PME.

WHAT DID CHATGPT "NOTICE" ABOUT PRIME NUMBERS? INSIGHTS FROM AN AI-GENERATED LESSON PLAY

Ban Heng Choy

National Institute of Education, Nanyang Technological University

The introduction of Artificial Intelligence (AI) into education has the potential to usher in an era of new possibilities for reimagining teacher learning. For instance, Gurl et al. (2024) have explored the use of ChatGPT as a lesson-planning assistant and have found it useful, provided that teachers critically reflect on the AI agent's output, which may contain errors and misconceptions that are difficult to notice when using it. Furthermore, these AI-generated lesson plans tend to be more teacher-centred in their suggested approaches. Given the importance of lesson planning, it is critical for teachers to think more deeply about the mathematical concepts they teach, anticipate their students' thinking, and consider different teaching approaches in response to student thinking. A lesson play—" an elaboration on a lesson or part of a lesson in the form of a script"—is a way for teachers to "imagine a classroom", "learners' reactions", and "their possible explanations" in response to a critical incident (Zazkis et al., 2013, p. 271). A lesson play can thus make visible what teachers *notice* about content, student thinking, and teaching approaches—critical aspects of teaching that serve as useful foci for teachers (Choy, 2016). In this presentation, I investigated the characteristics of a ChatGPT-generated lesson play by examining what it "noticed" about the mathematics content, student thinking, and teaching approaches. The presentation is framed by the following question: What did ChatGPT notice when prompted to write a lesson play? The lesson play presented and analysed in this paper was generated by ChatGPT (GPT-5.2) to address two main pedagogical aims: explain what prime numbers are and how to determine whether a given number is prime, using a prompt developed by Zazkis et al. (2013, p. 89). Analysis began with a line-by-line parsing to identify key segments of the discourse. I then analysed the discourse patterns embedded in the generated lesson play and coded it using Choy's (2016) foci for noticing. I will present findings on the characteristics of this AI-generated lesson play, invite feedback, and discuss possible directions for future research.

References

Choy, B. H. (2016). Snapshots of mathematics teacher noticing during task design. *Mathematics Education Research Journal*, *28*(3), 421-440.

Gurl, T. J., Markinson, M. P., & Artzt, A. F. (2024). Using ChatGPT as a Lesson Planning Assistant with Preservice Secondary Mathematics Teachers. *Digital Experiences in Mathematics Education*, *11*(1), 114-139.

Zazkis, R., Sinclair, N., & Liljedahl, P. (2013). *Lesson play in mathematics education: A tool for research and professional development*. Springer.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 469. PME.

FEELING AND UNDERSTANDING FRACTIONS: RELATIVE UNIT AND MUSICAL TIME IN TEACHER TRAINING

Alexander Conde[1], Sandra Parada[2], Vicente Liern[3]

[1]Universidad Complutense de Madrid, [2]Universidad Industrial de Santander, [3]Universitat de València

In mathematics education, the teaching and learning of fractions is often reduced to procedures that do not consolidate meanings about unity, equal division, and equivalence. This paper is part of the context of "Sensing and making sense of mathematics," showing how musical time (experienced physically, auditorily, and visually) offers a privileged terrain for sensing relationships (duration, repetition, half/double) and, based on this, giving meaning to fractional structures.

We present the results of a research program that explores cognitive links between music and mathematics through rhythmic experiences. We propose the notion of relative unit as a central construct: a reference of measurement that is fixed and transformed according to the rhythmic pattern, allowing durations to be compared, rescaled, and relationships verified (Liern, 2011). Through tasks, we analyze multimodal evidence from students and describe a trajectory of construction and use of relative unit at four levels: identification (discrimination of durations), categorization (regularities and comparisons), association (bridges between registers and explicitation of the unit), and signification (justification of equivalences, reasoning about equal distribution, and resolution of situations with fractions). The results show that the transition from performing the rhythm to a meaningful understanding of fractions is strengthened when the task requires changes in unit and verification procedures (Conde, 2013). Based on the results, we formulated a research agenda aimed at teacher training (Parada & Fiallo, 2022), which contains: (i) re-signifying the relative unit, (ii) designing tasks that articulate sensory experience and mathematical justification, (iii) orchestrating mediations (gestural, notational, and technological), and (iv) interpreting multimodal evidence as indicators of fractional thinking. We propose lines of work for initial and continuing training through reflection and action, with didactic designs and rubrics for analyzing multimodal evidence.

References

Conde, L. A. (2013). *La unidad relativa como vínculo cognitivo entre el tiempo musical y las fracciones* (Tesis doctoral). Departamento de Matemática Educativa, CINVESTAV-IPN, México.

Liern, V. (2011). Música y matemáticas en educación primaria. *SUMA*, 66, 107–112.

Parada, S. y Fiallo, J. (2022). *Comunidades de práctica de profesores de matemáticas que incorporan tecnologías digitales en el aula*. Ed: Ediciones UIS. ISBN impreso: 978-958-8956-43-5. En Colombia.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 470. PME.

MAKING ROOM FOR MATHEMATICAL PROFICIENCY

Priscila D. Corrêa

University of Windsor

A large body of mathematics education research highlights a shift away from schools' conventional lecture-based mathematics instruction toward innovative and flexible teaching practices that support diverse learners and improve mathematical confidence, understanding, and achievement. Research also identifies various classroom approaches that can promote this shift and develop students' conceptual understanding. Yet teachers need to be intentional in applying these methods, ensuring that they yield the intended outcomes. This is not an easy task, given teachers' prior experiences as learners, university training, and current teaching practices. Teachers may need to let go of some ties to further enable students to sense and make sense of mathematics, deepen their understanding, and build long-lasting mathematical knowledge. Inspired by this understanding, this research sought to identify and investigate potential classroom scenarios that favour meaningful teaching practices. Grade 4 and 5 mathematics classrooms were observed seeking opportunities to intentionally develop students' mathematical proficiency, namely, conceptual understanding, strategic competence, procedural fluency, adaptive reasoning, and productive disposition. Kilpatrick et al.'s (2001) model of mathematical proficiency was used as the theoretical framework for data analysis. The research methodology was grounded in action research (Stringer, 2007), involving me, the classroom teacher, the assistant teacher, and the elementary school pedagogical coordinator as research collaborators. We gathered to discuss, plan, and follow up on classroom practices and tasks aligned with the research theoretical framework. Classroom interventions were implemented to prompt, nurture and develop students' mathematical proficiency. Data was collected through field observations of classroom activities and from students' activity materials. Preliminary results show that students will naturally ask questions and create opportunities for teachers to explore and foster students' mathematical proficiencies. However, it is crucial to adjust original lesson plans or activities as needed to allocate sufficient time to delve into those opportunities. This oral communication will present examples of how classroom activities and situations were explored to support students' development of mathematical proficiency. Research findings indicate that teachers can be more flexible and adapt ingrained, stagnant teaching models to better assist students and meaningfully promote their mathematical learning.

References

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Stringer, E. T. (2007). *Action research*. Sage Publications.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 471. PME.

EPISTEMIC POSITIONING AND AGENCY IN AI-MEDIATED MATHEMATICS LEARNING AMONG GIFTED STUDENTS

Şahin Danişman[1], Emrah Bilgiç[2]

[1]Düzce University, [2]Sakarya University

Within sociocultural perspectives, tools are understood as mediators that reorganize thinking processes rather than merely supporting performance (Vygotsky, 1978). Recent discussions in mathematics education therefore focus on epistemic agency, referring to learners' role in directing knowledge construction (Scardamalia & Bereiter, 2006). Although artificial intelligence is often assumed to diminish learner agency due to its epistemic authority, little is known about how students conceptualize AI and how these conceptualizations relate to perceived control during problem solving. This study examines how mathematically gifted students position artificial intelligence epistemically and how these positions relate to perceived procedural agency in AI-supported mathematical activity. Guided by an interpretive perspective on epistemic positioning, the study employed a qualitative interpretive design to analyse students' written responses. The study involved 53 mathematically gifted students participating in an enrichment program. Data consisted of written metaphor responses and reflections on control during AI use. Metaphors were analysed qualitatively as indicators of epistemic positioning and categorized into seven roles (Instrumental Tool, Cognitive Assistant, Navigator, Collaborative Partner, Epistemic Authority, Cognitive Extension, and Opaque Intelligence). The analysis resulted in a set of epistemic agency positions describing how responsibility and control were distributed between students and AI, ranging from primarily human-dominant agency (occasionally negotiated), supported but learner-regulated agency, and guided agency, to distributed and negotiated agency, AI-dominant or externally validated agency, shared cognitive agency through externalization, and epistemic opacity. Results indicate that metaphorical positioning and perceived agency were not aligned along a single continuum. Students who described AI as a *teacher* or *expert* frequently reported maintaining control over questioning, decision-making, and evaluation processes, suggesting that attributing epistemic authority to AI does not necessarily reduce learner agency. Instead, agency appeared dynamically negotiated during interaction. These findings suggest that AI-mediated mathematical activity reorganizes rather than transfers epistemic responsibility between learner and technology.

References

Scardamalia, M., & Bereiter, C. (2006). Knowledge Building: Theory, Pedagogy, and Technology. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (pp. 97–115). Cambridge University Press.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Harvard University Press.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 472. PME.

MATHEMATICIANS' ASSESSMENT OF POTENTIAL WRITING CONVENTIONS FOR SYMBOLS IN STUDENTS' PROOFS

Nathania de Sena Maier, Silke Neuhaus-Eckhardt

Julius-Maximilians-Universität Würzburg

Writing proofs is challenging for first-year students, for example because they have to adhere to writing conventions that are often only implicitly given. These conventions can be understood as non-universal practices and vary across different proof contexts, for example blackboard or textbook proofs (Lew & Mejía-Ramos, 2019). It is therefore worthwhile to examine which conventions instructors apply to student proofs submitted as homework. Given that students frequently struggle with technical language, particularly with the use of symbols (Guce, 2017), we ask: *What writing conventions do instructors hold regarding the use of symbols in students' written proofs?*

To address this question, five instructors at a German university were asked to think-aloud while evaluating a handwritten and mathematically valid student proof on matrix eigenvalues containing (among others) four potential breaches of convention in the use of symbols in text. The potential breaches are *grammatically incorrect use of a symbol*, *a symbol at the beginning of a sentence*, the *use of a relation sign to connect a non-symbolic and a symbolic expression*, and the *use of an implication arrow as a logical connective*. In a subsequent discussion about breaches which were not evaluated, we asked whether the instructors would revise their feedback to make conventions explicit that are often applied implicitly.

Using qualitative content analysis on our data, we found that the use of symbols in text was scarcely evaluated. However, after discussing the breaches of conventions, most instructors considered them to be relevant confirming previous studies on such breaching experiments (Lew & Mejía-Ramos, 2019). The lecturers also gave several reasons why some conventions should be followed, such as avoiding misunderstandings, improving readability, and aesthetics. Interestingly, many lecturers who still regarded some of the included breaches as irrelevant nevertheless identified contexts in which they would find them disturbing, for instance in computer-written student proofs or in scientific publications.

References

Guce, I. K. (2017). Mathematical writing errors in expository writings of college mathematics students. *International Journal of Evaluation and Research in Education*, *6*(3), 233–242.

Lew, K. & Mejía-Ramos, J. P. (2019). Linguistic conventions of mathematical proof writing at the undergraduate level. *Journal for Research in Mathematics Education*, *50*(2), 121–155. https://doi.org/10.5951/jresematheduc.50.2.0121

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 474. PME.

THE PERSPECTIVE OF THEORY OF OBJECTIFICATION IN ALGEBRAICAL PROBLEM POSING BY GRADUATE STUDENTS

José Jorge de Sousa[1], Silvanio de Andrade[1], Jinfa Cai[2]

[1]Universidade Estadual da Paraíba, [2]University of Delaware

We intend to analyze students' knowing through problem posing tasks under a sociocultural lens. It considers the objectification and subjectification processes emerging during a problem-posing activity, framed by the theory of objectification that prioritizes collective, semiotically mediated work. The Theory of Objectification (TO) have been settled as a sociocultural approach to mathematics education that conceives learning as a historical, cultural, and collectively mediated process (Radford, 2021). It has been adopted as lens because it allows us not only to analyze the processes of *knowing* during the activity and the *becoming* (how students collectively assist and care for each other and learn through joint work). The study adopts a qualitative, interpretive approach. Data were collected from audio and video recordings, written paperwork and researcher's logbook during a workshop with 20 graduate students working in small groups on a problem-posing task based on an algebraic growing figure from audio and video recordings, written paperwork and researcher's logbook. The results indicate how objectification (*knowing*) unfolds through joint work. Students collectively negotiated the meaning of the figure, mobilizing mathematical ideas such as functions, equations, generalization, and arithmetic progression, as well as producing multiple interpretations (e.g., parked cars, a cross, a flower). Participants used speech, gestures, drawings, and material actions (e.g., modeling clay) to pose problems and discuss mathematical ideas. At the same time, these interactions involved processes of subjectification, as students listened to each other, negotiated interpretations, and showed concern for how their problem would be understood. Students debated whether the situation was best represented by $f(x) = 4x - 1$ or $f(x) = 4x - 3$. This disagreement was solved through an intense, collective dialogue between them. This highlights the importance and power of considering problem posing in the classroom as a collective and dialogical activity. Therefore, in this process, learning through problem posing involves a deep intertwining between culture and human action, including ways of thinking, acting, feeling, and positioning oneself in the world.

Additional information

This work was supported by CNPQ, Brazil.

References

Radford, L. (2021). *The Theory of objectification: A Vygotskian perspective on knowing and becoming in mathematics teaching and learning*. Brill.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 473. PME.

MODELING TEACHING, AFFECTIVE FACTORS, AND MATH ACHIEVEMENT: A MULTILEVEL PISA 2022 ANALYSIS

Isabelle Demonty[1], Joëlle Vlassis[2]

[1]Catholic University of Louvain, [2]University of Luxembourg

This study contributes to research modelling mathematics performance by integrating perceived teaching quality with students' motivational and emotional variables (Li et al., 2021; Liu et al., 2024). Mathematics achievement is conceptualized as a dynamic system in which these dimensions interact, with affective factors playing a central mediating role. Unlike prior work that typically examines teaching practices (Liu et al., 2024) and student characteristics separately (Du et al., 2021; Namkung et al., 2019), our model combines them within a single explanatory structure, revealing pathways of influence. The aim is to test, using a multilevel model, the direct and indirect effects of teaching-related variables (teacher support, cognitive activation) and students' motivational and emotional characteristics (engagement, self-efficacy, anxiety), accounting for the mediating roles of self-efficacy and anxiety and for school-level variance.

The analyses draw on PISA 2022 mathematics data from 4,875 students aged 15 in French-speaking of Belgium. Two-level multilevel models (students/schools) were estimated, retaining only significant relations. Results show that self-efficacy is the strongest positive predictor of performance, whereas anxiety exerts a substantial negative effect. Teacher support and student engagement influence achievement indirectly by strengthening self-efficacy and reducing anxiety. Cognitive activation displays a non-linear effect: beneficial when accompanied by sufficient student engagement and teacher support, but potentially counterproductive when intensified without adequate support. Differences between schools further confirm the value of a multilevel approach. This modelling shows that teaching practices influence performance primarily through motivational and emotional mechanisms. These results suggest that more effective mathematics learning environments rely both on supportive and cognitively stimulating teaching practices and on strengthening students' motivational and emotional resources.

References

Liu, X., Hansen, K. Y., Valcke, M., & De Neve, J. (2024). A decade of PISA: student perceived instructional quality and mathematics achievement across European countries. *ZDM–Mathematics Education*, 56(5), 859-891.

Namkung, J. M., Peng, P., & Lin, X. (2019). The relation between mathematics anxiety and mathematics performance among school-aged students: A meta-analysis. *Review of Educational Research*, 89(3), 459-496.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 475. PME.

INVESTIGATING THE RELATIONSHIPS BETWEEN COMPUTATIONAL AND ALGEBRAIC THINKING

Xixi Deng, Rui Ding

Northeast Normal University

Computational thinking (CT) has been widely recognized as an essential twenty-first-century literacy, and increasing efforts have been made to integrate CT into formal school curricula. Prior research suggests that algebraic thinking (AT) can be supported through CT-related activities (e.g., Bråting & Kilhamn, 2021). However, empirical evidence on how CT relates to students' mathematical thinking, particularly AT, remains limited. This study aims to quantitatively examine the extent to which elementary students' CT predicts their AT. Drawing on Brennan and Resnick's (2012) framework, CT is conceptualized in terms of core computational concepts common across programming languages, including sequences, loops, events, and conditionals. AT is framed based on Blanton et al. (2019), encompassing (1) equivalence, expressions, equations, and inequalities; (2) generalized arithmetic, and (3) functional thinking. As an abstract method of thinking, algorithmic thinking manifests in both mathematical and computational contexts, involving the formulation of systematic procedures through sequences of explicit instructions. Thus, we developed a mediation model in which algorithmic thinking serves as a mediator in the association between CT and AT. Quantitative analyses were conducted using data collected from 345 students in Grades 4-6 in China who responded to valid and reliable measures of CT, AT, and algorithmic thinking. Structural equation modeling indicated an acceptable model fit (CFI=0.94, TLI=0.93, RMSEA=0.028, SRMR=0.053). The model revealed that CT had a significant direct effect on AT ($\beta=0.70$, $p<0.001$). In addition, the indirect effect through algorithmic thinking was also significant ($\beta=0.46$, $p<0.001$; $\beta=0.20$, $p<0.01$), indicating partial mediation. These findings suggest that CT contributes to the development of AT both directly and indirectly via algorithmic thinking. The study provides support for incorporating CT practices into students' early algebra learning.

References

Bråting, K., & Kilhamn, C. (2021). Exploring the intersection of algebraic and computational thinking. *Mathematical Thinking and Learning*, *23*(2), 170-185.

Brennan, K., & Resnick, M. (2012). New frameworks for studying and assessing the development of computational thinking. In *Proceedings of the 2012 annual meeting of the American educational research association* (pp. 1-25).

Blanton, M., Isler-Baykal, I., Stroud, R., Stephens, A., Knuth, E., & Gardiner, A. M. (2019). Growth in children's understanding of generalizing and representing mathematical structure and relationships. *Educational Studies in Mathematics*, *102*(2), 193-219.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 476. PME.

USING ALGEBRA TILES TO FOSTER ALGEBRAIC THINKING IN 12- AND 13-YEAR-OLD STUDENTS

Ana Clara Díaz[1], Mathías A. López[2]

[1]Universidad de Montevideo, [2]Tufts University

One major challenge for algebra students is to use letters and numbers to construct and manipulate algebraic expressions that represent generalized quantities (Kieran, 2004). Standards-based guidance emphasizes the use of multiple representations, including physical manipulatives, to support students' understanding of algebraic expressions (NCTM, 2000). Algebra tiles offer an area-based representation that may bridge geometric and symbolic registers. Our goal is to answer the following question: What are the affordances and constraints of algebra tiles for 7th-grade students as a tool for representing and operating on algebraic expressions? We offer empirically grounded conjectures about how area-based representations support algebraic sense-making.

We used a design-based research (DBR) approach organized around a conjecture map (Sandoval, 2014), which links learning conjectures to design features and the mediating processes expected in students' work. We designed and implemented a 12-lesson sequence using algebra tiles with 25 students in 7th grade (ages 12–13) at a private institution in Uruguay. We video-recorded all lessons and collected written work. For analysis, we used the mediating processes from our conjecture map as initial codes, iteratively refining conjectures when we observed outcomes that diverged from predictions.

Our first pass through the full set of lesson videos and student work led us to refine two provisional conjectures in our conjecture map. Students linked generalized expressions, such as x or x^2, to the area of a specific shape, which they found useful when identifying similar monomials: students recognized that different shapes represent unlike terms. Students' prior strategies for adding integers interfered with their work when simplifying expressions. When asked to reduce like terms, most students relied on mental integer-addition techniques, so the teacher had to explicitly model tile-based reduction at the end of the lessons. We expect our refined conjecture map to inform the design of theoretically grounded algebra-tile lesson sequences.

References

Kieran, C. (2004). Algebraic thinking in the early grades: What is it? *The Mathematics Educator*, *8*(1), 139-151.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*.

Sandoval, W. (2014). Conjecture mapping: An approach to systematic educational design research. *Journal of the Learning Sciences*, *23*(1), 18-36.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 477. PME.

COMPUTATIONAL THINKING AND MATHEMATICS EDUCATION: A BIBLIOMETRIC ANALYSIS

Behiye Dinçer-Aksoy, Kenan Gökdağ

Ministry of National Education, Türkiye

Introduction

Computational Thinking (CT) is increasingly described as a set of epistemic practices rather than merely programming skills (Li et al., 2020), its relation to modelling and reasoning in mathematics education has been noted (diSessa, 2018), yet its structural position within mathematics education research remains unclear (Rich et al., 2024).

Method

This study used a bibliometric design with a structured Scopus TITLE-ABS-KEY search to build two datasets (computational thinking publications and mathematics–computational thinking intersections) and analyzed them via VOSviewer keyword co-occurrence mapping (van Eck & Waltman, 2010).

Results

CT research is primarily structured around K–12 education, STEM, programming, and teacher education, whereas mathematics education is anchored in modelling, reasoning, and problem solving; within their intersection, CT appears mainly in pedagogical and programming contexts, with modelling and problem solving serving as linking nodes, and although algorithmic thinking and debugging have gained recent visibility, CT remains structurally connected yet not conceptually central to the broader mathematical thinking literature. These findings provide a macro-level descriptive mapping of CT's positioning within mathematics education research.

References

diSessa, A. A. (2018). Computational Literacy and "The Big Picture" Concerning Computers in Mathematics Education. *Mathematical Thinking and Learning, 20*(1), 3–31. https://doi.org/10.1080/10986065.2018.1403544

Li, Y., Schoenfeld, A.H., diSessa, A.A. et al. (2020). Computational Thinking Is More about Thinking than Computing. *Journal for STEM Educ Res 3*, 1–18. https://doi.org/10.1007/s41979-020-00030-2

Rich, K.M., Yadav, A. & Fessler, C.J. (2024). Computational thinking practices as tools for creating high cognitive demand mathematics instruction. *J Math Teacher Educ* 27, 235–255. https://doi.org/10.1007/s10857-022-09562-3

van Eck, N. J., & Waltman, L. (2010). Software survey: VOSviewer, a computer program for bibliometric mapping. Scientometrics, 84(2), 523–538. https://doi.org/10.1007/s11192-009-0146-3

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 478. PME.

REFLECTIONS OF ALGEBRAIC HABITS OF MIND IN NON-ROUTINE ALGEBRA PROBLEMS

Hasret Doganlar, Berna Cantürk Günhan

Dokuz Eylul University

In recent years, the emphasis on mathematical thinking processes in mathematics curricula has made it a more visible goal for students to be able to use their knowledge flexibly in different problem situations and to produce generalisations (Poindexter, 2011). As a specific form of mathematical habits of mind, algebraic habits of mind constitute a framework that can be developed through classroom practices and discussions and that encompasses process actions such as organising information, predicting patterns, identifying repeated information in patterns, defining rules, describing change, and verifying rules (Driscoll, 1999; Eroğlu & Tanışlı, 2017). The purpose of this study is to examine middle school mathematics teachers' algebraic habits of mind and how these habits are reflected in the problem-solving processes of Grade 7 students with whom they have worked together for three years, in the context of a non-routine river-crossing problem. In the research conducted with a qualitative case study design, written solutions and clinical interview data were collected from two teachers and their students, and the data were analysed according to the components in Driscoll's framework. The findings show that participants mostly simplified the problem and structured the initial situation, and that they predicted the pattern by organising information and identifying repeated information using representations such as arrows, diagrams, and tables. However, it was also determined that difficulties experienced particularly in the components of organising information, predicting patterns, and identifying repeated information markedly reduced performance in the later stages of the solution, namely defining rules, describing change, and verifying rules. This situation may lead to an inability to implement the generalisation–verification cycle consistently and may hinder the development of algebraic thinking. Consequently, regularly integrating non-routine algebra problems into instruction and employing multi-representation-based guidance that makes early algebraic habits of mind visible seem critical for strengthening generalisation and rule verification.

References

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers, gr 6–10*. Heinemann.

Eroğlu, D., & Tanışlı, D. (2017). Integration of algebraic habits of mind into the classroom practice. *İlköğretim Online*, *16*(2), 566–583.

Poindexter, C. (2011). Teaching "habits of mind": Impact on students' mathematical thinking and problem-solving self-efficacy. In L. McCoy (Ed.), *Studies in Teaching 2011 Research Digest* (pp. 97–102). Wake Forest University.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 479. PME.

INVESTIGATING THE INTERPLAY: TEACHER KNOWLEDGE, INSTRUCTIONAL QUALITY, AND STUDENT LEARNING

Irene Duranczyk[1], Vilma Mesa[2]

[1]University of Minnesota, [2]University of Michigan

Research in ISCED 1-3 contexts show that instructional quality is positively associated with student learning and that teacher knowledge is positively associated with instructional quality, although establishing a clear connection among the constructs remains challenging (Hill et al., 2008). Having validated instruments for community college settings (ISCED 5) to measure (a) mathematical knowledge held by instructors who routinely teach algebra (Mesa et al., 2026); (b) the quality of instruction in algebra classrooms, Evaluating the Quality of instruction in Postsecondary Education (EQIPM; Ko, 2025); and (c) student performance on the Algebra and Precalculus Readiness-Community College (APCR-CC) assessment (Peralta, 2017), we report on an anomalous case. Ellie, an instructor whose standard score on the Quality of Instructor-Content Interaction construct of EQIPM was 0.26; the average gain from the beginning to the end of the term score on the APCR-CC for her students was 2.42 (the second highest in the sample of instructors); her knowledge standardized score was -0.16, allowed us to find consistency in the measures obtained, that is, the high scores in EQIPM and APCR-CC suggest that Ellie's instruction supported students' performance as the APCR-CC items required the foundational work presented in Ellie's lessons. Likewise, the contrasting knowledge measurement score does suggest weakness in Ellie's knowledge not captured by EQIPM or assessed in the APCR-CC. We believe that this detailed qualitative analysis helps us provide a better understanding of how these measures work together in the community college context.

References

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, *39*(4), 372-400.

Ko, I. (2025). *Dimensionality analysis of the Evaluating the Quality of Instruction in Postsecondary Mathematics measure of quality of instruction*. Technical Report. AI@CC 2.0: VMQI. University of Michigan.

Mesa, V., Ko, I., Duranczyk, I., & The VMQI Research Team. (2026). *The Mathematical Knowledge for Teaching Community College Algebra Instrument.* University of Michigan, Ann Arbor; University of Minnesota; Maricopa Community Colleges, Oregon State University. https://dx.doi.org/10.7302/27986

Peralta, Y., Kohli, N., Ström, A., Duranczyk, I., Mesa, V., & Watkins, L. (2019). A psychometric analysis of the Algebra and Precalculus Concept Readiness assessment. *Journal of Psychoeducational Assessment*, *38*(3), 1-16.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 480. PME.

PRE-SERVICE ELEMENTARY SCHOOL TEACHERS' ATTITUDES TOWARDS MATHEMATICS AND GENERALIZATION WHEN SOLVING FUNCTIONS PROBLEMS

María C. Cañadas, María D. Torres, Araceli Durán-Rubio

Universidad de Granada

This study is focused on the attitudes towards mathematics and generalization of Spanish pre-service elementary school teachers. Positive attitudes toward mathematics improve academic performance and reduce anxiety about the subject (e.g., Akin & Kurbanoglu, 2011). Algebraic sense involves relationships between variables. Attitudes is one element of socio-affective sense; and generalization of functions are connected to algebraic sense (Carraher & Schliemann, 2007). Teachers must help elementary students manage their attitudes, particularly to what algebraic sense concerns. Our specific research objectives are to describe: (a) their attitudes toward mathematics, (b) the generalization evidenced in problems involving functions, and (c) the relationship between their attitude toward mathematics and their success in generalization. Seventy three pre-service teachers completed an individual questionnaire, with (a) three closed questions about attitudes —"how do you feel about mathematics?", "what has your previous experience been like solving mathematical problems?", and "does liking mathematics depend on the teacher?" —; and (b) two function problems requiring generalization. The data analysis was mainly quantitative.

Most pre-service teachers recognize the influence of the teacher as a determining factor in their attitude toward mathematics (95.83%). The majority express an ambivalent attitude: they find mathematics interesting but, at the same time, perceive it as difficult. There were a minority of pre-service teachers who expressed the generalization correctly, with a tendency to express it through algebraic symbolism, even when it was required verbally. We found no significant differences between generalization and attitudes, nor between generalization and the specific problems provided.

Acknowledgement

This study is funded by the research projects PID2020-113601 GB-I00, and PID2024-157106NB-I00; financed by MCIN/AEI/10.13039/501100011033, and FEDER, UE.

References

Akin, A., & Kurbanoglu, N. (2011). The relationships between math anxiety, math attitude and self-efficacy: A structural equation model. *Studia Psychologica, 53*(3), 263-273.

Carraher, D. W., & Schliemann, A. D. (2007). Early algebra and algebraic reasoning. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 669-705). Information Age Publishing.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 481. PME.

ADULT READERS' ENGAGEMENT WITH DATA VISUALISATIONS WHEN INTERPRETING STATISTICS IN NEWS MEDIA

Ulrika Wikström Hultdin[1,2], Anneli Dyrvold[1,2]

[1]Department of Science and Mathematics Education, Umeå University
[2]Umeå Mathematics Education Research Centre, Umeå University

Ideally, schools should foster students' abilities to act as informed and active citizens. This includes the development of data literacy, which mathematics education is well positioned to support (OECD, 2025). Within this competence, data visualisation literacy (DVL) constitutes a specialised form of visual understanding essential for interpreting graphical data. However, to design teaching for DVL in real-life contexts, it is necessary to understand how data visualisations (DVs) are interpreted by citizens outside educational contexts (Börner et al., 2019).

With a focus on DVL, this pilot study explores experienced readers' strategies and what they take away from a text. Two adults, an HR coordinator (R1) and a medical science researcher (R2), read a newspaper article presenting statistical data in seven DVs integrated with the verbal narrative. Data were collected through eye-tracking and interviews, and the ways in which the readers constructed meaning were analysed using an analytical framework that qualitatively distinguishes between visualisation preferences, decoding strategies, and readers' propensity to adhere to textual guidance. Our findings show that although both readers regarded the statistical content as important, they used the DVs differently. While R1 prioritised the verbal narrative, R2 based their interpretation chiefly on the DVs, drew their own conclusions from them, and recognised the narrative as one of several possible alternatives. This pattern was evident in gaze patterns and further supported by interview data. Although these findings do not allow conclusions about which DVL abilities are required of a literate citizen, they clearly demonstrate the benefits of developing DVL skills and provide sufficient reason to consider the development of DVL in mathematics education for all students. Further studies will examine the gap between current teaching and the data literacy demands of contemporary public information.

References

Börner, K., Bueckle, A., & Ginda, M. (2019). Data visualization literacy: Definitions, conceptual frameworks, exercises, and assessments. *Proceedings of the National Academy of Sciences, 116*(6), 1857–1864.

Organisation for Economic Co-operation and Development. (2025). *Future-focused mathematics curricula: Empowering learners for the 21st century* (OECD Education Spotlights No. 18). OECD Publishing.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 482. PME.

IDENTITY+: FROM MATH TEACHER TO AI EDUCATOR

Rebecca Ellis[1], Daria Smylova[2], Shiyan Jiang[2,3], Jie Chao[1]

[1]The Concord Consortium, Concord, Massachusetts
[2]North Carolina State University, North Carolina
[3]University of Pennsylvania, Pennsylvania

Math teacher identity, shaped through practice, discourse, and participation in professional communities, also involves teachers' background and motivations (Akkerman & Meijer, 2011; Ntow & Adler, 2019). The process of developing an *AI Educator* identity, one who teaches students about AI, foregrounds teachers' epistemic authority to frame what counts as legitimate knowledge in their classrooms. Thus, becoming an AI Educator entails reauthoring one's professional role in relation to knowledge, agency, and instructional responsibility. Our team developed a validated research instrument to measure teachers' AI Educator identities (Jiang et al., under review). We applied this instrument to research the shift of math teachers' AI Educator identity over the course of a professional development workshop.

The sample comprises 11 secondary mathematics teachers who attended a workshop on how to teach a curriculum module that contextualizes AI learning from a disciplinary perspective. During the workshop, the teachers experienced the content firsthand and learned about the corresponding pedagogical aspects. To measure the impact of the workshop on the change in AI Educator identity, a pre- and post-survey was administered containing six identical questions.

Results demonstrated a significant increase in the overlap between teachers' existing identities and that of an AI Educator. Teachers saw greater alignment between AI and math education, reported increased confidence in their ability to successfully integrate AI teaching into their math classes, and anticipated allocating more class time to AI topics. Results suggest teachers came to view AI education as advancing core mathematical learning goals, not displacing them, signaling a shift in what they perceive as legitimate mathematical content. During the presentation, the details and implications of these results, their limitations, and next steps will be discussed.

References

Akkerman, S. F., & Meijer, P. C. (2011). A dialogical approach to conceptualizing teacher identity. Teaching and Teacher Education, 27(2), 308–319.

Jiang, S., Duan, S., Xiao, F., Ellis, R., Rosé, C. P., & Chao, J. (under review). From stretch to set: The Spring model of AI educator identity development. Educational Researcher.

Ntow, F. D., & Adler, J. (2019). Identity resources and mathematics teaching identity: An exploratory study. ZDM Mathematics Education. https://doi.org/10.1007/s11858-019-01025-z

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 483. PME.

INVESTIGATING STUDENTS' CREATIVE MATHEMATICAL REASONING: THE CASE OF THE TRAPEZOID

Elçin Emre Akdoğan, Çiğdem Alkaş Ulusoy, Ekin Balcı

TED University

Developing students' capacity for meaningful mathematical reasoning is a central goal of mathematics education, requiring not only procedural fluency but also the ability to construct explanations, generate novel strategies, and justify thinking. Within this perspective, Creative Mathematical Reasoning (CMR) involves constructing new and plausible solution paths grounded in mathematical principles rather than imitation (Lithner, 2008). Despite its importance, classroom practices often privilege imitative reasoning, limiting opportunities for meaningful sense-making. Moreover, as reasoning is interactionally constituted through classroom discourse, little is known about how CMR emerges within such interactions. This study addresses this gap by examining the following research question: How does students' CMR emerge within classroom interactions during engagement with a trapezoid-based proof task?

This study was conducted within a design-based research framework in a high school classroom in Türkiye with 14 students aged 15–16. Over a seven-week sequence of classroom observations on proof-based geometry tasks, the present analysis focuses on one week involving a trapezoid task, during which students engaged in classroom interaction. Data were analysed qualitatively by identifying episodes of students' reasoning from transcripts, which were then coded using Lithner's (2008) framework to examine instances of creative mathematical reasoning, particularly in terms of novelty, plausibility, and mathematical foundation.

The findings indicate that the task, together with interactional support, enabled students to move beyond procedural activity and construct novel solution pathways, particularly through geometric dissection and reconfiguration. These ideas were developed through classroom discourse, where students articulated, justified, and refined their reasoning in response to peers and teacher prompts. As a result, students' reasoning integrated novelty, plausibility, and mathematical foundation, drawing on geometric principles such as area conservation and properties of parallel sides. Overall, the study highlights the role of proof-based geometry tasks and classroom interaction in fostering creative and mathematically grounded reasoning.

Acknowledgment: This study was funded by the Scientific and Technological Research Council of Türkiye (TUBITAK) ARDEB 3501 under Grant Number 222K223.

References

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics, 67*, 255-276.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 484. PME.

DEVELOPING AN INSTRUMENT FOR ASSESSING (PRE-SERVICE) TEACHERS' BELIEFS IN ERROR SITUATIONS

Marieke Ende, Nils Buchholtz

University of Hamburg

How teachers deal with learners' errors depends on a variety of factors, including their beliefs. Therefore, beliefs towards errors are considered to be part of teachers' professional error-competence (Wuttke & Seifried, 2017). From a behaviourist perspective, errors represent deficits in the learning process, while from a constructivist point of view, errors are seen as valuable learning opportunities. Regarding error handling, using student-centred activities can be related to a strength-based framing of students, whereas in deficit-centred framing often a teacher-centred instruction is preferred (Scheiner, 2026). Although different Likert-based instruments on teachers' beliefs in error contexts already exist, these either only cover a single aspect of errors (e.g., error causes or a constructivist view on errors) or do not originate from a teaching context (e.g., employees). Therefore, this study aims to develop and evaluate a standardised instrument that comprehensively and operationally captures the various factors that influence teachers' beliefs regarding errors in mathematics teaching.

For this purpose, Likert-based items were deductively developed using existing items and related literature. The subscales cover a deficit- and a resource-oriented view on errors, beliefs on deficit- and strength-oriented error handling and beliefs on teaching-related and learner-related error causes. The items were reviewed by 9 experts, revised and tested in a pilot study (N = 100 pre-service teachers). The items' factorial structure was evaluated using exploratory factor analysis and reliability analysis.

In a first analysis the subscales *resource-oriented view on errors* (α = .766), *strength-oriented error handling* (α = .696) and *deficit-oriented error handling* (α = .767) showed satisfactory or sufficient reliabilities, partly after excluding single items. On the other hand, for the remaining three subscales only insufficient reliabilities were achieved or exploratory factor analysis revealed a deviating factor structure. These results indicate that some items already represent reliable subscales for assessing teacher beliefs in error situations, while others need further adjustments.

References

Scheiner, T. (2026). Transformative learning: noticing student strengths through frame shifting. *ZDM*, 58(1-2), 263–278. https://doi.org/10.1007/s11858-025-01715-x

Wuttke, E., & Seifried, J. (2017). Competence, Teacher Competence and Professional Error Competence: An Introduction. In E. Wuttke & J. Seifried (Eds.), *Professional Error Competence of Preservice Teachers* (pp. 1–14). Springer International Publishing. https://doi.org/10.1007/978-3-319-52649-2_1

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 485. PME.

USING CONTRADICTIONS TO ACCESS AND EXPAND STUDENTS' MATHEMATICAL THINKING IN WHOLE-CLASS DISCUSSIONS

Inger Eriksson[1], Marie Björk[2], Helena Eriksson[1], Gunilla Pettersson Berggren[2], Sanna Wettergren

[1] Stockholm University, [2]Stockholm City

The aim of this presentation is to exemplify and discuss how contradictions can be used as a didactical tool for accessing and expanding students' mathematical thinking in whole-class discussions. The study draws on learning activity theory and the El'konin–Davydov curriculum (Davydov, 2008). From this perspective, contradictions are understood not as individual errors, but as productive tensions that emerge within collective mathematical activity (see Radford, 2021). A contradiction arises when students' existing, often empirical, ways of reasoning are insufficient to resolve a mathematical problem, creating a need for a new way of understanding the object of learning. Such contradictions become visible through students' verbal contributions as well as through their embodied actions and interactions with mediating tools. For teachers, these moments provide opportunities to gain insight into students' current ways of thinking and to support the development of more conceptual ways of thinking.

The presentation is based on a design research study: Problem Situations, Learning Models and Teaching Strategies. Video-recorded data from collaboratively designed research lessons, focusing the positional system, were analysed with a focus on whole-class discussions in which contradictions surfaced in relation to central mathematical ideas. The analysis focused on how contradictions were expressed through students' talk and action with shared tools, as well as how teachers responded to and sustained these moments in instruction. The findings implicates that when contradictions are recognised and maintained in whole-class discussions, they can support collective exploration of underlying mathematical relationships. By posing provocative questions or juxtaposing different ideas teachers can use contradictions to make students' thinking visible and open for collective reflections in the classroom. From a didactical perspective, contradictions thus function not only as access points to students' thinking, but also as resources for creating a shared space for evaluating and developing mathematical ideas.

References

Davydov, V. V. (2008). *Problems of developmental instruction: A theoretical and experimental psychological study*. Nova Science Publishers. (Orig. work published 1986)

Radford, L. (2021). *The theory of objectification: A Vygotskian perspective on knowing and becoming in mathematics teaching and learning*. Brill.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 486. PME.

MOTIVATIONAL MECHANISMS SHAPING GENDERED PARTICIPATION IN MATHS-INTENSIVE FIELDS

Tanya Evans, Huayu Gao

University of Auckland

This presentation reports two conceptual replications examining how gender-segregated secondary schooling relates to students' mathematics achievement and participation in mathematics-intensive university pathways in New Zealand. New Zealand provides a distinctive context: 11.4% of students attend single-sex schools, and 83.6% of these are state or state-integrated (MoE, 2025), unlike the United States and Australia where single-gender schooling is more often private or Catholic. This setting allows a strong test of generalisability within a shared national system.

Study 1 replicates Docherty et al. (2020) using enrolment data from the University of Auckland (2016–2020), extending earlier work at the University of Canterbury. Logistic regression showed that girls from single-sex schools were overrepresented in Engineering: 42.0% of female Engineering students at Auckland had attended girls' schools, compared with ~33% in other programmes. The difference from Canterbury's 56.0% was small ($d = 0.35$), supporting the robustness of the association. No comparable pattern was observed for boys.

Study 2 extends Smith and Evans (2024) using TIMSS 2019 data and hierarchical linear modelling, framed by situated expectancy–value theory. Confidence positively predicted mathematics achievement across genders. Importance value showed no main effect once confidence was controlled; however, for girls only, school gender type moderated the value–achievement relation. At high levels of valuing mathematics, girls in single-sex schools outperformed their co-educational peers.

Together, the findings suggest that single-sex schooling in New Zealand is associated with girls' participation in mathematics-intensive pathways and, under specific motivational conditions, mathematics achievement, highlighting important contextual boundary conditions in gendered mathematics trajectories.

References

Docherty, P. D., Fox-Turnbull, W. H., Naswall, K., Homewood, A., Bradley, H., Zaka, P., & Chase, J. G. (2020). The contribution of gender segregated secondary education on the progression to engineering. *Australasian Journal of Engineering Education, 25*(1), 31-38.

MoE. (2025). *Education counts*. Retrieved from https://www.educationcounts.govt.nz/statistics/school-rolls

Smith, A., & Evans, T. (2024). Gender gap in STEM pathways: The role of gender-segregated schooling in mathematics and science performance. *New Z. J. of Ed. S., 59*(1), 269–287.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 487. PME.

MATHEMATICAL WELL-BEING AND STEM CAREERS INTEREST: AN EXPLORATORY PROFILE ANALYSIS

Haomin Fang[1], Qiaoping Zhang[2],Dan Lin[1]

[1]The Education University of Hong Kong, [2]The University of Hong Kong

While mathematics education is widely recognized as a critical foundation for students' engagement and success in STEM disciplines, students increasingly suffer from negative affective experiences, such as math anxiety and low engagement. Previous research on students' STEM aspirations has mainly focused on cognitive and motivational variables (Ferdinand et al., 2024), leaving students' holistic mathematical experiences underexplored. This study highlights the critical role of mathematical well-being (MWB), an emerging, multidimensional construct capturing students' subjective experiences of feelings and functioning in mathematics learning (Campbell, 2025). Specifically, this exploratory study investigates whether distinct MWB profiles are associated with variations in students' STEM career interest (STEM-CI). Drawing on a sample of 155 secondary school students in Hong Kong, the study employed a profile-based exploratory approach. MWB was assessed using a comprehensive 22-item scale (Zhang et al., 2025), alongside a validated STEM-CI survey (Kier et al., 2014). Participants were categorized into low, moderate, and high MWB profiles based on tertile distributions. A one-way ANOVA revealed a significant, gradient effect of MWB on STEM-CI: students in the high MWB group (M = 3.53) demonstrated significantly greater interest in STEM careers than those in the moderate (M = 2.96) and low (M = 2.81) MWB groups. Post Hoc comparisons confirmed significant differences between the high MWB group and the other two cohorts. These findings highlight the importance of multidimensional MWB in shaping students' STEM aspirations, which offering valuable insights for educators and policymakers aiming to broaden the STEM pipeline.

References

Campbell, T. (2025). Mathematical wellbeing: an emerging construct with exciting potential. *Mathematics Education Research Journal*.

Ferdinand, R., Malanchini, M., & Rimfeld, K. (2024). Mathematics interest, self-efficacy, and anxiety predict STEM career choice in emerging adulthood. *Npj Science of L.*, *9*(1).

Kier, M. W., Blanchard, M. R., Osborne, J. W., & Albert, J. L. (2014). The development of the STEM career interest survey (STEM-CIS). *Research in Science Education (Australasian Science Education Research Association)*, *44*(3), 461–481.

Zhang, Q., Fang, H., & Tang, X. (2025). Modeling mathematical well-being among undergraduates: An exploratory study in Mainland China. *International Journal of Educational Research Open*, *9*, 100507. https://doi.org/10.1016/j.ijedro.2025.100507

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 488. PME.

MIXED-METHODS ANALYSES IN PROBLEM SOLVING: IDENTIFYING STUDENTS' PROFILES

Zeina Fares

LDAR, Paris-Est Créteil; Paris Cité University

This oral communication presents preliminary findings on arithmetic problem solving and is based on a cohort study conducted in France during the transition from elementary to secondary school. It aims to identify students' profiles to be followed in a longitudinal study and to inform teacher professional development. The research question is: To what extent does a mixed-methods approach support the construction of students' profiles in arithmetic problem solving, and what convergence exists between qualitative and quantitative analyses in this process? This study draws on Vergnaud's Theory of Conceptual Fields (2009), Brousseau's Theory of Didactic Situations (1997), and categorizations of arithmetic problems, which guided both the design of the diagnostic test and the interpretation of students' responses, leading to the construction of students' profiles. A sequential mixed-methods design (QUAL → QUAN) was implemented (Bikner-Ahsbahs et al., 2015). The literature review (problem solving, profiles, mixed-methods), together with teachers' interviews and classroom observations, informed the design of a diagnostic test and an analytic grid, which were refined through a didactic a priori analysis and a pilot study (n = 74). The analytic grid combined quantitative and qualitative criteria (calculations, errors, reasoning, representations, etc.), used to code the data. The test was administered in January 2025 to 327 students (aged 10–12) across 13 teachers. Data were analysed both qualitatively (observations, interviews, and students' work) and quantitatively using R (R Project for Statistical Computing) through Multiple Correspondence Analysis (MCA) and Partitioning Around Medoids (PAM) clustering, and convergence between the two analyses was used to validate the students' profiles. Five distinct profiles were identified. This study shows that a mixed-methods approach produces robust and didactically meaningful students' profiles, with strong convergence between qualitative and quantitative results. In the cohort study, these profiles will be used to track students' development over time and to support the design of teaching resources. We will illustrate the method and the characteristics of the students' profiles using a sample problem and its analysis.

References

Bikner-Ahsbahs, A., Knipping, C., & Presmeg, N. (Eds.). (2015). *Approaches to qualitative research in mathematics education: Examples of methodology and methods*. Springer.

Brousseau, G. (2002). *Theory of didactical situations in mathematics* (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Eds. & Trans.). Kluwer Academic Publishers.

Vergnaud, G. (2009). The theory of conceptual fields. *Human Development*, *52*(2), 83–94.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 489. PME.

FOSTERING STUDENTS' QUANTITATIVE AND COVARIATIONAL REASONING IN FUNCTIONS USING BUILDING BLOCKS

Emine Fazli, Mesut Koşmaz, Saziye Tatli Uluşik, Merve Şevval Özkan, Gökhan Derin, Emin Aydin
Marmara University, Istanbul

Quantitative reasoning, particularly covariational reasoning, which involves coordinating how quantities change together, is central to understanding functions (Carlson et al., 2002). In this study, we report on a teaching experiment on students' quantitative and covariational reasoning using a building-block-based task. Our research question is: "In what ways do students engage in quantitative and covariational reasoning regarding functions while working with a physical building-block-based task?" We adopted a teaching experiment methodology with six low-achieving 11th-grade students. Data included activity sheets, observations, and audio recordings from an 80-minute session. The task involved a physical stage lighting model constructed from building blocks that students could manipulate by turning a handle to rotate gears and adjust a lamp's height. Data were analysed iteratively using episode-based analysis grounded in covariational reasoning frameworks (Carlson et al., 2002). We identified four recurring forms of reasoning in the first phase of the teaching experiment. First, students coordinated changes between quantities, identifying an inverse proportional relationship between gear teeth and rotations. Second, by measuring physical variations, students coordinated amounts of change and represented these relationships algebraically and graphically. In the third episode, students reasoned about rates of change. However, when attempting algebraic representations, they experienced conversion challenges and relied on teacher guidance to bridge conceptual reasoning with formal representations. Finally, moving beyond pairwise comparisons, students coordinated multiple quantities simultaneously, connecting rotations, height, and illuminated area to interpret illumination effectiveness. Overall, such tasks can provide opportunities for engaging students in quantitative and covariational reasoning, even among low-achieving students. The building-block-based design provided a shared manipulable representation that made the simultaneous variation of quantities visible and accessible for students. However, teacher guidance played a role in connecting intuitive to formal representations. In future iterations, we aim to refine task instructions to reduce teacher funnelling and promote more independent exploration, thereby strengthening the interpretive potential of the task designed with building blocks.

References

Carlson, M., Jacobs, S., Coe, T., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modelling dynamic events: A framework and a study. Journal for Research in Mathematics Education, 33(5), 352–378.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 490. PME.

EXPLORING THE CONSTRUCTION OF MATHEMATICAL MEANING THROUGH CORPUS-BASED LANGUAGE ACTIVITIES

Elisa Corino, Cecilia Fissore, Marina Marchisio Conte
University of Turin

This study builds on research that conceptualises language as a central resource for constructing mathematical meaning rather than merely expressing it (Radford, 2002; Planas et al., 2018). Linguistic choices can provide insight into how students articulate and stabilise mathematical meanings, especially when language is explicitly foregrounded in instruction. The study adopts a Data-Driven Learning approach, in which students engage in guided exploration of a corpus, understood as a structured collection of authentic texts (Corino et al., 2022). Our corpus consists of excerpts from secondary school textbooks related to the concept of function. Corpus-based activities are designed as language-focused tasks in which students identify recurring linguistic patterns and use them to complete mathematically meaningful statements, to foster reflection on mathematical meanings starting from linguistic forms. An eight-hour research activity was conducted with 38 Grade 10 students, combining (a) pen-and-paper tasks integrating procedural questions with explanation, and (b) corpus-based activities supported by automated formative feedback and collective discussion. A qualitative analysis focusing on recurring patterns where correct procedures coexisted with difficulties in linguistic articulation, and on lexical choices indicating conceptual uncertainty. The results reveal a recurrent asymmetry: while many students successfully perform procedures, they struggle to articulate relational meanings. Variation in lexical choices and non-responses indicates uncertainty in expressing relationships between mathematical objects. Corpus-based activities make these tensions more visible. Although students often recognise linguistic patterns, they encounter difficulties when using them to construct mathematically meaningful statements. The combination of immediate feedback, guided exploration, and collective discussion can create a space in which mathematical meanings are examined and progressively stabilised.

References

Corino, E., Fissore, C., & Marchisio, M. (2022). Data Driven Learning activities within a Digital Learning Environment to study the specialized language of Mathematics. In *2022 IEEE 46th Annual Computers, Software, and Applications Conference*, 167-176. IEEE.

Planas, N., Morgan, C., & Schütte, M. (2018). Mathematics education and language: Lessons and directions from two decades of research. *Developing res. in mathematics ed.*, 196-210.

Radford, L. (2002). Syntax and meaning. In M. J. Høines & A. B. Fuglestad (Eds.), *Proc. 28th Conf. of the Int. Group for the Psychology of Mathematics Education* (1), 161–166. Norway: Bergen University College.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 491. PME.

EXPERIMENTAL EVIDENCE FOR THE EFFECT OF TEACHER MATHEMATICS ANXIETY ON TEACHING BEHAVIOR

Madita Frühauf

Freie Universität Berlin, Germany

Primary school teachers are affected by mathematics anxiety (MA), which involves feelings of fear, apprehension, and tension when engaging with mathematics (Ashcraft, 2002) and is more common among female teachers (Artemenko, 2021). While correlational research suggests that teacher MA may transfer to teacher behavior in the classroom, experimental evidence remains absent. Drawing on interpersonal theory (Leary, 1957), this study conceptualized teacher behavior along two fundamental dimensions: warmth (responsiveness, communion) and directivity (guidance, agency), recognizing that both are critical for student outcomes. In a preregistered randomized experiment, 172 preservice primary school teachers (80% female, 70% aged 20-29 years) were asked to imagine teaching either a simple (teaching the area of rectangles; low-complexity condition) or a complex mathematical topic (teaching Euler's polyhedron formula; high-complexity condition) and to describe how warm and directive they would behave toward the class in either scenario. Given that anxiety involves preoccupation with anticipated threats and increased internal focus, it was hypothesized that MA would impair teachers' sensitive attunement to students, manifesting in reduced warmth. Further, to maintain control and to avoid revealing perceived limitations in their expertise, it was expected that MA would be associated with greater directivity, particularly among women who are faced with the negative stereotype about their group's mathematics-related capabilities. Manipulation checks confirmed significantly higher MA in the high-complexity condition ($F(1, 141) = 20.08$, $p = < .001$, $\eta^2 = .13$). Latent change score analysis revealed that preservice teachers exhibited significantly less warmth ($\beta = -.53$, $p = .021$) and, regardless of gender, more directivity ($\beta = .62$, $p < .001$) in the high-complexity condition relative to the low- complexity condition. These findings provide the first causal evidence that MA may influence interpersonal teaching behavior. They underscore the importance of addressing MA in teacher professional development.

References

Ashcraft, M. H. (2002). Math anxiety: Personal, educational, and cognitive consequences. *Current Directions in Psychological Science*, *11*(5), 181–185.

Artemenko, C., Masson, N., Georges, C., Nuerk, H.-C., & Cipora, K. (2021). Not all elementary school teachers are scared of math. *Journal of Numerical Cognition*, *7*(3), 275–294. https://doi.org/10.5964/jnc.6063

Leary, T. F. (1957). *Interpersonal diagnosis of personality*. Ronald Press.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 492. PME.

DISCURSIVE CHANGES IN A TEACHER'S NARRATIVES IN A NEWLY FORMED LESSON STUDY COMMUNITY

Yuka Funahashi[1], Shinichiro Matsumoto[2], Toshihito Sato[3]

[1]Nara University of Education, [2]Shizuoka University, [3]Iwate University

Lesson study is a fundamental system supporting teacher professional development in Japan (Fernandez & Yoshida, 2012). However, in lower secondary schools, subject specialization, declining student populations leading to single-teacher departments, and weakened district study groups limit opportunities for mathematics-focused lesson study. To address this issue, we established a cross-school lesson study community comprising mathematics teachers, university faculty, and representatives from the board of education. Within this project (Sato, 2025), the present study focuses on the first lesson study cycle and examines changes in the narratives of Mr. Fujita, who taught the research lesson, as he participated in the lesson study community.

This study adopts Sfard's (2008) commognitive framework, which conceptualizes learning as participation in discourse. From this perspective, professional development is analysed as discursive change, particularly as changes in the narratives through which teaching is structured and connected across lessons.

The community was launched on October 10, 2026. Following the kick-off, four additional online meetings were held to refine the lesson, resulting in approximately seven hours of discussion in total. Mr. Fujita taught the research lesson on spatial geometry for Grade 7 on February 17, 2027. Data consists of (a) transcripts of meetings and post-lesson discussions, and (b) four revised versions of Mr. Fujita's teaching plan. The analysis focused on identifying changes in his narratives across both spoken interactions and written planning.

Teachers from different schools used different words when discussing mathematics lessons. Clarifying the processes reified by these words became essential for constructing endorsed narratives within the community discourse. Through feedback from community members, Mr. Fujita became increasingly attentive to his word use, reflecting gradual discursive changes in how he framed his teaching.

References

Fernandez, C., & Yoshida, M. (2012). *Lesson study: A Japanese approach to improving mathematics teaching and learning*. Routledge.

Sato, T. (2025). Formation of a "lesson study community" for inquiry-based learning: A framework for inquiry-oriented lessons and its functions [in Japanese]. *Proc. of the 13th Spring Research Conference of the Japan Society of Mathematical Education*, 333–334.

Sfard, A. (2008). *Thinking as communicating*. Cambridge University Press.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 493. PME.

USING BIBLIOTHERAPY TO RELIEVE MATHEMATICS ANXIETY IN COLLEGE STUDENTS

Maria Fung, Alivia Glynn

Worcester State University

Mathematics anxiety is widely recognized as a consistent detriment to mathematics learning and performance (Luttenberger et al, 2018). Its presentations through avoidance, dissociation, memory issues, and panic appear to be very pronounced in developmental mathematics students and those taking general education college mathematics courses. Bibliotherapy is a personal growth tool that involves reading and responding to stories. It has four components that ensure its efficacy—relatability, normalization, modelling and reframing (EdTech Books, n.d.).

In this mixed methods study, we consider the research question: can a bibliotherapy story intervention about overcoming mathematics struggle (and specifically written for each type of course) aid in reducing mathematics anxiety for students in developmental algebra, mathematics for liberal arts, and algebra for elementary school teachers.

The AMAS (Worden, 2023) nine-question survey was administered as a pre- and post-instrument to a sample of 33 developmental students (17 matched), 27 liberal arts students (15 matched) and 14 pre-service teachers (12 matched). Students answered the same questions about their views and personal experiences with mathematics. Several weeks into the semester, students listened to and responded to their course's specific 3-minute bibliotherapy story, by reflecting on how the story was resonant, relevant, and able to provide them with a possible map to success in mathematics.

We saw a statistically significant decrease in the matched mean AMAS score across all three types of students, with the developmental group showing the largest decrease ($p=0.006$). 42.8% of students associated mathematics with anxiety, stress and confusion at the beginning, versus only 17.2% on the post-survey. There was a significant shift in seeing mathematics success as a process of perseverance, patience, and growth. We recommend further investigation into bibliotherapy's effects, as well as its use as a significant tool for reducing math anxiety among college students.

References

EdTech books. (n.d.). https://edtechbooks.org/addressing_wellbeing/bibliotherapy

Luttenberger, S., Wimmer, S., & Paechter, M (2018). Spotlight on math anxiety. *Psychology Research and Behavioral Management, 11*, 311-312.

Worden, E. (2023). An update on the abbreviated math anxiety scale: current evidence of Reliability. *Master's Theses.* 269. https://digscholarship.unco.edu/theses/269

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 494. PME.

THE IMPACT OF TEACHING THROUGH PROBLEM-SOLVING ON STUDENTS' ATTITUDES TOWARD MATHEMATICS

Éva Fülöp

Department of Pedagogical, Curricular and Professional Studies, University of Gothenburg

Teaching Through Problem-solving (Takahashi, 2021) situates a carefully designed problem as the starting point for instruction, through which targeted mathematical concepts emerge. Student-generated solutions, including partial responses, are deliberately selected and sequenced to surface key structures and relationships. Whole-class discussion is structured to compare strategies and progressively formalize mathematical ideas while maintaining high cognitive demand. Rather than adding problems to instruction, the problem itself functions as the driving engine of mathematical learning. In enacting this approach, the teacher in this study also incorporated selected practices aimed at enhancing student engagement and thinking, such as the use of vertical non-permanent surfaces to support collaborative problem-solving (Liljedahl, 2021).

This study is part of a larger project and focuses on how teaching optimization using derivatives through a Teaching Through Problem-solving approach in two upper secondary classes (N=27) influence students' attitudes toward mathematics. Measuring mathematical attitudes can be performed at different levels and through various instruments. This study used the widely recognized Attitudes Toward Mathematics Inventory (Lim & Chapman, 2013), including its short form, which was revised and adapted for Swedish upper secondary students. We aimed to create a version suitable for students who struggle in mathematics and show low motivation, administering it as a pre- and post-test to capture potential changes in attitudes. Preliminary results across four analytical categories show varying patterns of change. In the category *The tendency to participate in or avoid mathematics activities*, positively worded items showed a slight increase in looking forward to lessons and a small decrease in finding challenges appealing. Negatively worded items revealed clearer shifts, including reduced reluctance to answer questions and less nervousness about solving problems. These results will be discussed and illustrated further in the presentation.

References

Lim, S. Y., & Chapman, E. (2013). Development of a short form of the Attitudes Toward Mathematics Inventory. *Educational Studies in Mathematics*, 82, 145-164.

Liljedahl, P. (2021). *Building thinking classrooms*. Corwin Press.

Takahashi, A. (2021). *Teaching Mathematics Through Problem-Solving: A Pedagogical Approach from Japan* (1st ed.). Routledge. https://doi.org/10.4324/9781003015475

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 495. PME.

THE OTHER WHEN INTERPRETING UNDERSTANDING IN MATHEMATICS: A PHILOSOPHICAL VIEW

Jesús Gallardo, Verónica A. Quintanilla
University of Málaga, Spain

Interpreting students' mathematical understanding on the basis of their shared classroom activity remains an ongoing challenge, one that also involves specific aspects related to otherness (Abtahi et al., 2018). Among these are issues concerning students' intentionality and responsibility towards others, as well as the transformative effect of access to the understanding of the other (Guillemette & Dematte, 2022; Metz & Simmt, 2015). As a theoretical contribution, we examine the role of the other in interpreting mathematical understanding, drawing on Levinas's phenomenology of sociability and Ricœur's dialogical hermeneutics. Specifically, we broaden the ontological significance of understanding—as an internal cognitive phenomenon—and incorporate the responsibility for the other in the functional approach to interpreting understanding, defined in terms of the uses made of mathematical knowledge. We recognise the existence of a meaning inherent in understanding that is prior to—and irreducible to—intentionality and rational knowledge, and that points directly to otherness: understanding emerges when the self is called into question by the critical presence of the other, who continually addresses and concerns me. Thus, we propose an interpretation that provides a shared context appropriate to the call of the other, in which dialogue elicits responsibility, understood as the student's selfless willingness to approach the other and offer different responses to otherness—distancing, representation, and consent. This (ethical) responsibility enables, through dialogical consent with the other, the transformation that leads to an inclusive return to the student's mathematical understanding. Our interpretative proposal complements phenomenological methods of guided introspection developed for empathic access to students' mathematical experiences (Metz & Simmt, 2015). Moreover, it establishes integrative links between hermeneutics and ethics (Guillemette & Dematte, 2022) within the specific context of understanding in mathematics.

References

Abtahi, Y., Adler, J., Guillemette, D., Herheim, R., Lerman, S., Maheux, J.-F., & Valero, P. (2018). Otherness in mathematics education. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proc. 42nd Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 1, pp. 95-123). PME.

Guillemette, D., & Dematte, A. (2022). A discussion on Levinas's ethics of otherness, originals from history and teaching-learning mathematics. *For the Learning of Mathematics, 42*(2), 22-28.

Metz, M. L., & Simmt, E. S. M. (2015). Researching mathematical experience from the perspective of an empathic second-person observer. *ZDM, 47*(2), 197-209.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 496. PME.

EXPLORING MATHEMATICAL CONNECTIONS IN ARGUMENTATION WITH A RATIONAL-QUESTIONING GPT

Oğuzhan Gençaslan[1,2], Selin Urhan[2], Selen Galiç[3], Şenol Dost[2]

[1]Kilis 7 Aralık University, [2]Hacettepe University, [3]Johannes Kepler University

Although National Council of Teachers of Mathematics (2024) stresses the need to both develop and test generative artificial intelligence (GenAI) tools, current research predominantly focuses on testing them. The motivation of this study is based on the idea that rational questioning (Zhuang & Conner, 2022) expected of teachers during argumentation can also be provided to students through a GPT specifically developed for this purpose. This study aimed to develop and test a customized GPT for rational questioning, addressing the research question: How does such a GPT affect students' mathematical connections? Data were collected using a pattern generalization task designed to investigate mathematical connections involving participants' numerical understanding of the derivative concept. The study involved a university student in a mathematics education programme who had sufficient mathematical background to complete the task and experience using GenAI tools. The participant first worked individually and was then given the opportunity to use the customized GPT to complete or refine the solution. Data were analysed using Habermas' (1998) theory of rationality and García-García and Dolores-Flores' (2021) mathematical connections framework. Findings indicate that the customized GPT supported the participant in making correct mathematical connections but exhibited limitations in sustaining rational questioning, as its performance was influenced by the participant's questions. Future studies should explore the sustainability of GPT's questioning and test it with varied tasks and diverse participants.

Acknowledgment: This work was supported by The Scientific and Technological Research Council of Türkiye (TÜBİTAK) under the 2224-A Grant Program.

References

García-García, J., & Dolores-Flores, C. (2021). Exploring pre-university students' mathematical connections when solving Calculus application problems. *International Journal of Mathematical Education in Science and Technology*, *52*(6), 912–936.

Habermas, J. (1998). *On the Pragmatics of Communication*. The MIT Press.

National Council of Teachers of Mathematics (2024). *Artificial intelligence and mathematics teaching: A position of the National Council of Teachers of Mathematics.* https://www.nctm.org/standards-and-positions/Position-Statements/Artificial-Intelligence-and-Mathematics-Teaching/

Zhuang, Y., & Conner, A. (2022). Teachers' use of rational questioning strategies to promote student participation in collective argumentation. *Ed. St. in Mathematics, 111*(2), 345–365.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 497. PME.

HOW DO PRE-SERVICE EARLY CHILDHOOD TEACHERS ENVISION THE TEACHING OF MATHEMATICS?

Camilo Vergara-Sandoval[1,4], David Maximiliano Gómez[2,4,5], José Cantó-Doménech[3]

[1]U. de Santiago de Chile. [2]U. de O'Higgins, [3]U. de València,
[4]Núcleo Milenio MEMAT, [5]Centro de Investigación Avanzada en Educación CIAE

Early childhood education is critical for the development of mathematics skills, but research on the knowledge and conceptions of early childhood education (ECE) teachers about mathematics teaching and learning is scarce. Building upon Alsina and Delgado-Rebolledo's (2022) model of the knowledge of the ECE teacher to teach mathematics, we investigated how do pre-service ECE teachers envision the teaching and learning of mathematics in a Kindergarten classroom. In particular, we aimed to uncover their conceptions about what mathematics are taught in this educational level, and how. To do this, we asked 18 fourth-year pre-service ECE teachers from a Chilean public university to *draw a Kindergarten classroom in which the Mathematical Thinking Nucleus of the Chilean ECE curriculum is being developed*. Participants' drawings were analyzed using a filter integration method (Makramalla, 2018), considering a series of questions tapping on aspects such as: What mathematical concepts are evident in the classroom? How are these concepts presented? What objects/materials are in the classroom? Where is the class taking place (indoor/outdoor)? What are children expected to do? The answers to these questions allowed us to characterize the classrooms drawn. The analysis was done using Atlas.ti, double-coded by two of the authors and solving disagreements through discussion. The coding process led us to infer a gradient of five conceptions related to mathematics and its teaching and learning in ECE: classrooms focused on 1) numbers and arithmetic presented in a traditional format; 2) the names of numbers, figures, and objects, allowing for their manipulation; 3) the association between numbers and their related quantities, using concrete materials; 4) problem solving through guided play; and 5) the identification and generalization of visual patterns. We will discuss what kind of mathematical knowledge and how to teach it emerges in each of these classrooms, as well as the role of the teacher within them.

Acknowledgements. This research was supported by the Chilean Agency of Research and Development, through the funds Fondecyt 3250855, Milenio NCS2021_014, and CIAE CIA250005.

References

Alsina, Á. & Delgado-Rebolledo, R. (2022). ¿Qué conocimientos necesita el profesorado de Educación Infantil para enseñar matemáticas? *Matemáticas, Ed. y Sociedad*, 5(1), 18-37.

Makramalla, M. (2018). *Culture contextualisation of mathematics instruction. A draw a scientist test (DAST) analysis*. Unpublished MA thesis: The American University in Cairo.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 498. PME.

PRESERVICE TEACHERS' FLEXIBILITY IN NON-ROUTINE PROBLEM SOLVING

Isabelle Gretzschel

Martin-Luther-University Halle-Wittenberg

Solving non-routine problems often requires modifying or switching strategies, as initial approaches may not lead to a solution. This ability is referred to as flexibility in problem solving. Previous research has examined flexibility within and across problems differing markedly in surface features and structure, primarily focusing on students (e.g., Elia et al., 2009). Yet, non-routine problems in which the need for a change in strategy is not immediately apparent due to superficial similarities and subtle structural differences have received little attention in flexibility research. Teachers, as students' role models, are expected to solve such problems flexibly. However, it remains unclear how pre- and in-service teachers deal with non-routine problems that are superficially similar yet non-isomorphic, thereby requiring flexibility.

Therefore, this qualitative exploratory study aims to examine elementary school preservice teachers' strategy use and flexibility using a design based on such problems. A total of 48 participants worked individually on one of three series of systematically varied non-routine mathematical problems in videotaped semi-structured think-aloud interviews. Each series consisted of three problems that shared surface features (i.e., context, numerical values) and varied structurally in subtle ways, potentially obscuring these differences. Data were analyzed using a process-oriented qualitative content analysis focusing on participants' use, modification, and switching of strategies.

Overall, only a minority of participants exhibited flexibility exclusively across the problems of a series, primarily through modifying strategies. In many cases, flexibility emerged only after unsuccessful attempts, often by switching to a trial-and-error strategy. Furthermore, participants rarely engaged in further attempts once a solution had been reached, indicating limited flexible exploration of alternative strategies.

The findings reveal that series of problems with subtle structural variations can make preservice teachers' levels of strategic flexibility in problem solving visible in ways that might otherwise remain less apparent. In teacher education, flexibility should therefore be explicitly addressed, particularly by supporting preservice teachers in reflecting on their strategy use and their responses to unsuccessful attempts, fostering modifications of strategies, and considering further attempts beyond initial solutions.

References

Elia, I., van den Heuvel-Panhuizen, M., & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in non-routine problem solving by primary school high achievers in mathematics. *ZDM Mathematics Education*, *41*(5), 605–618.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 499. PME.

TEACHERS' ENACTMENT OF ASSESSMENT RUBRICS

Chiara Grisendi

University of Pisa

This study stems from an ongoing research project focusing on the transition from school to university, with particular attention to students' development of mathematical competencies. In preparing students for this transition, we foreground helping them grow toward independent and responsible learners. To support students in this objective, we design evidence-seeking activities with criterion-referenced rubrics—focusing on two mathematical competencies: "argumentation" and "representing and communicating"—and teachers' feedback throughout. Current research has examined the positive effects of rubrics with evidence-seeking activities (Nieminen et al., 2021) on students' learning. To meaningfully engage in evidence-seeking, teachers' own ways of identifying and articulating evidence become crucial, as they implicitly legitimate evidence in the discipline. Research in this field typically does not analyse how teachers enact these practices; this study addresses that gap. The analysis of teachers' use of rubrics is framed within the Documentational Approach to Didactics (Trouche et al., 2018), which studies how teachers enact—that is, appropriate and adapt—resources designed by others. On this basis, the research question is (RQ):

How do teachers enact the designed rubrics in their assessment practices as outlined by the kinds of evidence they identify and articulate?

Data came from a one-hour video-recorded session in which three teachers watched together three videos of student group presentations from a prior classroom implementation, assessed them using the rubrics, and discussed the evidence for their assessments. To address the RQ, we analysed the transcript through a stepwise process: examining how teachers appropriated rubric language, what they treated as evidence for their judgments, and which elements they adapted autonomously.

The findings provide insight into teachers' documentational work as a dynamic interplay of appropriation and adaptation, constituting an initial exploration of teachers' enactment of rubric-based assessment. As future work, the activity will be implemented in classroom settings.

References

Nieminen, J. H., Asikainen, H., & Rämö, J. (2019). Promoting deep approach to learning and self-efficacy by changing the purpose of self-assessment: a comparison of summative and formative models. *Studies in Higher Education, 46*(7), 1296–1311.

Trouche, L., Gueudet, G., Pepin, B. (2018). Documentational Approach to Didactics. In Lerman, S. (Eds.), Encyclopedia of Mathematics Education (pp. 237–247). Springer. https://doi.org/10.1007/978-3-319-77487-9_100011-1

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 500. PME.

REALITY AND GEOMETRY IN THE CURRICULUM

Douglas Ribeiro Guimarães, Rúbia Barcelos Amaral

São Paulo State University (UNESP)

This study builds on research in curriculum theory and Mathematics Education (Shimizu & Vithal, 2024) that examines the relationship between school knowledge and reality, particularly in Geometry. Previous studies highlight that Geometry can become more meaningful when connected to students' lived experiences, contributing, for instance, to their understanding of space and geometric objects (Kageyama, 2018). At the same time, critical perspectives on reality, such as those proposed by Kosík (1976), warn against reducing reality to its immediate appearance, while Freire (2000) emphasizes the need for education to foster critical consciousness and transformation. Within this framework, Geometry is understood not only as a tool for describing the physical world but also as a means of interpreting and critically engaging with it.

The study presents results based on a qualitative documentary analysis of the Brazilian curricular document Base Nacional Comum Curricular – BNCC (Brasil, 2018). Focusing on the Mathematics section, the analysis examined how reality is mobilized in relation to Geometry across different stages of schooling.

The results indicate that the BNCC consistently establishes connections between Geometry and reality, particularly through spatial experiences such as observation, location, and recognition of shapes in the environment. However, the analysis also reveals that the relationship between reality and Geometry is predominantly framed in functional terms. From a critical perspective, this may limit the potential of Geometry to promote deeper understandings of reality, as the social, cultural, and historical dimensions of real-world contexts are not always made explicit.

References

Brasil. (2018). *Base Nacional Comum Curricular* [National Common Core Curriculum]. Ministério da Educação.

Freire, P. (2000). *Pedagogy of the oppressed*. Continuum.

Kageyama, K. (2018). Criteria for knowing a geometrical object: The enactivist perspective. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proc. of the 42nd Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 3, pp. 187-194). PME.

Kosík, K. (1976). *Dialética do concreto* [Dialectics of the concrete]. Paz e Terra.

Shimizu, Y., & Vithal, R. (2024). School mathematics curriculum reforms: Widespread practice but under-researched in mathematics education. In Y. Shimizu & R. Vithal (Eds.), *Mathematics curriculum reforms around the world: The 24th ICMI Study* (pp. 3-21). ICMI.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 501. PME.

INDICATIONS OF SUBSTITUTE-RELATIONAL CONCEPTION IN YOUNG STUDENTS' TALK ABOUT THE EQUAL SIGN

Robert Gunnarsson[1], Emma Persson[2]

[1]School of Education and Communication, Jönköping University
[2]Handskerydsskolan, Nässjö Kommun

For many years, research on students' understanding of the equals sign (see, e.g., Kieran, 1981) focused on the operational (equal sign as a trigger of "to do" something) versus the relational (the equal sign describes the relation between numbers) aspect of the symbol. The conclusion has been that students need to embrace the relational understanding to manage algebraic expressions, particularly equations. In more recent research studies, students' understanding of the relational aspect of the equals sign has been more nuanced. For instance, Jones et al. (2013) discuss the relational conception in terms of a sameness-relational conception or a substitute-relational conception, and advocate making students aware of the substitute-relational conception when teaching the concept of the equal sign. Their study was made on students in the first years of secondary school (aged 11-12 years). However, the equal sign is typically taught to students from very young age. Our question therefore is: Can students, already in the early school years have the substitute-relational conception of the equal sign?

From a previous project (Gunnarsson & Persson, 2024), we have data with 12 students in grades 1-3 (7-10 years old) when they talk about the equal sign. The students were interviewed in pairs and shown different types of expressions with equal signs and discussed these. Hence, triggered by the study of Jones et al. (2013), our data was re-analyzed, now focusing on the different relational aspects of the equal sign by using codes from Jones et al. (*ibid*.).

In this short presentation, the result is described in brief, concentrating on how the young students' talk includes utterances about the different relational aspects of the equal sign. Many of the students give utterances that can be categorized as a sign of a relational conception of the equal sign. At least some of the students give utterances that can be indications of a substitute-relational conception.

References

Gunnarsson, R. & Persson, E. (2024). Om likhetstecknet som "blir" eller "är". [About the equal sign as "becomes" or "are"]. Presented at Madif 14, the fourteenth conference of the Swedish Society for Research in Mathematics Education. Örebro, March 19-20, 2024.

Jones, I., Matthew, I., Gilmore, C. and Evans, R. (2013). Teaching the substitutive conception of the equals sign. *Research in Mathematics Education*, *15*(1), 34-39.

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics*, *12*(3), 317-326.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 502. PME.

BRIDGING INDIVIDUAL DIFFERENCES IN MATHEMATICS: DESIGN AND EFFICACY OF INTELLIGENT PERSONALIZED HOMEWORK BASED ON COGNITIVE DIAGNOSIS

Jiahui Guo[1], Haonan Cheng[2], Na Li[1]

[1]School of Mathematics and Statistics, Central China Normal University

[2]Xinyu No.4 Middle School

As uniform homework overlooks individual cognitive differences, leveraging intelligent technologies for personalized design has become an essential reform. Specifically, empirical findings indicate that cognitive diagnosis models (CDMs) serve as the foundation for designing effective personalized interventions (Wu et al., 2022). However, since most existing studies are confined to constructing theoretical frameworks or macro-implementation strategies, systematic empirical evidence regarding their effectiveness in authentic teaching contexts remains scarce.

This research aims to empirically investigate the efficacy of personalized homework underpinned by intelligent cognitive diagnosis in providing precise learning support. Consequently, the study selects “Quadratic Functions” as the focal case, specifically because its abstract complexity often precipitates cognitive gaps, rendering uniform standardized homework ill-suited for students with varying cognitive needs. To accurately diagnose mastery, a cognitive diagnostic assessment was developed, the results of which directly informed the design of personalized homework assignments.

The sample comprises 59 Grade 9 students from two classes. An independent Kruskal-Wallis test confirmed baseline equivalence in pre-test performance. Subsequently, the classes were assigned to experimental condition (personalized homework) and a control condition (standard assignments) for a six-week intervention, followed by a post-test. The findings reveal that the experimental group achieved a superior cognitive progression rate (46.66% vs. 31.03%). Furthermore, they demonstrated a positive academic trajectory (Cohen’s d=0.35) contrasting with the control group’s decline, validating the intervention’s efficacy. Moreover, the long-term impact of diagnostic-based homework warrants longitudinal investigation, given the importance of sustained cognitive development. The presentation will further detail specific implementation strategies and pedagogical implications.

References

Wu, X., Zhang, Y., Wu, R., & Chang, H.-H. (2022). A comparative study on cognitive diagnostic assessment of mathematical key competencies and learning trajectories. *Current Psychology*, *41*(11), 7854 – 7866.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 503. PME.

COMPARING GRADE 5 MATHEMATICS FROM TÜRKİYE AND THE USA THROUGH A SYNTHESIZED MATHEMATICS LEARNING TAXONOMY

Özge Gün[1], Michael J. Bossé[2]

[1]Bartin University, [2]Appalachian State University

This study employs Gun and Bossé's (2025) Synthesized Mathematics Learning Taxonomy (ST) as its framework. The ST distinguishes between Knowledge Level (mathematical task (action) and mathematical learning) and Cognitive Level (mathematical processes (actor) and problem solving) taxonomies. The ST recognizes the levels of mathematical understanding: (1) Knowledge and Routine Procedures; (2) Conceptual Understanding and Relational Thinking; (3) Strategic Problem-Solving; (4) Analytical and Extended Abstract Thinking; (5) Real-World Application and Synthesis; and (6) Creative Problem-Solving and Innovation. Our research question is: How do the ST levels of 5th-grade mathematics learning outcomes under the Türkiye Century Maarif Model (TCMM2024) compare with the standards of North Carolina (NC2025)? From recent studies, gaps in the literature remain: most studies have analyzed curricula through at most one learning-level lens; no studies have yet compared NC2025 with TCMM2024; and few studies focus in detail on a single grade comparison. The ST addresses all three gaps simultaneously. Two researchers employed document analysis.

The findings are distilled into two themes: (a) considering the ST levels by outcomes and standards, (b) ST levels by themes and domains, and (c) a comparative analysis. (Data and results are hyperlinked here.) This study found that while TCMM2024's learning outcomes were heavier in ST Levels 2 and 4, NC2025's standards focused on Levels 2 and 3. TCMM2024 saw its highest share of Level 4 outcomes in the Algebraic Thinking with Operations, Numbers and Quantities, and Geometric Shapes. NC2025 demonstrated a relatively smaller set of Level 4 standards, with no Level 5 standards. Little difference was found in overall rigor. TCMM2024 more often anticipated students' abstract thinking, whereas NC2025 focused more on students' solving modeled word problems. This all leads to two implications: First, in TCMM2024, researchers can examine how learners transition from relational explanations (Level 2) to generalized propositions (Level 4). Second, in NC2025, researchers can study how learners' strategy selection, representation shifts, and reasonableness checks stabilize over time (Level 3) and when structural justification emerges (Level 4).

References

Gun, O., & Bossé, M. J. (2025). Synthesizing cognitive mathematics learning taxonomies. *Thinking Skills and Creativity, 57*, 101796. https://doi.org/10.1016/j.tsc.2025.101796

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 504. PME.

EXAMINING TEACHERS' DESIGN OF MODEL-ELICITING TASKS IN VIRTUAL FIELD TRIP CONTEXTS

Gizem Güzeller Sarıgöllü

TED University

Virtual Field Trips (VFTs) are often positioned as powerful learning environments that enhance students' engagement and sense of presence (Dede, 2009). Their interactive structures support active learning processes by enabling learners to investigate, respond, and make decisions within contextualized environments (Freeman et al., 2014). Grounded in place-based learning principles, such environments offer opportunities to situate mathematical thinking within socially and spatially meaningful contexts (Asakle, 2014). However, contextual immersion alone does not ensure mathematical sense-making. The Models-and-Modeling Perspective (MMP) (Lesh & Doerr, 2003; Lesh & Lehrer, 2003) offers a framework for conceptualizing modelling as the development of explicit, shareable, and revisable mathematical systems. From this perspective, learning is not the production of answers but the construction and refinement of models through iterative cycles of expression, testing, and revision (Lesh, 2006). Model-Eliciting Activities (MEAs), grounded in six interrelated design principles (Lesh et al., 2000), operationalize this view by requiring tasks to support model construction, documentation, self-assessment, shareability, reusability, and grounding in realistic contexts. This study investigates: (1) How do secondary mathematics teachers design ThingLink-based VFT tasks when aiming to integrate mathematical modelling? (2) To what extent and in what ways do these designs align with MEA design principles?

Adopting a qualitative design within a modelling-oriented professional development program, the study involves 12 secondary mathematics teachers selected through purposive sampling based on sustained engagement in the design process. Teachers were invited to design place-based VFT tasks using ThingLink, integrating mathematical modelling into digitally represented spatial environments. Data sources include teacher-designed task artefacts, embedded prompts within ThingLink environments, and written design rationales. Using the six MEA principles as analytic lenses, the study examines whether teacher-designed VFT tasks support model construction, documentation, shareability, reusability, and iterative refinement.

References

Asakle, S., Barak, M. Location-based learning and its effect on students' understanding of Newton's laws of motion. *J Sci Educ Technol 31,* 403–413 (2022).

Dede, C. (2009). Immersive interfaces for engagement and learning. *Science, 323*(5910), 66–69.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 505. PME.

FROM THE MATHEMATICAL OUT-OF-SCHOOL LAB TO THE CLASSROOM – EXPLORING THE UPTAKE OF CONTENT IN SCHOOL MATHEMATICS LESSONS

Ramona Hagenkötter[1], Kim Fenrich[2], Katrin Rolka[2]

[1]University of Duisburg-Essen, [2]Ruhr-University Bochum

Previous research on out-of-school learning particularly emphasises the importance of meaningfully linking outer-school learning, such as learning in out-of-school labs (OSLs), and inner-school learning to achieve long-term effects (e. g., Haidari et al., 2024). Against this background, we explore how the content of a mathematical OSL project is taken up in school mathematics lessons (research objective 1) and to what extent students are able to connect their experiences made during the OSL project with the classroom content (research objective 2). For this purpose, we used a qualitative approach and analysed data from the larger research project "MerLab" (Mathematik erleben im Lehr-Lern-Labor; experiencing mathematics in an OSL), funded by the Reinhard Frank-Stiftung. The data analysed here were obtained from written classroom observations of two 90-minute mathematics lessons of a 6th grade class and semi-structured interviews with six students in pairs from that class (four female, two male; M_{age} = 10.83, SD = 0.41) after participating in a day-long mathematics OSL project entitled "Discovering symmetries in a game and sports museum" at a large German university. The OSL project aimed at enabling students to engage with different types of symmetries by working in small groups on various hands-on tasks and with digital elements focusing on activities of constructing and analysing symmetrical figures as well as verbalizing symmetry properties. To analyse the data, we used thematic analysis (Braun & Clarke, 2006). With regard to research objective 1, we observed that the mathematics teacher explicitly took up selected assignments and excerpts from the lab booklet the students used during the OSL project in her mathematics lessons. Furthermore, she implicitly drew on the vocabulary introduced during the OSL project. Regarding research objective 2, we observed that the students primarily remembered the exploration of different figures for symmetry during the OSL project. In addition, two of the interviewed student pairs pointed out that they had reused an excerpt from the lab booklet in the school mathematics lessons.

References

Braun, V. & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*, 77-101. https://doi.org/10.1191/1478088706qp063oa

Haidari, S. M., Karakus, F. & Kanadli, S. (2024). Short and long-term effects of out-of-school learning activities on student achievement: A mixed-research synthesis. *Educational Research Review, 44*, 100628. https://doi.org/10.1016/j.edurev.2024.100628

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 506. PME.

AN OPENNESS SPECTRUM IN MATHEMATICAL MODELLING

Hadas Handelman, Ortal Nitzan-Tamar, Zehavit Kohen

Technion - Israel Institute of Technology

Mathematical modelling is often treated as inherently "open", as it typically involves making assumptions, selecting variables and representations, and iteratively interpreting and validating results (Stender & Kaiser, 2015). Although modelling literature distinguishes openness of the initial state, goal state, and transformation (Wiehe et al., 2025), modelling tasks are often defined in binary terms, i.e. as open or closed. This raises the question of whether the theoretical framework can be expanded to conceptualize degrees and types of openness along a spectrum that explicitly accommodates less-open modelling tasks. The current study raises the research question of whether and how a modelling task can be defined and coded along an openness spectrum?

The study follows an iterative design-based process combining construct synthesis, rubric construction, reliability testing, and validation, through task coding. Openness indicators were first derived by synthesizing modelling and open-task literature (e.g., Wiehe et al., 2025). They were then tested through independent coding of a corpus of 10 modelling tasks sampled from literature (e.g., Wess et al., 2021) using a designed rubric. Inter-rater reliability was examined, and descriptors that generated systematic disagreement were revised, enabling comparisons across task sets while preserving interpretability. Analyses of the tasks based on the designed rubric revealed multiple openness levels within tasks used in existing research, as well as different combinations of task characteristics that converge to the same overall openness degree. The study contributes theoretically by reframing openness in modelling as a spectrum that includes less-open and closed-ended modelling tasks; methodologically, by developing a rubric that yields an openness profile and overall quantitative degree of openness; and practically, by enabling clearer task design and comparison through distinguishing openness from guidance and scaffolding.

References

Stender, P., & Kaiser, G. (2015). Scaffolding in complex modelling situations. *ZDM*, 47(7), 1255–1267.

Wess, R., Klock, H., Siller, H. S., & Greefrath, G. (2021). Mathematical modelling. In *Measuring Professional Competence for the Teaching of Mathematical Modelling: A Test Instrument* (pp. 3-20). Cham: Springer International Publishing.

Wiehe, K., Schukajlow, S., Krawitz, J., & Rakoczy, K. (2025). Openness in mathematical modelling: Do experiences of competence and autonomy mediate the effects of an intervention on modelling problems on task values and cost? *ZDM*, 57(2–3), 519–534.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 507. PME.

CONCEPTUALIZING PHASES OF JUNIOR HIGH SCHOOL STUDENTS' PROGRAMMING-BASED MATHEMATICAL PROBLEM-SOLVING

Quanrong He[1], Yufeng Guo[1,2]

[1]School of Mathematical Sciences, Beijing Normal University

[2]National Research Institute for Mathematics Teaching Materials

The integration of programming into mathematics classes creates new opportunities for fostering both computational thinking (CT) and mathematical thinking (MT) in embodied digital environments. As CT represents a problem-solving methodology (Wing, 2006), it shares cognitive connections with mathematical problem-solving. However, prior studies have either focused on general problem-solving processes (Pólya, 1945) or universal CT practices (Shute et al., 2017), neglecting the specific trajectory of mathematical problem-solving in programming contexts. This study views programming as an embodied digital practice, aiming to characterize the cognitive trajectories of programming-based mathematical problem-solving.

The research questions are: (1) What distinct cognitive phases and transition paths emerge in embodied programming-based mathematical problem-solving? (2) How do CT and MT function and interact across the phases? Data were collected from a 16-lesson mathematics course at a Chinese junior high school, including 35 students' programming artifacts, recordings of coding processes and discussion transcripts.

Episodic segmentation and thematic analysis identified three core phases encompassing six sub-processes, which accounted for over 95% of students observed problem-solving behaviors. The mathematization phase includes symbolic abstraction and relational abstraction, dominated by the abstract nature of MT. The algorithmization phase involves module decomposition and logical sequence design, guided by MT-CT synergy. The automation phase focuses on execution validation and system optimization, driven by CT's practical dimension. State-transition analysis further revealed that 42% of the problem-solving processes followed linear paths, while complex problem-solving exhibited non-linear recursive loops (58%). These findings demonstrate how programming serves as an embodied cognitive mediator to support the meaningful integration of computational and mathematical thinking.

References

Pólya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton Uni press.

Shute, V. J., Sun, C., & Asbell-Clarke, J. (2017). Demystifying computational thinking. *Educational Research Review, 22*, 142–158.

Wing, J. M. (2006). Computational thinking. *Communications of the ACM*, *49*(3), 33–35.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 508. PME.

NOTICING THE SERVICEABILITY OF STUDENT WORK IN A SIMULATION OF PROBLEM-CENTERED INSTRUCTION

Patricio Herbst, Soobin Jeon, Amanda Brown

University of Michigan

We contribute to research on teacher learning from digital simulations. Problem-based instruction (PBI) uses problems as instruments to launch students into activity leading to new mathematical ideas. In PBI, teachers must notice whether student work has features serviceable for progressing toward instructional goals, though other features of student work (e.g., its correctness, its compliance with norms of the situation used to frame the work, or its responsiveness to the question asked) might also be noticeable (Herbst et al., 2023). To the extent that PBI is desirable, teacher learning environments should support development of disposition to notice serviceability. Our simulation series is situated in a lesson in which a problem of creating a circle tangent to two intersecting lines at two given points is used to develop the theorem that tangent segments are equidistant. We report on data from two simulations. One of them engages teachers in teaching the entire problem-based lesson and is used pre- and post-intervention. In this simulation the teacher needs to, among other things, select student work for a classroom discussion. The other, intervening simulation focuses teachers' attention on noticing student contributions through sorting samples of student work into bins. All simulations were completed by a group of 29 practicing teachers who had, on average, 16 years and 11 years of experience teaching high school mathematics and high school geometry respectively. We looked for evidence that individuals' noticing of serviceability in the intervention was associated with increased selection of serviceable work in the full lesson. A Principal Component Analysis allowed us to identify a variable to account for teachers' sorting activity that indicated noticing of serviceable work. Teachers who increased selection of serviceable work from pre- to post- performance in the full lesson tended to also score at higher values of that identified variable though teachers who did not show increase their selection of serviceable work were more widely distributed across the range of this variable. The analysis follows a pupil-mediation paradigm for research on teaching applied to teacher learning by identifying the strength to which participants' noticing of serviceability had a strong effect size predicting who would improve their serviceability scores.

Additional information: Work supported by grant 220020524 from the James S. McDonnell Foundation.

References

Herbst, P., Brown, A. M., Chazan, D., Boileau, N., & Stevens, I. (2023). Framing, responsiveness, serviceability, and normativity: Categories of perception teachers use to relate to students' mathematical work in problem-based lessons. *School Science and Math*, *123*(7), 398-413.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 509. PME.

HOW DO I DIVIDE TO GET THE RESULT I WANT?

Waqas Bin Sarfraz, Dave Hewitt, Ian Jones

Loughborough University

Szkudlarek et al. (2022) showed that young children have a sense of division before meeting the division sign. We are interested in what children can do with regard to multiplication and division in an environment that does not explicitly indicate either are involved, yet require both to be used. Our specific research question is: What issues do young children find with division when engaged with an app called *Stick and Split*?

Video recordings were taken of five pairs of 5-6 year olds and five pairs of 6-7 year olds using the app for the first time. Each pair had two 30-minute sessions. The app shows a target number above rods of certain lengths. The goal is to make a series of rods of the target length by either sticking rods of equal length together, or splitting rods into equal parts. The tasks started with rod lengths being factors of the target number to tasks where, for example, 24 had to be made with varying length rods (Figure 1). Analysis developed themes which occurred across the pairs of students.

Figure 1. Screen shot of *Stick and Split*

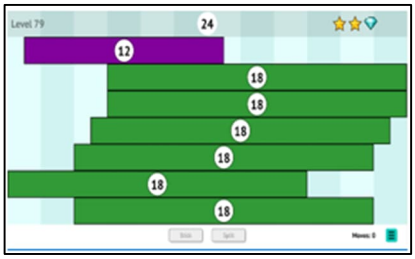
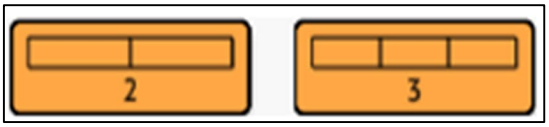


Figure 2: Split options



On tasks which required splitting of rods, even though students knew which rod length they wanted, they were unsure how many equal parts to divide a longer rod to achieve that. For example, one pair tried to split a 6 rod into three 2-rods. On clicking 'split' the app presented the option to split into two or three equal parts (Figure 2). The split-into-two option was chosen which, to their surprise, caused two 3-rods to appear instead of the three 2-rods they were expecting. We observed that students repeatedly confused the desired result with the divisor. We inferred that students showed awareness that rods can be split equally to their wanted length, but that they struggled to know which divisor would achieve that. This supports the notion that young students can have a sense of division by knowing that one number is a factor of another, but not know so easily which divisor is required to obtain that factor.

References

Szkudlarek, E., Zhang, H., DeWind, N. K., & Brannon, E. M. (2022). Young children intuitively divide before they recognize the division symbol. *Frontiers in Human Neuroscience*, *16*, Article 752190. https://doi.org/10.3389/fnhum.2022.752190

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 510. PME.

CHILDREN'S EARLY-STAGE PROPORTIONAL REASONING IN INTRODUCTORY LESSONS ON MULTIPLICATION

Keiko Hino[1], Hiraku Ichikawa[2], Hisae Kato[3]

[1]Utsunomiya University, [2]Miyagi University of Education,
[3]Hyogo University of Teacher Education

Given its educational importance, proportional reasoning has been widely studied, and its structure, key aspects, and developmental indicators have been extensively identified. Among these, unitizing, covariance, and invariance are consistently highlighted as central (e.g., Lamon, 2020). However, promoting the development of proportional reasoning in school settings remains a challenge. In particular, how teachers notice students' thinking and provide appropriate support in everyday classroom interactions continues to be an important issue. This study examines the kinds of early-stage proportional reasoning that children demonstrate through interactions with teachers and peers in a second-grade classroom (ages 7–8) where multiplication is introduced. Drawing on Jeannotte and Kieran (2017), proportional reasoning is conceptualized as a discursive activity in which narratives about two covarying quantities in proportional relationship are generated and developed through processes of exploration and validation. The term "early-stage" is used to inclusively capture children's reasoning, including approaches based on counting or additive relationships. Three lessons were designed and implemented on November 17, 18, and 21, 2022, in a class of 33 pupils at a university-affiliated elementary school in northern Japan. The lessons were grounded in a familiar context in which children recalled visiting a local temple shop and took on the role of shopkeepers explaining drink prices to customers. The unit price was fixed at 130 yen, regardless of drink type. Tasks were designed to promote unitizing with composed units ("○ bottles for □ yen"), supported by a photograph of price table used in the shop that highlighted both the constancy of the unit price and the relationship between the number of bottles and total cost. The lessons were video-recorded, transcribed, and analyzed qualitatively. Ten episodes on early-stage proportional reasoning were identified and examined from structural and process aspects (Jeannotte & Kieran, 2017). Children compared known information and justified their methods, leading to increasingly flexible use of composed units to make claims. The underlying idea of invariance ("all unit prices are the same") was gradually made explicit in their narratives; analysis of the price table served as an important opportunity. In both cases, the teacher's support—through the visual and coherent structuring of children's narratives—became evident.

References

Jeannotte, D., & Kieran, C. (2017). A conceptual model of mathematical reasoning for school mathematics. *Educational Studies in Mathematics*, *96*(3), 1-16.

Lamon, S. J. (2020). *Teaching fractions and ratios for understanding* (4th ed). Routledge.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 511. PME.

AUTHENTICITY OF MATHEMATICAL LITERACY OPPORTUNITIES IN TEXTBOOKS: A COMPARATIVE STUDY OF TAIWAN AND THE INTERNATIONAL BACCALAUREATE

Wan-Ling Hong, Ting-Ying Wang

National Taiwan Normal University

A central goal of mathematics education is to support students in connecting mathematical concepts to real-world situations. Yet many educational materials lack authenticity or are merely dressed in contextual language, leading students to disconnect mathematics from lived reality and fail to develop mathematical literacy (Incikabi, 2025). Taiwan, a high-achieving East Asian country long emphasizing formal mathematical knowledge, launched a new curriculum in 2019 promoting mathematical literacy for real-world application. However, whether this emphasis is reflected in the potentially implemented curriculum—textbooks—remains unclear. This study adopts the eight-aspect authenticity framework and scoring system proposed by Incikabi (2025) to examine the authenticity of a current Taiwanese mathematics textbook and to compare it with an International Baccalaureate (IB) textbook, selected for its emphasis on mathematical literacy. The trigonometry unit, given its strong connection to real-life applications, was chosen for analysis. A document analysis was conducted on 59 and 41 items drawn from Taiwanese and IB textbooks respectively. The framework evaluates eight aspects: event, question, language use, solution requirements, existence of information, realism of information, specificity of information, and affective purpose. Each item was rated on each aspect using a 3-point sufficiency scale (0 = insufficient, 0.5 = partially sufficient, 1 = sufficient). Based on total scores and the presence or absence of specific aspects, items were classified into three levels of authenticity: good-fit, stereotyped, and poor-fit. Only 39% of the Taiwanese textbook items qualified as authentic tasks, compared to 61% in IB textbooks. However, the number of good-fit items is comparable (TW: 13; IB: 15). In both textbooks, authentic items generally demonstrate relatively high sufficiency in language use, solution requirements, and existence of information. Nevertheless, more than half of the good-fit items in both textbooks are rated only as partially sufficient in realism of information and affective purpose. A notable discrepancy emerges in specificity of information: while only 13% of IB items fall below sufficiency, 46% of Taiwanese items do so. These findings highlight a gap between the curriculum's literacy-oriented goals and their realization in textbooks, and provide important implications for pedagogical practices and textbook design.

References

Incikabi, S. (2025). Investigating authentic nature of PISA mathematics items. *School Science and Mathematics.* Advance online publication. https://doi.org/10.1111/ssm.18402

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 512. PME.

REGIONAL VARIATIONS IN ACADEMIC RESILIENCE: MACHINE-LEARNING EVIDENCE FROM NORDIC AND EAST ASIAN COUNTRIES IN PISA 2022

Cheng-Yu Hsieh, Yoxiu Chen, Su-Wei Lin
National University of Tainan, Taiwan

PISA 2022 results indicate that socioeconomic status (ESCS) leads to a math achievement gap of four to five years of schooling, a disparity intensified in the post-pandemic era (OECD, 2023). However, certain disadvantaged students demonstrate exceptional persistence and motivation, achieving success far beyond expectations—a group defined as "Academically Resilient Students" (ARS) (Xenofontos & Mouroutsou, 2023). As resilience factors are deeply culturally embedded and pathways to success vary across systems (Agasisti et al., 2018), comparing East Asia and the Nordic region enables the identification of regional feature heterogeneities. This study defines ARS as students in the bottom 25% of the national ESCS index who rank in the top 25% globally for math performance (PVMATH1). Utilizing Random Forest algorithms to analyze 61 student-level features across East Asia (Taiwan, Hong Kong, Japan, Korea, Macau, and Singapore) and the Nordic region (Denmark, Finland, Iceland, Norway, and Sweden), model performance was evaluated using Receiver Operating Characteristic - Area Under the Curve (ROC-AUC). In intra-regional identification, the ROC-AUC reached 79.03% for the Nordic region (N=7,804; ARS ratio: 16.61%) and 83.13% for East Asia (N=8,606; ARS ratio: 45.34%). Inter-regional classification demonstrated even greater power; across the total sample of 16,410 disadvantaged students, the ROC-AUC for distinguishing between regions was 95.04%, rising to 96.36% for the ARS cohort. This indicates that inter-regional cultural structures exhibit higher discriminability and separability than intra-regional resilience characteristics. Although intra-regional performance is slightly lower, the models remain highly effective in defining academic resilience, providing robust evidence for its strong context-dependency. By leveraging big data mining to identify cultural heterogeneities, this study demonstrates that academic resilience is not a universal trait but a context-dependent construct, necessitating highly culturally-sensitive educational interventions.

References

Agasisti, T., Avvisati, F. B., & Agrawal, S. (2018). *Academic resilience: What schools and countries do to help disadvantaged students succeed in PISA* (OECD education working papers no. 167). https://doi.org/10.1787/e22490ac-en

OECD. (2023). *PISA 2022 results (Volume I): The State of Learning and Equity in Education.* OECD Publishing. https://doi.org/10.1787/53f23881-en

Xenofontos, C., & Mouroutsou, S. (2023). Resilience in mathematics education research: A systematic review of empirical studies. *Scandinavian J. of Ed. Research*, *67*(7), 1041–1055.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 513. PME.

EXPLORING ELEMENTARY STUDENTS' MATHEMATICAL COMMUNICATION SKILLS WITHIN THE PROBLEM-SOLVING CYCLE

Chia-Jui Hsieh, Yuan-Pei Tsai, Chia-Huei Lin

National Taipei University of Education

Mathematical literacy tasks require students not only to compute but also to communicate how they make sense of contextual information. In the PISA mathematics framework, these demands relate to the modelling cycle—formulating, employing, and interpreting/evaluating—and to communication as a key 21st-century skill (OECD, 2023). In Taiwan, the tools for assessing students' mathematical communication in literacy-oriented tasks are still scarce. This study investigates how Taiwanese sixth graders' mathematical communication difficulties vary across stages of the cycle. Mathematical communication is defined as three sub-skills: accessing/retrieving (A), integrating/interpreting (I), and representing/expressing (E).

Research questions. (1) How do students' A/I/E performances vary across stages of the cycle? (2) What recurring communication error types characterize each stage? We developed a questionnaire based on contextualized quantitative literacy situations, using a two-dimensional blueprint crossing problem-solving stages with communication sub-skills. Data were collected from 696 sixth-grade students. Responses included selected and constructed items and were coded using scoring guidelines (double scoring with adjudication). Analyses combined (a) a partial credit approach with IRT to estimate A/I/E abilities and (b) stage-focused profiling with qualitative summaries of recurring error types. The findings provide evidence for a stable "A-advantage" profile in literacy-oriented tasks; that is, students perform strongest in A (67.95), while I (53.68) and E (55.00) remain major learning needs. Stage comparisons reveal differentiated bottlenecks: (a) during formulating, students most often struggle to integrate information into a solvable mathematical problem; (b) during employing, weak integration is frequently accompanied by procedure-first attempts (e.g., number-plugging) with limited justification; (c) during interpreting/evaluating, expressing becomes the weakest, reflecting high demands in communicating meaning and reasonableness through representations and explanations. Error patterns suggest cumulative effects, where early misinterpretations and incomplete retrieval constrain later integration and expression. Implications include diagnostic feedback and targeted scaffolds for integration, justification, and explanation (interpreting/evaluating).

References

OECD. (2023). PISA 2022 Assessment and Analytical Framework. PISA, OECD Publishing. Paris. https://doi.org/10.1787/7ea9ee19-en

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 514. PME.

TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE FOR TEACHING LENGTH MEASUREMENT AND ESTIMATION

Hsin-Mei E. Huang[1], Jessica Hoth[2], Wan-Shin Chang[3], Silke Ruwisch[4], Aiso Heinze[5]

[1]University of Taipei, [2]University of Rostock, [3]National Academy for Educational Research, Taiwan, [4]Leuphana University of Lueneburg, [5]IPN Kiel

Teachers' pedagogical content knowledge (PCK) for involves skills of transforming content knowledge (CK) of particular topics into teaching practices. Such knowledge enables teachers to take proper actions in their teaching practices and contributes to their students' achievement. A cross-country comparison revealed that Taiwanese and German primary school students developed different profiles in their length estimation skills (Hoth et al., 2023). German students were found to have superiority in the situation where the to-be-estimated objects [TBEOs] were not small and not touchable, whereas Taiwanese students had better skills in the situations where the TBEOs were small as well as not small and touchable. It is assumed that teachers hold different culture-specific pedagogical views on classroom practices, shaped by their educational contexts (e.g., curricula, teacher training), which lead to variances in teaching practices and cause the different estimation skill profiles.

Sixty in-service primary school teachers (30 Taiwanese [T-Group]; 30 German [G-Group]) completed the PCK questionnaire on a) CK focusing on length measurement and estimation (LME) concepts, b) instructional actions for estimation activities, and c) approaches for teaching estimation strategies centring on levels of tendencies of using benchmarks and operation-and-touching TBEOs. The findings show significant differences between the two groups in all three sub-sections with medium to large effect sizes. The T-group had better CK of LME than the G-group. The result is consistent with those of Tatto et al. (2012). Although both groups preferred to choose TBEOs from classrooms and hall-ways, the G-group provided students more objects selected from school sport fields. Moreover, the G-group placed higher value on constructing benchmarks by memorizing the lengths of daily objects used frequently than the T-Group did. The differences in teachers' instructional actions and teaching strategies for estimation may help explain the results of Hoth et al.'s (2023) study.

References

Hoth, J., Heinze, A., Huang, H.-M. E., Weiher, D. F., Niedermeyer, I., & Ruwisch, S. (2023). Elementary school students' length estimation skills- Analyzing a multidimensional construct in a cross-country study. *Int. J. of Science and Math Ed.*, *21*(6), 1841-1864.

Tatto, M. T., Schwille, J., Senk, S. L., Ingvarson, L., Rowley, G., Peck, R., Bankov, K., Rodriguez, M., & Reckase, M. (2012). *Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics in 17 Countries* (TEDS-M). IEA.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 515. PME.

TEACHER PRACTICES ASSOCIATED WITH DIVERSE STUDENTS THRIVING IN MATHEMATICS

Jodie Hunter[1], Julia Hill[2], Generosa Leach[1]

[1]Massey University, [2]RMIT University

Developing equitable education systems in relation to mathematics is an international goal. Research studies have shown that teacher practices influence student mathematical outcomes (Lynch et al., 2017) and there is a relationship between general student wellbeing and mathematical achievement (Estipona & Delos Santos, 2025). This highlights the potential for the classroom environment and teacher practices to influence equitable outcomes in mathematics. In this presentation, we examine the influence of teacher practices on mathematical outcomes for diverse students. We ask: *What pedagogical practices are evident in classrooms where diverse students are thriving in mathematics?* An ongoing professional learning and development (PLD) research project, Developing Mathematical Inquiry Communities (DMIC), focuses on supporting teachers to shift mathematics teaching to better serve diverse students in New Zealand. Drawing on data from a larger longitudinal study including student questionnaires, achievement data, and classroom observations, we present an exemplary case study of one primary teacher of Year 5 students and her practices following the DMIC PLD. Students in this classroom had substantially higher levels of self-reported mathematical wellbeing than other Year 5 students and strong levels of mathematical achievement. We examine the pedagogical practices used by the teacher that supported her class of Māori and Pacific students to thrive mathematically.

Our findings show that the teacher consistently drew on five pedagogical practices in classroom observations for which she showed high-quality enactment throughout entire lessons. This differed from the observation results in other classrooms. These practices included the use of challenging mathematical tasks, displaying high expectations, supporting substantive mathematical conversations, facilitating mathematical practices, and developing social norms. We argue that given evidence of diverse students thriving in this classroom, examining the pedagogical practices of the teacher offers important insights for equity.

References

Estipona, E.P., & Delos Santos M. S. M. (2025). Linking wellbeing to success: life satisfaction on mathematics performance of Philippine public high school students. *Frontier Education*. 10

Lynch, K., Chin, M., & Blazar, D. (2017). Relationships between observations of elementary mathematics instruction and student achievement: Exploring variability across districts. *American Journal of Education*, *123*(4), 615-646.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 516. PME.

IMPROVING LOW MULTIPLICATION AND DIVISION FLUENCY THROUGH A DIGITAL GAME

Markus Hähkiöniemi, Heidi Harju, Jarno Rautiainen, Tuire Koponen

University of Jyväskylä

Some children continue to struggle with basic arithmetic skills, such as multiplication and division fluency, long after these topics have been taught in school (Psyridou et al., 2023). Because multiplication and division are foundational tools in many mathematical contexts, these children may benefit from targeted basic-skills training even when the curriculum moves on. Digital learning games offer one way to provide such training, though more research is needed on their effectiveness for children with learning difficulties (Alam et al., 2025). The aim of this study is to examine learning effects of the digital mathematics game REI-O for children with low multiplication and division fluency. REI-O game was designed to include different kinds of practice in adventurous gaming world. For example, it includes adaptive repeated practice with immediate feedback, conceptual instruction in form of a dialogue and practice, and minigames that require flexible consideration of different solution strategies.

Among voluntary participants in grades 4–6 (10–12 year-olds), children with dysfluency in multiplication and division skills were selected for the game intervention group to play REI-O for 12–15 minutes three times a week. In case of 4th graders, there was an enhanced control group receiving paper-and-pencil training. Multiplication and division fluency measure was implemented in two pre-tests and a post-test. After excluding incomplete cases, the game intervention group contained 85 and the control group 38 children. The data was analysed with repeated-measures ANOVA.

The increase in multiplication and division fluency during the intervention was statistically significant, with medium effect size. There was no difference between the game and paper-and-pencil groups for the 4th graders. The findings suggest that REI-O game can effectively support fluency development for children needing additional practice. Furthermore, teachers can choose between REI-O game and paper-and-pencil materials based on other considerations.

References

Alam, S.S., Gao, J., & Dubé, A.K. (2025). A systematic review and meta-analysis of digital mathematics interventions for K-12 students with mathematical learning disabilities. *International Journal of Science and Mathematics Education, 23*(8), 4099–4126.

Psyridou, M., Torppa, M., Tolvanen, A., Poikkeus, A.-M., Lerkkanen, M.-K., & Koponen, T. (2023). Developmental profiles of arithmetic fluency skills from grades 1 to 9 and their early identification. *Developmental Psychology, 59*(12), 2379-2396.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 517. PME.

WORDS FOR =: HOW DO THE WORDS WE USE RELATE TO THE MEANINGS WE HAVE

Jenni Ingram, Emily Macmillan

University of Oxford

The equals sign is used and interpreted in different ways in mathematics classrooms, intended to convey particular relationships associated with equality and equivalence. An understanding of the meanings associated with =, and flexibility in the uses of these meanings, are widely considered to be fundamental to algebraic thinking (Kieran, 2022). Distinctions are often made between an operational view, a relational view and a substitutive view (Jones & Pratt, 2012). Teaching students to recognise this polysemous nature of the equals sign is widely explored in existing research, but more recent work on using meaning-related language reveals how the words we use may support, or not, this teaching (Ward-Penny et al., 2026; Zentgraf & Prediger, 2024). In this study, we explore the relationship between the verbal representation of = and the meanings that mathematics teachers associate with these verbal representations.

An online survey was designed as a pedagogic activity for use during professional development workshops. It was based on the different examples of how the = sign is used in school mathematics, and the words used by teachers during their lessons, taken from a video study of mathematics teaching reported at PME 48. Where consent was given, the responses to the survey, were analysed using simple descriptive frequencies to examine relationships between the representations and the meanings. The results show that teachers associate particular words with specific relationships in many instances, whilst also using more ambiguous choices that could describe several relationships. Furthermore, some meanings of = were rarely referred to or distinguished from others. These choices have important implications for how we support students in understanding complex concepts such as equivalence and in making connections between words and symbols.

References

Jones, I., & Pratt, D. (2012). A substituting meaning for the equals sign in arithmetic notating tasks. *Journal for Research in Mathematics Education*, *43*(1), 2–33.

Kieran, C. (2022). The multi-dimensionality of early algebraic thinking: background, overarching dimensions, and new directions. *ZDM – Math Education*, *54*(6), 1131–1150.

Ward-Penny, R., Ingram, J., Erath, K., Kimber, E., & Planas, N. (2026). Equals in the wild: How mathematical equality is talked about in lessons. *The Mathematics Enthusiast*, *23*(1–2), 55–74. https://doi.org/10.54870/1551-3440.1680

Zentgraf, K., & Prediger, S. (2024). Demands and scaffolds for explaining the connection of multiple representations: Revisiting the bottle-filling task. *The Journal of Mathematical Behavior*, *73*, 101118. https://doi.org/10.1016/j.jmathb.2023.101118

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 518. PME.

TEACHERS' NOTICING GUIDED BY LEARNER OBSERVATION SHEET IN LESSON STUDY: AFRICAN EDUCATORS LEARNING JAPANESE PROBLEM-SOLVING LESSON

Hiroshi Ishii

Hokkaido University of Education

This study investigates how a learner observation sheet influences the development of teacher noticing—particularly attending to and interpreting students' mathematical thinking—among African educators participating in a Lesson Study–based professional development program in Japan. Grounded in the theoretical framework of teacher noticing, the study conceptualizes professional learning as a transformation in how teachers perceive and make sense of students' thinking during instruction. Eight educators from African countries engaged in a nine-day training program in Feb 2025, where they observed and conducted problem-solving–oriented mathematics lessons using a learner observation sheet adapted from the TRU framework to systematically document students' thinking processes, engagement, and collaborative activities. The analysis revealed a clear developmental progression in participants' noticing across the training phases. In the early stage, teachers primarily attended to observable student behaviors and lesson outcomes, with limited attention to underlying cognitive processes. Gradually, their focus shifted toward students' reasoning, problem-solving strategies, and peer interactions, accompanied by emerging interpretations of students' mathematical thinking. In the later stage, participants demonstrated a more sophisticated and analytical perspective, capturing the full learning process—from individual work to collaboration, integration, and reflection—and recognizing causal relationships between students' actions, instructional design, and the learning environment. As emphasized by Sherin et al. (2011), teacher noticing involves not only observing classroom events but interpreting their meaning within instructional contexts. The findings indicate that the learner observation sheet functioned as a powerful mediating tool that guided teachers' attention toward student-centered aspects of learning and supported a shift from descriptive observation to analytical interpretation. By structuring observation through shared criteria, the tool also facilitated deeper reflection and common understanding during post-lesson discussions. Overall, this study suggests that the use of learner observation sheets can enhance teacher noticing and promote a deeper understanding of problem-solving–oriented mathematics instruction, particularly in cross-cultural professional development contexts. Limitations include the small sample size and the short duration of the program, indicating the need for further longitudinal and comparative research.

References

Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (Eds.). (2011). *Mathematics teacher noticing: Seeing through teachers' eyes*. Routledge.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 519. PME.

AN ANALYSIS OF STUDENTS' IDEAS ON MULTIPLICATION PROBLEM SITUATIONS THROUGH REFLECTIVE THINKING

Kamonchanok Japa, Narumon Changsri, Khem Khenkhok

Khon Kaen University

Understanding students' ideas requires interpreting the reasoning underlying their responses rather than merely evaluating their correctness. Within reflective thinking, introspective thinking (IT) examines ideas that align with prior anticipation, while contemplative thinking (CT) reconstructs reasoning when students' responses diverge from what was anticipated (Emori, 2026). The transformative lesson study incorporating open approach (TLSOA) structures professional reflection through three interconnected processes: collaborative plan, do, and see (Inprasitha, 2022). However, how IT and CT function analytically in examining students' ideas in multiplication problem situations remains underexplored. Therefore, this study aims to analyze students' ideas through reflective thinking within the TLSOA model. This study purposively selected nine Grade 2 elementary students to solve three-lesson multiplication problem situations. One example of a problem situation that required 19 students to be arranged into groups of 2 or 3 students without leaving anyone alone. The data comprised the lesson plan, student worksheets, and classroom field notes. During the collaborative plan, the teachers anticipated possible ideas through the IT. During collaborative do, they observed the students' ideas in the classroom, and during collaborative see, they analyzed their written responses using IT and CT. Qualitative content analysis was employed to interpret the reasoning structures underlying students' solutions. The results revealed that most students produced valid groupings, such as $3\times5 + 2\times2$, coordinating with equal groups and members. These responses aligned with prior anticipation and were interpreted through IT. Some responses required CT: $19\times1=19$ ignored the grouping condition, whereas $(5\times2) + (9\times1)$ reached 19 but included groups of one member. These responses revealed structural miscoordination between the numerical calculation and contextual constraints. Although limited to three lessons, distinguishing between IT and CT provides a focused lens for examining students' ideas in multiplication problem situations.

References

Emori, H. (2026). Mathematical communication competencies for enhancing teaching: Focusing on anticipation, observation, and reflection. In M. Inprasitha (Ed.), *Proceedings of the 12th National Conference on Mathematics Education: Open class—The heart of the innovative transformative lesson study with the open approach (TLSOA) model in the professional development system* (pp. 23–52). Thailand Society of Mathematics Education.

Inprasitha, M. (2022). Lesson study and open approach development in Thailand: A longitudinal study. *International Journal for Lesson & Learning Studies, 11*(5), 1-15.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 520. PME.

EXAMINING THE RELATIONSHIP BETWEEN MATHEMATICAL WELLBEING AND STEM CAREER INTEREST IN JAPANESE HIGH SCHOOL STUDENTS

Qiaoping Zhang[1], Ziyi Jin[2], Takuya Baba[2], Yuichiro Hattori[3]

[1]The University of Hong Kong, [2]Hiroshima University, [3]Okayama University

Across school subjects, mathematics is widely viewed as a core foundation for other STEM domains (Goos et al., 2023). Students' experiences in learning mathematics can shape their long-term sense of competence and influence educational and career choices. However, mathematics is also often experienced as a source of pressure (Dowker et al., 2016). Students' affective responses to mathematics, their sense of classroom relationships, and their value judgments regarding mathematical learning are therefore important conditions that may shape their STEM interest and career intentions. Although prior research has highlighted the significance of mathematical wellbeing which represents an emerging, multidimensional construct capturing students' subjective experiences of feelings and functioning in mathematics learning (Campbell, 2025), systematic evidence linking it to students' STEM career interest and intentions remains limited.

The present study surveyed 309 senior high school students from two schools in Japan using a well-established, internationally validated questionnaire. We computed composite scores, including a core mathematical wellbeing index and an indicator of mathematical tension/helplessness. Students' STEM career interest was positively correlated with their core mathematical wellbeing ($r = .314$, $p < .001$) and actual tension/helplessness ($r = .280$, $p < .001$). A linear regression controlling for grade and gender indicated that both predictors remained significant, with model fit $R^2 = .150$ (adjusted $R^2 = .139$). Grade and gender were not significant predictors ($p > 0.05$). This suggests a complex relationship: while students' stronger STEM orientation co-occurs with more positive core mathematical wellbeing, it is simultaneously associated with greater tension and helplessness in mathematics.

References

Campbell, T. (2025). Mathematical wellbeing: an emerging construct with exciting potential. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-025-00532-5

Dowker, A., Sarkar, A., & Looi, C. Y. (2016). Mathematics Anxiety: What Have We Learned in 60 Years? Frontiers in Psychology, 7, Article 508. https://doi.org/10.3389/fpsyg.2016.00508

Goos, M., Carreira, S. & Namukasa, I. K. (2023). Mathematics and interdisciplinary STEM education: recent developments and future directions. *ZDM-Mathematics Education*, 55, 1199–1217. https://doi.org/10.1007/s11858-023-01533-z

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 521. PME.

VARIATIONS IN MATHEMATICAL COMPETENCIES AMONG HIGH-ACHIEVING STUDENTS

Helena Johansson[1], Jonas Jäder[1], Elisabet Mellroth[2], Magnus Österholm[3]

[1]Mid Sweden University, [2]Karlstad University, [3]Umeå University

High-achieving students in mathematics are often treated as a homogeneous group. However, using quantitative analysis, Prast et al. (2025) identified distinct subgroups among high-achieving students differing in cognitive, motivational, and achievement-related characteristics. Although their results show substantial heterogeneity, the identified differences primary concerns cognitive and motivational aspects rather than mathematical competence itself. Thus, there is a need to explore qualitative differences in how mathematical competencies are enacted by individual high-achieving students. This ongoing study investigates similarities and differences in individual mathematical competence profiles among high-achieving students. The study draws on the Mathematical Competency Research Framework (Lithner et al., 2010), which conceptualises mathematical knowledge as a set of competencies. To capture different aspects within each competency, three competency-related activities (CRA) are defined: Interpret, Do and Use, and Judge. The five competencies included in the Swedish mathematics curriculum: *conceptual*, *procedural*, *problem solving*, *reasoning*, and *communication*; are operationalised through these CRA. This enables a fine-grained, qualitative analysis of students' mathematical competence beyond grades or number of correct answers.

Eight high-achieving lower secondary students (age 13-15) individually solved up to 10 tasks, selected to allow demonstration of the different competencies. Written and oral solutions were analysed to create competence profiles, visualised as matrices across competencies and CRAs. Preliminary results show both homogeneity and heterogeneity among the students. Most students show strong performance in the productive activity "Do and Use", particularly in procedural competency. Clear variation emerges in reasoning and communication, especially in the analytical activity "Judge": some students engage in meta-reflection, while others show limited evaluation of methods or arguments. Our findings indicate that high achievement does not imply uniform development of mathematical competencies (cf. Prast et al., 2025). We contribute an empirically grounded mathematics-specific perspective on heterogeneity among high-achieving students, with implications for task design and differentiated instruction.

References

Lithner, J., Bergqvist, E., Bergqvist, T., Boesen, J., Palm, T., & Palmberg, B. (2010). Mathematical competencies: A research framework. In C. Bergsten, E. Jablonka & T. Wedege (Eds.), *Proceedings of MADIF 7 the Seventh Mathematics Education Research Seminar* (pp. 157-167). SMDF.

Prast, E.J., Hickendorff, M., & Van de Weijer-Bergsma, E. (2025). High-achieving students in mathematics: A heterogeneous group. *Learning and Ind. Differences*, *119*, 102629.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 522. PME.

UNPACKING COMPUTATIONAL THINKING THROUGH PROGRAMMING TASKS IN KOREAN INFORMATICS TEXTBOOKS

Won Jung, Kyungwon Lee, Oh Nam Kwon

Seoul National University

Computational thinking (CT) has increasingly been integrated into mathematics education, particularly as students develop solution procedures, represent mathematical relationships, and systematically verify their results (Brennan & Resnick, 2012). Textbooks play a central role in shaping classroom practice, yet how CT is realized in textbook tasks remains underexplored. Programming tasks in informatics textbooks frequently involve mathematical ideas and representations. Therefore, examining these tasks enables the identification of practical approaches to integrating CT into mathematics learning. We conducted a textbook analysis of 23 Korean informatics textbooks (middle school: 12, high school: 11). We extracted 197 tasks in which mathematics was explicitly embedded in the coding process or was required to complete the task. Each task was coded along two dimensions: (a) mathematics concepts and (b) CT concepts, drawing on the CT components synthesized by Ye et al. (2023). CT concepts were identified as required programming features (variables, conditionals, loops, sequences, events, subroutines, and operators). When coding CT concepts, we examined whether they were represented as individual concepts or as bundled sets of multiple concepts, with a focus on elucidating the modes of thinking required by these representational forms. The analysis identified two main areas of focus. First, the mathematics concepts of programming tasks were concentrated in the domains of algebra and geometry, reflecting frequent use of numerical calculations and geometric constructions or transformations. Second, CT concepts formed procedural patterns centered on variable manipulation and the sequential execution of mathematical procedures. These patterns structured how tasks stored and updated quantities while guiding repeated calculations or geometric constructions. This analysis provides insights into how CT can be enacted through programming tasks and offers implications for the design of tasks and instructional practices related to CT in mathematics education.

References

Brennan, K., & Resnick, M. (2012). New frameworks for studying and assessing the development of computational thinking. In *Proceedings of the 2012 annual meeting of the American Educational Research Association* (pp. 1–25).

Ye, H., Liang, B., Ng, O., & Chai, C. S. (2023). Integration of computational thinking in K-12 mathematics education: a systematic review on CT-based mathematics instruction and student learning. *International Journal of STEM Education*, *10*(1), 1–26.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 523. PME.

DESIGNING AUTOMATIC DIAGNOSIS OF STUDENTS' RATIONAL NUMBER STRATEGIES

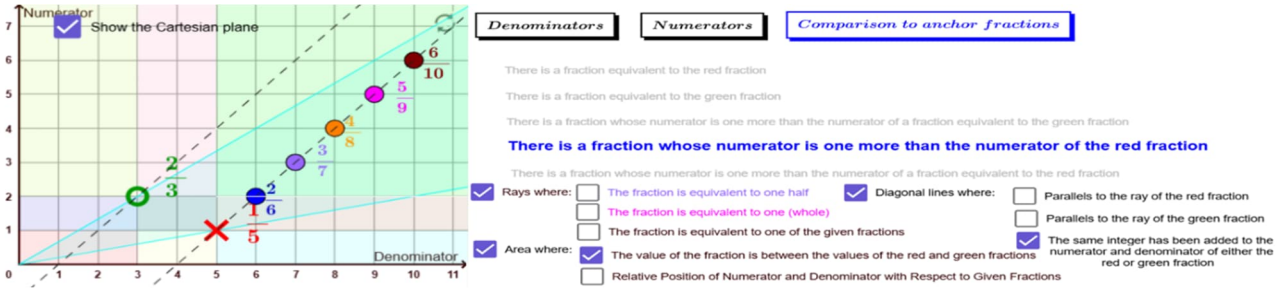
Amal Kadan-Tabaja[1,2], Daniel Chazan[2]

[1]University of Haifa, [2]University of Maryland

Open-ended fraction tasks provide access to students' reasoning by eliciting multiple responses, potentially making patterns visible (Watson & Mason, 2005; Yerushalmy, 2020). However, they create an interpretive challenge for teachers who must make sense of diverse responses. With AI support, we address this challenge with the goal of developing a tool to support teachers' interpretation of student-generated data.

We analyzed responses from 47 elementary students to seven example-eliciting tasks involving finding fractions between given bounds; their responses were represented on a Cartesian plane, making patterns in their work visible. AI helped us organize these patterns and design an interface for teachers. Figure 1 illustrates how this interface supports recognition of, for example, a strategy of adding 1 to both the numerator and denominator.

Figure 1: One dimension of feedback to teachers about a student strategy



The diagnostic framework, now embedded in software, organizes feedback on student reasoning along three dimensions: denominators, numerators, and comparison to anchor fractions (see Figure 1 for the comparison dimension). By grounding interpretation in observable patterns, the framework supports automated, interpretable reporting of reasoning. This work contributes a design perspective on how visual representations support the development of diagnostic tools for formative assessment.

References

Watson, A., & Mason, J. (2005). *Mathematics as a constructive activity: Learners' generating examples*. Lawrence Erlbaum Associates.

Yerushalmy, M. (2020). Example-eliciting tasks. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education*. Springer.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 524. PME.

SPATIAL REASONING THROUGH COLLECTIVE ENACTMENT ACROSS SPACE SCALES

Kazuya Kageyama

Hiroshima University

Recent mathematics education research has moved beyond treating spatial reasoning as an individual, measurable skill and has instead highlighted its emergent, contextually organized character. This study examines how students' spatial reasoning was reorganized through shifts between collective enactment in a gym and video-mediated analysis in a classroom. Building on Davis et al. (2015)'s spatial reasoning model and an embodiment perspective on figural formation (Benally et al., 2022), this study focuses on how different space scales make distinct aspects of collective movement perceptually available and analytically negotiable. The study involved 36 ninth-grade students across three lessons. Students were asked to transform a cross-shaped collective formation into a circle while moving counterclockwise in a gym. Their activity was video-recorded, including an overhead view from the second floor. In subsequent classroom work, students analyzed the recorded movement through discussion and individual worksheets while observing the video. Data for analysis included a gym video, transcripts of classroom discussions, student-drawn diagrams, blackboard records, and observers' field notes. The analysis shows that some students attempted to form a circle by using the student in front of them as a local reference and trying to maintain a constant distance from that student. However, from their floor-level perspective, they could not determine whether the overall formation was actually circular. In the classroom, however, the overhead video observation made aspects of the collective movement more available for discussion. In one episode, students distinguished between outer and inner movements of the cross-shaped formation. They argued that, although the outers could trace circular paths, the inners had to move in a stair-step or straight-line manner relative to the rotating formation. Thus, movement initially treated as "making a circle" was reinterpreted as a set of rule-governed local motions that collectively generated circularity. This shift suggests that spatial reasoning in collective, embodied geometry cannot necessarily be reduced to the formal geometric construction of a figure. Rather, shifts across space scales affected what students could notice, discuss, and coordinate.

References

Benally, J., Palatnik, A., Ryokai, K., & Abrahamson, D. (2022). Learning through negotiating conceptually generative perspectival complementarities: The case of geometry. *For the Learning of Mathematics*, *42*(3), 34–41.

Davis, B., & Spatial Reasoning Study Group. (2015). *Spatial reasoning in the early years: Principles, assertions, and speculations*. Routledge.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 525. PME.

ALGORITHMIC THINKING THROUGH COLORING ACTIVITIES IN PRIMARY MATHEMATICS CLASSROOMS

Wiramon Kanlayarat, Narumon Changsri, Maitree Inprasitha

Khon Kaen University

Algorithmic thinking (AT) is closely related to mathematical thinking, as both involve interpreting conditions, analyzing relationships, and constructing possible solutions. In primary mathematics classrooms, unplugged coloring activities can provide opportunities for students to make these reasoning processes visible (Changsri et al., 2024). This study explores how coloring activities support students' AT in primary mathematics classrooms under the Transformative Lesson Study incorporating Open Approach (TLSOA): collaborative plan, collaborative do with four steps of Open Approach, and collaborative see (Inprasitha, 2022). This classroom-based qualitative study was conducted in two phases. First, six sets of coloring activities were analyzed to examine how task sequences could support students' AT. Second, the activities were implemented with nine Grade 6 students over six consecutive lessons, each lasting approximately 80–90 minutes. Data were collected from students' worksheets, classroom protocols, and field notes, and analyzed using Araya's (2021) framework of AT: Searching, Decomposing Problems, Linking Commands, and Recursion. The findings showed that students demonstrated three components of algorithmic thinking: (1) Searching was shown through identifying and selecting objects that met given conditions by examining attributes such as color, quantity, and position. Students compared possible choices and determined which objects satisfied task requirements. (2) Decomposing Problems was demonstrated by breaking coloring tasks into smaller conditions and considering each requirement step by step before completing the overall solution. (3) Linking Commands was evident in students' ability to organize coloring actions in a logical sequence to satisfy multiple conditions within the same task. These findings suggest that well-sequenced coloring activities can support students' algorithmic thinking in primary mathematics classrooms.

References

Araya, R. (2021). Gamification Strategies to Teach Algorithmic Thinking to First Graders. In S. Nazir, T. Z. Ahram, & W. Karwowski (Eds.), *Advances in Human Factors in Training, Education, and Learning Sciences.* (Vol. 269, pp. 133–142). Springer.

Changsri, N., Inprasitha, M., Araya, R., & Isoda, M. (2024). Enhancing students' ideas of algorithm with coloring book in mathematics classroom. In T. Evans, O. Marmur, J. Hunter, G. Leach, & J. Jhagroo (Eds.), *Proc. 47th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 1, p. 117). PME.

Inprasitha, M. (2022). Lesson study and open approach development in Thailand: A longitudinal study. *International Journal for Lesson & Learning Studies, 11*(5), 1–15.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 526. PME.

THE STUDY ON GENDER DIFFERENCES IN MATHEMATICAL ANXIETY AND EMOTIONAL REGULATION AMONG MIDDLE SCHOOL STUDENTS IN TAIWAN AND SINGAPORE

Ching-wen Kao, Su-Wei Lin
National University of Tainan, Taiwan

Emotions are increasingly recognized as central to mathematics learning. Control-Value Theory (Pekrun et al., 2002) suggests that students' ability to regulate emotions supports achievement, while mathematics anxiety can hinder cognitive processing and problem solving (Beilock & Willingham, 2014). Previous studies have reported gender differences in mathematics-related emotions, but findings vary across cultural contexts. This study examines emotional control and mathematics anxiety as predictors of mathematics achievement and explores gender differences in two high-performing education systems. The research questions are:

- Do gender differences exist in emotional control and mathematics anxiety?
- How do emotional control and mathematics anxiety predict mathematics achievement?
- Do these relationships differ between Singapore and Taiwan?

Data were drawn from PISA 2022, including Singapore (n=6,606) and Taiwan (n=5,857). Emotional control and mathematics anxiety were measured using validated student questionnaire scales, and mathematics achievement was assessed through standardized mathematics literacy scores. A cross-sectional multilevel modeling approach was used to account for student- and school-level variation, and Bayesian estimation was applied to improve parameter estimation and address missing data. Results indicated that higher emotional control significantly predicted higher mathematics achievement, while higher mathematics anxiety significantly predicted lower achievement in both countries. Female students demonstrated significantly higher emotional control in both contexts. Overall, male and female students in Taiwan has higher mathematics anxiety than in Singapore. These findings highlight the importance of emotional regulation in mathematics learning and suggest that gender differences in emotional experiences are similar and need to explore culture context in the future. The results support the development of culturally responsive instructional and emotional support strategies to improve mathematics learning outcomes.

References

Pekrun, R., Goetz, T., Titz, W., & Perry, R. P. (2002). Academic emotions in students' self-regulated learning and achievement: A program of qualitative and quantitative research. *Educational Psychologist*, *37*(2), 91-105.

Beilock, S. L., & Willingham, D. T. (2014). Math anxiety: Can teachers help students cope? *Educational Leadership*, *72*(4), 10-15.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 527. PME.

DIDACTICAL TRANSFORMATION OF A FRACTAL STRUCTURE THROUGH LESSON STUDY: DESIGNING A SIERPIŃSKI-BASED MATHEMATICAL GAME

Oben Karahan Kanbolat[1], Duygu Arabacı[2]

[1]Faculty of Education, Erzincan Binali Yildirim University

[2]Faculty of Education, Düzce University

Fractals are mathematical structures that preserve their formal similarity under magnification and consist of recursively embedded substructures. However, such abstract structures cannot be directly transferred into instructional settings; they require pedagogical reconstruction. The framework of didactical transposition conceptualizes the process through which scholarly knowledge is transformed into an object of teaching (Chevallard, 1985). This study presents a qualitative case study examining the transformation of the Sierpiński fractal from a scholarly mathematical structure into an instructional object within a lesson study process. The participants were four prospective elementary mathematics teachers. Over a seven-week lesson study cycle, the participants designed, implemented, observed, and revised a mathematical game based on the Sierpiński fractal. Data sources included a game design bulletin, successive versions of the game, classroom implementation–observation notes, semi-structured interviews, and individual reflection reports. Data were analyzed within the framework of didactical transposition (Chevallard, 1985), tracing how fractal knowledge was reconstructed from its scholarly context into an instructional setting. Findings indicate that in the initial design, the fractal remained at the level of a visual representation. Following classroom implementation and observations, the scoring system was restructured to reflect the hierarchical and layered growth structure of the fractal. As a result of the revision process, the fractal evolved from a represented content element into the mathematical organizing principle of the game. Furthermore, the didactical transformation was found to be collectively constructed through mathematical reasoning within the lesson study process rather than individually enacted. These findings suggest that lesson study can function as a collective space of didactical transformation, enabling abstract mathematical structures to be reconstructed for instructional purposes while preserving their structural integrity rather than reducing them to superficial representations. Lesson study thus serves as a mediating space for the structural reorganization of mathematical knowledge.

References

Chevallard, Y. (1985). *La transposition didactique: Du savoir savant au savoir enseigné*. La Pensée Sauvage.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 528. PME.

SELF-EFFICACY, ANXIETY, AND LEARNING STRATEGIES ACROSS MATHEMATICS ITEMS OF VARYING DIFFICULTY

Jimmy Karlsson

Department of Mathematics, Karlstad University

Students' self-efficacy and learning strategies are important for mathematical development and predictive of achievement (Bandura, 1997). Social cognitive theory (SCT) proposes that students with higher self-efficacy employ more efficient learning strategies and exert greater persistence with challenging tasks. Furthermore, SCT suggests a negative relation between self-efficacy and anxiety, such that stronger self-efficacy beliefs are associated with lower anxiety and fewer negative emotions (Bandura, 1997). However, mathematics achievement is not a homogeneous outcome: test items vary in difficulty, and relations between self-related perceptions (e.g., self-efficacy), learning strategies, and achievement may differ across items of varying difficulty. The aim of this study is to examine how self-efficacy, anxiety, and students' use of rehearsal and self-regulation strategies are related to mathematics achievement across items of different difficulty. A total of 1009 upper secondary students in Sweden (age 16–17) completed a questionnaire measuring self-efficacy, anxiety, and learning strategies (rehearsal and self-regulation). Questionnaire data were linked to students' results on a test at the end of the mathematics course. Partial least squares structural equation modeling showed that higher anxiety was associated with greater use of rehearsal strategies ($\beta = 0.41$), whereas higher self-efficacy was associated with greater use of self-regulation strategies ($\beta = 0.46$) and overall achievement ($\beta = 0.49$). Item-level logistic regression analyses further revealed that anxiety was associated with lower odds ratios for success, especially for difficult items. Self-efficacy was positively associated with success across all item types, and the odds ratios increased with item difficulty. Rehearsal strategies showed decreasing odds ratios as item difficulty increased (mean 1.03 for easy items and 0.78 for difficult items), whereas self-regulation showed increasing odds ratios with increasing item difficulty (easy: 1.27, difficult: 1.38). Odds ratios were estimated for each item using standardized predictors. Items were ordered by success rate in the sample, and grouped into difficulty terciles. By distinguishing achievement outcomes by item difficulty, this study provides a more fine-grained account of how self-related perceptions and learning strategies are associated with mathematical achievement. The findings suggest that self-efficacy and self-regulation may be particularly important for success on more demanding items, whereas anxiety and rehearsal are associated with lower odds as difficulty increases.

References

Bandura, A. (1997). *Self-efficacy: The exercise of control*. W. H. Freeman.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 529. PME.

MATHEMATICS TEACHER EDUCATORS AS DESIGNERS

Signe E. Kastberg[1], Alyson E. Lischka[2]

[1]Purdue University, [2]Middle Tennessee State University

We are relational constructivist mathematics teacher educators (Steffe & D'Ambrosio, 1995) whose practices include curriculum design (Ackermann, 2012) for various goals in mathematics methods courses. Drawing from this perspective, we seek to improve our design practice through relationships with preservice teachers (PTs) alongside iterations of cycles of instructional activity design and researching those activities through PTs' responses to our designed activities (Zaslavsky & Sullivan, 2011). Drawing from Ackermann's designer dilemma we assume that PTs are autonomous and will interpret designed situations in unanticipated ways. Research on mathematics teacher educators (MTEs) as curriculum designers has identified general processes, categories for design elements, and influential factors. Less reported are insider perspectives of MTE development as curriculum designers. Our study of MTE design practice addressed the research question: How do MTEs design tasks to support PTs' development of curricular decision-making?

We use self-study, a practitioner focused methodology, and methods including dialogic analysis and reflections to develop awareness and attention to improve our design practice in the context of supporting preservice teachers' development as curriculum decision makers. Primary data sources from 2023-2025 included designed activities, reflections, and analytic dialogues. Secondary data sources included PTs' work samples, conference proposals, and proceedings encapsulating our design processes. Findings from dialogic analysis were supported by coded transcripts and reflections. Through dialogic analysis we developed awareness of constraints as a component of MTEs' curricular design. Although constraints initially were used to bound PTs' activity and resulting work samples, constructing models of PTs' curricular decisions resulted in our awareness of constraints as tools in activity design. Resulting intentional use of constraints provided PTs with situations that provoked discussion of curricular constraints. These discussions provided us with insight into PTs' rationales for curricular decisions in response to such constraints. Findings illustrate MTEs' awarenesses and attention to constraints as a component in MTE curricular design that supports PTs' development of curricular decision-making.

References

Ackermann, E. (2012). Construction and transference of meaning through form. In L. P. Steffe & J. Gale (Eds.) *Constructivism in Education* (pp. 341-354). Routledge.

Zaslavsky, O., & Sullivan, P. (2011). Setting the stage: A conceptual framework for examining and developing tasks for mathematics teacher education. In O. Zaslavsky & P. Sullivan (Eds.) *Constructing Knowledge for Teaching Secondary Mathematics*, 1–19.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 530. PME.

EQUALITY UNDER UNCERTAINTY: STUDENTS' REASONING ABOUT SMALL MEASUREMENT DISCREPANCIES

Elena Kazakevich, Nadav Marco

The David Yellin Academic College of Education

Recent research highlights an epistemic tension between school mathematics, centred on exact equality, and real-world mathematics, where results are approximate because they rely on measurement. In STEM activities, students must coordinate theoretical equality with empirical agreement based on measured data, even when small discrepancies appear. Klein's precision–approximation distinction remains useful for task design and mathematical literacy (Marco & Palatnik, 2024). Yet students often struggle to interpret minor numerical differences and decide whether they reflect meaningful effects or acceptable error (Wan, 2023). This study examines how students interpret small weight discrepancies when geometrical reasoning suggests equality.

We analysed 68 written submissions from 9th–11th-grade students in the Two Jewel Activity (Kazakevich & Marco, 2023), spanning four stages that required estimating and interpreting a difference in gold quantity. We identified four response categories; the two most salient were: (i) students who treated the weight discrepancy as negligible or explainable without undermining intended equal gold quantity (n = 29, ~43%), citing measurement imprecision, aging/dirt/material loss, or physical factors such as thickness, density, impurities, or voids; and (ii) students who treated the difference as critical (n = 15, ~22%), arguing that equal areas do not imply equal gold quantities and that the discrepancy undermines the geometrical proof.

Overall, the activity makes visible a productive epistemic tension between mathematical precision and empirical approximation. We argue that the design of the activity intentionally creates this tension and can support students' STEM reasoning by promoting coordination between theoretical proof and measurement-based interpretation. Such tasks may broaden students' conception of equality beyond strict numerical identity and strengthen their capacity to reason under uncertainty.

References

Kazakevich, E., & Marco, N. (2023, July). Mathematical proving for subversive critical thinking. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel., & M. Tabach (Eds.), *Proc 46th Conf Int Group Psychol Math Educ* (Vol. 3, pp. 147–162). PME.

Marco, N., & Palatnik, A. (2024). Teachers pose and design context-based mathematics tasks: What can be learned from product evolution?. *Educ Stud Math., 115*, 223–246.

Wan, T. (2023). Investigating student reasoning about measurement uncertainty and ability to draw conclusions from measurement data in inquiry-based university physics labs. *Int J Sci Educ, 45*(3), 223–243. https://doi.org/10.1080/09500693.2022.2156824

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 531. PME.

HOW DO PRIMARY SCHOOL MATHEMATIC EDUCATORS CONNECT SCHOOL MATHEMATICS TO UNIVERSITY MATHEMATICS?

Azimehsadat Khakbaz; Bu-Ali Sina University

One challenge in mathematics teacher education curricula is that the mathematics that is taught to future teachers at university differs from the mathematics they are expected to teach as primary school teachers which has first explained by Felix Klein at the beginning of 20th century. He believed teachers experienced a "double discontinuity". The first discontinuity occurs when student-teachers transition from learning school mathematics, which is intuitive and practical, to studying university mathematics, which is abstract and theoretical. The second discontinuity arises when these future teachers attempt to return to the school classroom and use what they have learned at university to teach school mathematics effectively. Dreher et. Al (2018) believed in a mutual relationship between school mathematics and academic mathematics since academic mathematics precedes school mathematics which serving as a framework for its structure and school mathematics precedes academic mathematics as it provides a path to understanding it. However, research showed academic courses lack opportunities for future teachers to acquire the knowledge and skills to construct a didactic ''bridge'' between university mathematics and student-oriented primary mathematics (Wasserman, Buchbinder & Buchholtz, 2023). On the other hand, student teachers may have low motivation to learn academic mathematics if they cannot see its direct application to school teaching (Guse, Kienhues & Jucks, 2023). Moreover, mathematics teacher educators, as the main agents of teacher education curricula, adopt strategies to fill this gap in their professional practice, which the present study aimed to explore. This study aims to introduce these strategies by examining the lived experiences of 12 mathematics teacher educators at four German universities, employing a phenomenological approach. For this purpose, data were collected through semi-structured interviews and analyzed using coding and categorization. Data analysis revealed four general strategies: top-down connections (applications of academic mathematics in school mathematics), bottom-up connections (background mathematical concepts in school mathematics textbooks), multiple representations and solutions, and predicting and interpreting students' errors and misunderstandings.

References

Dreher, A., Lindmeier, A., Heinze, A. et al. What kind of content knowledge do secondary mathematics teachers need? *Journal for Mathematics Didactic, 39,* 319-341.

Guse, S., Kienhues, D., & Jucks, R. (2023). Math is cool—for school? Insights into preservice teachers' views on the mathematics discipline between an academic and a job-related focus. *Teaching and Teacher Education, 132*, 104247.

Wasserman, N. H., Buchbinder, O., & Buchholtz, N. (2023). Making university mathematics matter for secondary teacher preparation. *ZDM–Mathematics Education, 55*(4), 719–736.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 532. PME.

MATHEMATICAL IMAGINATION WITHIN THE CLASSROOM

Francesca Ferrara, Giulia Ferrari, Kalina Maria Kijewska

Università di Torino, Torino

Despite the relevance of imagination for mathematics learning (Kotsopoulos & Cordy, 2009), research on this topic is still scarce (e.g. Lajos et al., 2025). Imagination is a complex process; aspects central to it are its dynamic nature, temporality, the abstract-concrete tension, the interaction of action and perception, the interplay of actual and possible (Gfeller & Zittoun, 2021; Rucińska, 2024). Imagination evokes visual perception without being something that is actually depicted or seen, so it eludes linear accounts or contours. We see imagination as emerging in utterances and embodied interactions that may be situated and limited in space and time, but that may unfold covertly in a wider time span and may be dispersed across collectives rather than pertaining to the individual. We take imagination to be materially enmeshed with any learning assemblage and we suggest that, along these lines, characterizing mathematical imagination would be more like knowing the texture of a piece of cloth by smelling it, touching it, and/or hearing it move, than looking at its label.

We attempt to grasp the complex nature of mathematical imagination by analysing a classroom episode engaging grade 3 children with a pattern activity that was designed in the context of a larger intervention study aimed to develop early algebraic thinking. Using video data, we focus on how new mathematical relationships and regularities come to matter, making the imaginative process increasingly mathematical, as well as increasingly collective. We also highlight how different imaginative paths might have been elicited by different choices inside the classroom, speaking directly to the material, embodied, enactive and social dimension of mathematical imagination.

References

Gfeller, F., & Zittoun, T. (2021). The embodied dimension of imagination. Expanding the loop model. *Integrative Psychological & Behavioral Science, 55*(1), 73–88. https://doi.org/10.1007/s12124-020-09550-3

Lajos, J., Soto, H., & De Jesus Pagan, F. (2025). Imagination as an embodied space for engaging with abstract algebra concepts. *The Journal of Mathematical Behavior, 80*, Article 101275. https://doi.org/10.1016/j.jmathb.2025.101275

Kotsopoulos, D., & Cordy, M. (2009). Investigating imagination as a cognitive space for learning mathematics. *Educational Studies in Mathematics, 70*(3), 259–274. https://doi.org/10.1007/s10649-008-9154-0

Rucińska, Z. (2024). Enactive Imagination: Its Roots and Contemporary Horizons. In G. Wellner, G. Dierckxsens, & M. Arienti (Eds.), *The Philosophy of Imagination: Technology, Art, Ethics* (pp. 203–215). Bloomsbury.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 533. PME.

ANALYSIS OF STUDENTS' COMMUNICATION IN LESSONS ON SIMILARITY USING DYNAMIC GEOMETRY SOFTWARE

Yeonha Kim, Bomi Shin

Chonnam National University

Similarity instruction tends to emphasize algorithms, which may result in student misconceptions. This study investigates mathematical communication facilitated by the DGS AlgeoMath's manipulability. We categorize learning into: similarity ratios, triangle similarity conditions, and the congruence-similarity comparison to derive pedagogical implications. Grounded in the communicative perspective of Sfard (2008) and theoretical framework established by Yu and Song (2013), 27 eighth-graders performed similarity tasks using the DGS's dilation, rotation, and reflection. Lesson data (transcripts, screen captures, worksheets) were analyzed on the basis of Table 1.

Table 1: Mathematics Communication Analysis Framework

Sub-domain	Communication Types (Codes)
Understanding Expression	Understanding of Term (U1), Misuse (U2)
Judging Others' Strategies	Questioning (J1), Counter-questioning (J2), Doubt (J3), Affirmation (J4), Comprehension (J5)
Reflection on Thinking	Revision (R1), Reflection (R2), Justification (R3), Clarification (R4), Generalization (R5), Inquiry (R6)

The students initially confused similarity ratios with area ratios. However, AlgeoMath's visual feedback led them to doubt (J3) these hypotheses and reconstruct meanings by reflectively re-examining (R2) expansion ratios based on side lengths. They comprehended (J5) why the two overlapped triangles were congruent and generalized (R5) AA similarity from ASA congruence. Furthermore, they refined their discourse by comparing the concepts that similarity involves overlapping after dilation/rotation (U1), whereas congruence requires the same size (R4). According to the above, AlgeoMath lessons expanded mathematical discourse, fostering student growth. These outcomes are significant when DGS features are combined with teacher questioning and noticing that stimulate student thinking.

References

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourse, and mathematizing*. Cambridge: Cambridge University Press.

Yu, S., & Song, S. (2013). Process analysis on mathematical communication and analogical thinking through trapezoid's area obtaining activity. *The Mathematical Education, 23*(2), 253–267. (In Korean).

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 534. PME.

NEGOTIATING AUTONOMY AND PRESCRIPTION: ENACTING THE TRR CURRICULUM IN SWEDISH CLASSROOMS

Tuula Koljonen

Department of Behavioural Sciences and Learning, Linköping University

Recent efforts to introduce more structured approaches to mathematics instruction have renewed interest in how teachers interpret and enact prescriptive curricula. In Sweden, where curriculum materials have traditionally been optional and descriptive rather than mandatory, the introduction of the TRR (Thinking, Reasoning, and Reckoning) curriculum represents a marked shift. TRR consists solely of a teacher's guide and promotes activity-based learning through detailed lesson sequences. This shift creates a tension between prescriptive instructional design and a system characterised by strong teacher autonomy, reflecting a broader international challenge of balancing standardisation with professional judgement. Despite international research on teacher agency and curriculum enactment, we still know little about how Swedish teachers work with highly structured activity sequences in practice. This pilot study addresses this gap by examining one teacher's work with TRR over a six-month period. Drawing on nine video-recorded lessons and two interviews, the study explores the research question: How does a teacher balance professional autonomy with the prescriptive elements of the TRR curriculum in classroom practice? The analysis combines Priestley et al.'s (2012) ecological model of teacher agency with the Consolidated Framework for Implementation Research (CFIR) to clarify how intervention characteristics, contextual conditions, and teacher judgement interact during enactment. Preliminary findings suggest that the teacher actively negotiates the degree of adherence to TRR, alternating between following the prescribed activity sequences and adapting them to meet student needs, time constraints, and existing routines. Rather than simply complying or resisting, the teacher engages in situated, moment-to-moment decision-making that reflects both the affordances and constraints of TRR. The structured nature of TRR generates tensions—given Sweden's weak tradition of prescriptive materials—while also supporting lesson organisation and mathematical focus. These findings illustrate how teacher agency operates through moment-to-moment instructional decisions and how these decisions reshape the TRR model in practice.

References

Priestley, M., Edwards, R., Priestley, A., & Miller, K. (2012). Teacher agency in curriculum making: Agents of change and spaces for manoeuvre. *Curriculum Inquiry, 42*(2), 191-214.

Nilsen P. (2015). Making sense of implementation theories, models and frameworks. *Implement Science, 10*(53), 1-13.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 535. PME.

SENSING THE PROCEDURE, MISSING THE SENSE: EPISTEMOLOGICAL OBSTACLES IN PRE-SERVICE TEACHERS' UNDERSTANDING OF RADICAL EXPRESSIONS

Himmet Korkmaz[1], Mahmut Kertil[2], Ali İhsan Mut[3]

[1]Ordu University, [2]Marmara University, [3]Dicle University

Several mathematical concepts have historically emerged from moments of epistemic crisis, one of which is radical numbers. Originating from attempts to resolve mathematical inconsistencies and conceptual tensions, these concepts carry traces of their historical struggles into contemporary classrooms. In mathematics education, such historically rooted difficulties have been described as epistemological obstacles. Research consistently indicates that students experience persistent challenges in learning and making sense of radical expressions (Fischbein et al., 1995; Sirotic & Zazkis, 2007). The present study aims to examine whether pre-service mathematics teachers are able to meaningfully interpret and justify the procedures they perform when working with radical expressions, thereby exploring the extent to which procedural fluency aligns with conceptual understanding. This qualitative case study involved 40 third-year pre-service mathematics teachers. A two-dimensional open-ended test designed to elicit both procedural and conceptual reasoning was administered, followed by semi-structured interviews with five participants to gain deeper insight into their reasoning. The collected data were analysed using descriptive and content analysis techniques. The findings revealed a clear discrepancy between procedural success and conceptual understanding. Although approximately 90% of participants correctly solved operation-based questions involving radical expressions, the majority struggled to explain why the procedures were mathematically valid or what underlying meanings justified them. From a procedural and conceptual learning perspective, the results indicate that correct procedures were not supported by strong conceptual understanding. Moreover, the parallel between the historical emergence of radical numbers and the conceptual struggles observed among future teachers suggests that epistemological obstacles may continue to shape contemporary learning. Given that these pre-service teachers will soon assume responsibility for teaching such topics, the study highlights the importance of fostering deeper conceptual engagement with mathematically and historically complex concepts that inherently embody epistemological tensions.

References

Fischbein, E., Jehiam, R., & Cohen, D. (1995). The concept of irrational numbers in high-school students and prospective teachers. *Ed. Studies in Mathematics, 29*(1), 29-44.

Sirotic, N., & Zazkis, R. (2007). Irrational numbers: The gap between formal and intuitive knowledge. *Educational Studies in Mathematics, 65*(1), 49-76.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 536. PME.

FROM OPERATIONAL USE TO STRATEGIC LEADERSHIP: A T3 MODEL–BASED APPROACH FOR MATHEMATICS TEACHERS' DIGITAL TRANSFORMATION

Berna Aygün[1], Merve Koştur[2]

[1]Süleyman Demirel University, [2]Başkent University

In the rapidly digitizing education ecosystem, it is not considered sufficient for teachers to use technology merely at a tool-based level; they are expected to redesign it in a way that creates pedagogical transformation. This study investigates mathematics teachers' digital transformation through the lens of the T3 Model (Magana, 2017). The T3 framework offers a hierarchy ranging from Translational (T1) (efficiency-focused instrumental use) to Transformational (T2) (where cognitive shifts occur through student-led production) and Transcendent (T3) (focusing on self-directed inquiry and social entrepreneurship). This research specifically examines how a structured professional development program can transition mathematics teachers from operational users to strategic leaders capable of redesigning digital learning environments.

Using a qualitative research design, the study involved 10 mathematics teachers from private and public schools. The professional development program was implemented through a model-based webinar series designed to support teachers' progression through the T3 stages by integrating both field-independent (e.g., assessment tools) and field-specific (e.g., dynamic geometry software, online manipulatives) digital tools. Teachers explored AI-supported applications and innovative assessment methods, adopting a teacher-entrepreneur perspective. Data collection involved pre-interviews, teacher lesson plans, classroom video recordings, field notes, and semi-structured interviews. The data obtained were analyzed using thematic analysis.Findings suggest that the T3-based professional development effectively shifted teachers' practices from translational to transformational and strategic level. In particular, it has been determined that participants moved away from using technology solely as a tool for efficiency and instead focused on developing pedagogical designs that restructured mathematics teaching. Furthermore, teachers exhibited increased competence in adapting AI tools to enrich pedagogical environments. This study provides evidence for the T3 Model's applicability in mathematics education and offers insights for designing professional development that supports sustainable digital transformation.

References

Magana, S. (2017). *Disruptive Classroom Technologies: A Framework for Innovation in Education*. Corwin Press.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 537. PME.

FROM UNDERSTANDING TO ENACTMENT: HOW PRE-SERVICE PRIMARY TEACHERS IMPLEMENT FORMATIVE ASSESSMENT IN ELEMENTARY MATHEMATICS LESSONS

Tereza Kottová, Darina Jirotková

Faculty of Education, Charles University

Formative assessment (FA) is widely recognized as a powerful approach to improving pupils' learning and fostering deeper mathematical understanding. However, in mathematics, teachers may struggle to move from procedural evaluation to feedback that addresses pupils' specific misconceptions. This longitudinal study traces how five pre-service primary teachers develop their understanding and classroom enactment of FA in mathematics , drawing on Wiliam and Leahy's (2015) framework. The study addresses two research questions: (1) How does the understanding of FA in mathematics develop over time? and (2) Which FA tools do they enact in mathematics lessons to address pupils' thinking?

The study adopts a qualitative longitudinal multiple-case design following five participants over two years, from initial training to their first year as in-service teachers. Data were collected across four phases and included video-recorded mathematics lessons (15 lessons in total), semi-structured interviews with post-lesson reflections, and an online questionnaire. To analyze classroom practice (RQ2), a deductive coding scheme based on Wiliam and Leahy's (2015) five strategies and 29 subcategories was used as an observation protocol.

Regarding RQ1, analysis revealed a conceptual shift. Initially, participants viewed FA mainly as "kinder verbal feedback" or a descriptive alternative to marks. Over time, their accounts shifted toward a process-oriented view linking explicit learning goals, success criteria, and pupils' responsibility. Regarding RQ2, lesson analyses showed a gradual but uneven expansion of FA repertoires. After the university course, teachers introduced tools like mini-whiteboards and random-selection techniques to make mathematical thinking visible. However, explicit work with success criteria remained weak. For instance, when addressing a recurring error in written subtraction (52–48), a teacher successfully used mini-whiteboards to surface the specific misconception (forgetting to decrease the tens column after regrouping). Yet, the formative cycle remained incomplete, suggesting that participants were more successful in eliciting pupils' mathematical thinking than in turning such evidence into stable formative routines.

References

Wiliam, D., & Leahy, S. (2015). *Embedding formative assessment: Practical techniques for K-12 classrooms*. Learning Sciences International.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 538. PME.

USING STUDENT SOLUTIONS TO PROMOTE DISCUSSION OF MATHEMATICS TEACHING AND LEARNING

Daniela Kovalčíková, Ingrid Semanišinová

Pavol Jozef Šafárik University in Košice

Teachers who strive to understand how students approach a task and base their teaching decisions on this knowledge create a positive learning environment. One way teachers can gain insight into students' thinking is by analysing their solutions. In this paper, we present research aimed at encouraging productive discussions about student thinking among in-service mathematics teachers (ISTs). Some of them are Interpreters and some of them are Evaluators (Gehrtz, Hagman, & Barron, 2024). To promote such discussion, we created a research tool consisting of a geometric task from a mathematics competition for 14-year-olds and seven student solutions to this task.

The research took place at a summer school workshop designed to prepare students for a maths competition, with 26 teachers participating. The workshop consisted of three phases. First, the ISTs solved the problem individually. Second, they had to rank seven student solutions from best to worst. Third, they engaged in a collective discussion to evaluate the individual student solutions. The seven students employed five different strategies: two used an inductive strategy, while five used four different deductive strategies. These strategies were based on various properties of triangles and exhibited differing levels of argumentation. Two students reached the expected result.

Our findings indicate that the research tool effectively promotes discussion about mathematics teaching and learning during the third phase of the workshop. The ISTs tried to interpret how students approached the task and to consider possible reasons for errors or shortcomings in their solutions (Interpreters). This positively influenced those who focused mostly on the correctness of the result (Evaluators). Furthermore, some ISTs pointed out inconsistencies in the assessments. For example, some rated a solution containing an uncalculated numerical expression more highly than one with an incorrect calculation, and a solution without a visual representation more highly than one with an inaccurate sketch. Therefore, the discussion mobilised mathematics teacher's specialised knowledge of the participating ISTs (Carrillo et al., 2018).

References

Gehrtz, J., Hagman, J. E., & Barron, V. (2024). Engagement with student written work as an instantiation of and proxy for how college calculus instructors engage with student thinking. *The Journal of Mathematical Behavior, 76*, 101187.

Carrillo-Yáñez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., & Ribeiro, M. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education, 20*(3), 236–253.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 539. PME.

PROBLEM POSING AND SELF-EFFICACY IN MATH: DO BASIC NEED EXPERIENCES MEDIATE INTERVENTION EFFECTS?

Janina Krawitz[1], Lars-Meyer-Jenßen[2], Katrin Rakoczy[3]

[1]University of Cologne, [2]University of Erfurt, [3]University of Gießen

Self-efficacy—students' beliefs in their capabilities to organize and execute actions required to attain learning goals—is a central construct in motivation research and a strong predictor of engagement and achievement (Bandura, 1997). According to Self-Determination Theory (SDT), experiencing autonomy, competence, and relatedness supports motivational beliefs such as self-efficacy. From this perspective, self-efficacy is shaped not only by performance outcomes but also by the quality of motivational experiences in instructional contexts. Problem posing constitutes a promising instructional practice in this regard (Cai & Leikin, 2020). By allowing learners to generate their own mathematical problems, problem posing enables meaningful choice, ownership of learning content, and social interaction, which may support experiences of autonomy, competence, and relatedness, which in turn improve self-efficacy. Although previous studies have reported positive effects of problem posing on self-efficacy, little is known about the mechanisms through which these effects emerge. In the present study, we tested a model that connects problem posing as an autonomy-supportive intervention, experiences of basic needs during the intervention, and self-efficacy in mathematics. We conducted an experimental study with N = 194 tenth-grade students. We compared a four-lesson intervention in which students either posed and solved their own mathematical problems or solved given problems. After each lesson, students reported their experiences of autonomy, competence, and relatedness. Self-efficacy in mathematics was assessed before and after the intervention. Results showed no total effect of posing and solving own problems on self-efficacy in mathematics. However, posing and solving own problems increased students' experiences of autonomy and relatedness (but not competence). Experiences of autonomy, competence, and relatedness were positively associated with self-efficacy, and autonomy and relatedness mediated the effect of problem posing on self-efficacy. Prior self-efficacy did not moderate intervention effects on basic needs. The findings refine motivational accounts of problem posing by highlighting autonomy-supportive mechanisms that can indirectly foster self-efficacy.

References

Bandura, A. (1997). *Self-Efficacy: The exercise of control*. Freeman.

Cai, J., & Leikin, R. (2020). Affect in mathematical problem posing: Conceptualization, advances, and future directions for research. *Educational Studies in Mathematics, 105*(3), 287—301.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 540. PME.

PARTICIPATION WITHOUT MATHEMATICAL AGENCY

Pablo Duran Oliva[1], Rebecca Butler[1], Peter Karanevich[2], Nancy Kress[2]

[1]Virginia Commonwealth University, [2]University of Colorado Boulder

Undergraduate mathematics instruction has shifted in recent decades from lecture-based practices toward more frequent use of active, inquiry-based strategies. While these shifts may increase student agency, without explicit attention to classroom power structures and positionalities, these contexts may inadvertently perpetuate traditional participation patterns (Hagman, 2021). Applying positioning theory (Drageset & Ell, 2024) and Esmonde's framework for Opportunities to Learn (OtL) (2009) we investigate how participation, identity, and communication shape mathematical authority for students in small groups in an undergraduate precalculus classroom. We examined data consisting of conversational-turn transcripts from three consecutive class sessions, focusing on one focal student, a peer, and their instructor. Discourse was analysed at the turn-of-talk level, distinguishing between inquiry-oriented contributions (e.g., explanation, justification) and validation-oriented moves (e.g., seeking or confirming correctness) to understand the focal student's positionality within the mathematical community. Our findings show that the nature of the focal student's communications differed dramatically from those of their peer. While the focal student contributed over half the conversational turns (56%), these focused largely on seeking validation. The peer student had fewer conversational turns (44%), but those contributions consisted primarily of inquiry and providing validation. These patterns suggest a stable expert–novice dynamic, potentially limiting the focal student's opportunities to move toward more agentic participation. Our findings indicate that without explicit interventions in negotiating mathematical authority, students' roles can remain fixed, resulting in inquiry-based classrooms where social inclusion co-exists with mathematical marginalization.

Acknowledgements

This work was partially supported by the National Science Foundation (NSF) under Grant No. 2400902, No. 2400903. Any opinions and conclusions expressed are those of the authors.

References

Drageset, O. G., & Ell, F. (2024). Using positioning theory to think about mathematics classroom talk. *Educational Studies in Mathematics*, *115*, 353–385.

Esmonde, I. (2009b). Mathematics Learning in Groups: Analyzing Equity in Two Cooperative Activity Structures. *Journal of the Learning Sciences*, *18*(2), 247-284. https://doi.org/10.1080/10508400902797958

Hagman, J. E. (2021). The Eighth Characteristic for Successful Calculus Programs: Diversity, Equity, & Inclusion Practices. *PRIMUS*, *31*(1), 70–90.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 541. PME.

IMPLEMENTING THINKING CLASSROOM PRACTICES IN A UNIVERSITY CALCULUS REMEDIAL COURSE

Bjarnheiður Kristinsdóttir

University of Iceland

Previous research indicates that university STEM students are more likely to succeed when teachers use active teaching methods (Freeman et al., 2014), yet most university teaching still follows teacher-led instruction (Børte et al., 2023). This was also the case in Calculus I at the University of Iceland in fall 2024. Of 340 enrolled students, 257 signed up for the exam and 142 passed the course. The 115 students who failed were invited to participate in a remedial course in spring 2025. The remedial course was designed using thinking classroom practices (Liljedahl, 2020) to support students' active participation, shared thinking and individual sense-making. Students attended two weekly sessions for ten weeks (minimum 80% attendance) working in visibly random groups of three on vertical whiteboards, engaging with curricular tasks, and receiving formative feedback through viewing other groups' boards as well as through questioning, hints, and whole-class consolidation led by the two instructors. Weekly interactive check-your-understanding worksheets with immediate feedback and links to further exercise problems supported independent practice. In this case study, we looked for potential changes in students' perceived value of attending exercise classes and academic success. Of the 86 students who completed the remedial course, 69% passed the Calculus I equivalent examination. A post-course survey (37% participation) indicated increased study engagement and strong student support for the thinking classroom approach. Qualitative responses emphasized collaborative problem solving, peer explanation, and increased willingness to express uncertainty as key learning affordances. As a result of these outcomes, Calculus I exercise classes in fall 2025 were redesigned and tutors received training to adopt thinking classroom teaching practices. This talk reports further findings from the case study and discusses implications for course design, remediation structures, and the use of active learning pedagogies in first-year university mathematics.

References

Børte, K., Nesje, K., & Lillejord, S. (2023). Barriers to student active learning in higher education. *Teaching in Higher Education*, *28*(3), 597–615.

Freeman, S., Eddy, S. L., McDonough, M., Smith, M. K., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences*, *111*(23), 8410–8415.

Liljedahl, P. (2020) *Building Thinking Classrooms in Mathematics*. Corwin Press.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 542. PME.

FACILITATORS' STATEMENTS ON DIFFERENTIATED TASKS IN MENTAL CALCULATION STRATEGIES OF SUBTRACTION

Marlene Krüger, Birte Pöhler

Institute of Mathematics, University of Potsdam

Due to the heterogeneity of students in mathematics classrooms, student focus and instructions providing tasks for different competency levels are important. Students at the basic level with fragile conceptual understanding must be a focus, as gaps can hinder further learning. Therefore, teachers must use tasks aligned with students' levels. These tasks can be categorised as supporting, fostering, or challenging (Prediger et al., 2023; Vygotsky, 1978). Supporting compensates for short-term missing prerequisites by reducing cognitive load. Fostering targets the long-term development of basic concepts, whereas challenging involves tasks slightly above current abilities. Future professional development (PD) research should optimise approaches to overcome the prevalence of supporting practices and strengthen fostering aimed at learning progress (Prediger et al., 2023). Within the nationwide German QuaMath program, a teacher PD is designed to foster differentiated, student focused secondary mathematics instruction. Facilitators play a key role in mediating this content and must make informed statements about tasks for different learning levels. Against this background, this study investigates how facilitators evaluate task suitability for supporting, fostering, or challenging a basic-level student in mental calculation strategies of subtraction. To this end, 15 facilitators assessed three tasks for a Year 5 student. Data were analysed using inductive qualitative content analysis. Two patterns emerged: half of the facilitators identified the fostering task as most appropriate for the student's conceptual development, while others prioritized supporting or challenging tasks. The fostering task was rejected due to cognitive overload from additional work on the concrete level, a perceived mismatch between demands and learning level, and unuseful material use. In contrast, the supporting task was justified by its immediate useful function, and the challenging task by its role in promoting reflection or motivation, sometimes with reduced solution expectations. Findings indicate that some facilitators associate fostering with increased task difficulty rather than the development of conceptual understanding. This suggests a need for facilitator PDs to explicitly address the distinctions between supporting, fostering, and challenging.

References

Prediger, S., Dröse, J., Stahnke, R., & Ademmer, C. (2023). Teacher expertise for fostering at risk-students' understanding of basic concepts: conceptual model and evidence for growth. *Journal of Mathematics Teacher Education*, *26*(4), 481–506.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes* (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Eds. & Trans.). Harvard University Press.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 543. PME.

TRANSLANGUAGING AND AI IN EXPLANATORY VIDEO PRODUCTION

Jessica Kunsteller, Simeon Schwob

University of Muenster

This study investigates how multilingual primary students use artificial intelligence (AI) during explanatory video production in mathematics. In linguistically diverse classrooms, meaning is negotiated across languages, representations, and digital tools. While translanguaging supports meaning-making (García & Wei, 2014), little is known about how learners engage with AI-generated output. We conceptualize explanation as an interactional, multimodal process (Barwell, 2018). AI is understood as a mediating artifact that generates probabilistic linguistic proposals which must be interpreted in context. From this perspective, learner agency (Noren, 2015) becomes visible in how students accept, modify, or reject AI output. The study follows an interpretative qualitative design with multilingual students (grades 3–4) working on "Reach 1000" and producing explanatory videos; data include recordings, transcripts, and chat logs.

In one episode, a student uses AI to translate an explanation from Arabic into German. The output introduces a semantic shift ("minutes" instead of numbers) that does not fit the mathematical context and is rejected. After prompt reformulation, the output improves, yet shared understanding is not achieved, showing that correct translation does not guarantee mathematical understanding. Meaning emerges through negotiation. Learners display agency by critically evaluating AI output, while interaction supports reflection on language through prompt refinement.

These findings suggest that AI-supported translanguaging is an interpretative accomplishment rather than a transfer process. AI introduces an additional layer of mediation that can shift meanings in mathematically problematic ways, making learners' evaluative work central. Explanatory video production provides a productive setting to observe these dynamics (Oechsler & Borba, 2020).

References

Barwell, R. (2018). From language as a resource to sources of meaning in multilingual mathematics classrooms. *Journal of Mathematical Behavior*, *50*, 155-168.

García, O. & Wei, L. (2014). *Translanguaging. Language, Bilingualism, and Education*. Palgrave Macmillan.

Noren, E. (2015). Agency and positioning in a multilingual mathematics classroom. *Educational Studies in Mathematics*, *89*(2), 167-184.

Oechsler, V., & Borba, M. (2020). Mathematical videos, social semiotics and the changing classroom. *ZDM Mathematics Education*, *52*(2), 377–389.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 544. PME.

DIALOGISM OF MATHEMATICS: RETHINKING THE MATHEMATICAL EXPERIENCE

Antoine Lachance

Simon Fraser University

Mathematics Education has extendedly theorized and studied the activity of doing mathematics: what it means to do mathematics, what it yields in terms of outcomes for the person and what the broader experience of mathematical activity itself is. While prior research emphasizes learning through doing mathematics, less attention has been given to the real time situated "happeningness" of mathematics. This study mobilizes a didactical perspective to theorize the process of meaning-making in mathematical practices. More precisely, it explores a multi-faceted perspective constituted of three main constructs: debugging/*bricolage* (Papert, 1980), ambiguity (Byers, 2007) and dialogic (Bakhtin, 1975/1981). Rather than attempting a comprehensive synthesis, this study adopts a focused articulation of these realities, aiming to capture mathematics as lived practices in which every mathematical act can be seen as opening a space between perspectives and making meaning from differences. Yet, the lived interaction of these dimensions in real-time practice remains to be examined. The research questions are: *How is the interplay of debugging, ambiguity and dialogism lived in mathematical practices? How does this dialogical reality shape mathematical meaning-making?*

Mobilizing a task-based interview protocol on the theme of spherical geometry, preliminary results of a pilot study reveal how dialogical structures of meaning-making emerge in mathematical activity. In particular, iterative debugging processes and situations of mathematical ambiguity appear to function as productive local sites of meaning negotiation. For instance, tasks pertaining to spherical triangles have led participants to live different versions of the mathematical notion of line straightness on spheres: both as locally straight from a phenomenological/embodied standpoint of someone walking *and* as globally curved from external geometric perspective. Here, the meaning of straight lines appeared to be less lived as converging toward a single stabilized understanding, and more as a situated alignment of differing perspectives that may remain, at least partially, irreducible to one another. Such observations lead to a view of mathematical activity as one of continual rearticulation in which meaning is constituted through the dynamic maintenance of tensions rather than their removal.

References

Bakhtin, M. M. (1981). *The dialogic imagination: Four essays.* (C. Emerson & M. Holoquist, Trans.). University of Texas Press. (Original work published in 1975)

Byers, W. (2007). *How mathematicians think: Using ambiguity, contradiction, and paradox to create mathematics.* Princeton University Press.

Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas.* Basic books.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 545. PME.

IMPLEMENTATION OF THE RESPONSE TO INTERVENTION MODEL (RTI): THE EFFECTS OF TIER-2 INTERVENTION IN MATHEMATICS FOR AT-RISK FIRST GRADE STUDENTS

Noémie Lacombe[1], Anne-Françoise de Chambrier [2]

[1]University of Fribourg, [2]University of Teacher Education of Lausanne

Longitudinal data show that difficulties in mathematics identified at school entry tend to persist throughout schooling and into adulthood (Morgan et al., 2009). However, early and targeted interventions in mathematics, particularly RTI Tier 2, can significantly improve performance and reduce the proportion of students experiencing difficulties (Jitendra et al. 2021). This research therefore aims to test the effectiveness of implementing Tier 2 intervention in first-grade classes in Switzerland. These models are still relatively uncommon in Europe compared to North America. The research questions are as follows: a. How effective is the Tier 2 intervention program ? b. What is the perceived social validity of the intervention?

This randomized controlled trial (RCT) study was conducted with 24 classes from 9 schools with 356 students (age of 83.3 months SD=4.13) in French speaking part of Switzerland. All student were tested in mathematics and pupils with scores <0.75 standard deviation below the mean were considered at risk. In the study, 43 students were randomly assigned to the experimental group and 39 to the control group. Fifty-six scripted lessons were developed for the Tier-2 program and conducted by teachers to small groups of 4 to 6 students, 3-4 times a week for 14 weeks. Finally, the teachers completed a questionnaire in order to mesure social validity. The results showed that in the post-test, the difference in scores between the experimental group and the control group was significant with a moderate effect size $F(1.79)=6.16$, $p=.02$, $d=0.29$. In addition, none of the children in the experimental group were in the pathological range at the end of the intervention, compared to 13% in the control group. Secondly on a 6-point scale, teachers report that Tier 2 program is useful ($M=5.06$, $SD=0.65$), usable ($M=5.25$, $SD=0.86$), and acceptable ($M=5.33$, $SD=0.64$). Thus, the positive effects observed on learning, combined with strong social validity, indicate that structured, explicit, and scripted interventions can be meaningfully integrated into teachers' regular practices to support at-risk students.

References

Jitendra et al. (2021). The Effects of Tier 2 Mathematics Interventions for Students With Mathematics Difficulties: A Meta-Analysis. *Exceptional Children, 87*(3), 307–325.

Morgan, P. L., Farkas, G., & Wu, Q. (2009). Five-year longitudinal associations between initial disability status and subsequent academic and behavioral functioning. *Journal of Learning Disabilities, 42*(3), 197–214

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 546. PME.

VARIATION IN SIXTH GRADERS' MULTIPLICATION ACROSS STRUCTURES: THE INTEGER–FRACTION TRANSITION

Yen-Ting Lai[1], Chia-Hsin Kuo[2], Erh-Tsung Chin[1]

[1]National Changhua University of Education, [2]National Chi Nan University

This study explores how sixth graders' multiplication performance varies as they move from integer to fraction multiplication across multiplicative structures. While students often develop procedural fluency with whole-number multiplication, this knowledge does not readily transfer to fractions, which require coordination of quantities (Siegler et al., 2011). In addition, multiplication problems differ in their underlying structures, which impose distinct conceptual demands on learners (Greer, 1992), while task characteristics further shape students' problem solving (Vessonen et al., 2026). However, little research has examined how this transition varies systematically across multiplicative structures.

To address this gap, the present study examines differences in performance between integer and fraction multiplication across four structures: equal groups, multiplicative comparison, rectangular area, and Cartesian product. Thirty-five sixth-grade students from three rural primary schools in Taiwan completed a researcher-developed paper-and-pencil test including integer and fraction items across the four structures, with parallel tasks allowing comparison across number domains. Accuracy rates were compared across structures and number domains to identify structure-related patterns.

Results showed that students performed better on integer than on fraction multiplication across all structures. However, the magnitude of this difference varied by structure. The smallest gap was observed in equal group problems, while the largest gap occurred in rectangular area problems. Multiplicative comparison and Cartesian product showed intermediate patterns. These findings suggest that the difficulty of fraction multiplication is not uniform but depends on the underlying multiplicative structure. Structures involving continuous quantities (e.g., area) appear to impose greater conceptual demands than discrete structures such as equal groups.

References

Greer, B. (1992). Multiplication and division as models of situations. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 276–295). Macmillan.

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Journal of Educational Psychology, 103*(2), 273–296.

Vessonen, T., Hellstrand, H., Aunio, P., & Laine, A. (2026). Pictures in mathematical word problems: A systematic review on the role of task and student characteristics. *Educational Psychology Review, 38*, Article 20.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 547. PME.

THE AFFECTIVE EXPERIENCES OF UPPER ELEMENTARY STUDENTS WHEN PLAYING CARD AND DICE GAMES INTENDED TO DEVELOP BASIC FACT FLUENCY

Cherie Lamont, Armando Paulino Preciado Babb

University of Calgary

Mathematics can be challenging for many students, and negative emotional experiences may hinder achievement. From Immordino-Yang's (2016) cognitive-affective perspective, emotion and cognition are inseparable, so students' feelings about mathematics shape how they engage with and learn mathematics. Likewise, Plass et al. (2015) describe game-based learning as the interaction of cognitive, motivational, affective, and social processes. Together, these perspectives suggest that mathematics games can become meaningful experiences that build fluency, confidence, and understanding.

- What are the experiences of upper elementary students when playing Sweet Sixteen and Tick Tock Roll a Clock to develop basic fact fluency?

This qualitative study examined how 28 Grade 4 students experienced these card-and-dice games while practicing fact fluency. Using a constructivist approach, data from videography, interviews, student drawings, and field notes were analyzed thematically to explore how gameplay influenced students' confidence, engagement, and mathematical understanding across individual and collaborative settings.

The findings suggest that mathematics games can create a positive affective learning environment by promoting enjoyment, confidence, and sustained engagement, although students' experiences remained diverse. Many students described the games as more motivating than traditional worksheets, with the social nature of gameplay reducing pressure and encouraging persistence. Collaboration, strategic choice, and manageable challenge appeared to strengthen students' sense of agency while transforming moments of stress into productive struggle.

A key contribution of the study is its focus on the diversity of students' affective experiences over a two-month period. These insights may help inform future game design and reinforce the importance of attending to affect in creating more supportive mathematics learning environments.

References

Immordino-Yang, M. H. (2016). *Emotions, learning, and the brain: Exploring the educational implications of affective neuroscience*. W. W. Norton & Company.

Plass, J. L., Homer, B. D., & Kinzer, C. K. (2015). Foundations of game-based learning. *Educational Psychologist*, *50*(4), 258-283.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 548. PME.

TOOL-BASED FRAMEWORK FOR SENSE AND SENSING IN EMBODIED MATHEMATICS TEACHER EDUCATION

Huk-Yuen Law

The Chinese University of Hong Kong

Embodied cognition research (Lakoff & Núñez, 2000; Abrahamson, 2014) suggests that meaningful mathematical understanding emerges when students' sensorimotor experiences are intentionally connected to abstract reasoning. However, task design in mathematics education often separates procedural mastery ("sense") from conceptual understanding ("sensing"), which can hinder deeper learning. The Embodied Mathematical Imagination and Cognition (EMIC) framework addresses this gap by supporting pre-service teachers (PSTs) in designing tasks that purposefully integrate both dimensions. This study explores whether structured design tools can help PSTs bridge the sense-sensing gap in their mathematics lessons. Two research questions guide the inquiry: (1) How do design tools shape PSTs' pedagogical reasoning and collaborative task design? (2) How do tool-mediated tasks support students' movement from procedural activity to conceptual justification? A design-based study was conducted with four PSTs and mentor teachers in three Hong Kong schools over two months. PSTs were introduced to four principled design tools: Anchor-Explain-Generalize Template, Mode-Shift Planner Matrix, Controlled Dissonance Prompt, and Concreteness Fading Blueprint. Working in teams, they designed and implemented 2–3 lesson sequences. Data sources included completed design artefacts, lesson videos, and teacher reflections. Qualitative analysis focused on how the tools influenced design discussions and classroom interactions, particularly moments when students transitioned from perceptual anchors to verbal or gestured explanations, or when perceptual dissonance prompted justification. It is anticipated that the toolkit will help PSTs shift from using ready-made activities to engineering principled, embodied tasks. Tasks that employ perceptual dissonance are expected to stimulate student inquiry and support transitions from sensorimotor engagement toward formal mathematical reasoning. This research shows that well-designed tools, grounded in embodied cognition, can operationalize EMIC for PSTs. The project provides a replicable toolkit for teacher education, highlights perceptual dissonance as a key design principle, and offers classroom exemplars. Ultimately, it aims to empower future teachers to integrate embodied learning systematically into mathematics instruction.

References

Abrahamson, D. (2014). Building educational activities for understanding: An elaboration on the embodied-design framework and its epistemic grounds. *International Journal of Child-Computer Interaction*, *2*(1), 1-16.

Lakoff, G., & Núñez, R. (2000). *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being* (Vol. 6, p. 489). New York: Basic Books.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 549. PME.

EXAMINING MATHEMATICS TEACHERS' CONCEPTIONS OF EQUITY THROUGH SPECULATIVE FICTION

Generosa Leach[1], Jodie Hunter[1], Tracy Helliwell[2]

[1]Massey University, [2]University of Bristol

Internationally, there has been increasing recognition of the need to focus on equity in mathematics education (Vithal et al., 2024). Teachers play a key role in attending to and developing equitable mathematical learning contexts through their pedagogical practices (Gutiérrez, 2011). However, it appears that there are limited studies that focus on in-service teacher conceptions of equity. Vithal et al. (2024) contend that the concept of equity in mathematics education is difficult to define and there are both theoretical and methodological differences in conceptualising equity. We ask, how do teachers contextualise and frame the notion of equity in mathematics teaching and learning?

This presentation draws on data from four New Zealand teachers' conceptions and framings of equity in mathematics education using a methodological tool of speculative fiction. Speculative fiction is an umbrella term for stories that imagine worlds beyond our current reality, exploring possibilities that have not yet occurred, often focusing on a taken-for-granted norm or structure and considering what if it were *not* the case (Helliwell & Ng, 2022). Participating teachers were invited to write a story (a piece of speculative fiction) in response to a prompt, imagining a mathematics classroom of the future (in 10, 20, or even 50 years). It is these stories that form the basis of our analysis.

Our findings illustrate how the methodology drew out dimensions of equity, shifting teachers away from gap-gazing to imagining future worlds where all children embodied powerful mathematical identities. We contend that utilising speculative fiction as methodology enabled a group of teachers to voice what equity in mathematics education could be, going beyond those of dominant discourses. While each teacher was situated within unique and individual contexts, what emerged were distinct, yet overlapping articulations of a future world where equity flourished.

References

Gutiérrez, R. (2011). Context matters: How should we conceptualize equity in mathematics education? In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in discourse for mathematics education: Theories, practices, and policies* (pp. 17-33). Springer.

Helliwell, T., & Ng, O. L. (2022). Imagining possibilities: Innovating mathematics (teacher) education for sustainable futures. *Research in Mathematics Education*, *24*(2), 128–149.

Vithal, R., Brodie, K. & Subbaye, R. (2024). Equity in mathematics education. *ZDM Mathematics Education, 56*(1), 153–164.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 550. PME.

“I FEEL WORRIED”: SENSING MATHEMATICS AS A THREAT: YOUNG CHILDREN’S EMODIED EXPERIENCES OF MATHEMATICS ANXIETY

Agatha Lebethe, Chante Waggie, Linda Bosman

Stellenbosch University

Mathematical anxiety (MA) involves cognitive worry, negative emotions, and physical responses like sweating, rapid heartbeat, and muscle tension (Luttenberger, Wimmer & Paechter, 2018). From an embodied cognition view, these physical reactions are integral to the mathematical experience—they reflect the body's sensing of mathematics as a threat even before cognitive understanding occurs (Varela, Thompson & Rosch, 2017). Despite the importance of early bodily sensing, the lived experiences of South African Grades 1-3 learners regarding MA are largely understudied, and children rarely serve as authoritative voices on their own mathematical-emotional experiences. This study explores these experiences from the children's perspective, with the main question: How do children perceive and express mathematics anxiety? It also investigates what insights children's voices might offer for developing suitable interventions. An interpretivist, qualitative multiple-case study was conducted at two after-school centres, involving 19 Foundation Phase learners. Data collection included the Maths Anxiety Scale, semi-structured interviews, focus groups, drawings, and individual reflection sessions. Analysis was conducted thematically, using the constant comparative method and inductive coding. The learners described MA through embodied language — helplessness, shame, and physiological reactions like sweating and freezing — showing awareness of their somatic responses to math threat. Child-informed coping strategies were shared, framing learners as active agents rather than passive recipients. The body's perception of math as a threat disrupted early sense-making. Recognizing children as experts in their own anxious experiences and using multimodal, child-centred methods — including drawing as embodied data — improves methodological and substantive understanding of early MA. Creating Foundation Phase maths environments where the body feels safe rather than threatened may be essential for mathematical sense-making.

References

Harari, R. R., Vukovic, R. K., & Bailey, S. P. (2013). Mathematics anxiety in young children: An exploratory study. *Journal of Experimental Education, 81*(4), 538–555.

Luttenberger, S., Wimmer, S., & Paechter, M. (2018). Spotlight on math anxiety. *Psychology Research and Behaviour Management, 11,* 311–322.

Varela, F. J., Thompson, E., & Rosch, E. (2017). *The embodied mind, revised edition: Cognitive science and human experience.* MIT Press.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 551. PME.

SENSING AND MAKING SENSE OF MATHEMATATICS THROUGH MATHEMATICAL HABITS OF MIND: A CROSS-CULTURAL STUDY OF GRADE 3 CLASSROOMS

Duangmanee Leelalawan[1], Sampan Thinwiangthong[1], Hiroki Ishizaka[2]

[1]Khon Kaen University, [2]Naruto University

This initial study employs the construct of mathematical habits of mind to reconceptualize mathematical understanding as students' participation in coherent, epistemically grounded reasoning. Despite its theoretical significance, limited empirical research has examined how these habits are enacted and supported in everyday classroom practice, particularly in cross-cultural contexts. This study addresses that gap by investigating how mathematical habits of minds are cultivate in primary classrooms, thereby contributing empirical insight into how students develop the capacity to make sense of mathematics as a coherent system of ideas.

This study presents a comparative investigation of Grade 3 mathematics classrooms in Japan and Thailand, focusing on the development of mathematical habits of mind. The participants group were students in Grade 3 at the Attached School of Naruto University of Education in Japan and at the Demonstration School of Khon Kaen University in Thailand. Data were collected through video recordings, photographic documentation, and field notes. The data were analysed using a descriptive approach, which were based on Cuoco, Goldenberg and Mark (1996)'s framework.

The findings reveal different forms of mathematical engagement across the two Grade 3 classrooms. In Japan, students interpreted problem situations, identified multiplicative structures through distributive representations, and explained their reasoning to the class. In Thailand, students connected mathematical ideas to everyday contexts and articulated the meaning of a right angle conceptually. These contrasts illustrate how classroom practices shape students' opportunities for mathematical sense-making.

Grounded in mathematical habits of mind, this study compares Grade 3 classrooms in Japan and Thailand to examine how instructional structures shape students' opportunities for mathematical sense-making. The findings offer cross-cultural insight into how primary mathematics can be organized as a disciplined form of reasoning.

References

Cuoco, A., Goldenberg, E. P., & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curriculum. *Journal of Mathematical Behavior, 15(4)*, 375–402.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 552. PME.

COGNITIVE ACTIVATION AS PERCEIVED BY STUDENTS AND TEACHERS, AND ITS RELATION TO STUDENTS' MOTIVATION

Piia Lehtola, Markus Hähkiöniemi, Jonna Pulkkinen

University of Jyväskylä

Cognitive activation is one dimension of instructional quality, and it refers to teachers' practices, which promote students higher-order thinking; for example, providing challenging tasks and asking students to discuss and explain their answers (Lipowsky et al., 2009). There is evidence that it is related to students' mathematics related educational outcomes like achievement and motivation, but evidence varies depending, for example, on measurement approaches (Charalambous et al., 2025).

In this study, we examine how cognitive activation in classroom is related to Finnish 8th graders' liking of learning mathematics and confidence in mathematics at student- and class level. We utilize the student and teacher questionnaires from TIMSS 2023 (Trends in Mathematics and Science Study) to measure cognitive activation as perceived by students and teachers.

In our sample, there were 5 808 students and 392 classrooms. Two-level structural equation models revealed a strong relationship between student reported cognitive activation and their liking of learning mathematics both at student ($\beta = 0.366$, $p < 0.001$) and class level ($\beta = 0.662$, $p < 0.001$). Similar results were found for students' confidence in mathematics at student ($\beta = 0.311$, $p < 0.001$) and class level ($\beta = 0.502$, $p < 0.001$). However, teacher reported cognitive activation was not related to students' motivational outcomes at class level.

These results highlight the importance of cognitive activation in fostering students' motivation but also leads to question about how cognitive activation should be measured (Kunter & Baumert, 2006). In the presentation the results are presented in more detail and varying results with different measures of cognitive activation are discussed.

References

Charalambous, C. Y., Sergiou, S., & Georgiou, G. K. (2025). Do all students benefit equally from the supply and use of cognitively activating teaching? Introducing students' cognitive processing skills into the equation, *International Journal of Educational Research*, *134*.

Kunter, M., & Baumert, J. (2006). Who is the expert? Construct and criteria validity of student and teacher ratings of instruction. *Learning Environments Research*, *9*(3), 231–251.

Lipowsky, F., Rakoczy, K., Pauli, C., Drollinger-Vetter, B., Klieme, E., & Reusser, K. (2009). Quality of geometry instruction and its short-term impact on students' understanding of the Pythagorean Theorem, *Learning and Instruction*, *19*(6), 527–537.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 553. PME.

TALENT DEVELOPMENT IN INCLUSIVE MATHEMATICS CLASSROOMS: PD FACILITATORS' PERSPECTIVES

Janina Lenhart, Nele Spillner, Marcus Nührenbörger

University of Münster

The discourse on inclusion has reoriented giftedness research from a focus on the "identified gifted few" to talent development for the "talented many" in inclusive settings (Maker & Bahar, 2024). In mathematics education, this requires developing mathematical talents within regular classrooms. Addressing this challenge, our teacher professional development (PD) programme qualifies PD facilitators who lead networks of primary mathematics teachers. Building on principles for talent development in mathematics classrooms (e. g., Prediger & Schnell, 2017), our programme focuses on (1) providing individualised support for students' learning and developmental processes and (2) designing adaptive learning environments. The study asks: What characteristics of successful talent development in mathematics classrooms do PD facilitators describe, and to what extent are these descriptions mathematics-specific? *N* = 11 primary-level mathematics PD facilitators were interviewed via Zoom in nine interviews (two conducted in tandems; 33–55 min) at the end of their first qualification year. Data were analysed using content-structuring qualitative content analysis with a deductive category system derived from the two principles, yielding seven subcategories. With regard to the first principle (individualised support), facilitators emphasised social participation (mentioned in 8 of 9 interviews), variety of tasks (3/9), and process orientation (2/9). For the second principle (adaptive learning environments), qualitative differentiation (7/9) and openness (7/9) were most prominent, followed by interaction (6/9) and collaboration (4/9). Across subcategories, mathematics-specific concretisation remained largely implicit. Targeted use of materials and representations, substantial tasks that initiate cognitively demanding mathematical processes, and orientation towards mathematical structures were rarely articulated. Overall, PD facilitators' alignment with both principles tends to remain at a general inclusive-pedagogical level, with mathematics-specific concretisations articulated only selectively. This risks reducing "talent development for all" to surface features and points to the need for more explicit support in developing mathematics-specific interpretations of both principles within PD facilitator qualification.

References

Maker, C. J., & Bahar, A. K. (2024). Talent development in inclusive classrooms: An analysis of student growth. *The Australasian Journal of Gifted Education*, *33*(2), 28–55.

Schnell, S., & Prediger, S. (2017). Mathematics enrichment for all: Noticing and enhancing mathematical potentials of underprivileged students as an issue of equity. *Eurasia Journal of Mathematics, Science and Technology Education*, *13*(1), 143–165.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 554. PME.

MATHEMATICS TEACHERS' AI LITERACY: A SYSTEMATIC REVIEW

Jiayu Yan, Shuhui Li, Qianhui Zhao

East China Normal University

With the rapid evolution of artificial intelligence (AI), mathematics teachers face growing challenges: they must understand AI technologies and critically evaluate the mathematical accuracy of their outputs (NCTM, 2024). Yet, AI literacy remains largely absent from formal teacher education (Sperling et al., 2024), and research focusing specifically on mathematics teachers is still limited. Given that mathematics is grounded in logic, reasoning, and precision, a discipline-specific conception of AI literacy is essential. Since the release of ChatGPT in late 2022, AI has significantly influenced mathematics classrooms, but a systematic understanding of how teachers develop and apply AI literacy remains scarce. This study, therefore, asks: What are the recent developments in research on mathematics teachers' AI literacy since 2023?

We conducted a systematic literature search across ScienceDirect, SpringerLink, ERTC and Google Scholar for studies published between January 2023 and January 2026. Using the search string ("Mathematics" OR "Maths") AND ("Teachers") AND ("AI" OR "Artificial Intelligence") AND ("Literacy" OR "Competency"), we identified 228 records, from which 39 met the inclusion criteria. Two researchers independently coded the 39 studies through content analysis with reliability checks.

The review reveals that current research conceptualizes AI literacy as multidimensional but lacks mathematics-specific depth. Pre-service and in-service exhibit theoretical confidence yet limited practical competence, while in-service teachers often remain cautious observers. AI can act as a "creative partner" for lesson design, yet teachers frequently over-trust their outputs and overlook mathematical inaccuracies. Existing training programs encourage reflective engagement but inadequately address mathematical verification, emphasizing technical fluency over disciplinary precision. Overall, cultivating mathematics teachers' AI literacy faces multiple challenges. Future professional development should prioritize mathematically grounded, responsible AI integration.

References

National Council of Teachers of Mathematics. (2024). Artificial intelligence in mathematics education. NCTM.

Sperling, K., Stenberg, C. J., McGrath, C., Åkerfeldt, A., Heintz, F., & Stenliden, L. (2024). In search of artificial intelligence (AI) literacy in teacher education: A scoping review. *Computers and Education Open, 6*, Article 100169.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 555. PME.

MATHEMATICS SELF-EFFICACY, ANXIETY, AND LITERACY: A MULTILEVEL STUDY OF TAIWANESE PISA 2022 STUDENTS

Yi-Jhen Li, Su-Wei Lin

National University of Tainan, Taiwan

Previous research has suggested that self-efficacy and learning anxiety are associated with students' academic literacy (Weissgerber et al., 2022). However, little is known about how these psychological factors jointly relate to mathematical literacy across student- and school-level contexts. Drawing on PISA 2022 data, this study investigated the relationships among mathematics self-efficacy, mathematics anxiety, and mathematical literacy at both levels by applying hierarchical linear modeling (HLM). At the student level, indices of socioeconomic status, gender, mathematics self-efficacy, and mathematics anxiety were included as predictors; at the school level, aggregated socioeconomic status and mathematics self-efficacy and anxiety were considered. Missing data were treated using Bayesian estimation. As for the results, the intraclass correlation coefficient (ICC) from the null model was .425, indicating substantial between-school variation and justifying multilevel modeling. At the student level, mathematics self-efficacy was positively associated with mathematical literacy and weakened the association between socioeconomic status and literacy, whereas mathematics anxiety was negatively related, even after controlling for gender. This pattern suggests that strong self-efficacy beliefs may buffer the disadvantages associated with lower socioeconomic status in mathematics learning across student groups. At the school level, average mathematics self-efficacy further reduced the influence of school-level socioeconomic status on mathematical literacy. Interestingly, when mathematics anxiety was included alongside self-efficacy at the school level, the previously negative association between anxiety and literacy became positive, implying that, in school contexts characterized by high self-efficacy, a certain level of anxiety may coexist with or even signal higher achievement. These findings highlight the complex interplay between self-efficacy, anxiety, gender, and socioeconomic factors across levels. By simultaneously modeling student- and school-level processes, this study extends prior work by clarifying how psychological factors and structural inequalities intertwine to shape mathematical literacy outcomes. The results suggest that, in addition to enhancing students' mathematics self-efficacy, educators may benefit from incorporating appropriately challenging tasks that elicit manageable levels of anxiety and from fostering school climates that support confidence in mathematics, thereby promoting the development of mathematical literacy.

References

Weissgerber, S. C., Grünberg, C., Neufeld, L., Steppat, T., & Reinhard, M. (2022). The interplay of math anxiety and math competence for later performance. *Social Psychology of Education: An International Journal, 25*(4), 977-1002.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 556. PME.

ENHANCING MATHEMATICAL DISCOURSE AND PRACTICE AMONG PRESERVICE TEACHERS

Emmanuel Azwidowi Libusha

University of Johannesburg

This study draws on Community of Practice (Lave & Wenger, 1991) and Mathematical Discourse, complemented by Gee's (2014) theory of Discourse, to conceptualise mathematics learning as participation in a socially situated community where meaning is constructed through language, interaction, and shared practices. Within this framework, mathematical proficiency extends beyond procedural fluency and conceptual understanding to include the ability to communicate reasoning using recognised mathematical language. The study examines how preservice teachers' explanations of whole-number subtraction reflect their participation in the mathematical community of practice. It addresses the question: How does preservice teachers' mathematical discourse reveal varying levels of alignment between procedural knowledge, conceptual understanding, and participation in the mathematics community? A qualitative study was conducted with 86 first-year Bachelor of Education (Foundation Phase) preservice teachers at a South African university. Data from written explanations of the standard subtraction algorithm were analysed, with three exemplar responses selected to represent key discourse patterns: procedural, emerging conceptual, and discursively aligned. Analysis was guided by the community of practice and discourse theory.

Findings reveal three patterns: most preservice teachers showed procedural fluency with informal or inconsistent language, indicating limited discourse engagement; some demonstrated emerging conceptual understanding but struggled to communicate coherently; and only a few showed strong alignment between conceptual understanding and precise mathematical language. These results indicate that correct answers alone do not reflect full participation, as discursive competence is essential. The study concludes that mathematical discourse is central to specialised content knowledge for teaching. Preservice teachers' ability to explain ideas using appropriate language reflects their participation in the mathematical community. The findings highlight the need for teacher education programmes to develop discourse alongside procedural fluency, positioning it as a key indicator of mathematical understanding.

References

Gee, J. P. (2014). *An introduction to discourse analysis: Theory and method* (4th ed.). Routledge.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, UK: Cambridge University Press

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 557. PME.

EXPLORING PRE-SERVICE ELEMENTARY TEACHERS' CLASSROOM LANGUAGE ACROSS BACKGROUNDS AND TEACHING PRACTICE: A TEXT MINING ANALYSIS

Chia-Huei Lin, Chia-Jui Hsieh, Yuan-Pei Tsai

National Taipei University of Education

Traditional evaluations of elementary mathematics teachers' classroom discourse are often subjective and difficult to scale (Tseng et al., 2018). To address this limitation, this study applies natural language processing and text mining to objectively analyze the lexical features of teachers' discourse across different instructional performance levels. The findings aim to verify the feasibility of automated text analysis for objective teaching evaluation and provide an empirical basis for future AI-assisted assessment. We analyze 335 ten-minute mathematics teaching demonstration videos conducted by pre-service teachers, simulating the formal teacher screening model in Taiwan. The study investigates how gender, college affiliation (Science, Humanities, Education), mathematics background, and teaching practice influence instructional language. Based on expert evaluations, participants were categorized into three instructional performance groups: Outstanding (***O***; n=55), Standard (***S***; n=257), and Needs Improvement (***NI***; n=23). To identify distinctive classroom discourse patterns, video transcripts were processed via text mining pipeline encompassing transcript extraction, CKIP-based Chinese word segmentation, keyword extraction, TF and TF-IDF analysis. The results reveal lexical differences across teacher backgrounds and performance levels. Regarding gender, female group more often used logical scaffolding terms such as "所以[so] " and "可以[can] ", whereas male group more often used "然後[then]" and addressing terms "老師[teacher]", "同學[classmate]". Education college group used "can" more often and de-emphasized teacher authority by ranking "teacher" lower, while Science college group more frequently used "teacher" and "多少[how much]". Regarding academic background, mathematics majors used concrete nouns "披薩[pizza]", "蛋糕[cake]", while non-majors relied on procedural terms "等於[equals]", "答案[answer]". Regarding performance, the ***NI***-group used "then" exceptionally often (double the TF-IDF of others) and pronouns like "我們 [we] " and "你們 [you, plural]" the least. Lexical data for the ***O***-group and ***S***-group were difficult to distinguish by terminology alone. Overall, these findings demonstrate that text mining provides objective, quantifiable insights into instructional discourse and teaching quality.

References

Tseng, C.-W., & Chou, J.-J., & Tsai, Y.-C. (2018). Text mining analysis of teaching evaluation questionnaires for the selection of outstanding teaching faculty members. *IEEE Access, 6*, 72870-72879.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 558. PME.

HETEROPERCEPTION IN COLLABORATIVE 3D GEOMETRY: THE CASE OF BODY-SCALE POLYHEDRON CONSTRUCTION

Catalina Lomos[1], Alik Palatnik[2], Dor Abrahamson[3]

[1]Luxembourg Institute of Socio-Economic Research
[2]Hebrew University of Jerusalem, [3]University of California, Berkeley

Group activities in mathematics education are often treated as advantageous contexts for conceptual learning, particularly when participants share resources and coordinate action to create common products (Ma, 2017). Yet a persistent pedagogical dilemma remains: groups may collaborate effectively and complete demanding tasks, while individual participants develop divergent understandings that remain invisible to instructors and even to themselves. In body-scale construction tasks, this issue becomes especially salient, as participants may simultaneously manipulate the same material elements and appear to share attentional focus. But does coordinated action imply shared perception?

We propose the construct of *heteroperception* to characterize cooperating agents bearing different perceptual organizations of a shared domain, even while jointly acting on the same objects (Abrahamson, et al., in press). Drawing on interactional perspectives on cognition, we consider how perception and action are co-constituted in collaborative mathematical activity, and how divergence in perceptual organization may play a productive role in conceptual development.

Four adult participants co-assembled a body-scale icosahedron using wooden dowels and links, with minimal structural guidance. The session was audio–video recorded and was followed by individual stimulated recall interviews (SRI).

Results indicate that participants coordinated effectively while attending to different structural features. For example, one participant monitored the number of edges meeting at a vertex, while another organized the same configuration as a “roof” composed of triangular faces. These divergent perceptual constructions did not surface during the interaction and did not hinder task completion. Yet, during the SRIs, some participants detected these differences and enriched their insights.

Joint action need not require shared perception. Divergence may remain implicit in action, suggesting that collaborative geometry tasks should engage reflection.

References

Abrahamson, D., Lomos, C., Hilkey, S., Apsari, R. A., & Palatnik, A. (in press). Heteroperception in joint action. *Learning, Culture, and Social Interaction*.

Ma, J. Y. (2017). Multi-party, whole-body interactions in mathematical activity. *Cognition and Instruction*, *35*(2), 141–164.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 559. PME.

HOW MANY FEET DO FOUR CHICKENS HAVE? MAKING SENSE OF KINDERGARTNERS' FUNCTIONAL THINKING

Mathías A. López[1], Susanne Strachota[1], Johyeon Chang[1], Bárbara M. Brizuela[1], Despina A. Stylianou[2], Maria Blanton[3], Angela Murphy Gardiner[3], Rena Stroud[4]

[1]Tufts University, [2]The City College of New York, [3]TERC, [4]Merrimack College

Functional thinking is central to early algebra. Identifying patterns in particular cases (whether recursive or functional) builds toward generalizing relationships (Blanton et al., 2015). To study how young learners coordinate quantities in particular cases, we asked: *what responses do kindergartners give to a prompt about the number of feet for four chickens, and how can we make sense of their responses?* We analyzed the video data of 742 semi-clinical interviews. Interviewers presented a two-column table showing chickens (1, 2, 3) and feet (2, 4, 6) and asked: "How many feet would four chickens have?" We tallied numerical answers and took a grounded approach to analyze focal interview videos, interpreting how students constructed the answers. Four answers were most frequent: eight (n=191, 25.7%), five (n=136, 18.3%), four (n=92, 12.4%), and six (n=80, 10.8%). The following ways of reasoning corresponded to the four most frequent answers. Functional: Coordinated across columns (e.g., 4+4=8). Recursive: Skip-counted the feet column to eight. Next-number: Read the chicken column as a counting sequence continuing to five. Input-matching: Noticed repeated 2s and assumed input four resulted in output four. Add-two: Applied the "up by two" rule from the feet column (2, 4, 6) to the four chickens (4+2=6). We observed that what children noticed in the table and the prior knowledge they brought to the task worked together to shape their answer (Levin, 2018). Ultimately, our contribution frames the ideas children bring to a task as initial, legitimate ways of functional thinking.

Acknowledgements: This research is supported by the National Science Foundation's DRK-12 Award #2404984. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

References

Blanton, M., Brizuela, B. M., Gardiner, A. M., Sawrey, K., & Newman-Owens, A. (2015). A learning trajectory in 6-year-olds' thinking about generalizing functional relationships. *Journal for Research in Mathematics Education*, *46*(5), 511–558.

Levin, M. (2018). Conceptual and procedural knowledge during strategy construction: A complex knowledge systems perspective. *Cognition and Instruction*, *36*(3), 247–278.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 560. PME.

PRE-SERVICE TEACHERS' USE OF GESTURE IN THEIR BILINGUAL TEACHING

Tarryn Lovemore, Mellony Graven
Rhodes University

Multimodal communication can support teaching and learning mathematics in bilingual settings. It includes verbal explanation, written and drawn representations and gesturing, to help learners access meaning by drawing on their multiple semiotic resources. Pre-service teachers (PSTs) are not necessarily prepared for taking full advantage of these multiple ways of communicating (Prediger, 2019), which is especially useful in contexts where learners' home language is not the language of learning and teaching (LoLT). We therefore aim to explore, 'How do PSTs use gesture in their bilingual teaching of the Jump Strategy?'. This study is framed by Vygotsky's social cultural theory (1978), acknowledging that learning takes place in social contexts through language, and recognising the value of mother tongue in learning. The qualitative case study took place in a South African University, with first-year PSTs. The mathematics teacher educator (MTE) demonstrated the Jump Strategy on the empty number line (ENL) by following scripted lessons in a teacher guide with supporting demonstration videos. PSTs then worked in pairs to video-record their own pretend teaching of the strategy to an imagined learner whose home language is not the LoLT (English). Here we focus on three PSTs who opted to pretend teach a subtraction calculation bilingually in English and isiXhosa, a previously marginalised language in South Africa. Videos were transcribed, translated into English, and broken down into steps to code, including the demonstration video as a baseline. We coded gesture as including writing (W), drawing (D), pointing (P) and showing jump direction and size (J). The PSTs' videos aligned closely to the demonstration video with implementing the writing (calculations and numbers), drawing (ENL and jumps), and pointing; however, the gesturing of the jump direction and size in the PSTs' videos was less prevalent than in the demonstration video. This suggests a missed opportunity to convey meaning. Implications include the value of MTEs and PSTs carefully analysing their pretend teaching in the form of videos (and comparing to a baseline). This enables critical reflection on their implementation fidelity, making explicit the pedagogical value of each multimodal component for enhancing access to meaning for their bilingual learners. The presentation will explore the PSTs' videos and transcripts in more detail.

References

Prediger, S. (2019). Investigating and promoting teachers' expertise for language-responsive mathematics teaching. *Mathematics Education Research Journal, 31*(4), 367–392.

Vygotsky, L. (1978). Mind in society: The development of higher psychological processes. Harvard University Press.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 561. PME.

PROBLEM POSING FOR INTEGRATED STEM: A STUDY OF PROSPECTIVE ELEMENTARY SCHOOL TEACHERS

Marta T. Magiera

Marquette University

Advocates for integrated STEM education argue that integrated STEM experiences help students develop an understanding of the characteristics of STEM disciplines as forms of human knowledge and inquiry. Such integration increases students' awareness of how STEM fields shape human environments and empowers them to take ownership of their learning (English, 2016; Roberts et al., 2022). Furthermore, proponents recognize that meaningful STEM integration relies on teachers who can successfully navigate the unique instructional challenges inherent in this approach.

This study examined opportunities for integrated STEM learning embedded in problems posed by 22 prospective elementary school teachers. Drawing on a conceptual framework that characterizes integrated STEM as encompassing authentic contexts, engineering design thinking, STEM content integration, STEM practices, 21st-century competencies, and exposure to STEM careers (Roehrig et al., 2021), the analysis provides a nuanced portrait of how prospective elementary school teachers enact STEM integration through problem posing. The results document that many prospective teachers successfully framed their problems around human needs, constraints, and system-level considerations, generating opportunities for engagement in engineering design, STEM practices, development of 21st-century skills, and exposure to STEM careers.

The findings also reveal specific patterns of STEM disciplinary connections within the posed problems, with mathematics and science being the most frequent pairing. These patterns reflect the dominant positioning of mathematics and science within elementary curricula and teacher preparation, as documented in previous research (English, 2016). Ultimately, these results underscore concerns raised in the STEM education literature and call for a greater emphasis on technology and engineering in teacher training (Roehrig et al., 2021).

References

English, L. (2016). STEM education K-12: Perspectives on integration. *International Journal of STEM Education, 3*(1), 1–8. https://doi.org/10.1186/s40594-016-0036-1

Roberts, T., Maiorca, C., Jackson, C., & Mohr-Schroeder, M. (2022). Integrated STEM as problem-solving practices. *Investigations in Mathematics Learning, 14*(1), 1-13.

Roehrig, G. H., Dare, E. A., Ellis, J. A., & Ring-Whalen, E. (2021). Beyond the basics: A detailed conceptual framework of integrated STEM. *Disciplinary and Interdisciplinary Science Education Research***,** 3:11. https://doi.org/10.1186/s43031-021-00041-y

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 562. PME.

PROSPECTIVE SECONDARY TEACHERS' MATHEMATICAL KNOWLEDGE: EVIDENCE FROM ENTRY-LEVEL ASSESSMENTS IN A MASTER'S PROGRAMME

Albert Mallart-Solaz, Jordi Deulofeu Piquet

Faculty of Education, Autonomous Barcelona University (UAB)

Research on mathematics teacher education highlights that effective teaching requires solid mathematical knowledge closely linked to teaching practice (Carrillo-Yañez et al., 2018); however, students entering the Master's Programme for Secondary Teacher Education (MUFPS) show persistent difficulties in problem solving, reasoning, and conceptual understanding, particularly in specialised and horizon knowledge, even after completing their practicum experiences (Hine & Thai, 2018). This study aims to show that many students entering the MUFPS in Spain do not possess the mathematical knowledge required to fully benefit from the programme's didactic training, as well as to identify these gaps and propose supplementary support measures. The study analyses the initial mathematical knowledge of 48 students entering the MUFPS through a one-hour diagnostic test consisting of four pre-university-level problems (geometry, calculus, matrix algebra, and linear programming). The results indicate low performance, with mean scores below 4 out of 10 in three of the four problems (3.60; 3.91; 2.84), while only linear programming achieved a mean score above the pass mark (5.14). Indicator-based analysis reveals that, compared to acceptable results in basic content (means between 6.25 and 7.08), tasks involving differentiation, solving irrational equations, and classifying extrema obtained mean scores below 2, pointing to significant shortcomings in analytical procedures essential for teaching at secondary and upper-secondary levels. The findings show that many students enter the MUFPS without the expected level of mathematical knowledge, particularly in calculus and algebra. These shortcomings call into question the adequacy of the current teacher education model and support the need for entry-level diagnostic assessments and reinforcement measures.

References

Carrillo-Yañez, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., … Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education, 20*(3), 236–253.

Hine, G., & Thai, T. (2018). Readiness to teach secondary mathematics: A study of pre-service mathematics teachers' self-perceptions. En J. Hunter, P. Perger, & L. Darragh (Eds.), *Making waves, opening spaces* (Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia, pp. 392-399). MERGA.

Acknowledgements: Study developed within project PID2023-149580NB-I00 (MICINN).

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 563. PME.

CONNECTING TEACHER AND STUDENT PERSPECTIVES IN AN UNDERGRADUATE MATHEMATICS COURSE

Sigal-Hava Rotem[1], Ofer Marmur[2]

[1]Utrecht University, [2]University of Auckland

Despite ongoing efforts to improve university mathematics instruction, challenges remain in aligning teaching approaches with student learning needs. In particular, in the prevalent lecture-based teaching, there is often a disconnect between what lecturers and students perceive as meaningful teaching and learning moments (e.g., Lew et al., 2016). Our research investigates this gap by examining how lecturers' views of their teaching align (or misalign) with students' learning experiences, and how insights from this relationship might inform undergraduate mathematics teaching. Theoretically, we draw on two constructs that offer insight into meaningful classroom episodes. From a teacher perspective, critical events refer to moments when students' thinking becomes apparent to teachers, offering opportunities to deepen mathematical understanding through teachers' ability to identify, interpret, and respond to these moments (Rotem & Ayalon, 2022). From a student perspective, memorable events are conceptualised as learning moments that hold significance and meaning for the student who experienced them, and are typically accompanied by strong emotions (Marmur, 2019). Taken together, these offer complementary lenses, capturing the cognitive and affective significance of classroom events from both teacher and student viewpoints. The study is part of an ongoing wider research project. Here, we report on data from a large undergraduate mathematics course (approximately 120 students). Data sources include four lecture observations, pre- and post-lesson interviews with the lecturer, and questionnaires completed by students. Preliminary analysis reveals both similarities and tensions between learning moments identified as meaningful by the lecturer and by the students, and the ways in which they were characterised. In the presentation, we will share selected moments from the lecturer's and students' reflections, and use these to examine how lecturer planning and noticing may shape students' mathematical and emotional experiences in undergraduate mathematics classrooms.

References

Lew, K., Fukawa-Connelly, T., Mejia-Ramos, J. P., & Weber, K. (2016). Lectures in advanced mathematics: why students might not understand what the mathematics professor is trying to convey. *Journal for Research in Mathematics Education*, *47*(2), 162–198.

Marmur, O. (2019). Key memorable events: a lens on affect, learning, and teaching in the mathematics classroom. *The Journal of Mathematical Behavior, 54*, 100673.

Rotem, S. H., & Ayalon, M. (2022). Building a model for characterizing critical events: Noticing classroom situations using multiple dimensions. *The Journal of Mathematical Behavior, 66*, 100947.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 564. PME.

PROBLEM POSING AS A PROCESS: A CASE STUDY

Aitana Martín-Ferraz, Alexánder Hernández, Matías Camacho-Machín

Universidad de La Laguna

This study examines how pre-service teachers move through the problem-posing process when working on structured tasks in a technology-rich environment. Our analysis is guided by Cai and Rott's (2024) model, which distinguishes four phases: orientation, connection, generation and reflection, and makes it possible to trace how participants' activity develops across the process. The study addresses the following question: How do pre-service teachers move through the problem-posing process in a technology-rich environment?

A qualitative case-study approach was adopted. The participants were a pair of mathematics undergraduates enrolled in a teacher education course who engaged in two problem-posing tasks explicitly organized around the four phases of the process. Data consisted of the pair's written productions, records of their work during the tasks, GeoGebra constructions, and interactions with AI chatbots. The analysis focused on how they interpreted the task, organized ideas, formulated candidate problems, and revised their proposals, while also considering the role of GeoGebra and AI chatbots during both problem solving and problem posing. In orientation, they tried to interpret the situation, prompt, and purpose of the task, although they initially approached it from a problem-solving perspective. In connection, they identified relationships, concepts, and possible directions for problem posing, building on ideas that emerged while solving the original problem and through GeoGebra exploration; this made the phase central for later work. During connection and generation, AI supported the examination and validation of emerging ideas. In generation, they proposed, compared, and selected candidate problems; in reflection, they evaluated them and considered revisions. Throughout the process, digital tools supported the pair's work, but the main interest of the study lies in how they negotiated what to do, which ideas to pursue, and how to pose a meaningful problem. The study shows how a phase-based analysis can make visible how participants' activity develops throughout problem posing. For teacher education, the findings point to the need to support future teachers in recognizing problem posing as a mathematical activity in its own right.

Acknowledgements

This research was partially funded by the research project PID2022-139007NBI00.

References

Cai, J., & Rott, B. (2024). On understanding mathematical problem-posing processes. *ZDM Mathematics Education*, 56, 61–71. https://doi.org/10.1007/s11858-023-01536-w

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 565. PME.

RECONSTRUCTING AN INTEGRATED LEARNING TRAJECTORY OF ANGLE CONCEPTS: DEVELOPMENTAL COORDINATION IN GRADE 3–4 STUDENTS

Yuki Masuda

Saitama University of Education

Angle is a fundamental geometric concept involving both static (e.g., corners and openings) and dynamic (e.g., turning and rotation) aspects and is introduced in Grades 3 and 4 in Japan. Despite its importance, prior research shows that students struggle to develop a unified understanding of angle, often treating "corners," "openings," and "turns" as separate categories and experiencing difficulties in inclination and rotation contexts (Mitchelmore & White, 2000; Huang et al., 2018). This study examines how students coordinate geometric and quantitative interpretations of angle across contexts and how this coordination develops along a learning trajectory. Clements and Sarama (2021) proposed separate learning trajectories that conceptualize angle as a measurable quantity and part of geometric development. This study integrates these trajectories into a unified framework of eight stages, enabling contextual difficulties to be interpreted not as isolated recognition problems but as indicators of incomplete integration. A questionnaire survey was conducted with Grade 3–4 students to examine their positions along this trajectory.

Results showed that students more readily recognized angles when two lines were explicitly visible, whereas selection rates were lower in inclination and rotation contexts. Beyond contextual difficulty, students struggled to coordinate geometric interpretations with inclination and rotation as dynamic–quantitative interpretations, which require constructing an implicit reference line and interpreting angle as the measurable amount of turning. These findings suggest that the central challenge lies not merely in recognizing angles in specific contexts, as previously reported, but in coordinating geometric structure with dynamic–quantitative meaning within a unified concept of angle, a coordination essential for later mathematical development.

References

Clements, D. H., & Sarama, J. (2021). *Learning and teaching early math: The learning trajectories approach* (3rd ed.). Routledge.

Huang, H.-M. E., Hsieh, C.-C., & Tu, S.-M. (2018). *Children's performance of recognizing angles of inclination, turning, and rotation*. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 5, p. 69). Umeå, Sweden: PME.

Mitchelmore, M. C., & White, P. (2000). Development of angle concepts by progressive abstraction and generalization. *Educational Studies in Mathematics, 41*(3), 209–238.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 566. PME.

EXPLORING THE VALUES OF A LEARNING COMMUNITY USING PHENOMENOLOGICAL METHODS

Mitsuru Matsushima

Kagawa University

In mathematics learning, values are defined as characteristics of specific sociocultural contexts rather than personal beliefs, and as the meaning-making mechanisms underlying discourse and interactional norms in mathematics learning (Seah & Wong, 2012). However, the process by which values are formed remains unclear. This research aims to explore the process of value formation in mathematics learning and to provide an example of a method for such exploration.

This study aims to clarify the process of value formation using phenomenological methods. Phenomenology is a philosophical approach that emphasizes discerning the essence behind individual phenomena from a subjective perspective (Brinkmann & Friesen, 2018). In 2022, the author conducted continuous observation and analysis using phenomenology for one hour each week in a class of fourth-grade elementary school students in Japan. Observations revealed 12 episode descriptions related to students' values regarding mathematics learning. These episodes consist of Ep.1 through Ep.12.

Each of the 12 episode descriptions is embedded with words, actions, and emotions related to Teacher A's support and the students' values. Examining this series of episode descriptions reveals a shift in the learning community's values from student-teacher collaboration to student-to-student collaboration. In Episode 1, the value of Teacher A supporting students and facilitating the entire learning process to deepen their understanding becomes clear. However, in Episode 9, the students object to including Teacher A in their discussion, reflecting their value system of wanting to build new knowledge independently. This suggests that as the students continued to learn under Teacher A, they came to realize the value of building mathematical knowledge on their own. This study partially clarified the process of value formation using phenomenological methods.

Acknowledgement

This work was supported by JSPS KAKENHI Grant Number JP22K13708.

Reference

Brinkmann, M. & Friesen, N. (2018). Phenomenology and education. P. Smeyers (Ed.), *International Handbook of Philosophy of Education* (pp. 591-608), Springer.

Seah, W. T. & Wong, N. Y. (2012). What students value in effective mathematics learning: a 'Third Wave Project' research study. *ZDM Mathematics Education*, 44, pp.33-43.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 567. PME.

POSSIBILITIES FOR QUALITY MATHEMATICS TEACHING: FOCUS ON FUNCTIONS

Hlamulo Wiseman Mbhiza

University of South Africa

This paper extends Prediger et al.'s (2022) framework by proposing explicit recognition rules for analyzing high-quality mathematics teaching. Using a Grade 10 lesson episode on hyperbolic functions, I illustrate the coding process and examines enacted pedagogy.

Around the world, there is increasing acknowledgement that high-quality mathematics teaching must be a central educational priority, considering its role in enabling students' epistemological access to powerful mathematical knowledge and, by extension, meaningful participation in academic, civic, and economic spaces (Prediger et al., 2022). Ensuring such access requires more than curriculum coverage; it demands teaching practices that deliberately support learners in engaging with, making sense of, and productively using mathematical ideas. One crucial strategy for advancing this agenda lies in the development of comprehensive theoretical frameworks that make it possible to examine, with analytic depth, the quality of teaching that unfolds in classrooms. These frameworks not only allow researchers to methodically characterize pedagogical practices, but also provide practical guidance for teachers, helping them adapt their teaching in ways that foster richer learning opportunities. One such framework is that developed by Prediger et al. (2022), which consists of five interconnected principles: conceptual focus, cognitive demand, longitudinal coherence, student focus and adaptability, and enhanced communication. Prediger et al.'s framework outlines principles for high quality mathematics teaching but offers limited guidance on recognizing and grading them in practice. This paper proposes detailed recognition rules, illustrates their use through a Grade 10 lesson on functions in South Africa, and examines teaching quality patterns and the analytic value of the extended framework.

References

Prediger, S., Götze, D., Holzäpfel, L., Rösken-Winter, B., & Selter, C. (2022). Five principles for high-quality mathematics teaching: Combining normative, epistemological, empirical, and pragmatic perspectives for specifying the content of professional development. *Frontiers in Education, 7*, 69212. https://www.frontiersin.org/articles/10.3389/feduc.2022.969212/full

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 568. PME.

COGNITIVE DEMAND OF TASKS IN PRIMARY MATHEMATICS TEXTBOOKS: AN ANALYSIS OF MEASURES AND SHAPE AND SPACE IN IRELAND

Gráinne McHugh[1,2], Hamsa Venkat[1], Paul Grimes[1]

[1]Dublin City University, [2]Educational Research Centre

Findings from the Trends in International Mathematics and Science Study (TIMSS) have consistently indicated relative weaknesses among Irish students in the domains of Measurement and Geometry. This study focuses on one component of a doctoral project examining alignment across curriculum, instruction, and assessment by investigating the content focus and cognitive emphasis of tasks in primary mathematics textbooks. Specifically, it analyses tasks from three Fourth Class (Fourth Grade) mathematics textbooks within the Measures and Shape and Space strands.

This study draws on two frameworks: the revised Bloom's taxonomy of educational objectives (Krathwohl, 2002), used to classify cognitive emphasis, and Porter's Survey of Enacted Curriculum (Porter & Smithson, 2002), used to examine alignment across content and cognitive dimensions. The cognitive levels in the adapted Bloom's framework include remember, understand, apply, analyse, evaluate, and create.

Using the above frameworks, all tasks within the selected textbooks were classified and the resulting data were transformed into matrices representing frequencies and proportional distributions. Visual representations, including tables and heatmaps, were generated to illustrate patterns across the textbooks. Porter's SEC was then used to calculate the alignment between each of the textbooks.

Across all three textbooks, a clear concentration of tasks at the apply level is evident, particularly within the Measures strand. Higher-order tasks involving evaluation, or creativity remain scarce. Alignment scores indicate moderate consistency between the three textbooks in terms of content focus and cognitive demand.

These findings suggest that the textbooks emphasise procedural practice over higher-order mathematical thinking. This raises important questions about the extent to which students are provided with opportunities to engage in the kinds of reasoning and problem solving reflected in large-scale assessments such as TIMSS.

References

Krathwohl, D. R. (2002). A Revision of Bloom's Taxonomy: An Overview. *Theory Into Practice*, *41*(4), 212-218. http://www.jstor.org.dcu.idm.oclc.org/stable/1477405

Porter, A., & Smithson, J. (2002). *Alignment of Assessments, Standards and Instruction Using Curriculum Indicator Data*.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 569. PME.

CONTRASTING TEXTBOOK CONCEPTIONS AND STUDENTS' RESPONSES TO PIVOTS QUESTIONS

Ayla Carvalho[1], Vilma Mesa[2], Elijah Cooley[2], Meet Patel[3]

[1]Sao Paulo State University, [2]University of Michigan, [3]Rutgers University

The PROTEUS project (Mesa et al., 2023) developed interactive open questions for open-source calculus and linear algebra textbooks housed in the Runestone Academy learning analytics platform. The responses to these questions, designed to be answered prior to formal instruction, allow us to map how students reason about specific mathematical topics. Using Balacheff's cK¢ model (Balacheff & Gaudin, 2009), we contrasted the control structures that can be inferred from the textbook with the control structures present in students' responses to textbook questions about pivots. The data come from 38 students from three different teachers in the U.S.

We found three conceptions in the textbook related to the question of determining how many solutions a consistent linear system has. The control structures supported by the textbook were: 1) Pivots on the columns (students could check whether every column of the coefficient matrix contains a pivot leads to a unique solution), 2) Free variables, (identifying at least one free variable supports the conclusion of infinitely many solutions), and 3) Parametric description (expressing variables in terms of basic variables leads to the conclusion of infinitely many solutions). In the student responses, we identified two additional control structures, Degree of freedom and Relationship between number of pivots and variables. Most responses (n=17) used Free variables or a combination of two control structures (n=12).

The emergence of control structures not explicitly supported by the textbook suggests that students' mathematical validation processes cannot be fully inferred from curricular propositions alone. In addition, we can observe that, despite students developing control structures beyond what is explicitly presented, there is a noticeable influence of the textbook content in their responses, reflecting how curricular materials might shape the strategies students employ.

Acknowledgment. Funding for this work was provided by the National Science Foundation. Any opinions, findings, and conclusions or recommendations are those of the author(s) and do not necessarily reflect the views of the NSF.

References

Balacheff, N., & Gaudin, N. (2009). Modeling students' conceptions: The case of function. *Research in Collegiate Mathematics Education*, 16, 183–211.

Mesa, V., Beezer, R., Austin, D., Miller, B., & Judson, T. (2023). *PreTeXt-Runestone: Open Textbooks Engaging Undergraduates in STEM (PROTEUS)* (National Science Foundation DUE Awards 2334767, 2334768, 2334769) [Grant]. NSF. University of Michigan.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 570. PME.

FEELING FIT: NEGOTIATING MODELLING PRACTICES

Elizabeth Metts, Cory A. Bennett, Wendy Ruchti

Idaho State University

The prominence of data has intensified calls to integrate data science into K–12 mathematics (NCTM, 2024). Integrating mathematical and data perspectives can support students in taking up and extending disciplinary ways of knowing, but also raises pedagogical challenges, as domain-specific practices involve distinct forms of reasoning and ways of structuring and evaluating knowledge (Lehrer & Schauble, 2021). Drawing from a larger design-based research project studying teachers' integration of data and mathematics, this analysis focuses on a lesson in which an experienced Algebra 1 teacher, whom we call Veronica, worked to coordinate mathematical ideas of rate of change and nonlinear relationships with data modelling. We explore how mathematical and data-oriented modelling practices interactionally emerged and were coordinated or brought into tension to interpret nonlinear data. Drawing on the construct of disciplining perception (Stevens & Hall, 1998)—a deliberate practice of seeing in ways that are socially organized and relevant to a particular discipline—we used interaction and discourse analysis on multi-perspective classroom video from the focal lesson to understand how language and gesture were used to evaluate models across mathematics and data science. In the lesson, Veronica asked students to construct linear models of nonlinear data that "have the most dots on the line," introducing a provisional criterion for fit that supported models that might work locally but not across the entire distribution. Through affective evaluations (e.g., "that sucks"), she destabilized this criterion, positioning aesthetic judgment as central to evaluating models without emphasizing disciplinary notions of fit. Different ways of seeing emerged and were taken up, including going through points, representing rate of change, and balancing error. Students felt when linear models were insufficient, motivating the need for piecewise models. However, this ambiguity was not productive for all students. In moments of breakdown, these ways of seeing came into conflict, resulting in Veronica prioritizing the mathematical structure of rate of change. These findings highlight the complexity of fostering epistemic diversity, as efforts to integrate and sustain multiple ways of seeing can shift toward privileging specific disciplinary criteria.

References

Lehrer, R., & Schauble, L. (2021). Stepping carefully: Thinking through the potential pitfalls of integrated STEM. *Journal for STEM Education Research, 4*(1), 1–26.

National Council of Teachers of Mathematics (NCTM). (2024). *Teaching Data Science in High School: Enhancing Opportunities and Success*.

Stevens, R., & Hall, R. (1998). Disciplined perception: Learning to see in technoscience. *Talking Mathematics in School: Studies of Teaching and Learning*, 107–149.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 571. PME.

PUPILS' CRITERIA FOR EVALUATING MATHEMATICAL ARGUMENTS

Michael Meyer

University of Cologne

Evaluating arguments is a key objective in mathematics education, yet pupils often struggle in this area. Although there are theoretical frameworks for proof competence, these tend to focus on methodological knowledge rather than the specific criteria pupils use in practice (e.g., Heinze & Reiss, 2003). This study takes an entirely empirical approach to identifying the criteria applied by pupils when evaluating mathematical arguments. A qualitative study was conducted with 37 eighth-grade pupils (aged 13–14) to investigate these criteria. The participants were asked to evaluate 15 different arguments. These arguments varied in terms of their representation (verbal, symbolic or iconic), as well as in terms of the completeness and correctness of their structural elements (Toulmin, 1969). The pupils assigned a grade to each argument and provided a justification for each grade given. The data were analysed using an interpretative approach based on hermeneutics to reconstruct the underlying criteria and corresponding categories.

The analysis revealed a comprehensive list of 15 distinct criteria used by the pupils, covering everything from content to recipient orientation. Some examples of the categories of criteria are: correctness and logic (pupils focused on the correctness of the result and argument, as well as the conclusiveness and internal logic of the argument), communication and structure (criteria such as understandability, linguistic expression, and the quality of visualisation or representation), metacognitive aspects (pupils distinguished between their perceived understanding of the author and the traceability of the explanation provided), formal and structural elements (completeness, mathematical formality, and the generality of the argument). The findings demonstrate that pupils possess multidimensional perspectives and are capable of evaluating arguments from various perspectives including pedagogical and subject-specific ones. The identified criteria provide a valuable foundation for discussing what constitutes a "good" argument in mathematics education for pupils. The findings also provide a starting point for classroom discussion about which criteria are useful for evaluating (mathematical) arguments and how these criteria can be applied when constructing one's own arguments.

References

Heinze, A. & Reiss, K. (2003). Reasoning and Proof. In M. A. Mariotti (Ed.), *International Newsletter of Proof*, 4-6.

Toulmin, S. E. (1969). *The Uses of Argument*. Cambridge UP.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 572. PME.

CONCEPT CARTOONS IN THE TEACHING OF MATHEMATICS EDUCATION RESEARCH FINDINGS ON GEOMETRY

Verónica Molfino

Consejo de Formación en Educación

Understanding how prospective mathematics teachers (PMTs) interpret research findings is a key challenge in Mathematics Education. While mathematics teaching generates extensive research, the pedagogy of teaching these findings to PMTs remains underexplored, with teacher educators often relying on a "show and tell" approach. Drawing on Ochoviet & Parodi (2024), this paper presents preliminary findings from an ongoing study that employs concept cartoons (Naylor & Keogh, 2013) to engage PMTs with Wilson's (1990) research on inconsistent ideas about geometric definitions.

The study adopted a qualitative exploratory case study design, involving seven PMTs enrolled in a final-year teacher education course. Data generation centred on a sequence of three concept cartoons depicting PMTs discussing authentic student productions on geometrical definitions. Participants' written responses to the CCs and transcriptions of the audio-recorded plenary discussions were analysed following a thematic approach: open coding was first employed to identify emergent patterns, after which axial coding enabled the grouping of these patterns into coherent categories, which were subsequently interpreted in light of Wilson's (1990) framework.

Analysis of data reveal how PMTs progressively identified five key difficulties: transition between definition and representation; set inclusion; deeply-rooted conceptual prototypes; construction of definitions; and testing of definitions. These closely mirror Wilson's (1990) sources of student inconsistency: implications in definitions, imprecise language, strong prototypes, and logical reasoning difficulties, yet emerged spontaneously from PMTs' interpretations. The findings suggest that CCs constitute a productive context for eliciting pedagogical content knowledge, offering a low-stakes alternative to traditional practices that deepens PMTs' sensitivity to student cognition.

References

Naylor, S. & Keogh, B. (2013). Concept cartoons: what have we learnt? *Journal of Turkish Science Education, 10*(1), 3–11.

Ochoviet, C. & Parodi, S. (2024). Viñetas conceptuales para la enseñanza de la didáctica de la Matemática. *Revista Binacional Brasil-Argentina: Diálogo entre as ciências, 13,* 386-413.

Wilson, P.S. (1990). Inconsistent Ideas Related to Definitions and Examples. *Focus on learning problems in mathematics, 12,* 31-47.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 573. PME.

GENDERED VISUO-SPATIAL REASONING IN FIFTH GRADE: THE ROLE OF THE DIDACTIC CONTRACT

Serena Monica

University of Ferrara

Spatial reasoning is a central component of mathematical sense-making in geometry, although literature documents a persistent gender gap in visuo-spatial tasks, with males often outperforming females as early as primary school (Lauer et al., 2019). In this context the study is framed through the notion of the didactic contract, understood as the system of implicit expectations that defines legitimate mathematical activity (Brousseau, 1986). These expectations can privilege explicit and verbally justifiable strategies, potentially constraining intuitive spatial reasoning, which has been shown to develop differently across genders during the primary school years (Lauer et al., 2019). The study adopts a mixed-methods design combining large-scale assessment data with an exploratory qualitative analysis of students' written justifications on a visuo-spatial item involving non-standard orientation. Preliminary quantitative results indicate a gender effect favouring males, while qualitative analyses provide initial interpretative insight into this pattern. Female students' responses show recurring tendencies such as reliance on school-familiar but geometrically irrelevant properties, visually unstable reasoning and hesitation in mentally manipulating spatial configurations. In some cases, initially correct intuitions appear to be abandoned when they cannot be aligned with expected forms of justification. In contrast, such patterns are not systematically observed in male students' responses, which more often rely directly on spatial configurations and tend to preserve initial intuitions. Overall, these findings suggest that gender differences in visuo-spatial performance may be related to the interaction between spatial sense-making and classroom norms. The didactic contract operates privileging compliance with rules and verbal control over intuitive exploration. For students, the pressure for formal justification and procedural correctness can lead to a suspension of agency in tasks that require generative and autonomous visualization. Recognizing these socio-didactic constraints is essential for designing practices that validate visual intuition and foster more equitable conditions for mathematical sense-making. This study contributes to ongoing research by providing empirical insight into the interplay between gender differences in visuo-spatial reasoning and the didactic contract and opens the way to further work aimed at strengthening and refining this relationship through a broader mixed-methods design.

References

Brousseau, G. (1986). Fondements et méthodes de la didactique des mathématiques. *Recherches En Didactique Des Mathématiques, 7*(2), 33–115.

Lauer, J. E., Yhang, E., & Lourenco, S. F. (2019). The development of gender differences in spatial reasoning: A meta-analytic review. *Psychological Bulletin, 145*(6), 537–565.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 574. PME.

TASK COMMITMENT IN MODELLING SHOWED BY MATHEMATICALLY TALENTED STUDENTS

María López-Moreno[1], Lidia Torrente-Imbernon[2], Jesús Montejo-Gámez[1]

[1]Universidad de Granada, [2]Centro de Magisterio La Inmaculada

This study arose to stimulate mathematical talent of gifted students. Several phycological models emphasise the importance of personality-related factors in stimulating and expressing talent associated with specific domains. One of them is Task Commitment (TC), included in the Three Ring Model (Renzulli & Reis, 2021) which encompasses characteristics such as persistence, involvement in facing challenges, and the ability to organise ideas and plan strategies. These are connected to some of the features of mathematical talent pointed out in the literature on mathematics education (e. g, Leikin, 2018), which highlights the importance of research on TC for promoting mathematical talent in secondary school students. Despite this importance, there have not been enough studies analysing the characteristics of TC in mathematics shown by gifted students. Nevertheless, authors such as Ferrando et al. (2024) found that these students approached modelling tasks with greater motivation than their non-gifted peers. This raises the research question: *To what extent do modelling tasks foster TC among mathematically talented secondary school students?* To address this question, two secondary school students (aged 14 and 15) with high scores on the g-R factor and EFAI-3 tests were selected. They tackled two tasks, focusing on simplification and structuring, and on validation competencies, respectively. These students' modelling processes were qualitatively analysed in terms of *a priori* categories derived from the literature on TC. The results showed that both students demonstrated their ability to effectively organise the data in the task focused on structuring and to persist in searching for a pattern. In the second task, they demonstrated their ability to evaluate their own problem-solving processes. These results corroborate the potential of modelling to stimulate TC, and by extension mathematical talent.

References

Ferrando, I., Segura, C., Castillo, J. (2024). Regular and Talented Students' Behaviour When Solving Modelling Tasks: Are There Differences? In H. S. Siller et al., *Researching Mathematical Modelling Education in Disruptive Times* (pp. 401-411). Springer.

Leikin, R. (2018). Giftedness and High Ability in Mathematics. In *Encyclopedia of Mathematics Education* (pp.1-11). Springer International Publishing.

Renzulli, J. S., Reis, S. M. (2021). The Three Ring Conception of Giftedness: A Change in Direction from Being Gifted to the Development of Gifted Behaviors. In RJ Sternberg, D Ambrose, *Conceptions of giftedness and Talent* (pp. 335-355). Palgrave Macmillan.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 575. PME.

MAKING SENSE OF MATHEMATICS THROUGH A BOUNDARY ARTEFACT IN VOCATIONAL NURSING EDUCATION

Michele G. Fiorentino[1], Antonella Montone[1], Pauline Vos[2]

[1]University of Bari, Italy, [2]Western Norway University of Applied Sciences

In vocational mathematics education, many students are demotivated to learn mathematics, which to them looks detached from their desired professions (FitzSimons & Boistrup, 2017). To address this disconnect, we investigated how students in nursing education can make sense of mathematics through interaction with a blood-pressure measurement device. Underlying our research is (1) the Theory of Semiotic Mediation and its concept of didactical *artefact* that evokes mathematical meaning (Bartolini Bussi & Mariotti, 2008), and (2) the concept of *boundary object* that preserves its identity across disciplines (Akkerman & Bakker, 2011). We combined these concepts into a new concept, *Boundary Artefact* (BA), which is a mediating tool for meaning making in both the vocation and in mathematics. The classroom activities, in which students measured their blood pressure using the BA, were co-orchestrated by a mathematics and a vocation teacher. The video-recorded data were analysed on how students' interaction with the BA gave rise to personal signs, such as numerical values being considered as "normal" or "problematic" and gestures indicating increase or decrease. In the ensuing classroom discussion, these evolved into shared meanings from the mathematical and vocational domains. Students' meanings related to measurement, variability, and thresholds, which remained closely connected to professional judgement and institutional norms. Our study reveals that a BA, such as a blood-pressure measurement device, can enable students in vocational education to engage with mathematical meanings emerging from professional practices. In this semiotic mediation, the BA functions as a generator of signs and then progressively mediates mathematical and vocational meanings simultaneously. Our study contributes to PME research by offering a theoretically grounded analysis of mathematical sense-making in vocational contexts, where mathematics does not announce itself (FitzSimons & Boistrup, 2017).

References

Akkerman, S. F., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research, 81*(2), 132–169. https://doi.org.10.3102/0034654311404435

Bartolini Bussi, M. G., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artefacts and signs after a Vygotskian perspective. In L. D. English (Ed.), *Handbook of international research in mathematics education* (pp. 746–783). Routledge.

FitzSimons, G. E., & Boistrup, L. B. (2017). In the workplace mathematics does not announce itself: Towards overcoming the hiatus between mathematics education and work. *Educational Studies in Mathematics, 95*(3), 329–349.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 576. PME.

MATHEMATICAL MODELLING KNOWLEDGE AND ATTITUDES OF FIRST-YEAR SOCIAL SCIENCE STUDENTS

Miranda Moodley, Wisani Hlangwani, Jacob Jaftha, Duncan Mhakure

University of Cape Town

Research in mathematics education has established mathematical modelling (MM) as a key practice for developing conceptual understanding and connecting mathematics to real-world contexts (Marchisio & Roman, 2021). To date, MM research has largely focused on science, technology, engineering, and mathematics (STEM) disciplines with limited attention to social science and art students, particularly at the transition to university contexts (Marchisio & Roman, 2021). Addressing this gap, this study investigates the MM knowledge and attitudes of first-year social science students at a South African university thereby establishing a necessary foundation for the informed and responsive research on mathematical modelling in undergraduate social science contexts. The study is guided by the research question: *What are the levels of mathematical modelling knowledge and attitudes among first-year social science students* Using a descriptive, quantitative cross-sectional design, survey data were collected from 156 first-year psychology students at a South African university. Adapted versions of the Mathematical Modelling Knowledge Scale and the Mathematical Modelling Attitude Scale (Asempapa, 2016) were administered, and descriptive statistics and regression analyses were used to analyse the data. Findings indicate moderate levels of modelling knowledge, with students demonstrating strong recognition of modelling as a translation process and the use of multiple representations, but weaker understanding of the distinction between modelling and problem-solving and the role of technology in modelling. Attitudes toward modelling were generally positive, particularly with respect to real-world relevance, constructivist learning beliefs, and motivation. However, students reported lower confidence in their own understanding of modelling. While broadly aligning with earlier studies that used the same instruments, the present findings extend this work by demonstrating that these patterns also hold for first-year social science students, highlighting the relevance of mathematical modelling beyond traditionally studied STEM populations.

References

Asempapa, R. S. (2016). *Developing an instrument to assess teachers' knowledge of the nature of mathematical modeling and their attitude toward such modeling*. Ohio University.

Marchisio, M., & Roman, F. (2021). Basic mathematical modelling competencies for non-stem higher education. In *Proceedings of the 15th International Conference International Conference e-Learning*, Online, 2021, pp. 54 - 61.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 577. PME.

MATHEMATICAL MODELLING AND EARLY ALGEBRA: CHANGES IN TEACHERS' SENSE-MAKING IN A CONTINUING PROFESSIONAL DEVELOPMENT PROCESS

Antonio Moreno[1], María Florencia Cruz[2], Karla Bastías[1]

[1]Universidad de Granada, [2]Universidad de Córdoba

Mathematical modelling (MM) and early algebra have a prominent place in primary education and curricular guidelines. However, within mathematics education research, empirical evidence on MM in primary education—particularly its potential to foster algebraic thinking at early ages—remains limited (Anhalt, 2021). Given that implementation largely depends on teachers' interpretations and decisions, this study analyses the senses primary education teachers construct about MM to enhance early algebraic work in a continuing professional development course in Chile. We conducted a qualitative case study with primary education mathematics teachers in this course, focused on early algebraic work and the use of MM to promote it. To explore changes in the senses attributed to MM, we examined initial narratives and final group interviews through the lens of Esteley and Cruz (2021). The results show changes in the senses attributed to MM for enhancing algebraic thinking. Initially, teachers mainly linked algebra to equations and MM to contextualised single-solution problems. They later recognised MM's potential to promote Blanton's (2008) algebraic practices: generalisation, representation, justification, and reasoning. They also value the open nature of the tasks, particularly the selection and discussion of variables. For example, they note that pupils "discover new variables within a problematic situation and go beyond a simple result". By the end, they attribute a more open role to MM, highlight its richness for early algebraic work, and design and implement classroom proposals aligned with this orientation.

Acknowledgements: This study is funded by the research project PID2024-157106NB-I00; financed by MCIN/ AEI/10.13039/501100011033, and FEDER, UE.

References

Anhalt, C. O., Cortez, R., & Aguirre, J. M. (2021). Mathematical modeling thinking: A construct for developing mathematical modeling competency. In J. M. Suh, C. Anhalt, & R. Cortez (Eds.), *Exploring mathematical modeling with young learners* (pp. 45-66). Springer.

Blanton, M. L. (2008). *Algebra and the elementary classroom: Transforming thinking, transforming practice*. Heinemann.

Esteley, C., & Cruz, M. F. (2021). Producción de sentidos sobre modelización: el caso de un grupo de futuras profesoras. *Quadrante, 30*(2), 269-292.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 578. PME.

THE NEW NEEDS FRIENDS

Ayush Mukherjee

Simon Fraser University

What we call ‘new’ in math education is often linked with how students construct relevant math identities (Gee, 2000; Gutiérrez , 2013). These identities, further, are studied as sociopolitical constructs, with a focus on issues like ‘agency’ and ‘power’. However, the student also emerges at a more fundamental and ontological level: at the level of Being or bare subjectivity (see Brown, 1996). At this level, we assume that what (or who) emerges within ‘proper’ math classrooms must be a math student, since that is our curricular intent or the aims of (math) education. In other words, we justify ‘the new’ because *it is new*, thus fetishizing ‘the new’. To ontologically justify the new, we must resist the idea that math classrooms always construct math students and trace how *mathematical subjectivities* emerge.

This emergence is a temporal problem. In this study, I thus ask: “In what time do mathematical subjectivities emerge?”, or “When does the new appear?” Conventional answers might suggest that the new appears only in futures intended by the curriculum. Instead, within assignment submissions from 24 pre-service teachers enrolled in a course on math anxiety that I taught, I find evidence that my students also constructed themselves (as math subjects) in the past and the present, as well as in the future.

From their assignments, I recognized three distinct ways that the ‘new’ emerges and the their three corresponding modes of mathematical engagement. The ‘new of the past’ appears when the student considers herself already constructed as a math student and *refuses* to be re-constructed during the current lesson. The ‘new of the present’ appears when the student *intuits* her own construction as a math student as it happens. The ‘new of the future’ happens when the student *speculates* her construction as a math student and uses empty gestures to represent a future non-representable self. I read their temporal emergence through various continental philosophers and advocate a different role for the teacher. Instead of a guide, mediator or facilitator, I argue for a teacher-friend: one who travels into various ‘times’ to gather her students.

References

Brown, S. I. (1996). Towards humanistic mathematics education. In A. Bishop et al. (Eds.)., *International Handbook of Mathematics Education* (pp. 1289–1321). Kluwer Academic Publications.

Gee, J. P. (2000). Identity as an analytic lens for research in education. *Review of Research in Education, 25*, 99–125.

Gutiérrez, R. (2013). The sociopolitical turn in mathematics education. *Journal for Research in Mathematics Education, 44*(1), 37–68.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 579. PME.

EXPLORING RELATIONAL THINKING IN TEACHERS' WORKING WITH CONNECTED ADDITION TASKS

Angel Mukuka, Hamsa Venkat

CASTeL, Dublin City University

This study draws on research on relational thinking, defined as "using fundamental properties of number and operations to transform mathematical expressions rather than simply calculating an answer following a prescribed sequence of procedures" (Carpenter et al., 2005, p.54). Connected tasks afford such thinking by enabling prior results to be adapted when solving subsequent problems. However, South African research highlights gaps in instruction that promote sense-making and relational thinking for efficiency (Askew, 2019). We therefore examined how eight primary teachers, participating in task-based interviews preceding a WhatsApp-based professional development programme linked to their teaching after-school Maths Clubs in low-attainment contexts, responded to two connected addition tasks (A: 53 + 20; B: 53 + 19), in terms of solving for themselves and for supporting learners. Our analysis involved coding teachers' interview response video clips for the extent of relational thinking in their solving the two tasks. Relational thinking was identified when teachers used the structure or result of the first task to transform the second. For instance, seeing 53 + 19 as 53 + 20 − 1, or as one less than 73, rather than solving it as an independent calculation.

Across the two tasks, a clear pattern emerged in teachers' strategy use. While Task A was mostly solved mentally using jump strategies, place-value decomposition, or column algorithm, Task B was largely treated as a more complex and independent problem, which was solved using written column algorithms or Dienes' block representations for both self and learners. No instances of relational thinking were observed in teachers' own solutions, as teachers did not use the result of Task A to transform Task B, even when invited to consider the tasks collectively. Only one teacher indicated the possibility of using relational thinking in her teaching, using rounding and adjusting from 53 + 20. Teachers favoured written column algorithms and Dienes' blocks for their perceived accessibility to a wide range of learners. Overall, teachers' responses suggested that Task B was almost always recomputed as an independent task. This indicates limited uptake of the relational affordances intentionally built into our connected task pair.

References

Askew, M. (2019). Mediating primary mathematics: measuring the extent of teaching for connections and generality in the context of whole number arithmetic. *ZDM – Mathematics Education, 51*(1), 213–226. https://doi.org/10.1007/s11858-018-1010-9

Carpenter, T. P., Levi, L., Franke, M. L., & Zeringue, J. K. (2005). Algebra in elementary school: Developing relational thinking. *ZDM, 37*(1), 53–59.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 580. PME.

TASK DESIGN PRINCIPLES FOR SUDENTS' DEFINING

Shogo Murata

Nippon Sport Science University

In mathematics, definition and defining are essential, as with proof and proving. Some frameworks that capture students' defining have been proposed (Kobiela & Lehrer, 2015; etc.), and revealed diverse processes when students construct or refine definitions of mathematical concepts. Based on the accumulated research, the present study aims to develop classroom interventions especially focusing on mathematical tasks that students can engage in constructing and refining definitions. I am tackling the research question: How can we design tasks to promote students' defining? The present study has been conducted using the methodology of topic-specific design research. I constructed a conceptual framework to promote activities where students themselves construct their tentative definitions and refine them into mathematically appropriate definitions. This framework consists of two phases: (a) purposefully selecting a definition of a familiar concept or giving a name of a new concept, (b) refining tentative definitions into appropriate ones. Given these aspects in the framework, I established learning the goal to engage defining in the context of mathematical inquiry and designed the task for university students (preservice teachers) that is the extending inclusion relations of special quadrilaterals. This task involves students constructing definitions by examining examples and properties of quadrilaterals they know the names (e.g., trapezoids, kites) and refining their definitions into more correct and more sophisticated ones. Using this task, classroom interventions that consist of three sessions were implemented with 35 students. The lessons were transcribed and analyzed qualitatively with the help of a research assistant. Through the analysis, I identified several scenes in the classroom lesson episodes where students selected their definitions of special quadrilaterals and refined them by discovering or non-examples of the concepts. Students were able to not only define and classify squares, rectangles, rhombuses, parallelograms, and trapezoids, but also extend the inclusion relation of them to include kites, isosceles trapezoids, and other quadrilaterals that have special properties. The results suggest that the designed tasks are effective for promoting students' defining. I concluded that the following two task design principles for defining are formulated: (1) To present questions about an incomplete system that generates students' doubts, (2) To suggest a consideration about the relationship between multiple concepts in order to construct their definitions.

Acknowledgements

This work was supported by the JSPS, Grant Numbers 23K12778 and 24K00410.

References

Kobiela, M., & Lehrer, R. (2015). The codevelopment of mathematical concepts and the practice of defining. *Journal for Research in Mathematics Education, 46*(4), 423–454.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 581. PME.

PRE-SERVICE MATHEMATICS TEACHERS' NOTICING OF CRITICAL EVENTS ASSOCIATED WITH ARGUMENTATION

Samaher Nama[1], Michal Ayalon[2], Amal Sharif-Rasslan[1]

[1]The Academic Arab College for Education, [2]University of Haifa

Argumentation is increasingly recognized as essential in mathematics classrooms, yet teachers often struggle to capitalize on opportunities for its development. Given the multifaceted nature of argumentation, its successful facilitation in the classroom depends on teachers' ability to recognize moments that can foster the constriction, critique, and refinement of mathematical ideas. We refer to such moments as Critical Events Associated with Argumentation (CEAs) – instances identified by teachers as opportunities for argumentation to develop and to use them as a lens for analyzing teacher noticing (Ayalon & Nama, 2025).

This study focuses on preservice mathematics teachers (PSTs) and examines the CEAs they identify while observing classrooms during their teacher education program. Focusing on self-selected events offers insight into what PSTs perceive as potential for argumentation at early stages of professional learning. The research question is: What characterizes the CEAs identified by PSTs during classroom observations?

The study sample comprised nine secondary PSTs. During the program, they learned about argumentation in mathematics classrooms and conducted classroom observations to identify CEAs. Events were coded as structural or dialogic, with the structural aspect focusing on argument construction and justification, and the dialogic aspect on peer interactions. Participants identified 21 events: 11 structural, 10 dialogic, with two involving both aspects. Most structural events reflected challenges in argument construction, including constructing invalid arguments (6/11) and using inadequate justifications (4/11). Dialogic events frequently highlighted difficulties in participation, such as non-participation or withdrawal from discourse (4/10) and disrespectful responses to peers' arguments (3/10). The same number of events (3/10) focused on productive discourse that led to mathematical progress.

Taken together, the events identified by PSTs highlight multiple ways in which argumentation is recognized in classroom practice, encompassing both reasoning processes and interactional dynamics. While based on a small sample, these findings suggest that examining the events PSTs select can offer insight into their emerging understandings of what counts as an opportunity for argumentation.

References

Ayalon, M., & Nama, S. (2025). Mathematics teachers' noticing of critical events associated with argumentation. *Journal of Mathematics Teacher Education, 28*, 1077-1104.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 582. PME.

WHAT KINDS OF STUDENTS DOES MATHEMATICS ASSESSMENT PRODUCE?

Juuso Henrik Nieminen

Deakin University

Mathematics assessment is often conceptualised as an objective practice of measurement (Nieminen et al., 2023). This dominant paradigm has been challenged by views of assessment as a socio-historical practice that shapes how students are 'known' in mathematics education (Yolcu & Popkewitz, 2019). Namely, mathematics assessment creates and reifies the 'constructs' it purports to measure (Wiliam et al., 2004, p. 45). What kinds of students are mobilised by mathematics assessment, then?

In this study, this objective is explored by analysing curriculum and policy documents regarding mathematics assessment in Finland. The analysis relies on Popkewitz' policy analysis tools that call us to focus on those practices of mathematics education that make it possible to generate truth statements about who children are, who they are becoming, and how they differ from each other (Yolcu & Popkewitz, 2019). Assessment produces precisely such truth statements about who students 'are'. Through a socio-politically oriented document analysis of Finnish mathematics assessment, this study contributes to assessment research by making visible the 'ideal' learners of mathematics that are mobilised by these policy frameworks.

The policy analysis shows that the desired student subject is mobilised by assessment as individualistic and objectified. To summarise, mathematics assessment (and summative assessment in particular) aims to target the individual student's mathematical abilities and in doing so creates a particular historical idea of an individual knower and doer of mathematics (cf. Nieminen et al., 2023). On the other hand, through assessment, (and formative assessment in particular), students are made into objects to be managed, controlled, and 'put into shape' (cf. Yolcu & Popkewitz, 2019). This study calls for further attention to the (un)desired ways of being a student that are made possible by mathematics assessment.

References

Nieminen, J. H., Bagger, A., Padilla, A., & Tan, P. (2023). Student positioning in mathematics assessment research: A critical review. *Journal for Research in Mathematics Education, 54*(5), 317-341.

Wiliam, D., Bartholomew, H., & Reay, D. (2004). Assessment, learning, and identity. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 43-61). Kluwer.

Yolcu, A., & Popkewitz, T. S. (2019). Making the able body: School mathematics as a cultural practice. *ZDM, 51*(2), 251-261.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 583. PME.

ACTIONABLE DISPOSITIONS: RETHINKING MATHEMATICAL AFFECT THROUGH MATHEMATICAL HABITS OF MIND

Jihwa Noh[1], Elena Prieto-Rodriguez[2], Nan Huh[3], Ho Kyoung Ko[4]

[1]Pusan Nat'l University, [2]University of Newcastle, [3]Kyonggi University, [4]Ajou University

Traditionally research emphasizes latent affective traits like interest and confidence, which are difficult to observe or influence directly. While mindset theories (Dweck, 2006) provide a valuable framework for growth, mindset remains an internal construct that poses challenges for teachers in terms of direct assessment. This study addresses this limitation by proposing a framework that treats mathematical affect as actionable dispositions. We posit that by repurposing Mathematical Habits of Mind-the intellectual behaviors used by mathematicians (Cuoco et al., 1996)-we can create a visible diagnostic lens for student affect. We conceptualize this mathematical mind as a system requiring the interaction of two pillars: Habits of Action and Habits of Critical Thinking. Without Action, engagement becomes passive observation; without Critical Thinking, effort becomes aimless trial-and-error.

To develop this framework, we validated a pool of Mathematical Habits of Mind items as indicators of actionable affect using a sample of 400 students (200 Korean, 200 Australian). Following item purification, iterative Exploratory Factor Analysis and higher-order factor analysis were performed to identify the hierarchical structure and confirm cross-cultural validity.

The empirical analysis confirmed that mathematical affect, as an actionable disposition, is structured into two primary meta-cognitive pillars: 1) Habits of Action (A generative orientation involving seeking structure, experimenting, and cognitive persistence), and 2) Habits of Critical Thinking (An analytical orientation involving justification, reasoning, and meta-cognitive monitoring). These findings offer a visible alternative to latent affective measures by shifting focus from 'how students feel' to 'how they act,' providing a cross-culturally validated bridge between internal affect and classroom behavior. This framework empowers teachers to move students from passive observation or aimless trial-and-error toward a functional, diagnostic 'mathematical mind.'

References

Cuoco, A., Goldenberg, E. P., & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curricula. *Journal of Mathematical Behavior*, *15*(4), 375-402.

Dweck, C. S. (2006). *Mindset: The new psychology of success*. Random House.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 584. PME.

PROSPECTIVE TEACHERS' USE OF REPRESENTATIONS IN A LEARNING STUDY ABOUT THE COMPLEMENT PRINCIPLE

Rimma Nyman, Pernilla Mårtensson, Angelika Kullberg

University of Gothenburg

This study investigates what prospective teachers (PTs) learned when teaching the relationship between addition and subtraction in Grade 1 during a learning study. The aim is to identify how the use of representations changed from the first to the second lesson in the iterative process. While representations can enhance mathematical learning (Schoenherr et al., 2024), their effectiveness depends on how they are aligned with the object of learning. The study was conducted within a mathematics course in primary teacher education at two universities, where 100 PTs participated in a learning study (Pang & Runesson Kempe, 2019). A case from one group working with the complement principle (if a+b=c, then c−b=a) is presented. The theoretical framework of the study is variation theory (Marton, 2015). Data include video recordings from Lesson 1 (L1) and Lesson 2 (L2), in which PTs used variation theory to design and plan the lessons and identified critical aspects for pupils' learning. One identified critical aspect was the same part-whole relation between the numbers for addition and subtraction. In L1, the prospective teachers used the bar model to illustrate the part-whole relation. A torn A4 sheet was used to help pupils experience a part-whole structure, but the structural relationship was not explicitly foregrounded. After revision, a more fine-grained use of the bar model was present, highlighting new critical aspects of the complement principle. The bar model in L2 was coordinated with symbolic expressions, strengthening connections between visual and numerical representations to increase understanding of the complement principle. In L1, the paper bar model was used as a general model, while in L2 it was used to illustrate a specific relation (5/4/1), accompanied by the question: "How can we make this a subtraction?" The bar model was used to find the missing parts (L2) instead of the whole (L1). In L2, the bar model was used to work with subtractions $5-4=1$ and $5-1=4$. PTs visualised the invariance of the whole, keeping 5 constant while varying the missing part, explicitly discerning how the same part–whole structure underpins both operations. The findings illustrate how participation in a learning study can support PTs in developing a theoretically grounded and purposeful use of visual representations in teaching relationships between addition and subtraction in early arithmetic.

References

Marton, F. (2015). *Necessary conditions of learning*. Routledge

Pang, M. F., & Runesson Kempe, U. (2019). The Learning Study: Recent trends and developments. *International Journal for Lesson and Learning Studies*, *8*(3), 162–169.

Schoenherr, J., Strohmaier, A. R., & Schukajlow, S. (2024). Learning with visualizations helps. *Educational Research Review*, *45*.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 585. PME.

MATHEMATICAL CONCEPTS AS FUNCTIONAL PROPERTIES

Christoph Oberbucher, Christina Krause

University of Graz

Learning mathematics within an enactive-ecological radical embodiment (EERE) framework is characterized as 'living' in the relationships and interaction of the learner and the world around them. Meanwhile, traditional cognitivist frameworks understand conceptual understanding and concept formation as part of the cognitive structure of a learner, merely being influenced by this interaction. Designing learning opportunities for conceptual understanding within EERE now needs to clarify and explicate what we are actually aiming for in terms of learning outcomes, problematizing what we understand as a mathematical concept within radical embodiment to take into account the entanglement of cognized and cognizer (Baggs & Chemero, 2021).

This theoretical contribution explores this question by discussing the nature of concepts on the basis of a central tenet of radical embodiment—that of *affordances.* Affordances are possibilities for action that arise as relational properties between an individual and features of an environment. Engaged in loops of perception and perceptually guided action, individuals become aware of certain affordances, which allow for coordinated interaction. Inspired by Thompson (2023), we propose a concept of a mathematical concept with the selective focus on affordances and the accompanying possibility of exclusion at its core: Assuming that changes in the layout of affordances can be perceived (Chemero, 2003), a certain affordance A1 can be distinguished from non-A1. The affordance A1 serves as a template, leading to a dichotomy of A1 and non-A1 through the process of exclusion, not based on similar properties of the objects being engaged with, but on similar sensorimotor responses in the course of goal-oriented tasks. In this sense, concepts are not 'some-things' but highlighting aspects of dynamic processes that "are a functional part of situations and have a participatory role in structuring them" (Thompson, 2023, p.147). From an educational standpoint, it is hypothesized that learning opportunities are designed so equivalence classes of sensorimotor response can be built by learners. Our idea of a mathematical concept will be discussed in the context of empirical illustrations.

References

Baggs, E., & Chemero, A. (2021). Radical embodiment in two directions. *Synthese*, *198*(S9), 2175–2190.

Chemero, A. (2003). An Outline of a Theory of Affordances. *Ecological Psychology*, *15*(2), 181–195.

Thompson, E. (2023). What's in a Concept? Conceptualizing the Nonconceptual in Buddhist Philosophy and Cognitive Science. In C. Coseru (Ed.), *Reasons and Empty Persons: Mind, Metaphysics, and Morality* (Vol. 36, pp. 165–210). Springer.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 586. PME.

INFORMAL AND FORMAL MODES OF PROOF-RELATED REASONING IN EUCLIDEAN GEOMETRY TASKS IN SOUTH AFRICAN TEXTBOOKS

Matome E. Matsepe, Ugorji I. Ogbonnaya

University of Pretoria

Proof is a central practice in mathematics, particularly in Euclidean geometry, where students are expected to organise knowledge using a system of definitions, axioms, and theorems. However, Tall (1998) argues that students are often introduced to proof as a finished product characterised by formal, symbolic language beyond their conceptual grasp. Consequently, they tend to memorise procedures rather than make sense of the logical relationships that justify each step. In response, contemporary research (e.g., Zhang & Qi, 2019) conceptualises proof within a framework of proof-related reasoning (PrR) that includes informal and formal modes. Proof is therefore understood not only as a formal deductive endpoint but as part of a continuum linking informal reasoning to structured deduction. Following Zhang and Qi's (2019) framing, informal modes, such as *identifying patterns*, *conjectures*, *non-proof arguments*, and *evaluating claims*, function as precursors to formal proofs. Formal proof (*demonstrate, generic examples, counterexamples*) requires theorem-based justification (Zhang & Qi, 2019). In this study, we adopted this framework to examine how modes of PrR are embodied in expository and exercise tasks and to determine their pedagogical orientation toward proof. We conducted a qualitative content analysis of nine purposively selected mathematics textbooks for Grades 10-12 that are widely used in public schools. A total of 1993 tasks were analysed. In South Africa, textbooks are among the primary resources for teaching and learning, particularly in rural and under-resourced schools. They therefore shape how mathematics is learnt. The results indicate that while textbooks provide substantial engagement with PrR (67.93%, 1354 of 1993 tasks), they emphasise formal proofs. Minor opportunities (11%) for informal reasoning are evident in Grade 10 but diminish progressively in Grades 11 and 12, where a strong formal pedagogical orientation prevails. As a result, the continuum from informal exploration to formal deduction is not consistently presented across the grades. This suggests a misalignment between theoretical models of proof development and their integration in the sampled textbooks. To facilitate sense-making, it is suggested that the sampled textbooks integrate both formal and informal modes of PrR across the grades.

References

Tall, D. (1998). The cognitive development of proof: Is mathematical proof for all or for some? In *Conference of the University of Chicago School Mathematics Project*.

Zhang, D., & Qi, C. (2019). Reasoning and proof in eighth-grade mathematics textbooks in China. *International Journal of Education Research, 98*, 77-90.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 587. PME.

COGNITIVE PROFILES AND PROBLEM-SOLVING IN AUTISTIC STUDENTS: PRELIMINARY FINDINGS FROM A SCOPING REVIEW

Daniela Olivares

Universidad de La Serena

Cognitive diversity is reshaping how we think about mathematics learning, yet autistic students remain underrepresented in education research (Petersson-Bloom & Holmqvist, 2022). Autistic students present distinctive cognitive profiles that influence their participation in mathematics lessons, particularly in the context of problem-solving. (Cox & Root, 2020), including the use of compensatory strategies that may inform the design of didactic orientations grounded in cognitive diversity. However, these differences remain largely invisible in mathematics education research. A scoping review was conducted to map the existing literature to address these questions: (1) What does the literature reveal about cognitive profiles and mathematical problem solving in autistic students? (2) What role has mathematics education research played in this field? The review was conducted searching in Web of Science, Scopus, and ERIC, for literature published between 2015 and 2025. Three complementary search equations addressed cognitive profiles and mathematical problem solving in autistic students. After deduplication, 773 records were screened by two independent reviewers. The scoping review identified 208 studies meeting the inclusion criteria. Most research examines autistic students' cognitive characteristics—such as executive functioning, working memory, and visuospatial processing—and relates these to mathematical problem solving through experimental or standardized tasks. Findings highlight heterogeneous cognitive patterns and indirect links to problem-solving processes. Only a small subset addresses strategic or compensatory approaches. Mathematics education research remains marginal, with limited attention to classroom practices and disciplinary frameworks. The findings reveal that research predominantly frames mathematical problem solving in autistic students through cognitive psychology, with limited contribution from mathematics education. This imbalance highlights the need for discipline-specific frameworks to better understand problem-solving processes and inform inclusive instructional practices.

References

Petersson-Bloom, L., & Holmqvist, M. (2022). Strategies in supporting inclusive education for autistic students—A systematic review of qualitative research results. *Autism & developmental language impairments, 7*.

Cox, S. K., & Root, J. R. (2020). Modified schema-based instruction to develop flexible mathematics problem-solving strategies for students with autism spectrum disorder. *Remedial and Special Education, 41*(3), 139-151.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 588. PME.

LEARNING AI IN MATHEMATICS EDUCATION

Runar Lie Berge[1], Joakim Olsson[1], Andreas Brandsæter[1,2]

[1]Volda University College

[2]NTNU – Norwegian University of Science and Technology

This study investigates how AI literacy can be developed within secondary mathematics education. Rather than treating AI primarily as a tool, the study argues that school mathematics is well suited to support students' understanding of what AI is, how it functions, and where its limitations lie. Through a theoretical analysis, the study shows how core AI concepts can emerge naturally through mathematical problem solving.

The analysis is grounded in Long and Magerko's (2020) AI literacy framework, which identifies a set of 17 competencies related to understanding, interacting with, and critically evaluating AI systems. The framework extends beyond technical skills to include decision making, learning from data, and the human role in AI. These competencies are used to analyze a teaching session centered on the question: *Can a machine predict the next digit in a "random" binary sequence produced by a human with better-than-chance accuracy?*

In the teaching session, students develop prediction algorithms from scratch rather than applying prebuilt models. Four increasingly sophisticated strategies are analyzed: frequency-based prediction, alternation-based prediction, conditional prediction based on recent context, and dynamic adaptation. Despite their simplicity, these strategies achieve relatively high predictive accuracy on students' input. Across a series of exploratory classroom experiments conducted in lower and upper secondary schools in Norway in late 2025 and early 2026, this outcome frequently surprises students. This foregrounds the idea that human behavior is often more predictable than the students have assumed, even when individuals deliberately attempt to act randomly. This insight invites reflection on how patterned data enable powerful inferences, both in the classroom and in everyday digital contexts. Through developing and refining prediction strategies, students engage with fundamental AI ideas such as learning from data, probabilistic reasoning, model training, adaptation, and algorithmic decision making. Overall, the study demonstrates how embedding AI literacy in mathematics education can strengthen mathematical understanding while also supporting students' capacity to critically engage with AI systems that increasingly shape everyday life.

References

Long, D., & Magerko, B. (2020). What is AI Literacy? Competencies and Design Considerations. *Proceedings of the 2020 CHI Conference on Human Factors in Computing Systems*, 1–16.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 589. PME.

INVESTIGATION OF GIFTED STUDENTS' COLLABORATIVE PROBLEM-SOLVING SKILLS IN ONLINE ENVIRONMENT

Deniz Özen Ünal, Derya Ak

Aydın Adnan Menderes University

Today, many problems require teamwork; therefore, collaborative problem-solving (CPS) is regarded as a critical 21st-century skill (Graesser et al., 2018; OECD, 2017). OECD (2017) defines CPS as the capacity of two or more individuals/agents to build a common understanding by integrating their knowledge, skills, and efforts to produce solutions to the problems they encounter. With the development of technology in recent years, the ability to collaborate in online environments has come to the fore and is considered among the critical skills of the future (OECD, 2017). The presence of collaboration skills is of great importance in gifted students, who are expected to be at the forefront of the development of countries and the solution of global problems (Renzulli, 2012).

In this study, it was aimed to examine the CPS skills exhibited by gifted students in an online environment. Three gifted middle school students continuing their education at a Science and Art Center in the province of Aydın, Turkey, participated in the study. During the data collection process, six open-ended mathematics problems in an eco-mathematics context (Chesky & Milgram, 2021) were used. These problems require the use of mathematical relations and operations in the solution of complex problems such as global warming and energy issues. The PISA 2015 CPS Matrix was utilized in the analysis of the data. It was observed that the participants most frequently exhibited skills related to establishing a common understanding, while they least exhibited skills aimed at establishing team organization during the problem-solving process. It was determined that the participants did not exhibit skills A1, A2, and A3 belonging to the 'Exploring and Understanding' step throughout the process. In this study, it is concluded that the participants exhibited at least one skill from the three collaborative competencies and were able to conduct the CPS process in an online environment.

References

Chesky, N., & Milgram, J. (2021). *Eco-mathematics education: K-8 lesson plans for ecological and social change* (Vol. 8). Brill.

Graesser, A. C., Fiore, S. M., Greiff, S., Andrews-Todd, J., Foltz, P. W., & Hesse, F. W. (2018). Advancing the science of collaborative problem solving. *Psychological Science in the Public İnterest*, *19*(2), 59-92. https://doi.org/10.1177/1529100618808244

OECD (2017). *PISA 2015 collaborative problem-solving framework.* OECD Publishing.

Renzulli, J. S. (2012). Reexamining the role of gifted education and talent development for the 21st century: A four-part theoretical approach. *Gifted child quarterly*, *56*(3), 150-159.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 590. PME.

FROM PERCENTAGES TO SOCIAL JUSTICE: MEANINGS CONSTRUCTED BY MATHEMATICS TEACHERS WHEN INTEGRATING ECONOMIC AND FINANCIAL EDUCATION

Sandra Parada[1], Alexander Conde[2], Vicente Liern[3]

[1]Universidad Industrial de Santander; [2]Universidad Complutense de Madrid, [3]Universitat de València

The integration of Economic and Financial Education (EFE) in Mathematics Education is often justified in terms of everyday usefulness. Here, we examine it from a social justice perspective, focusing on how such integration redistributes, recognizes, and represents knowledge in the classroom. The study addresses the following question: What meanings do mathematics teachers construct, when reflecting on the integration of EFE, to promote mathematical activity (MA) from a social justice perspective? The study articulates three theoretical strands: (i) The conception of social justice (redistribution, recognition and representation) of Fraser (2008), used to interpret access to learning opportunities, validation of students' experiences without moralization, and the definition of curricular priorities; (ii) the Reflection and Action model (RyA) of Parada y Fiallo (2022), which characterizes teaching practices as situated reflection leading to action taken, allowing decisions about tasks, resources, interaction, and assessment to be interpreted as expressions of professional meaning-making; and (iii) EFE understood as a school field that mobilizes mathematical notions as well as positions regarding consumption, saving, credit, and indebtedness. A descriptive–exploratory study was conducted through a questionnaire administered to mathematics teachers in public schools. The instrument included closed items and an open question about financial situations addressed in class. The analysis combined synthesis across categories of practice with thematic categorization of reported examples to reconstruct meanings associated with mathematical activity and social justice. Teachers' reports suggest that EFE enters the classroom unevenly and without sustained development: it gains presence when teachers design tasks based on familiar situations but weakens when institutional conditions are required. Interpreted through RyA, four meanings of mathematical activity with EFE emerge: using mathematics to calculate and interpret; comparing options to make decisions; viewing EFE as a right linked to unequal opportunities; and opening a critical perspective in development, particularly when arguing and discussing credit and indebtedness.

References

Fraser, N. (2008). *Scales of justice: Reimagining political space in a globalizing world*. Columbia University Press.

Parada, S. y Fiallo, J. (2022). Comunidades de práctica de profesores de matemáticas que incorporan tecnologías digitales en el aula. Ed: Ediciones UIS. En Colombia.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 591. PME.

INTERPRETING STUDENTS' MATHEMATICAL THINKING INVOLVING NATURAL NUMBER BIAS

Sebastián Parodi

Universidad Tecnológica del Uruguay

Natural number bias (NNB) has been widely documented as a persistent difficulty in learning rational numbers. However, less is known about how prospective mathematics teachers (PMTs) interpret student responses, in which this bias becomes evident when working with fractions and decimals. This paper examines how PMTs reconstruct the reasoning underlying such responses by asking: What characterizes PMTs' interpretations of student productions involving manifestations of NNB?

The study draws on two complementary frameworks. First, professional noticing (PN), focusing on interpretation as inferring students' mathematical understanding from the details of their productions and from knowledge of mathematical thinking (Jacobs et al., 2010). Second, conceptual change research provides the basis for understanding students' NNB as a persistent reasoning framework arising from the overgeneralization of natural number properties to rational numbers (Van Dooren et al., 2015).

Nine PMTs participated in a qualitative exploratory case study within a teacher education course. The study was based on a sequence of two teacher-role concept cartoons (TRCC) whose characters are PMTs discussing authentic student productions involving NNB. Data consisted of audio-recorded plenary discussions. Segments containing explicit inferences about students' thinking were analysed and coded into three levels (Jacobs et al., 2010): lack, limited, and robust evidence (LaE, LiE, RoE).

Analysis of the data showed the levels to be characterized as follows. At LaE, errors were reduced to procedural shortcomings; NNB was explicitly ruled out. At LiE, errors were attributed to the inappropriate transfer of learned rules or analogies, with NNB implicitly suggested but not conceptualized as an underlying framework. At RoE, errors were interpreted as coherent results of reasoning with rational numbers as if they were natural numbers, reconstructing NNB as an overgeneralization of natural number properties. These levels capture qualitative differences in interpretive depth rather than developmental stages, positioning TRCC as a resource for inquiring into PMTs' noticing of NNB in students' mathematical thinking.

References

Jacobs, V., Lamb, L., & Philipp, R. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169–202.

Van Dooren, W., Lehtinen, E., & Verschaffel, L. (2015). Unraveling the gap between natural and rational numbers. *Learning and Instruction, 37,* 1–4.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 592. PME.

REPRESENTATIONS AND GENERALIZATION PROCESS IN FIFTH-GRADE

María C. Pérez-Martos, Antonio Moreno, María D. Torres, María C. Cañadas

Universidad de Granada

This study addresses generalization in the functional context of algebraic thinking. Torres et al. (2021) proposed a model of the generalization process that comprises three successive stages: abduction, induction, and generalization. The existing research on this process has primarily focused on the early years of elementary education. Our interest here is the generalization process in fifth-grade students by observing the tabular and graphical representations they use in their progress toward generalization. Additionally, there is limited research on the effects of utilising diverse representations on this model of generalization process (Torres et al., 2021), especially with regard to the use of Cartesian graphs. In this study, the aim was defined through three specific goals: (a) to identify the different stages occurring in the student's generalization process, (b) to identify the representations used to document the structure in each stage of the process, and (c) to determine whether the representations utilised during the process facilitated the student's generalization.

Through a case study, we interviewed a student to provide an in-depth analysis of the representations used during the generalization process. We analyzed a semi-structured interview with the student in which they worked on a task that included various representations, in particular Cartesian graph. We selected this student because their interview was representative of an intermediate level, reflecting the typical performance of the fifth-grade class; and because of their active participation.

The research findings encompassed the student progressed through the three phases of the model (Torres et al., 2021). On the other hand, verbal, primarily, and gestural representations were the most prevalent throughout the process. The novelty of this study is that in this instance, the representation which most facilitated progression towards generalization was Cartesian graph, in conjunction with the gestural one, primarily due to its visual characteristics and the procedural manner of its construction.

Acknowledgment

This study was conducted with the support of Projects PID2020-113601GB-I00 and PID2024-157106NB-I00 financed by MCIN/AEI/10.13039/501100011033 and Fondos FEDER. Grant PRE2021-097391 financed by MICIU/AEI/10.13039/501100011033.

References

Torres, M. D., Moreno, A., & Cañadas, M. C. (2021). Generalization process by second grade students. *Mathematics, 9*(10), 1109.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 593. PME.

ENACTING THE "WE": COLLECTIVE IDENTITY IN UNIVERSITY ROLE-BASED COLLABORATIVE PROBLEM SOLVING

Luca Picariello, Annamaria Miranda, Cristina Coppola

University of Salerno

Within university mathematics education, undergraduate students' identity dynamics play a crucial role in shaping their academic experience. Against this backdrop, collaborative problem solving has proven to be a space in which students can explore mathematicians' problem-solving practices, thus providing a valuable setting for investigating these dynamics. Building on previous research examining university students' identity dynamics during role-based collaborative problem solving and focusing on the interplay between students' identities and the institutional discourse surrounding mathematical activity (Picariello et al., in press), this study aims to extend this line of research by investigating the configuration of collective identities in university collaborative problem solving. This study draws on data from a role-based collaborative problem-solving activity conducted in a second-year General Topology course within a mathematics degree programme in southern Italy, involving eight groups of students. The activity was designed to place students in a space where they could embody different ways of engaging in mathematical activity, allowing them to align with or question institutionally shaped norms of doing mathematics. Drawing on Judith Butler's understanding of collective identity (2015), the analysis sought to trace the plural actions around which collectivities, represented by groups engaged in role-based collaborative problem solving, are constituted.

Findings indicate actions enacted in and through the activity itself configuring a collectivity around shared theory of success for university mathematics and implicit norms that frame mathematical activity fundamentally articulated around rigour, in which the affective component is deemed as less necessary. The study highlights how narratives, implicit institutional norms surrounding mathematics, and individual identity are consolidated, shared, and enacted at the group level, thus adding a new layer to our understanding of the identity dynamics in which undergraduate mathematics students are involved.

References

Butler, J. (2015). *Notes Toward a Performative Theory of Assembly*. Harvard University Press.

Picariello, L., Liljedahl, P., Miranda, A., & Coppola, C. (in press). *"Left to myself, I wouldn't have done it": Dagen H in mathematics education*. 31st International Mathematical Views Conference (MAVI31).

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 594. PME.

VISUAL MEDIATORS IN LECTURES: BRIDGING COMOGNITIVE CONFLICT

Manuela Piñeros Rodríguez

École Polytechnique Fédérale de Lausanne (EPFL)

Research on instructors' practices in the university mathematics lecture has expanded over the past decade, but research on how students understand the lecture has received less attention (Melhuish et al., 2022). However, evidence suggests that communication breaches exist that may hinder students' access to the meaning conveyed by the instructor (Lew et al., 2016). The commognitive framework (Sfard, 2008), which views mathematics as a discourse, suggests that these breaches may be due to incompatible mathematical discourses (commognitive conflict). One proposed aspect that could help bridge this conflict is the use of visual mediators. Having access to visible objects on which to conduct operations can potentially lead to agreement on the objects of discourse, enabling more effective communication in situations where there are discursive mismatches.

This contribution explores the role of visual mediators in shaping students' commognitive engagement with a first-semester Real Analysis lecture. We analysed data from two distinct lectures where the definition of limit was introduced, along with stimulated-recall interviews with two or three student pairs per instructor. Our analyses focussed on the visual mediators used by the two instructors during their presentations and those invoked by students during their discussion of the definition episode.

Preliminary results indicate that instructors employ visual mediators differently in their lectures. Moreover, when discussing the definition excerpt, students rely on both the visual mediators introduced by the instructor and additional mediators they produce themselves. This suggests that comprehension of the lectures may depend on students' ability to interpret the visual mediators used by their instructors, potentially shaped by the additional mediators spontaneously available to them. We explore the implications of these findings for the effectiveness of university mathematics lectures.

References

Lew, K., Fukawa-Connelly, T. P., Mejía-Ramos, J. P., & Weber, K. (2016). Lectures in Advanced Mathematics: Why Students Might Not Understand What the Mathematics Professor Is Trying to Convey. *J. for Research in Mathematics Education*, *47*(2), 162–198.

Melhuish, K., Fukawa-Connelly, T., Dawkins, P. C., Woods, C., & Weber, K. (2022). Collegiate mathematics teaching in proof-based courses: What we now know and what we have yet to learn. *The Journal of Mathematical Behavior*, *67*, 100986.

Sfard, A. (2008). *Thinking as Communicating: Human Development, the Growth of Discourses, and Mathematizing*. Cambridge University Press.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 595. PME.

BAR MODELS IN EMBODIED MATHEMATICAL REASONING

Claire Poh

Charles University

This study is grounded in an embodied and semiotic perspective on mathematical cognition, which conceptualizes reasoning as emerging through the coordination of perception, action, and representation. From this perspective, thinking is understood as a multimodal and embodied process, mediated through gestures, language, and artifacts (Luis Radford, 2009). Within this framework, bar models are understood as visual-spatial representations that mediate embodied reasoning, enabling learners to structure and interpret mathematical relationships. The study aims to investigate how bar models support embodied mathematical reasoning in students' problem-solving processes. It focuses on how learners coordinate gesture and visual representation to attend to, structure, and communicate mathematical relationships. A single-case study was conducted focusing on a Primary 3 student from Singapore with prior experience using bar models for the four basic operations (addition, subtraction, multiplication, and division) but no formal instruction in fractions. As part of the research design, an instructional sequence was developed around a word problem task completed on a tablet device. The tablet interface captured the student's incorrect solution. This solution was then placed alongside a valid solution provided by the teacher-researcher, creating a side-by-side comparison intentionally designed to stimulate reflection and explanation, using comparison between solutions as a guiding reference for the student to articulate their reasoning verbally and through gesture. Video recordings of teacher–student interactions served as the primary data source for analysis.

The analysis of the embodied episodes shows that bodily actions played two complementary roles. First, they directed attention to salient elements of the bar model, including structural similarities. Second, they externalised structural relationships including sequences, part-whole connections or comparisons. Building on these episodes, evidence of embodied visual-symbolic cognition appears to emerge toward the end of the observation, as the student increasingly coordinates visual structuring in the bar model with symbolic calculation, mediated through gesture. This suggests a developmental progression from earlier embodied activity to symbolic reasoning. The findings offer preliminary evidence that teachers can be supported to attend to embodied strategies as indicators of reasoning not yet verbalised, recognising how students' gestures may reveal structural insights or signal underlying misconceptions.

References

Radford, L. (2009). Why do gestures matter? Sensuous cognition and the palpability of mathematical meanings. *Educational Studies in Mathematics, 70*, 111–126.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 596. PME.

SEMIOTIC TOOLS FOR RELATIONAL THINKING IN EARLY GRADES: PRELIMINARY DATA FROM A LONGITUDINAL STUDY

Elena Polotskaia[1], Elise Abdallah[1], Viktor Freiman[2], Nathalie Silvia Anwandter Cuellar[1]

[1]Université du Québec en Outaouais, [2]Université de Moncton

The interest in early relational thinking as a precursor for algebraic thinking is growing (e.g., Cabral et al., 2022). Based on Davydov's idea of quantitative relationships (Davydov, 2008), we define relational thinking as the noticing, communication, and use of quantitative relationships to solve contextual problems (e.g., word problems in primary school). In our longitudinal study, we aim to understand and describe the possible pathways of relational thinking development in students from grade 1 up to grade 4. Our participants are 33 students from two schools in Canada. At the end of grade 1, we conducted individual interviews where the participants solved 2 word problems: one presenting an additive composition relationship and the other presenting an additive comparison relationship. Based on Radford's objectivation theory (2016) and our previously developed bidimensional frame (Polotskaia et al., 2024), we analyzed the students' use of language, drawings and gesture in 63 video-enregistered cases. In 30 cases, students produced some drawings, which we categorized into several types. Only 7 of these drawings present a relational thinking character. In 17 cases, students produced some oral arguments which can be associated with relational thinking. In 5 cases, we identified gestures associated with relational thinking. We aim to discuss the variety of semiotic tools available for learners solving word problems. We also plan to present the Grade 2 data that will be available in May 2026 and discuss the progress.

References

Cabral, J., Mendes, F., & Oliveira, H. (2022). Noticing first graders' relational thinking: A study with preservice teachers. *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education*, Spain.

Davydov, V. V. (2008). *Problems of developmental instruction. A theoretical and experimental psychological study*. Novapublisher.

Polotskaia, E., Anwandter Cuellar, N. S., Savard, A., & Robert, V. (2024). *Learning Routes for Algebraic Thinking in Preschool* Proceedings of the 47th Conference of the International Group for the Psychology of Mathematics Education, Aukland, New Zealand.

Radford, L. (2016). The theory of objectification and its place among sociocultural research in mathematics education. *Int. J. for Research in Mathematics Education*, 6(2), 187206.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 597. PME.

STUDYING 11TH-GRADE STUDENTS' UTILIZATION OF PSEUDO-OBJECTS IN PROOF BY CONTRADICTION

Kostas Probonas

National and Kapodistrian University of Athens

The role of Dynamic Geometry Environments (DGEs) in proof by contradiction has been studied mainly through the notion of the pseudo-object. Pseudo-objects are geometrical figures that display conflicting properties and may degenerate due to conflicts between direct invariants—imposed by construction commands—and indirect invariants—emerging as Euclidean consequences of the construction. Experiencing a pseudo-object can support students in visualizing and engaging with proof by contradiction (Baccaglini-Frank et al., 2018). However, it remains unclear under what conditions pseudo-objects are utilized productively and how the coordination of figural and conceptual elements, central to geometric reasoning, operates in this context. To address this, we draw on the theory of figural concepts (Fischbein, 1993), which views geometric objects as integrating these elements, and investigate how students reinterpret the dynamic experience of a pseudo-object by exercising theoretical control—imposing and manipulating theoretical elements onto a figure. The research question is: How does the pseudo-object contribute to upper secondary students' productive geometric reasoning in proof by contradiction? Three interventions were conducted in an 11th-grade geometry classroom. Students worked on non-constructability tasks designed to promote the emergence of pseudo-objects, using both a DGE and paper-and-pencil environment. The analysis focuses on how students integrate figural and conceptual aspects of a figure in their argumentation. Our results suggest that when invariant conflicts remain purely figural, pseudo-objects fail to support students' reasoning. This contrasts with the pseudo-object framework, which posits that perceiving such conflicts suffices for pseudo-object actualization. Conversely, pseudo-objects proved productive when students interpreted these conflicts within Euclidean theory through theoretical control. This coordination involved attributing conceptual elements to figural ones and was mediated by transitions between dynamic and paper-and-pencil environments and peer interaction. The intertwined contribution of these settings—so far unexplored—emerged as central to the productive use of pseudo-objects, underscoring the need for further investigation.

References

Baccaglini-Frank, A., Antonini, S., Leung, A., & Mariotti, M. A. (2018). From pseudo-objects in dynamic explorations to proof by contradiction. *Digital Experiences in Mathematics Education, 4*(2), 87-109.

Fischbein, E. (1993). The theory of figural concepts. *Educational Studies in Mathematics, 24*(2), 139-162.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 598. PME.

EXPLORING RELATIONAL UNDERSTANDING IN PRIMARY ALGEBRA LEARNING

Shuyue Qu

University of Bristol

In primary algebra learning, relational thinking is recognised as an essential component of algebraic reasoning. To explore the manifestation of students' relational thinking in algebraic activities, this study is guided by two research questions: RQ1: How do students move between quantitative relationships and algebraic expressions during algebraic problem-solving? RQ2: How does relational thinking manifest when students evaluate and justify equivalence in mathematically incoherent situations (MIS)? MIS are context-based tasks that present structurally similar expressions for different quantitative relationships (e.g., b = a - c and b = a + c) without an immediate question to answer (Savard & Polotskaia, 2017). To address the research questions, nine Grade 6 students from a primary school in China participated in a first interview, a one-week learning session using a digital rod tool to support relational thinking in algebra, and a second interview involving parallel tasks. Students' written work from the interviews was also collected. The first and second interviews were designed to examine how students moved between quantitative relationships and algebraic expressions across the learning activities. Based on a sociocultural perspective, the analysis draws on Radford's (2021) theory of objectification, conceptualising relational thinking as a collective objectification process mediated by multimodal semiotic resources (e.g., explanations, written inscriptions, tool actions). The study found that students' movement between quantitative relationships and algebraic expressions was coordinated in their speech, written inscription, and actions. This coordination became stable and consistent across the two interviews, and in the second interview, relational thinking was expressed in a more settled symbolic form. Further, in MIS, relational thinking seems to help students view an algebraic expression as a quantitative relationship, enabling them to identify a relational mismatch and justify the correct expression. These initial findings show how relational thinking can become visible in algebra activities. The next stage of this study will further explore how relational thinking unfolds through students' speech, gestures, and actions in algebra classroom practices.

References

Radford, L. (2021). *The theory of objectification: A Vygotskian perspective on knowing and becoming in mathematics teaching and learning.* Brill Sense.

Savard, A., & Polotskaia, E. (2017). Who's wrong? Tasks fostering understanding of mathematical relationships in word problems in elementary students. *ZDM Mathematics Education, 49*(6), 823–833. https://doi.org/10.1007/s11858-017-0865-5

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 599. PME.

FROM SCAFFOLDING TO SUBSTITUTION: STUDENTS' SENSE-MAKING OF STATISTICAL CONCEPTS USING GENERATIVE AI

Samuel A. Quiroz[1], Tommy Tanu Wijaya[2], Dennis Lee Jarvis Ybañez[1]

[1]University of the Philippines Open University, [2]Beijing Normal University

The growing integration of Generative Artificial Intelligence (AI) in mathematics education presents both opportunities and challenges, as evidenced by an expanding body of literature (e.g., Rahmatika et al., 2025). In teaching statistics, this integration has been associated with both enhanced student performance and concerns about diminished critical thinking. These contrasting possibilities invite us to understand learning as the dynamic formation of connections within networks of human and non-human nodes, such as digital tools and intelligent systems. From this perspective, this study aims to explore how students use generative AI to make sense of statistical concepts, with a focus on measures of central tendency and variability. Using a qualitative descriptive design, 23 high school students were asked to complete a set of statistical tasks in a 90-minute session, during which they were allowed to use any generative AI tool to help them solve the problems. Thematic analysis was used to examine the nuances in how the students prompted AI and transformed the AI feedback into their handwritten solutions.

Three sense-making behaviors characterized students' use of AI in mathematics learning. First, some students used AI as a recall tool, prompting it to revisit previously learned concepts before attempting to solve problems independently based on the responses. Second, other students engaged in a more guided approach by uploading images of tasks and asking AI to teach them how to solve similar problems through alternative examples. Third, a number of students relied on AI for direct solutions by submitting task images and prompting the tool to solve them. Across these behaviors, AI functioned as a conceptual scaffold and cognitive clarifier, supporting understanding by breaking down ideas and processes. In some cases, students demonstrated procedural delegation with partial sense-making, allowing AI to handle computations while they attempted to interpret results. However, instances of full substitution were also evident, where students depended entirely on AI outputs with little critical engagement. These patterns highlight how AI can become embedded in students' learning networks, emphasizing the need for pedagogical strategies that maintain productive engagement with generative AI tools.

References

Rahmatika, W., Mursalin, Saputra, E., & Fonna, M. (2025). The influence of the use of AI in mathematics learning at the high school level: A systematic literature review. *Electronic Journal of Education, Social Economics and Technology, 6*(2), 1–8.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 600. PME.

PROMPTED, NOT TOLD: UNLOCKING CONCEPTUAL FRACTION COMPARISON WITH A CHATBOT

Eirini Rachmanidi, Christina Misailidou

Department of Primary Education, National and Kapodistrian University of Athens

Fraction comparison can be a challenging topic in primary school because students often apply whole-number reasoning to numerators and denominators rather than coordinating them in relation to the same referent whole (González-Forte et al., 2023). To address this difficulty, we designed and implemented *FractionsCoach*, a pedagogically controlled conversational assistant with intentionally constrained AI. This chatbot does not provide full solutions, but delivers graduated formative feedback (brief prompts/targeted explanations), requiring students to state an explicit comparison criterion and supporting evidence. Its adaptive pathway is organised into performance-gated phases that support differentiated pacing, from representation work to targeted comparison types, and then to mixed comparisons and justification. The research reported here aims to examine FractionsCoach's contribution to fraction comparison teaching and learning. Specifically, we investigate how the tool shapes mathematical justification (stating criteria, using representations as evidence) and whether the students' achievement is affected after using it. Our study employed an experimental pre-test/post-test control group design. It was conducted in two typical Grade-4 Greek classes (N=33; aged 9-10 years; intervention group n=17; control group n=16) and data were collected from tests, system logs, audio-recordings, group interviews and field notes. The intervention group improved significantly (M=56.82 to 92.12, $p<.001$); ANCOVA on post-test scores (pre-test as covariate). The results showed no statistically significant group effect ($p=.135$), though adjusted means favoured the intervention (92.96 vs 85.30). Log data indicate high accuracy on first attempts and greater difficulty in same-numerator than same-denominator comparisons (92.9% vs 96.5%), while qualitative evidence suggests that the tool strengthened criterion-based justification and collaborative regulation without providing full solutions. Overall, the findings reveal not only measurable achievement gains but also a qualitative improvement of the students' mathematical justification skills. As a result, this study contributes to the literature by introducing a pedagogically controlled, constrained-AI assistant which promotes mathematical argumentation and reasoning rather than simply aiding task completion.

References

González-Forte, J. M., Fernández, C., Van Hoof, J., & Van Dooren, W. (2023). Incorrect ways of thinking about the size of fractions. *International Journal of Science and Mathematics Education, 21*, 2005–2025.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 601. PME.

EYE MOVEMENTS OF 5- AND 6-YEAR-OLDS IN SPONTANEOUS FOCUSING ON NUMEROSITY WITH NATURALISTIC STIMULI

Satu Rajalakso[1], Markku Hannula[1], Ilona Södervik[1], Minna Hannula-Sormunen[2]

[1]University of Helsinki, [2]University of Turku

Spontaneous FOcusing on Numerosity, SFON, is a process in which a child spontaneously focuses their attention on the exact number of items and incidents and exploits the knowledge in action (Hannula & Lehtinen, 2005). Although a lot is known about SFON and how it supports learning, there is limited knowledge on the cognitive and attentional basis of SFON. Eye tracking has proven to be a valuable tool to study children's enumeration processes (Schindler et al., 2020). However, previous research has only used explicitly mathematical tasks. In this study, we investigated the variation in children's average number of fixations when they spontaneously focused on numerosity and enumerated sets of objects in naturalistic pictures versus when they did not and, thus, sought indications of the link between visual attention and SFON. We looked at set sizes two and four to distinct sets in subitizing and counting range. Total of 62 children aged 5 to 6 years completed a Memory card game task. The child looked at eight pictures on a screen for five seconds each while their eye movements were tracked with screen-mounted Tobii Pro Spectrum eye tracker. After each picture, the child was asked to describe what they saw as carefully as they could remember, and we registered for which targets they stated numerosity verbally or non-verbally. The pictures included sets of 1-6 familiar objects in varying arrangements. A 2 (SFON / no SFON) x 2 (set size: 2 / 4) repeated measures ANOVA revealed significant main effect of SFON on the number of fixations on the object sets, $F(1, 19) = 9.97$, $p = .005$, $\eta_p^2 = .34$, but no effect of set size or interaction of SFON and set size. Paired samples *t*-tests confirmed that there were significantly more fixations on average when the child spontaneously focused on numerosity compared to no SFON with sets of two objects $t(31) = -4.51$, $p < .001$, $d = -.80$, but not with four objects. The results indicate differences in eye movements when the child focuses on numerosity and when they do not. Since some children only enumerated one of the two set sizes, the samples in *t*-tests were not identical. Other background variables, such as enumeration skills, may explain why SFON related to the number of fixations only for the sets of two objects.

References

Hannula, M. M. & Lehtinen, E. (2005). Spontaneous focusing on numerosity and mathematical skills of young children. *Learning and Instruction, 15(3)*, 237– 256.

Schindler, M., Schovenberg, V. & Schabmann, A. (2020). Enumeration processes of children with mathematical difficulties: An explorative eye-tracking study on subitizing, groupitizing, counting, and pattern recognition. *Learning Disabilities: A Contemporary Journal, 18(2)*, 193–211.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 602. PME.

WHY DO SOME STUDENTS PROGRESS AND OTHER DO NOT? SELF-REGULATED LEARNING IN STATISTICAL REASONING

Rahmi Ramadhani Umar[1,2], Soeharto[1,3,4], Zsolt Lavicza[1]

[1]Linz School of Education, Johannes Kepler University of Linz, [2]Universitas Potensi Utama, [3]Research Center for Education, National Research and Innovation Agency, Indonesia; [4]Research Center of Educational Technologies, Azerbaijan State University of Economics

Statistical reasoning skills are important for students facing the challenges of a data-driven society. However, many students have difficulty understanding and interpreting data from graphical displays to make appropriate decisions due to a lack of confidence and dependence on teacher examples. Statistical reasoning in this study refers to students' ability to interpret, analyze, and draw conclusions from data based on Garfield & Ben-Zvi (2008). Self-regulated learning is conceptualized as students' ability to plan, monitor, and evaluate their learning process based on Zimmerman (2002). This study addresses the following question: *How does self-regulated learning relate to students' statistical reasoning development before and after instruction in a sociocultural context?* This study was conducted over 12 classroom meetings with 152 high school students in Nias Island, Indonesia. The students took an initial test to measure their skill level, and then they received statistics lesson in classroom context. Afterward, data were collected again for comparative analysis of statistical reasoning ability and self-regulated learning using the Stacking method via Rasch analysis. The findings showed that an increase in self-regulated learning led to an increase in statistical reasoning ability. Students with high levels of self-regulation were able to advance their statistical reasoning level from level 2 (verbal) to level 5 (integrated process). However, some students remained at level 2 (verbal) due to moderate self-regulation, highlighting that low self-regulation can hinder improvement in statistical reasoning. Interviews with these students revealed that, although they attempted to solve problems independently, they failed to reflect on or evaluate the difficulties they encountered. This indicates the importance of providing support for self-regulated learning to improve statistical reasoning skills. These findings provide insight into how self-regulated learning supports students' progression in statistical reasoning within sociocultural-based learning contexts and giving the individual center analysis perspective using Rasch analysis.

References

Garfield, J., & Ben-Zvi, D. (2008). *Developing students' statistical reasoning: Connecting research and teaching practice*. Springer Science & Business Media.

Zimmerman, B. J. (2002). Becoming a Self-Regulated Learner: An Overview. *Theory Into Practice*, *41*(2), 64–70. https://doi.org/10.1207/S15430421TIP4102_2

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 603. PME.

ADDRESSING MATHEMATICAL PROMISE IN TEACHER EDUCATION: PRE-SERVICE TEACHERS' EXPERIENCES

Miranda Rampe[1], Eva-Lena Erixon[2]

[1]Kristianstad University, [2]Stockhom University

Although pre-service teachers (PSTs) are expected to address diverse learners, mathematically promising students (MPS) risk marginalisation in contexts where inclusion is primarily framed in remedial terms. Mathematical promise is here understood as a dynamic and developable potential shaped by cognitive, affective, and social factors and nurtured through opportunity, challenge and support (Sheffield, 2003). Whereas prior research has focused on PSTs' individual conceptions of MPS. In contrast, this study adopts a systemic perspective, analysing how tensions within teacher education shape how MPS are understood and addressed, as manifested in PSTs' experiences following a targeted seminar and practicum. Drawing on cultural-historical activity theory (CHAT) (Engeström, 1987), teacher education is conceptualised as a culturally mediated activity system in which PSTs act as subjects. The object concerns how MPS are understood and addressed in mathematics teaching, mediated by tools, rules, division of labour, and community. Semi-structured interviews were conducted with eight voluntary PSTs (grades 4–6) after participation in a targeted three-hour seminar on mathematical promise and a subsequent five-week practicum in mathematics. The analysis was focused on identifying contradictions within the activity system as expressed in PSTs' experiences. Four interrelated contradictions were identified across the data. A primary contradiction emerged within the object itself, as remediation-oriented interpretations of student diversity clashed with the development-oriented perspective on MPS introduced in the seminar. Secondary contradictions arose between pedagogical tools and rules concerning MPS presented in teacher education and their enactment during practicum. A quaternary contradiction became visible between the activity systems of teacher education and school practice, as seminar-informed approaches to nurturing MPS conflicted with established school routines and priorities. Rather than indicating failure, these contradictions signal a potential reorientation of the object, teaching MPS, making previously implicit tensions visible. The study thus shows how systemic contradictions can function as developmental resources and illustrates the dynamic nature of the object in an activity system, which may be temporarily reoriented through deliberate changes in its mediating components.

References

Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research.* Helsinki: Orienta-Konsultit.

Sheffield, L. J. (2003). *Extending the challenge in mathematics: Developing mathematical promise in K–8 students.* Corwin Press.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 604. PME.

THE ROLE OF MOTIVATION IN SELF-REGULATED LEARNING OF MATHEMATICS

Georgina Reimer[1], Andreas Obersteiner[2], Frank Reinhold[1], Anselm Strohmaier[1]

[1]University of Education Freiburg Germany,[2]Technical University of Munich

Self-regulated learning (SRL), defined as learners' active regulation of motivation, cognition, and behaviour, is a key predictor of academic achievement and is closely connected with motivation (Zimmerman, 2008). This study examines motivational and self-regulatory processes during the instructional format of *FSA* (German "Freie Stillarbeit", *free silent work*). FSA provides students (during grades 5 to 7 in secondary school) with autonomy regarding learning content, timing, and pace, while instructional support is offered only upon request. This emphasizes self-regulation processes such as help-seeking, goal-setting, and time management. FSA differs from interventions like those in previous studies in terms of its duration. In the school year 2025/26, 50 fifth-grade, 40 sixth-grade, and 37 seventh-grade students at a German secondary school took part. Data collection comprised a paper-pencil pre-test assessing prior mathematical knowledge, questionnaires assessing intrinsic motivation, perceived instructional support (Deci & Ryan, 2003; Reinhold et al., 2021), and strategy use, as well as a post-test at the end of the school year evaluating the learning gain. The post-test is accompanied by a questionnaire about learning experience, time management, and satisfaction with the provided material. In this paper, the findings of the pre-tests and the first questionnaires are included. The performed ANOVA shows a significant difference in motivation during the FSA between grades (p = .023). The Bonferroni post-hoc test reveals a significantly lower motivation during FSA in grade 7 than in grade 5. Whereas in grades 5 and 7, there is no significant difference between regular lessons and FSA regarding motivation, there is one in grade 6 (p = .012). Those findings imply that there could be a specific grade where SRL is particularly effective in fulfilling learners' needs of perceiving autonomy and competence.

References

Deci, E. L. & Ryan, R. M. (2003). *Intrinsic Motivation Inventory*. https://selfdeterminationtheory.org/wp-content/uploads/2022/02/IMI_Complete.pdf

Reinhold, F., Hoch, S., Schiepe-Tiska, A., Strohmaier, A.R., & Reiss, K. (2021). Motivational and Emotional Orientation, Engagement, and Achievement in Mathematics. A Case Study With One Sixth-Grade Classroom Working With an Electronic Textbook on Fractions. *Frontiers in Education 6*. https://doi.org/10.3389/feduc.2021.588472

Zimmerman, B. J. (2008). Investigating Self-Regulation and Motivation: Historical Background, Methodological Developments, and Future Prospects. *American Educational Research Journal, 45*. 166-183. https://doi.org/10.3102/0002831207312909

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 605. PME.

INTERWEAVING ETHICAL AND MATHEMATICAL REASONING IN MATHEMATICS TASKS

Luke Reinke, Michelle Stephan, Premkumar Pugalenthi

UNC Charlotte

Critical mathematics educators and researchers (e.g. Skovsmose, 1994) seek to teach mathematics while also developing students' critical consciousness. In the study reported here, we situate critical consciousness within the broader field of ethics, meaning the moral principles that govern behavior, because as Atweh & Brady (2009) posit, ethics includes and transcends considerations of social justice and by situating social justice within the larger idea of ethics, justice becomes not something done out of charity but rather a moral obligation to do what is right. Critical mathematics curriculum designers frequently struggle to integrate mathematical and critical reasoning effectively within instructional design, often grounding their social justice components in established theory, but rarely incorporating instructional design theories to develop mathematical ideas. To address this imbalance, we integrate Realistic Mathematics Education (RME), a theory that anchors mathematical learning in realistic contexts and supports the gradual development of abstract reasoning (Gravemeijer, 1994) in our instructional design. We collaborated with two teachers and an administrator to develop an instructional sequence on ratio, proportion, and percent change within the context of critical and ethical reasoning about teen nicotine vaping. After a post-hoc qualitative document analysis of the instructional sequence, we set out to revise the sequence, with the primary goals of (a) integrating aspects of ethical reasoning beyond critical consciousness earlier on in the sequence and (b) developing tasks that simultaneously engage students' proportional, critical, and broader ethical reasoning. In this oral communication, we will highlight excerpts of the revised sequence aimed at simultaneously engaging mathematical, critical and ethical reasoning and share results of piloting these tasks. We hypothesize that such tasks are important because they provide opportunities for future mathematical professionals to develop their ability in mathematics to reason ethically and critically about the potential impact of their decisions.

References

Atweh, B., Brady, K. (2009). Socially response-able mathematics education: Implications of an ethical approach. *Eurasia Journal of Mathematics, Science & Technology Education*, *5*(3), 267-276.

Gravemeijer, K. (1994). *Developing realistic mathematics education*. Freudenthal Institute.

Skovsmose, O. (1994). Towards a critical mathematics education. *Educational Studies in Mathematics*, *27*, 35–57.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 606. PME.

PROPORTIONAL STRUCTURE IN INTEREST CALCULATION: THE INTEREST BAR AS A COHERENT REPRESENTATION

Simon Roth, Birte Friedrich

University of Potsdam

Interest calculation is often introduced in mathematics classrooms as an application of percentages, yet it is central to financial education. Mathematically, interest calculation can be described as a proportional structure in which capital (or credit), interest rate, and interest are linked by multiplicative relationships. These must be explicitly coordinated to ensure conceptual understanding (Lamon, 2007). Instruction frequently emphasizes formula-based procedures, leaving the proportional structure implicit and the quantities treated as isolated elements. This pattern was also confirmed in a diagnostic study conducted by the authors involving 61 students: interest problems were predominantly solved using memorized formulas, while the underlying proportional structure was rarely considered. Mismatches and unstable calculations indicate that the underlying relational dependencies are not systematically coordinated. These connections within the proportional structure can be supported by a representation. Specifically, the use of an interest bar in a learning environment developed in design-based research (Bakker, 2018) will be investigated to answer the research question: How does the interest bar, as a coherent representation, support the explicit coordination of proportional thinking and the use of formulas in interest calculation? Based on the principles of conceptual focus and longitudinal coherence (Prediger et al., 2022), the interest bar was selected as the central representation in the learning environment, as it visually structures the proportional relationships between capital (or credit), interest rate, and interest. The learning environment is iteratively implemented and refined. Ultimately, the aim is to use the interest bar to integrate procedural processes with relational understanding and reduce decontextualized routines.

References

Bakker, A. (2018). *Design research in education: A practical guide for early career researchers*. Routledge. https://doi.org/10.4324/9780203701010

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629–667). Emerald Publishing Limited.

Prediger, S., Götze, D., Holzäpfel, L., Rösken-Winter, B., & Selter, C. (2022). Five principles for high-quality mathematics teaching: Combining normative, epistemological, empirical, and pragmatic perspectives for specifying the content of professional development. *Frontiers in Education, 7*. https://doi.org/10.3389/feduc.2022.969212

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 607. PME.

PRE-SERVICE TEACHERS' ASSESSMENT LITERACY IN MATHEMATICS: NEGOTIATING FORMATIVE IDEALS AND DISCIPLINARY DEMANDS

Carolina Ruminot

University of Ottawa

Assessment is increasingly framed as integral to learning rather than merely certificative. In mathematics, this implies attending to students' reasoning and supporting instructional regulation. Yet persistent tensions remain between formative ideals and institutional routines. How pre-service teachers negotiate these tensions in mathematics during practicum remains underexplored. This study examines how pre-service elementary teachers enact assessment literacy in mathematics and how institutional, disciplinary, and equity-related constraints shape this enactment. Drawing on DeLuca et al.'s (2019) model of assessment literacy and research on integrating assessment and mathematics teaching (Suurtamm, 2018), assessment literacy is conceptualized as situated professional judgment.

A qualitative interpretive design was used. Eleven pre-service teachers were interviewed at the end of their practicum. Data were coded inductively and organized across four dimensions: functions, processes, equity, and validity. Four patterns emerged. Participants endorsed formative assessment, yet summative practices remained predominantly paper-based, revealing tension between discourse and institutional norms. Equity and differentiation were valued but constrained by time and accountability pressures. Concerns about validity—especially criteria and curriculum alignment—were intensified by the epistemic complexity of mathematics. Practicum functioned as a mediating space where assessment knowledge was reconfigured, often through mentor influence and in tension with coursework. Assessment literacy in mathematics thus appears negotiated at the intersection of formative ideals, disciplinary demands, and institutional constraints. The study positions practicum as a critical site where evaluative judgment becomes embedded in the epistemic structure of mathematics teaching.

References

DeLuca, C., Coombs, A., & LaPointe-McEwan, D. (2019). Assessment mindset: Exploring the relationship between teacher mindset and approaches to classroom assessment. *Studies in Educational Evaluation, 61*, 159-169.

Suurtamm, C. (2018). Enhancing Mathematics Teaching and Learning Through Sound Assessment Practices. In A. Kajander, J. Holm, & E. J. Chernoff (Eds.), *Teaching and Learning Secondary School Mathematics : Canadian Perspectives in an International Context* (pp. 473-482). Springer International Publishing.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 608. PME.

OPPORTUNITIES AND CHALLENGES IN GENERATIVE AI-BASED INSTRUCTION FOR FORMULATING STATISTICAL INVESTIGATIVE QUESTIONS

Kyoungmin Ryu, Bomi Shin

Chonnam National University

The 2022 Revised Mathematics Curriculum in South Korea emphasizes "formulating Statistical Investigative Questions (SIQ)" for statistical literacy. While this is a critical starting point, students often face challenges in refining their inquiries into structured questions with specific variables and populations. Generative AI (GenAI) serves as a cognitive tool addressing these limitations (Lee & Jho, 2024). This study explores the quality of SIQ formulated with GenAI, students' conceptual understanding of these questions, and their prompting strategies. Questions were assessed on a 3-level scale using three criteria refined from Arnold (2013): clarity of variables and population, and requirement for variability. Student-AI interactions were examined via seven prompting strategies (e.g., "Specify clear instructions", "Split complex tasks") based on prompt engineering principles (Bozkurt & Sharma, 2023).

Participants were 20 middle school students in five groups using "Wrtn" to formulate SIQs and data collection plans. Data included GenAI dialogue logs, student worksheets, and video/audio recordings of class and group activities. Each group's final SIQ achieved Level 3 in variable and population clarity, significantly supported by GenAI. However, some students confused "population" with "sample" during discussions, despite the clear SIQs formulated by GenAI. Students naturally used "Specify clear instructions" but faced a breakdown in interaction due to their limited understanding of data types, failing to refine prompts effectively. While GenAI scaffolds structuring students' inquiries, its output does not guarantee their deep conceptual understanding. Teachers must bridge this conceptual understanding gap by fostering "prompt-based communication competence" and providing targeted feedback on statistical concepts.

References

Arnold, P. (2013). *Statistical investigative questions* (Doctoral dissertation). The University of Auckland.

Bozkurt, A., & Sharma, R. C. (2023). Generative AI and prompt engineering. *Asian Journal of Distance Education*, *18*(2).

Lee, J., & Jho, H. (2024). The impact of an AI-based feedback system on enhancing elementary students' formulation of statistical investigative questions: The moderating effects of AI perception and feedback self-efficacy. *Brain, Digital, & Learning*, *14*(3), 459-473.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 609. PME.

WHAT SHOULD TEACHERS NOTICE IN COLLABORATIVE PROBLEM-SOLVING SITUATIONS?

Farzaneh Saadati
Universidad de Chile

Teacher noticing is widely conceptualized as the capacity to perceive, interpret, and decision making in respond to students' thinking (Jacobs et al., 2010). Empirical studies of teacher noticing have mainly examined how teachers attend to the reasoning of individual students. Yet collaborative problem-solving (CPS) situations challenge this framing. In CPS situations, what teachers must notice is more than individual cognition. They must attend to the emergent, distributed reasoning of the group, the interactional and social processes that shape understanding, and the impact of their own interventions on collective reasoning. This paper as part of a research project focused on understanding teacher noticing in problem solving situations invites a reconsideration of the question: What do teachers consider important to notice in CPS contexts? Drawing on teachers' written responses to a specific classroom scenario portraying CPS episode with teacher intervention, the analysis aims to interrogate how they conceptualize the scope and boundaries of noticing in collective settings. Based on the preliminary exploratory analysis of the responses, two conceptual tensions emerge. First, teachers' responses suggest a shift from noticing individual cognition to attending to the collectively negotiated understanding of the problem, concepts, and strategies that emerges through group interaction and discussion. The participants describe noticing as skills to perceive how ideas circulate, transform, and stabilize across participants. Second, teachers' responses point to a reflexive noticing, in terms of noticing not just to students' cognition, but also to the teacher's own influence or role on the group. Participants emphasized the importance of noticing changes in the task's cognitive demand, for example, whether a question or hint makes the problem too easy or maintains productive challenge. Noticing, in this sense, includes attention to how instructional actions reorganize collaborative system itself. As short, in problem solving when collaboration is central, the unit of analysis shifts: from student thinking to collective reasoning involves attending to collective mathematical activity as well as reflecting on the teacher's role in shaping this collaborative system. These findings point to the importance of both conceptual and methodological development: conceptually, in understanding how noticing operates at problem solving situations, and methodologically, in exploring ways to study and capture such distributed and interaction-mediated noticing in the group level.

Acknowledgment: This research supported by FONDECYT Project No. 1250933.

References

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for research in mathematics education, 41*(2), 169-202.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 610. PME.

EMBODIED MENTAL MODELS IN UNDERGRADUATE STUDENTS' TOPOLOGICAL REASONING

Şule Şahin

Hitit University, Turkey

Topology is among the most abstract branches of mathematics, yet learning it is deeply sensory: students draw diagrams, mentally deform objects, and use spatial language like "close enough" and "around it" to anchor formal definitions in bodily experience. Despite this, research on how students construct mental models for topological concepts remains scarce. Embodied cognition holds that abstract mathematical concepts arise through metaphorical extensions of sensory-motor schemas such as containment, proximity, and motion (Lakoff & Núñez, 2000). Topology is particularly salient here, since concepts like open set, neighbourhood, and ball inherently invoke spatial notions of "being inside" and "being near." Drawing on this perspective with Johnson-Laird's (1983) mental model theory, this study investigates how embodied spatial intuition and formal reasoning interact in the mental models that mathematics majors construct for topological concepts (Şahin, 2024).

Six cognitive dimensions—topological reasoning, set-theoretic explanation, symbolic logic, visual representation, geometric reasoning, and dimensional comprehension—were examined across seven topological concepts. Nine third-year mathematics students participated in two rounds of video-recorded, think-aloud interviews; data were analysed following Miles and Huberman's qualitative procedure.

Six mental models emerged on a gradient from formal abstraction to embodied grounding: a *Topological Mental Model* integrating all dimensions; primitive *Set-Theoretic* and *Symbolic* models; a *Geometric-Topological* model; and perceptually grounded *Euclidean-Geometric* and *Spatial* models. The formal model was rarely attained; most students relied on diagrams, Euclidean dimensional intuitions, and spatial metaphors that simultaneously *enabled* sense-making and *constrained* abstraction, generating misconceptions such as equating topological with metric spaces. These findings show that sensory resources are constitutive of topological meaning-making; instruction should explicitly engage and extend embodied intuitions to support the transition from spatial sensing to formal sense-making.

References

Johnson-Laird, P. N. (1983). *Mental models*. Cambridge University Press.

Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from*. Basic Books.

Şahin, Ş. (2024). *Investigation of misconceptions and mental models of mathematics major students about topological concepts* (Doctoral dissertation). Middle East Technical University.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 611. PME.

MAPPING PRESERVICE TEACHERS' INTEGRATION OF CULTURALLY RELEVANT MATHEMATICS TEACHING (CRMT) THROUGH LESSON PLANNING

Wesam Salem, Lara Condon

University of Memphis

Culturally Relevant Mathematics Teaching (CRMT) (Zaval & Aguirre, 2024) has gained traction among U.S. mathematics teacher educators as research increasingly highlights its effectiveness for supporting culturally and linguistically diverse learners and addressing persistent achievement disparities. National organizations in the U.S (e.g., NCTM) reinforced this momentum by urging teachers to integrate students' cultural identities and experiences into mathematics instruction. This study examines how lesson plan assignments utilized as approximations of practice in a mathematics methods course, support PSTs in developing CRMT approaches. Using Epistemic Network Analysis (ENA), we analysed 10 preservice teachers' (PSTs') lesson plans. ENA modelled the complex connections among the three dimensions of the CRMT rubric: Knowledge & Identities, Rigor & Support, and Power & Participation. The research questions: a) How do PSTs integrate dimensions of the CRMT rubric in their plans for mathematics instruction? b) Where do PSTs' lesson plans provide space and opportunities for culturally responsive mathematics teaching elements?

Our analysis and continued reflection on our practices highlighted the pressing challenge of supporting PSTs in integrating CRMT practices, such as Affirming Multilingualism, and in addressing Power and Participation. The lesson plan assignment provided the greatest space for PSTs to include CRMT practices, showing significant connections across all three CRMT dimensions. However, this configuration revealed a disconnect in how PSTs bridged their mathematical knowledge for teaching with effective instructional strategies. The study noted limited inclusion of CRMT practices related to the Power and Participation dimension, suggesting a lack of focus on teaching mathematics with a social-justice orientation. Additionally, the sub-dimension of Affirming Multilingualism was largely absent, pointing to a gap in instruction on meeting the needs of multilingual students. These findings resonate with international research showing that CRMT emerges differently across global contexts due to variations in cultural norms, teacher preparation, and curriculum structures. The study holds implications for MTEs to situate problems in community collaboration, multimodal representation, language use, and iterative module design.

References

Zavala, M. del R., & Aguirre, J. M. (2024). Cultivating Mathematical Hearts: Culturally Responsive Mathematics Teaching in Elementary Classrooms. Corwin Press.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 612. PME.

THE DEVELOPMENT OF PRESERVICE PRIMARY SCHOOL TEACHERS' RATIONAL NUMBER KNOWLEDGE DURING A MATHEMATICS DIDACTICS COURSE

Pauliina Salonen, Jake McMullen, Juulia Lahdenperä

University of Turku

Oliveira and Hannula (2008) state that aspiring mathematics teachers aim to learn mathematics and to learn how to teach it. Difficulties in mathematical content knowledge are especially pronounced in rational numbers, with misconceptions persisting even into adulthood (Vamvakoussi et al., 2012). Effective instruction therefore requires teachers to have a solid understanding of rational numbers themselves. Earlier studies on preservice teachers' rational number knowledge have been rather cross-sectional in nature or focused on a limited number of aspects of this knowledge. To address this gap, the present study examines six different aspects (adaptive rational number knowledge, conceptual knowledge of density and number system, arithmetic procedural knowledge, and procedural and conceptual knowledge of representations) during a mathematics didactics course.

The longitudinal quantitative study examines preservice teachers' (N=102) rational number knowledge at the beginning of the course and its development during the course. Differences at course entry were examined across six aspects of rational number knowledge, and development was analysed focusing on the four aspects most likely to change.

A cluster analysis identified three distinct profiles of rational number knowledge — low, medium, and high — which differed both in performance and in how they developed during the course. Although the mathematics didactics course primarily focused on pedagogical knowledge, preservice teachers' mathematical content knowledge improved over time. These findings suggest that mathematics didactics courses should place greater emphasis on supporting preservice teachers' mathematical content knowledge, particularly in rational numbers, while maintaining a strong integration of pedagogical and mathematical content knowledge.

References

Oliveira, H., & Hannula, M. S. (2008). Individual prospective mathematics teachers: Studies on their professional growth In K. Krainer & T. Wood (Eds.), *International handbook of mathematics teacher education* (pp. 13-34). Sense Publishers.

Vamvakoussi, X., Van Dooren, W., & Verschaffel, L. (2012). Naturally biased? In search for reaction time evidence for a natural number bias in adults. *The Journal of Mathematical Behavior, 31*(3), 344–355. https://doi.org/10.1016/j.jmathb.2012.02.001

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 613. PME.

COMPARING PRE-SERVICE MATHEMATICS TEACHERS' SELF-EFFICACY ACCORDING TO TEACHING PRACTICES

Brigitte Johana Sánchez Robayo

Universidad Distrital Francisco José de Caldas

Teacher self-efficacy is linked to teaching practices in different ways. On one hand, teacher self-efficacy impacts students' motivation and outcomes as well as teachers' performance, commitment, and interactions with students. On the other hand, field experiences influence preservice teacher self-efficacy. There are few studies in Latin America focused on the relationship preservice mathematics teacher training and self-efficacy. However, studying such a relationship may provide insights into how initial teacher training can be improved. Our research question was: Are there differences in preservice mathematics teachers' self-efficacy across different pedagogical practices?

Understanding teacher self-efficacy as teachers' beliefs in their ability to influence students' learning, we studied preservice mathematics teachers' self-efficacy in a program characterized by a predominant focus on teaching practices. The program has six educational criteria: non-segregating ethos, mathematical activity, problematization and research, proactive attitude, critical attitude, and collaborative work. The program's curriculum includes a core of practices. Progressively, student teachers transition through different pedagogical practices starting with research practices, moving through supervised teaching practices, and finishing with mentored teaching practices. We adapted and applied a teacher self-efficacy scale (Avilés-Canché & Marbán, 2024) that best fit the program's educational criteria. We applied the instrument to 84 preservice mathematics teachers. 20 were in the research practices, 31 in the supervised, and 33 in the mentored practices. We run an ANOVA after verifying normality and homoscedasticity. Assuming practice types as the factor, we found significant differences between groups in five educational criteria. Then, teacher self-efficacy varies depending on the type of practice. For instance, there are significant differences in teacher self-efficacy for including all the students in their class (ETHOS) between student teachers in research practices, supervised and mentored teaching practices ($F(2,81) = 8.38$, $p < .001$). This and other results allow us to conclude that different types of field experiences as part of a mathematics teacher training program increase the level of teacher self-efficacy in terms of mathematical activity, inclusive and questioning practices, a proactive attitude, and collaborative work.

References

Avilés-Canché, K. I., & Marbán, J. M. (2024). Validación de escalas de autoeficacia docente desde la perspectiva del conocimiento especializado para la enseñanza de las matemáticas. *Res Mobilis*, *53*(3), 247–256. https://doi.org/10.17811/rifie.20508

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 614. PME.

FRACTION AS OPERATOR: REPRESENTATIONAL PERFORMANCE BEYOND ARITHMETIC SUCCESS

Diana Herreros-Torres, Maria T. Sanz, Carlos B. Gómez-Ferragud

Universitat de València

Fractions are a challenging domain of school mathematics. Among their meanings, the fraction-as-operator is central to multiplicative reasoning in Arithmetic Word Problems (AWPs), yet research reports persistent difficulties in coordinating proportional relationships and quantity transformations (Tossavainen & Helenius, 2024). This study examines the representational adequacy of graphical productions in fraction-as-operator tasks, focusing on cases with correct arithmetic results. It identifies recurrent strategies, analyses their adequacy across problem variables compares primary students and prospective teachers, and characterises representational difficulties despite procedural success.

The study involved 750 primary students (Grades 4–6) and 263 prospective teachers (3rd–4th year). Participants solved three AWPs varying in quantity type and context within the fraction-as-operator meaning, providing both arithmetic and graphical responses. Analysis considered only correct arithmetic cases, classifying graphical productions as adequate, partially adequate, or inadequate through a qualitative category system. Descriptive statistics compared patterns across groups and contexts.

Preliminary analyses show a systematic gap between arithmetic correctness and representational adequacy in primary students. In Grade 4, 58 students correctly solved the continuous “fraction of a natural number” problem, yet only 8 produced an adequate graphical representation. Similar patterns appear across grades, with particularly low adequacy in fraction-of-a-fraction tasks. Prospective teachers show higher adequacy; however, arithmetic success does not consistently align with structurally coherent representations. Analysing both groups allows examination of the fraction-as-operator across two connected stages—its development in schooling and its consolidation in teacher education—suggesting that representational fragilities may persist beyond initial instruction. Ongoing analyses explore variations across educational levels.

References

Tossavainen, A., & Helenius, O. (2024). Student Teachers' Conceptions of Fractions: A Framework for the Analysis of Different Aspects of Fractions. *Mathematics Teacher Education and Development*, *26*(1), 4. https://orcid.org/0000-0002-0609-7064

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 615. PME.

OPERATIONALIZING MATHEMATICS TEACHERS' RESPONSIVENESS IN AI SIMULATIONS

Gil Schwarts[1], Reut Parasha[1,2], Raz Harel[3], Nadav Marco[3]

[1]Weizmann Institute of Science, [2]Orot Israel Academic Teachers' College
[3]David Yellin College

Mathematically Responsive Teaching (MRT)—grounding instruction in students' mathematical thinking—remains insufficiently theorized. Our AI-based teaching simulations, in which teachers interact with an agent-student, provide a controlled yet interactive setting for examining such responsiveness. We developed an AI-based teaching simulation platform, that includes a scenario featuring a middle school student who incorrectly adds probabilities instead of multiplying them in a two-event independence task. Whereas prior work has conceptualized responsiveness primarily as a set of instructional moves (e.g., Jacobs & Empson, 2016), we conceptualize MRT as a multi-dimensional construct. In this context, we ask: What can teacher–bot interactions in AI-based teaching simulation reveal about prospective teachers' MRT? Building on prior research (Schwarts, 2025), we developed a holistic framework that distinguishes commonly conflated MRT dimensions, comprising: (D1) identifying error, (D2) eliciting student perspectives, (D3) leveraging error as a resource, (D4) building on student perspectives, (D5) interpersonal responsiveness, and (D6) promoting mathematical reasoning. We applied the framework using a 0–3 scale to code seven prospective teachers' simulation logs ($M \approx 12$ teacher turns per simulation).

Both Eliciting (D2) and Using Error as a Resource (D3) had the lowest means (0.86), indicating these dimensions were most challenging for teachers. D5 was high ($M = 2.71$), reflecting strong interpersonal engagement in the simulation. Our analysis shows that the MRT framework distinguishes eliciting (D2) from building on student perspectives (D4; $M = 2$): two participants scored 0 on eliciting, yet closely followed the student's logic (coded 3 on D4) and did not introduce a tree representation. This underscores a key contribution of the framework: MRT is multifaceted and not reducible to talk counts; eliciting and building are analytically distinct.

References

Jacobs, V. R., & Empson, S. B. (2016). Responding to children's mathematical thinking in the moment: An emerging framework of teaching moves. *ZDM – Mathematics Education, 48,* 185–197. https://doi.org/10.1007/s11858-015-0717-0

Schwarts, G. (2025). Moving beyond moves: Redefining mathematically responsive teaching as a humanistic stance. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.), *Proc. 48th Conf. of the Int. Group for the Psychology of Mathematics Education: Research Reports* (Vol. 2, pp. 267–274). PME.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 616. PME.

FACILITATORS´ DISCUSSION-LEADING AND ADAPTATION PRACTICES DURING IMPLEMENTING A PD COURSE

Tjorven Lea Seibold, Bettina Roesken-Winter

University of Münster

When implementing a professional development (PD) course for mathematics teachers, facilitators have multiple responsibilities, such as *adapting PD materials* and *leading discussions* (Prediger et al., 2022). To ascertain how facilitators manage these tasks, six interviews were conducted with each of two facilitator tandems, reflecting on *adapting PD materials* and *leading discussions*. Each interview took place after one of the three sessions of a PD course had been delivered by the respective tandem as part of the QuaMath Project, which aims to improve the mathematics teaching quality across a range of topics. In order to conceptualize the discussion leading practices employed by the facilitators, we adapted the framework for leading mathematics discussions by Shaughnessy et al. (2021) from the classroom to the PD level. Four discussion-leading practices could be defined: *making contributions*, *eliciting teacher thinking*, *probing teacher thinking* and *orienting teachers towards the thinking of others* (Shaughnessy et al., 2021). Initially, both tandems focused solely on *making contributions*, emphasising the highlighting of relevant teacher statements and their link to the PD course content. During the subsequent PD sessions and follow-up interviews, the focus remained on *making contributions*, but *eliciting* and *probing teacher thinking* became a more prominent theme in the facilitators' reflections on their discussion-leading practices. However, *orienting the teachers towards the thinking of others* did not occur in either tandem's discussion-leading practices as the facilitators found this aspect particularly challenging. Regarding the task of *adapting PD materials*, the facilitators' practices remained consistent throughout the three sessions: One tandem kept the provided materials largely in their original state, while the other tandem omitted some of the provided conversation starters and activities for the teachers, resulting in a constrained discussion culture. The facilitators mostly justified their discussion-leading and adaptation practices by referring to the limited time available. As many PD courses have a constrained timeframe, a careful planning in terms of adaptation and discussion-leading, with a focus on best supporting teacher learning, should form a more intensive part of facilitator preparation.

References

Prediger, S., Roesken-Winter, B., Stahnke, R. & Pöhler, B. (2022). Conceptualizing content-related PD facilitator expertise. *J. of Mathematics Teacher Education, 25*(4), 403–428.

Shaughnessy, M., Garcia, N. M., O'Neill, M. K., Selling, S. K., Willis, A. T., Wilkes, C. E., II., Salazar, S. B., & Ball, D. (2021). Formatively assessing prospective teachers' skills in leading mathematics discussions. *Educational Studies in Mathematics, 108*, 451–472.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 617. PME.

MIDDLE SCHOOL STUDENTS' USE OF GENERATIVE ARTIFICIAL INTELLIGENCE IN PROVING PROCESSES

Hyosup Shim, Kyungwon Lee, Oh Nam Kwon

Seoul National University

This study aims to explore the use of generative artificial intelligence (genAI) in middle school students' proving processes related to argumentation in geometry. At the middle school level, proving primarily focuses on constructing arguments about geometric properties, which students often find difficult due to the demands of logical reasoning and formal justification. Yoon et al. (2024) proposed a framework illustrating the processes of genAI use in proof construction within an undergraduate context. Building on this work, this study seeks to refine understanding of how genAI is used in school-level settings, particularly in relation to argumentation in geometry. Therefore, this study investigates the ways in which students use genAI when engaging in proving tasks involving argumentation in geometry at the middle school level. An exploratory approach was adopted in this study, and clinical interviews were conducted with eight Korean eighth-grade students who had learned geometric properties. Seven problems related to argumentative tasks in geometry at the middle school level were presented, and students were asked to select two problems to solve freely. Considering the difficulties students encounter in proving, they were instructed to explain their solutions using dynamic geometry software. They then constructed proofs while autonomously using genAI through verbal interaction. Data collected included students' interaction logs with dynamic geometry software and genAI, written proofs, and verbal utterances. Based on these data, students' use of genAI during proving was analyzed using an open coding approach. Students used dynamic geometry software to confirm conclusions at an intuitive level and to justify those conclusions using geometric properties, thereby laying the groundwork for recognizing key components of proving. Students primarily used genAI to obtain or verify key ideas for constructing their proofs. When constructing written proofs, students focused on key ideas generated by genAI rather than copying detailed explanations verbatim or examining all provided content. During this process, students demonstrated varied strategies, such as setting constraints in prompting and varying the timing of genAI use. Findings suggest that genAI functions as a mediational resource in students' proving by supporting the generation of key ideas, shaping prompting strategies, and influencing the timing of proof construction.

References

Yoon, H., Hwang, J., Lee, K., Roh, K., & Kwon, O. N. (2024). Students' use of generative artificial intelligence for proving mathematical statements. *ZDM – Mathematics Education*, *56*(7), 1531–1551. https://doi.org/10.1007/s11858-024-01629-0

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 618. PME.

EXPLORING PERCEIVED CURRICULM OF AN EXPERIENCED TEACHER THRUOGH A RESEARCH-LESSON DESIGN

Yoshinori Shimizu

Institute of Human Sciences, University of Tsukuba

The current study aims to examine the alignment of different aspects of school mathematics curriculum from the eyes of experienced teachers. The study has a particular focus on the relationship between the "intended" and the "implemented" curriculum in the process of implementing the new national curriculum standards. A theoretical framework of curriculum alignment in the earlier TIMSS curriculum analysis (Valverde et al., 2002) was used to conceptualize the role of teachers across three different aspects of curriculum. By taking an ecosystem approach to curriculum (OECD, 2020), we address the research questions: How an experienced teacher interprets the "intended" curriculum? To what extents a research lesson design in school-based lesson study helps researchers to understand the "perceived" curriculum?

Data were collected at a public elementary school in an urban area in Japan. An experienced teacher was invited to conduct a research lesson in school-based lesson study in 5th grade classroom in the unit of "geometrical figure and angles" that is treated differently between the previous and the current curriculum standards. The research lesson and the post-lesson discussion were video-taped and then analysed qualitatively. Data sources included a detailed lesson plan, verbatim transcriptions of video-recorded lesson and the post-lesson discussion, and the fieldnotes of researchers.

The results of analysis reveal that the teacher interpreted the curriculum standards in the way that his own perspectives can be manifested on the values of school mathematics, in general, and on the development of mathematical reasoning, in particular. It was also found that he intended to change the sequence of the appearance of figure types different from those in the textbook to "customize" the instructional materials to fit with his students. Through the analysis the importance of textbooks was discussed both as a "potentially implemented curriculum" and as a platform for the teacher to exemplify what is officially intended in his classroom.

Acknowledgement: This research was supported by JSPS KAKENHI 24K00408.

References

OECD (2020). Curriculum (re) Design: A series of thematic reports from the OECD Education 2030 project, Paris: OECD.

Valverde, G. A., Bianchi, L. J, Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. The Netherlands: Kluwer Academic Publishers.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 619. PME.

A CROSS-GRADE ANALYSIS OF STUDENTS' STRATEGIES IN DIRECT, INVERSE, AND PSEUDO-PROPORTIONAL PROBLEMS

Ana Isabel Silvestre[1], Hélia Jacinto[2]

[1]ISPA - Instituto Universitário, [2]UIDEF, Institute of Education, University of Lisbon

Proportional reasoning is understood as the ability to identify and coordinate multiplicative relationships between quantities, distinguish proportional from other relations, and use different representations while recognising invariant structures, rather on mere use of rules (Silvestre, 2012). Research on proportional reasoning shows that students often succeed procedurally without demonstrating conceptual understanding (Lamon, 2007). Difficulties persist across schooling, raising questions about how students' reasoning, strategies, and justifications evolve over time. In Portugal, direct proportion is taught in 6th and 7th grades, while inverse in 9th grade. This study analyses 6th- and 9th-grade students' solutions to one pseudo-proportional and three proportional problems (direct and inverse) to examine whether their reasoning reflects genuine proportional understanding or mainly procedural approaches. It involved 74 6th-grade and 78 9th-grade students from four schools whose teachers agreed to participate. Both correct and incorrect responses were analysed. Correct responses were code by strategy type, distinguishing additive and multiplicative reasoning, including relational thinking between and within measures and algorithmic rules, while incorrect ones were code by error type. In the direct proportion missing value problem, 6th graders tend to use a scalar strategy, while 9th graders mainly rely on the rule of three. In a pseudo-proportional problem, 6th graders show a higher success rate, although both groups reveal misuse of direct proportionality. In the inverse proportionality problem, a slightly better performance is observed in 9th grade, but half of the 6th graders are already able to solve it before formal instruction using a scalar strategy based on inverse covariation. For the ratio comparison problem, both groups performed similarly, with a predominant use of ratio comparison, whereas additive reasoning was the most frequent error. Performance did not increase with schooling and seams linked to understanding multiplicative relationships, as shown by 6th graders' success with inverse proportionality. Difficulties reflect additive reasoning, linearity assumptions, and indiscriminate use of the rule of three, evident when intentionally juxtaposing problem types.

References

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629-668). Information Age Publishing.

Silvestre, A. I., & Ponte, J. P. (2012). Missing value and comparison problems: What pupils know before the teaching of proportion. *PNA*, *6*(3), 73-83.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 620. PME.

TEACHING FOR LINGUISTIC AND CONCEPTUAL TRANSFER: CROSSLINGUISTIC TRANSLANGUAGING IN MULTILINGUAL PRIMARY MATHEMATICS CLASSROOMS IN ENGLAND

Jo Skelton

Oxford Brookes University

Translanguaging is a pedagogical approach, which facilitates flexible use of learners' full linguistic repertoires and has gained increasing interest from mathematics education researchers in recent years (Planas & Chronaki, 2021). Cummins'(2021) crosslinguistic translanguaging theory (CTT) provides a theoretical framework, which advocates teaching for the transfer of concepts, skills and learning strategies across languages. From this perspective, languages are viewed as separate but connected, dynamic and fluid, rather than unitary and undifferentiated (Cummins, 2021). CTT is garnering increasing interest within mathematics education research, such as Farrugia et al. (2026), for connecting languages and mathematical concepts. In this discussion, I present examples of mathematical teaching and learning which draws from two studies based in Oxfordshire, UK. The first study, part of a PhD, analysed examples of translanguaging in maths lessons in a French-English bilingual classroom. In this setting, the co-teachers working with 8-9-year-old learners created specific opportunities for crosslinguistic and cross-conceptual transfer between the languages. The findings identified how CTT could be used to support the development of conceptual understanding through exploration of mathematical structure, representation and variation. The second study explored how teachers from English-medium schools trialled similar strategies with their learners, some of whom were multilingual. The preliminary findings from this study reveal that CTT can afford opportunities for pupils to utilise their linguistic repertoires, even where teachers do not share the learners' home languages. Furthermore, the findings indicate the possibilities for translanguaging across multiple languages within the same classroom, using representation, structure and variation as the core foci for teaching and learning.

References

Cummins, J. (2021). *Rethinking the Education of Multilingual Learners.* Multilingual Matters.

Farrugia, M.T., Vella, L.A., Muscat, D., & Casha Sammut, M. (2026). Mathematics in Maltese: The compilation of a bilingual English-Maltese glossary to support mathematics teaching and learning in Malta. *The Mathematics Enthusiast, 23*(1), 15-36.

Planas, N., & Chronaki, A. (2021). Multilingual Mathematics Learning from a Dialogic-Translanguaging Perspective. In N. Planas, C. Morgan & M. Schütte (Eds.), *Classroom Research on Mathematics and Language: Seeing Learners and Teachers Differently* (pp. 151-166). Routledge.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 621. PME.

PROSPECTIVE SECONDARY MATHEMATICS TEACHERS' CRITERIA FOR EVALUATING SUBJECT-SPECIFIC VS. INTERDISCIPLINARY TEACHING MATERIALS

Melissa Skerhutt, Daniel Sommerhoff, Kristin Litteck, Maren Koberstein-Schwarz, Anika Radkowitsch

IPN Leibniz Institute for Science and Mathematics Education

Interdisciplinary teaching offers the potential to motivate learners and connect and apply mathematical content across different subjects, but also relies on careful lesson planning. For that process, teachers often draw on existing or AI-generated teaching materials, which must be carefully evaluated to ensure their justified selection. Evaluating teaching materials should, for example, be based on objective quality criteria which mirror established criteria for teaching quality, such as instructional quality (e.g. Praetorius et al., 2018). While evaluating teaching materials is challenging for prospective secondary teachers (Cevikbas et al., 2024), there is little research on the evaluation of materials in interdisciplinary teaching contexts and it is unclear whether it differs from that in subject-specific contexts. To gain insights into prospective mathematics teachers' knowledge and skills regarding the criteria-based evaluation of teaching materials, our study addresses the research question: Which quality criteria do prospective secondary mathematics teachers a) mention and b) use when evaluating subject-specific and interdisciplinary teaching materials?

Using a within-subject design, 54 prospective secondary mathematics teachers in Germany were asked to provide criteria to evaluate (i) subject-specific teaching materials on basic concepts of functions and (ii) interdisciplinary teaching materials on basic concepts of functions and photosynthesis in randomized order. Subsequently, they received concrete teaching materials and were asked to evaluate these. Data were analysed using a qualitative content analysis.

Overall results show only few differences in the frequency of mentioned criteria between subject-specific and interdisciplinary teaching materials. However, the frequency of some criteria differed significantly between teaching materials, for example use of representations. The ongoing analyses focus on participants' use of criteria when evaluating subject-specific and interdisciplinary teaching materials and compare them with the criteria they previously mentioned.

References

Cevikbas, M., König, J., & Rothland, M. (2024). Empirical research on teacher competence in mathematics lesson planning: recent developments. *ZDM*, 56(1), 101–113.

Praetorius, A.-K., Klieme, E., Herbert, B. (2018) Generic dimensions of teaching quality: the German framework of Three Basic Dimensions. *ZDM*, 50, 407–426.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 622. PME.

CHILDREN'S USE OF DURATIONAL QUANTITATIVE PROXIES

Amy Smith[1], Darrell Earnest[2]

[1]Stetson University, [2]University of Massachusetts Amherst

Durational reasoning, or operating on and with the abstract quantity of "time", extends beyond the clock reading and elapsed time calculations of early schooling. Young children begin conceptualizing duration as a quantity before such schooling through their attention to other measurable attributes of experience, such as the amount of Lego pieces or the distance traveled to grandma's house (Smith, 2021). These other attributes serve as quantitative proxies for the invisible quantity of "time" and allow young children to begin reasoning about the durations of their lived experiences. We interviewed 4-, 5-, and 6-year-olds from across the United States (n=32) about their durational experiences and how they worked to mathematize this intangible, invisible quantity. Each child was interviewed twice, once focusing on reflections on past experiences, the second quantifying common experiences through tangible means. Using constant comparison analysis (Glaser & Strauss, 1967), we identified moments when participants worked to quantify durational aspects of their experiences. We found that young children conceptualized duration indirectly, treating other measurable attributes of experience, such as distance travelled or gross quantity, as quantitative proxies for time. Consistent with Piaget's (1969) notion of intuitive time, our participants seemed to reason about duration through these covarying, perceptible attributes, coordinating them in measurable ways (e.g., the time of vacuuming 3 rooms versus completing a puzzle with 20 pieces). In doing so, their activity reflects Thompson's (2011) definition of quantification, conceptualizing an attribute with a unit of measure and operating on the assumption of a proportional relationship—here, between their experience and a durational proxy, such as distance or accumulation.

Additional information

This work was supported by a Spencer Research Grant on Education: Small.

References

Glaser, B., & Strauss, A. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Mill Valley, CA: Sociology Press.

Piaget, J. (1969). *The child's conception of time*. New York, NY: Ballantine Books.

Smith, A. (2021). *"Time is when you wait:" An exploration of children's conceptions of duration as an attribute of their lived experiences*. (Publication No. 28418237) [Doctoral dissertation, University of Colorado Denver]. ProQuest Dissertations Publishing.

Thompson, P. W. (2011). Quantitative reasoning and mathematical modeling. In L. L. Hatfield, S. Chamberlain & S. Belbase (Eds.), *New perspectives and directions for collaborative research in mathematics education.* WISDOMe Mongraphs (Vol. 1, pp. 33-57). Laramie, WY: University of Wyoming.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 623. PME.

TEACHER LEADERS' MOVES TO FOSTER TEACHERS' NOTICING OF ARGUMENTATION IN MATHEMATICS CLASSROOMS

Horacio Solar[1], Sara Rivera[2], María, Andrés Ortiz[3]

[1]Pontificia Universidad Católica de Chile, [2]Universidad de Alicante, [3]Universidad Católica de la Santísima Concepción

Several countries have incorporated argumentation into mathematics curricula to promote a culture in which knowledge construction is viewed as a situated, critical, and reflective activity involving group participation. Despite its curricular relevance, teachers often experience difficulties fostering argumentation in classroom, are essential to develop teachers' competences to support argumentation in mathematics classrooms with a focus on teachers' noticing of argumentation (Ayalon, 2024). While Professional Development Programs (PDPs) in teachers' noticing have increased (Amador et al., 2024), less attention has been paid to how Teacher Leaders (TLs) enact moves in PDPs. This gap is particularly relevant with respect TLs' moves enacted in PDPs aimed at teacher noticing to foster argumentation in mathematics classrooms. Accordingly, this study explores the moves enacted by TLs during a PDP designed to developing teachers' noticing to foster argumentation in mathematics classrooms.

The study adopts a qualitative descriptive approach based on the analysis of nine video-recorded coaching sessions led by two experienced TLs. The analysis identified two main types of TL moves: those aimed at challenging teachers' practices and those aimed at eliciting teachers' noticing of argumentation-related classroom events. Important differences emerged between the two TLs. One TL tended to orient teachers toward predetermined distinctions, resulting in largely procedural participation. In contrast, the other TL promoted collective interpretation by inviting teachers to revisit their initial analyses in light of classroom evidence and conceptual resources. The analysis further suggests that collective, evidence-based elicitation may play a particularly relevant role in supporting teachers' noticing of argumentation. Although based on a limited number of cases, these preliminary findings point to the need for further research on how different TLs' enactments may influence teachers' professional learning in PDPs focused on argumentation.

References

Amador, J. M., Gillespie, R., Choppin, J., & Carson, C. D. (2024). Characteristics of mathematics coaches' suggestions to teachers. *Mathematical Thinking and Learning*, 27(2), 242–268. https://doi.org/10.1080/10986065.2023.2300862

Ayalon, M. (2024). A model for investigating and developing mathematics teachers' noticing of argumentation. *Teaching and Teacher Education*, 148, 104683.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 624. PME.

MATHEMATICAL STYLE: AN ANALYTICAL FRAMEWORK

Susan Staats, Erin E. Baldinger, Paul Anderson, Amelia Rosenhamer

University of Minnesota

While learners' intellectual output is a primary focus of mathematics' education research, their mathematical insight is always expressed through a communicative manner. A conjecture can be stated with a smile, a grimace, an apology, a gesture that is magnanimous or hesitant. At times, these mannerisms of doing mathematics become patterned and therefore sufficiently distinctive that they could be considered as a mathematical style. In sociolinguistics, style refers to "a *system of distinction* in which a style contrasts with other possible styles that can produce functionally similar social outcomes (Irvine, 2002, p. 22, original emphasis). Styles use multimodal resources to signal affiliation or disaffliation with others and frequently carry a sense of value, beauty, or distaste (Machin & van Leeuwen, 2005). In this communication, we consider what a "system of distinction" might mean for mathematical activity. We propose a framework for analyzing distinct, styled manners of doing mathematics through a two-axis system. One axis identifies whether a style is invoked by naming the style or by performing it. The other axis tracks whether co-participants, including researchers, identify the distinctiveness of a style as locally or socially scoped. We layer sociolinguistic analysis upon Kollosche's model of styles of reasoning to ensure that we attend to the mathematical nature of sociolinguistic style moments (2021). We share examples of mathematical style among teacher candidates during their methods course, and describe ways in which analyzing moments of mathematical style can deepen our understanding of their developing pedagogical practice. We suggest that scholarship on sociolinguistic style can restore the richness of students' mathematical experiences to our research record, and more broadly, contribute to understanding how expressive practices shape mathematical engagement.

References

Irvine, J. T. (2002). "Style" as distinctiveness: The culture and ideology of linguistic differentiation. In J. R. Rickford & P. Eckert (Eds.), *Style and sociolinguistic variation* (pp. 21–43). Cambridge University Press. https://doi.org/10.1017/CBO9780511613258.002

Jani-Demetriou, B., Bodó, C., & Boros. (2023). Stylisation, voice and crossing in the classroom. In J. I. Heltai & E. Tarsoly (Eds.), *Translanguaging for equal opportunities: Speaking Romani at school* (pp. 287–303). De Gruyter Mouton. https://doi.org/10.1515/9783110769609

Kollosche, D. (2021). Styles of reasoning for mathematics education. *Educational Studies in Mathematics, 107*(3), 471–486. https://doi.org/10.1007/s10649-021-10046-z

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 625. PME.

REFINING A TOOL FOR EVALUATING STUDENTS' MATHEMATICAL PROBLEM POSING

Anna Krisztina Stirling[1], Csaba Szabó[1,2], Janka Szeibert[1,2]

[1]MTA-ELE Theory of Learning Mathematics Research Group

[2]Edutus University, Budapest

Problem posing (PP) is increasingly recognized as a core component of mathematical proficiency, fostering creativity and higher-order cognitive functions. In this generative process, students construct personal interpretations of concrete situations to formulate meaningful problems (Stoyanova & Ellerton, 1996). Despite its benefits, PP remains marginalized in curricula as teachers often hesitate to integrate it due to a lack of familiarity, time, or practical frameworks (Cai & Hwang, 2023). To provide teacher support and facilitate classroom implementation, we examined Leavy and Hourigan's (2022) eight-dimensional Framework for Posing Elementary Mathematics Problems (F-PosE). Our study employed mixed methods to evaluate the applicability of F-PosE in a secondary education context with 16 high-achieving 10th-grade students. Over two weeks, groups developed problem sets covering algebra and geometry. The resulting 20 tasks were independently scored by two raters using F-PosE indicators. To ensure objectivity, a specialized form of inter-rater reliability (IRR) was applied to compare the scores and identify discrepancies in the framework's interpretation. The IRR analysis revealed mixed results regarding objectivity. While the raters were statistically consistent in scoring certain dimensions, other factors proved highly subjective, leading to differing interpretations. Based on these findings, we conducted a thorough investigation into the causes of these discrepancies. Consequently, we propose a refined F-PosE framework that retains the consistent indicators while broadening the definitions of others to include both contextual and intrinsic mathematical motivation. The goal is to provide a more teacher-independent feedback tool that bridges the gap between PP theory and formative assessment in the classroom.

References

Cai, J., Hwang, S. (2023). Making Mathematics Challenging Through Problem Posing in the Classroom. In: Leikin, R. (eds) Mathematical Challenges For All. Research in Mathematics Education. Springer, Cham. https://doi.org/10.1007/978-3-031-18868-8_7

Leavy, A., & Hourigan, M. (2022). The framework for posing elementary mathematics problems (F-PosE): Supporting teachers to evaluate and select problems for use in elementary mathematics. Educational Studies in Mathematics, *111*(1), 147-176.

Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students' problem posing. In P. Clarkson (Ed.), *Technology in mathematics education* (pp. 518–525). Mathematics Education Research Group of Australasia.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 626. PME.

EXAMINING FIDELITY OF IMPLEMENTATION IN A KINDERGARTEN EARLY ALGEBRA STUDY

Despina A. Stylianou[1], Rena Stroud[4], Mathías A. López[2], Susanne Strachota[2], Maria Blanton[3], Johyeon Chang[2], Bárbara M. Brizuela[2], Angela Murphy Gardiner[3]

[1]The City College of New York, [2]Tufts University, [3]TERC, [4]Merrimack College

Mathematics educators argue for an early introduction of algebra in school curriculum (NCTM 2000). However, further research is necessary to understand how this shift affects students' later mathematical development. To explore this topic, we implemented a large-scale, cluster-randomized study to test the effectiveness of a Grades K-2 early algebra intervention. We focus here on the fidelity of implementation (FOI) and examine how variation in FOI is related to student performance. We use data from Year 1 of the intervention.

Our study took place in three school districts, involving 40 schools, 189 classrooms and 2144 students. Classrooms were randomly assigned to treatment or control groups. We randomly selected 40 classrooms for FOI analyses and collected digital lesson logs, classroom video-recordings of specific lessons and teacher interviews. Students received a pre- and post-implementation digital test on algebra.

Lesson log and interview data suggested that, overall, teachers followed the curriculum as intended. Video coding and analysis revealed a subtly different picture: our coding aligned with teachers self-reporting regarding the logistical aspects of lessons (e.g., use of manipulatives, completing tasks, getting students to work in groups), but deviated on the prevalence of teachers' use of algebraic practices. After controlling for pre-test performance, Hierarchical Linear Modelling (HLM) showed that Logistical Curriculum Implementation was not a significant predictor of post-test scores ($p = .79$), whereas Algebraic Instructional Practice showed a trend toward significance ($p = .08$).

Overall, we noticed that teachers need support, not only to implement the new curriculum, but also to understand and appreciate the algebraic ideas at hand. As this was only the first year of the implementation, we expected teachers to focus first on the logistical aspects, particularly in Kindergarten, and on the bigger algebraic ideas.

Acknowledgements

This research is supported by the National Science Foundation's DRK-12 Award #2404984. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

References

NCTM (2000) Principles and Standards in School Mathematics. Reston, VA.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 627. PME.

THE GREAT LEARNING & THE CONTEMPORARY RELEVANCE FOR MATHEMATICS LEARNING

Emily Sum

Faculty of Education, The University of Melbourne

This theoretical paper seeks to clarify Confucian methods of learning through a textual analysis of Chu Hsi's commentaries on the *Great Learning* and other works. To illustrate the contemporary relevance, *the Learning Compass for Mathematics* proposed by the OECD (2023) is used as an example. I argue that Confucian learning is fundamentally experiential – *the unity of knowledge and action*.

> When you know something but don't act on it, your knowledge of it is still superficial. After you've personally experienced it, your knowledge of it will be much clearer, and its significance will be different from what it used to be (Chu, 1990/1270, p.116).

In the process of learning, knowledge is acquired through *investigation of things* and *extension* to perfection. "Action is more significant" (*ibid*) because the objective of learning is human functioning and flourishing to ensure a healthy society and collective well-being. Confucian's approach to learning mathematics, I would argue, is to investigate the principles in those things that learners encounter, such as identifying patterns in data and solving real-world problems. To take responsible actions thereby to have a deeper understanding. To apply mathematical knowledge in different contexts for an extensive period to develop genuine knowledge naturally. *Great Learning* is not rote memorization but by experiencing the practical implications of knowledge personally. Learners study for themselves and not for the sake of examinations (*ibid*). Confucian epistemology is comparable to the OECD's (2023) epistemic knowledge, which emphasises knowing how to think and act like a partitioner. Chu sees knowledge as tentative and subject to change over time; therefore, an intellectual virtue of open-mindedness is valued in learning (1990/1270). In the context of school mathematics, some ideas have to be rejected as scholarly knowledge is constantly evolving. Confucian iterative learning process also resembles the OECD's approach (2023) whereby learners continuously think critically and reflect upon their actions and those of others. Confucianism may have been misinterpreted in the past as the pedagogy and principles, such as investigation, experiential learning, and open-mindedness, are often neglected in the cultural influences on mathematics learning.

References

Chu (1990/1270). *Chu Hsi learning to be a sage: Selection from the conversations of master Chu, arranged topically*. (D. Gardner, Trans.). University of California Press.

OECD (2023). *The future of education and skills: OECD learning compass for mathematics, the future we want*. OECD Publishing, Paris

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 628. PME.

THE EMERGENCE OF EMBODIED OBJECTIFICATION UNDER CONDITIONS OF APHASIA: SEMIOTIC FLEXIBILITY IN A COVARIATIONAL TASK

Neruja Suriakumaran

University of Bremen

Research on covariational reasoning often privileges verbal articulation. Less attention has been paid to how mathematical meanings emerge epistemically as embodied processes when language cannot be taken for granted. Drawing on the Theory of Objectification (Radford, 2021), this study explores how embodied objectification unfolds under conditions of aphasia. Hence, the research question is: How does a learner objectify a covariational relation through semiotic resources under conditions of aphasia? Following a design research approach, a teaching experiment with a 39-year-old learner with aphasia was retrospectively analysed, focusing on key moments of objectification in a covariational "piggy bank" task (Radford, 2021). After modelling up to week 5, the interviewer (I) asked the learner (L) to determine week 10 without the red and black chips.

1	L:	Exactly the number five calculate (*smiles*), double two for the end – ten months [weeks] adjusted (*points with finger to week 5, then looks at the interviewer*).
2	I:	And how could I write that, or how can I remember what I have to do?
3	L:	(*counts red chips from week 5 silently with finger*) One, two, three, four, five. One, two, three, four, five. Ten.
4	I:	What would it be in week ten?
5	L:	The number (*whispers; blinks; then recounts silently with finger*) One, two, three, four, five. That is ten. And one of those (*looks at black coin of week 5*), yes. Ten, double number, yes.

When asked for week 10, the learner expressed "double" through gesture and minimal speech, and, when prompted, re-anchored the relation by counting the coins. Relational meaning (5→10 as doubling) thus stabilized through coordinated multimodal resources. In this case, embodied enactment emerges as constitutive of objectification rather than compensatory. The oscillation between relational articulation and material grounding indicates semiotic flexibility and positions inclusion as analytically relevant to understanding the dynamics of objectification rather than as mere contextual background. In the presentation, I will further discuss the implications of these findings for understanding the dynamics of embodied objectification in inclusive contexts.

References

Radford, L. (2021). *The theory of objectification: A Vygotskian perspective on knowing and becoming in mathematics teaching and learning* (Vol. 4). Brill.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 629. PME.

PRE-SERVICE MATHEMATICS TEACHERS' EXPERIENCES IN A GAMIFIED UNIVERSITY COURSE: A THEMATIC ANALYSIS

Sára Szörényi, Csaba Szabó, Janka Szeibert

Eötvös Loránd University

A growing body of research demonstrates the potential of integrating thoughtfully designed gamified systems into education. While gamification is increasingly recognised as a legitimate pedagogical tool, empirical studies continue to report mixed or even negative outcomes – underscoring that its effectiveness is highly contingent on design, context, and implementation (Zainuddin et al., 2020). This highlights a critical gap: we know whether gamification can work, but we know far less about how students experience it, particularly in high-stakes, emotionally charged contexts like the transition to university mathematics. Understanding the learner's perspective is therefore essential to move beyond performance metrics and design systems that are not only effective but also meaningful and sustainable. This study explores pre-service mathematics teachers' experiences in a point-based gamified Number Theory course during the sudden shift to online education in 2020. Students gained points through weekly quizzes, weekly assignments and active participation in lectures and practices. In addition to the point system, other game elements were implemented to make the course engaging. While quantitative results of the experiment (Muzsnay et al., 2025) showed significant gains in achievement and decreased dropout rates, this study reveals how students perceived the course design. The thematic analysis of 20 reflections followed Braun and Clarke's (2006) approach, using inductive, semantic coding. Students reported that they felt "seen", as their effort was recognised and mistakes were framed as developmental, not punitive. The course design helped them "see where they stood," enhancing certainty and reducing anxiety. Though some noted issues with pacing or clarity, the overall experience was empowering, and many plan to adopt similar systems in their future teaching. Our analysis reveals that gamification can help transform mathematics learning from a source of fear into an experience of agency.

References

Braun, V. & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77–101.

Muzsnay, A., Szörényi, S., Stirling, A. K., Szabó, Cs. & Szeibert, J. (2026). Enhancing First-Year Mathematics Achievement Through a Complex Gamified Learning System. *Education Sciences, 10(1).*

Zainuddin, Z., Wah Chu, S. K., Shujahat, M., & Perera, C. J. (2020). The impact of gamification on learning and instruction: A systematic review of empirical evidence. *Educational Research Review, 30, 100326.*

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 630. PME.

POETIC DIAGRAMMING: AN EXPLORATORY METHOD IN TRACING EMBODIED MATHEMATICS

Sheena Tan[1], Aehee Ahn[2], Pauline Tiong[3]

[1]Simon Fraser University; [2]Massey University;
[3]National Institute of Education, Nanyang Technological University

Mathematical embodiment posits that thinking and doing are entwined, and mathematical meaning emerges through the coordination of bodily actions with material forms (de Freitas & Sinclair, 2012). Following this view, we argue that acts of gesturing and diagramming constitute the doing of mathematics, and not mere representations of mathematics ideas. Consequently, these processes require a different way of transcribing and analysing, to better attend to how mathematical meaning and the distribution of agency emerge from them. Specifically, we propose that the approach of poetic diagramming can address this need. While poetic inquiry in mathematics education has mostly focused on the repetition and parallelism present in words, phrases, syntax and sounds (Staats, 2008), we see a potential to extend the poetic function in linguistic data to non-linguistic data by attending to repetitive visual patterns and moments of mobility in the making of the diagrams. Thus, in this paper, we focused on students' gestures and diagrams and explored how they might reflect poetic structures that communicate students' sense-making of mathematics and the nature of mathematical agency. Methodologically, we also experimented with different ways to "transcribe" the diagrams and gestures to illuminate these poetic expressions.

The data consists of video captures from a university course focused on problem solving where students worked on vertical whiteboards with non-permanent markers to solve problems collaboratively. Using the students' actual diagramming process in the video data as the basis, we created poetic diagrams to illuminate the poetic form and function of their diagramming and gesturing motions over time. These poetic diagrams allow us to see how the solution and students' mathematical understanding of a specific problem are constructed through the entwinement of rhythmic diagramming and gesturing. It also helps us to attend to the emergence of distributed agency across numerical relations. Overall, by attending to the visual and temporal structures, the poetic diagrams make mobility visible in ways that linear and verbal transcripts cannot. In the presentation, examples and findings will be discussed.

References

de Freitas, E., Sinclair, N. (2012). Diagram, gesture, agency: Theorizing embodiment in the mathematics classroom. *Educational Studies in Mathematics, 80*(1/2), 133–152.

Staats, S. (2008). Poetic lines in mathematics discourse: A method from linguistic anthropology. *For the Learning of Mathematics, 28*(2), 26–32.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 631. PME.

STUDENTS' DESCRIPTIONS OF ARITHMETIC SYMBOLS

Magnus Österholm, Tomas Bergqvist, Frithjof Theens, Ewa Bergqvist

Department of Science and Mathematics Education, Umeå University

Learning to use and understand symbols is crucial for learning mathematics. Using symbols can include how to perform calculations, for example, 5+3=8. Understanding symbols can include the ability to see "through" symbols (Alibali et al., 2014) to their referents (i.e., what the symbols stand for), for example, by linking the symbol "5" to a set of five objects. When linking symbols to referents, language can play a central role, for example, when "five" is used to describe both the numeral and the quantity. However, Powell and Fluhler (2020) found that it is more difficult for students to describe the meaning of arithmetic symbols than to use the symbols, but we do not know if their descriptions develop as they become more familiar with the symbols. Our study investigates the following research question: How do students' descriptions of referents to symbols vary depending on how familiar the symbols are to them? We asked 66 students (grades 3–9) how they would explain different symbols to a friend. Students' level of familiarity with the symbols was based on when they are introduced in school. The symbols, here sorted in decreasing level of familiarity, were, 3, =, +, %, π, and √. All symbols were presented within mathematical statements. We identified what basic parts of speech (verbs, nouns, and adjectives) students used when describing the meaning of each symbol. Signs of a more advanced mathematical discourse can be the use of more nouns than verbs, due to objectifications (Pimm & Wagner, 2003) and more adjectives, to focus also on properties (Davies et al., 2022). There was no correlation between familiarity with the symbols and types of words used, suggesting that the discourse about symbols is established early and maintained through grades 3–9. Further research that simultaneously analyses the discourse in teaching and the discourse of students could perhaps explain this lack of progression.

References

Alibali, M. W., Stephens, A. C., Brown, A. N., Kao, Y. S., & Nathan, M. J. (2014). Middle school students' conceptual understanding of equations: Evidence from writing story problems. *International Journal of Educational Psychology*, *3*(3), 235-264.

Davies, C., Syrett, K., Taylor, L., Wilkes, S., & Zuniga-Montanez, C. (2022). Supporting adjective learning across the curriculum by 5–7 year-olds: Insights from psychological research. *Language and Linguistics Compass*, *16*(11), e12476.

Pimm, D., & Wagner, D. (2003). Investigation, mathematics education and genre: An essay review of Candia Morgan's Writing Mathematically: The Discourse of Investigation. *Educational Studies in Mathematics*, *53*(2), 159–178.

Powell, S. R., & Fluhler, S. K. (2020). A brief measure of mathematics symbols: Performance at grades 1, 3, and 5. *Assessment for Effective Intervention*, *45*(4), 11.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 632. PME.

A COMPARATIVE CASE STUDY ON FORMATIVE FEEDBACK IN PROBLEM-BASED TEACHING

Yasmin Theile, Benjamin Rott

University of Cologne

Problem solving is a key part of primary school maths lessons. Previous research shows that problem-based teaching can be divided into different phases, based on Pólya's ideas (Rott, 2020), with the *Look back* phase being particularly important. At the same time, feedback practices vary depending on the timing or phase of a lesson (Smit et al., 2024). However, the relationship between the structure of the phases and feedback practices throughout a lesson remains unclear.

To this end, we conducted a comparative case study on teacher behaviour in problem-based teaching combining two different analyses and their respective instruments: Teachers *closely managing* their students' ideas by highlighting correct solutions vs. teachers *emphasizing* their students' ideas and strategies (Rott, 2020), combined with a detailed coding of teachers' feedback behaviour, classifying every teacher-student interaction (Theile & Rott, 2026). We assume that the structure of the *Look back* phase relates to the type of feedback provided in preceding phases. To investigate this, we analyse one lesson from each of two primary classes where students worked on a non-routine problem. The lessons were selected based on the design of the *Look back* phase, with one being closely managed and the other emphasizing students' strategies.

In the lesson featuring a strategically oriented *Look back* phase, responsive feedback predominates, focusing primarily on students' understanding of the task's structure, their own reflection, and various approaches to solving the problem. In the lesson featuring a closely managed *Look back* phase, invasive feedback occurs more frequently, serving primarily to ensure the correct result is achieved at an early stage. It also emerges that the same categories of feedback (e.g., offering partial solutions) serve different functions depending on the lesson design. The results suggest a link between the structure of the *Look back* phase and the teacher's feedback behaviour in the earlier phases of the lesson, even though the direction of causality remains unclear.

References

Rott, B. (2020). Teachers' Behaviors, Epistemological Beliefs, and Their Interplay in Lessons on the Topic of Problem Solving. *International Journal of Science and Mathematics Education, 18*(5), 903–924. https://doi.org/10.1007/s10763-019-09993-0

Smit, R., Bachmann, P., Dober, H., & Hess, K. (2024). Feedback levels and their interaction with the mathematical reasoning process. *The Curriculum Journal*, *35*(2), 184–202.

Theile, Y., & Rott, B. (2026). Teacher feedback in problem-oriented mathematics teaching – theoretical considerations and empirical findings. *Frontiers in Education*, 11.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 633. PME.

GEOMETRIC ABILITIES OF FIRST GRADE STUDENTS IN PROBLEM SOLVING APPROACH

Sampan Thinwiangthong, Naphaporn Woranetsudathip, Duangmanee Leelalawan

Khon Kaen University

Geometry represents a fundamental domain of mathematical knowledge that individuals encounter and apply consistently in their daily lives (Clements et al., 2018). Consequently, it is essential to introduce geometric concepts to students at an early stage of their academic development, particularly during the lower primary years.

This study was designed to investigate the geometric reasoning abilities of first-grade students through implementation of a problem-solving instructional approach. The research utilized qualitative methodology, employing video documentation and analytical description as primary data collection and analysis techniques. The participant group consisted of 36 first-grade students enrolled at Khon Kaen University Demonstration Elementary School during the first semester of the 2025 academic year. The students received instruction based on a problem-solving pedagogical framework across six sequential lessons. Data analysis was conducted using the theoretical framework developed by Clements et al. (2018).

The research findings demonstrated that first-grade students exhibited four core geometric abilities, (1) ability in shape recognition by correctly identifying circles, triangles, squares, and rectangles, (2) ability in shape reproduction, successfully recreating geometric shapes from memory, (3) ability in spatial orientations, engaging in discussions about whether shapes were positioned vertically or horizontally, and (4) ability in pattern, organizing figures according to identified patterns.

The study revealed two noteworthy findings beyond the initial framework. First-grade students exhibited the ability to recognize and articulate relationships between different geometric shapes. Moreover, they demonstrated capacity for modelling real-world objects through geometric representation, providing reasonable explanations and justifications for their reasoning. These findings suggest that first-grade students possess more advanced geometric understanding than conventionally anticipated, particularly in their ability to connect abstract geometric concepts to concrete real-world applications.

References

Clements, D. H., Sarama, J., Swaminathan, S., Weber, D., & Trawick-Smith, J. (2018). Teaching and learning geometry: Early foundations. *Quadrante, 27*(2), 7-31. https://quadrante.apm.pt/index.php/quadrante/article/view/18

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 634. PME.

PERSPECTIVES ON LEARNING TO TEACH MATHEMATICS FROM A VISUALLY IMPAIRED PRESERVICE TEACHER

Helen Thouless, René Hartmann, Zahara Hussain

St Mary's University

The theme of this conference is sensing and making sense of mathematics, but what happens when one sense is impaired? In this study we examined the following research questions: What aspects of our BA (Hons) Primary Education Core Mathematics sessions that were accessible to a preservice teacher (PST) with a visual impairment (VI)? What could have made the module more inclusive? What did the PST wish her lecturers knew about teaching and learning mathematics with a VI?

This study took a participatory research framework, where the PST with the disability was involved with the creation, production, and dissemination of the research (Zarb, 1992). The PST was an active contributor throughout the research process, discussing the ethical implications of the research, developing the interview questions, engaging in a semi-structured interview with her mathematics education lecturers, and participating in the dissemination process. After the interview did a membership check about the accuracy of the transcript and analysed the data to find themes.

This PST found the supportiveness of her mathematics lecturers an important aspect of the accessibility of the course. They were supportive by pre-teaching both the concepts and the use of manipulatives, by ensuring that the PST did not feel singled out because all the PSTs had access to the same manipulatives, and by adapting the materials for the PST. The use of an online mathematics audit was one of the least inclusive portions of the course, with multiple visual images taken from the Key Stage 2 National Curriculum Test and a small font size. The PST wished that her lecturers understood that students who learn mathematics through Braille are having to function in multiple languages and so giving them the same content, but fewer questions would be fairer. She also noted that consistent lighting is important, but being sat at the front or having the font size increased was not necessarily helpful. Students with VI are a heterogeneous group with regards to both the type and degree of their VI and their achievement in mathematics (Klingenberg et al., 2019), therefore these findings cannot be generalised to all PST's with VI.

References

Klingenberg, O.G., Holkesvik, A.H., & Augestad, L.B. (2019). Research evidence for mathematics education for students with visual impairment: A systematic review, *Cogent Education, 6*.

Zarb, G. (1992). On the road to Damascus: first steps towards changing the relations of disability research production, *Disability & Society, 7*(2), 125-138.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 635. PME.

TOWARDS SIMULATION COMPETENCE IN STEM-EDUCATION: VALIDATING THE NATURE OF SIMULATION

Christoph Till

PH Ludwigsburg University of Education

Using and understanding digital simulations, and fostering these competencies in STEM education, is becoming increasingly important, as many societal decisions are informed by economic, ecological, and scientific simulations. Preliminary research based on expert interviews suggests that the Nature of Simulation (NoSim) can be described as a three-dimensional construct including understanding models, modelling processes, and system thinking (Bleymehl, 2025). Building on Bleymehl's study, variability and covariational reasoning are added here as theoretically relevant dimensions, as both may support students in understanding stochastic and parameter-based simulations. The goal of the SIMBA-MINT project is an initial validation of the NoSim construct as a foundation for developing effective simulation tools and learning environments. In a first step, the following research questions are addressed:

- How many core dimensions underlie the NoSim construct?
- How is NoSim related to covariates such as digital experience and attitudes, cognitive abilities, understanding of the Nature of Science (NoS), and computational thinking?

To address these questions, a pilot online survey (approximately 30 minutes) is being conducted in April and May 2026 with N = 100 students aged 14–15 in southern Germany. The questionnaire combines multiple-choice competence tasks and a small number of self-report items across the five proposed NoSim dimensions and several covariates. It further includes an additional scale capturing students' epistemic understanding of simulations, for example their conditional character, limits, and the role of repeated runs in stochastic simulations. This initial validation step explores whether NoSim can be identified as a five-dimensional construct in an exploratory factor analysis and whether its dimensions show meaningful but distinct relations to the covariates. A better understanding of simulation competence can help to develop tools and learning environments that are better aligned with students' ways of thinking and working with simulations. In this way, students may come to see simulations as valuable tools for investigating complex and important real-world phenomena.

References

Bleymehl, S. J. E. (2025). *Explorative Studie zum Simulationsverständnis von Schülerinnen:* Identifizierung von Teilkompetenzen und ihre Evaluation in der Praxis [Dissertation, Pädagogische Hochschule Ludwigsburg]. OPUS PH Ludwigsburg. https://phblopus.phlb.de/frontdoor/index/index/docId/1425

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 636. PME.

GIVING A VOICE TO THE SILENT MATHEMATICAL SYMBOLS

Sandra Hughes[1], Sheena Tan[1], Pauline Tiong[2]

[1]Simon Fraser University
[2]National Institute of Education, Nanyang Technological University

With an increasing emphasis for communication as an important aspect in mathematics learning, research has looked into how students could be "seen as producers of mathematical language rather than merely as consumers" (Morgan, 1998). However, the notion of communication in the mathematics classroom is often interpreted as talking to learn mathematics, while the role of writing as a mode of communication to learn mathematics has not been as widely discussed, practised and researched. Specifically, writing in the mathematics classroom is mostly associated with the writing of mathematical equations, notations and symbols as the focus is primarily placed on the presentation of final solutions rather than the process of getting to the solutions. Consequently, this form of writing is usually depersonalised and decontextualized, and often leads to student work that is universally similar. Yet, the process of writing with characteristics (*integrative, connective* and *self-rhythmed*) which are deemed akin to effective learning strategies can potentially empower students in the learning process as they engage and coordinate their different senses to make sense of mathematics (Emig, 1977; Morgan, 1998). Hence, we are keen to explore how writing in the mathematics classroom could better focus on students' (different) processes or experiences of doing mathematics. Based on the data collected from an exploratory teaching experiment conducted in the grade 11 mathematics classroom, we examined how the process of writing mathematics, through the structure of a double-entry journal – with examples recorded on the left, supported by interpretations and explanations on the right – could help students attend to both the mathematics and their sense-making processes. Students' writings over a period of three days were analysed for the purpose of this study. In particular, we noticed how these students were intentionally engaging different senses (e.g., visual, temporal) to organise their examples, steps and explanations/reasoning. We could also see and hear students' voices emerging through their examples and style of writing as the depersonalised and decontextualized mathematical notations are now embedded with the meanings students afforded them. We will discuss more about the teaching experiment and the findings during the presentation.

References

Emig, J. (1977). Writing as a mode of learning. *College Composition and Communication*, *28*(2), 122–128. https://www.jstor.org/stable/356095

Morgan, C. (1998). *Writing mathematically: The discourse of investigation* (1st ed.). Routledge. https://doi.org/10.4324/9780203017715

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 637. PME.

MAKING SENSE OF INTERCULTURAL MATHEMATICS PRACTICES OF THE TIKMŨ'ŨN BRAZILIAN INDIGENOUS

Vanessa Sena Tomaz, Lucio Maxakali

Universidade Federal de Minas Gerais (UFMG)

In our research, we investigated how Tikmũ'ũn Brazilian Indigenous, named Maxakali, teach, understand, and use mathematics in environment restauration practices of their territories. To Maxakali (2025), tikmũ'ũn school education should be guided by their ancestral educational practices and the flow of their spiritual entities (*yãmĩyxop*). However, *tikmũ'ũn* schools have prioritized Western mathematical concepts and procedures that can be expressed in Maxakali language: the use of money, numbers, and operations algorithms, only as a visual representation (Tomaz & David, 2018).

Since 2023, we have been focused on the environmental restoration practices and planting of agroforest systems in their villages, with the participation of 30 agroforest agents, 16 nurseywomen, and 32 teachers and undergraduates. This project became central in Tikmũ'ũn's lives as their territory has less than 12% forest and the water courses are polluted and stilted up, leading to extremely poverty.

We selected from a two-year period the empirical material composed of videos of the collective practices and participants' field diaries. We identified numeracy practices (Fonseca, 2020), theoretically and methodologically grounded in the tikmũ'ũn knowledge system, transmitted by chants, drawing, see-listen-do with the whole body (Maxakali, 2025) and agroforest knowledge from non-indigenous. The findings show the appropriation of intercultural numeracy practices by the indigenous, for instance, maps for the reforestation area, use of de measures and proportionality when mixing soil and fertilizer, which support Indigenous well-being, including languages and mathematics are rooted in relationships to land and intergenerational cultural practices (Cajete,1994). These practices have been introduced into schools, strengthening their connection with the culture and redefining mathematics for the Tikmũ'ũn people.

References

Cajete, G. (1994). *Look to the mountain: An ecology of Indigenous education*. Kivaki Press.

Fonseca, M.C. (2020) Numeracy in youth and adult basic education: Syntactic, semantic, and pragmatic dimensions of a discursive practice. *ZDM, 52*, 395-406. https://rdcu.be/b30hk.

Maxakali, L. F. C. (2025). Ixtot ũxuktux tappet pet yõg, tappet pet ũgmũ yõg yã pãhã. Conta a história da escola tikmũ'ũn: a escola para nós é qualquer lugar. [Unpublished master's thesis]. Universidade Federal de Minas Gerais, Brazil.

Tomaz, V. S. & David, M. M. (2018). Mathematical artifacts as communicational facilitators between Indigenous and non-Indigenous cultures. *For the Learning of Math.*, *38*(3), 2-10.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 638. PME.

CO-DEVELOPMENT OF MATHEMATICAL AND COMPUTATIONAL THINKING VIA VARIABLES IN SCRATCH-SUPPORTED GEOMETRY AND MEASUREMENT TASKS

Hümeyra Türkmen[1,2], Hatice Akkoç[2]

[1]Cengiz Topel Middle School, [2]Marmara University

The integration of Computational Thinking (CT) into mathematics instruction has become increasingly important in recent years. However, the simultaneous development of CT and mathematical thinking (MT) in problem-solving processes has not yet been sufficiently explored (Ye et al., 2023). This study aims to examine the co-development of *mathematical variables* and *variables* as a CT concept, as 5th-grade students solved geometry and measurement problems in the Scratch environment. Accordingly, the research question is: How did lower-secondary school students develop the concept of variables in mathematics and CT contexts while solving geometry and measurement problems in the Scratch environment? A class of 30 fifth-grade students engaged in Scratch-supported geometry and measurement problem-solving tasks. Two of them were selected based on their knowledge of variables. The data sources consisted of students' Scratch projects, worksheets, and interview transcripts. We qualitatively analyzed students' transitions between tables of values, pseudocode, and Scratch scripts. The findings show that Scratch tasks provided an environment that enabled students to reinterpret mathematical variables during algorithmic operations. Students established relationships among mathematical variables using tables of values and translated these relationships into pseudocode, enabling them to approach mathematical operations in a more step-by-step manner. As these relationships became clearer, the process of creating algorithms became easier, supporting the mutual development of MT and CT. However, students experienced significant difficulties when converting their pseudocode into scripts in the Scratch environment, which were related to how they used *mathematical variables* and *variables* as a CT concept. Testing and debugging in Scratch enabled students to reflect on their solution steps. During this process, students questioned not only the algorithm but also the mathematical operations, variables, and generalizations they used, revealing the co-development of mathematical and algorithmic representations. These findings indicate that the co-development of mathematical and CT variables took place through the processes of transposition (converting pseudocode into scripts in the Scratch environment) and debugging.

References

Ye, H., Liang, B., Ng, O. L., & Chai, C. S. (2023). Integration of computational thinking in K-12 mathematics education: A systematic review on CT-based mathematics instruction and student learning. *International Journal of STEM Education, 10*(1), 3.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 640. PME.

DIFFERENCES IN TURKISH ELEMENTARY AND MIDDLE SCHOOL STUDENTS' LENGTH ESTIMATION SKILLS

Belma Türker Biber[1], Jessica Hoth[2]

[1]Aksaray University, [2]University of Rostock

Length estimation skill is a multidimensional construct influenced by object size and accessibility (Hoth et al., 2023). There is indication that estimation skills vary across distinct dimensions: estimating the length of small objects, not small and touchable objects, and not small untouchable objects. While the 2024 Turkish Education Model emphasizes length estimation skills as a relevant learning goal of mathematics education, research on how these skills develop across grade levels remains sparse. In this regard, the primary goal of this study is to examine how Turkish elementary and middle school students' length estimation skills differ.

A total of 601 students (53,1% boys, 21 classes, four different schools) solved the adapted length estimation test developed by Hoth et al. (2023). The sample of elementary school students comprises $n = 334$ students from Grade 4, while the sample of middle school students holds $n = 267$ students from Grades 5 and 6. Students' estimates were scored with regard to their deviation from the objects' actual length. The scale's reliability was acceptable ($\alpha = .79$). For all children, the mean score was determined for the three dimensions of estimation skills.

Results show that the mean scores of elementary and middle school students differ in all three dimensions of length estimation in favour of the middle school students. However, differences were only significant for the Small Objects dimension as well as the not small and touchable dimension. The results suggest that developmental gains in length estimation are dimension-specific and may reflect curriculum emphasis on concrete manipulation.

References

Hoth, J., Heinze, A., Huang, HM.E., Weiher, D. F. & Ruwisch, S. (2023). Elementary School Students' Length Estimation Skills—Analyzing a Multidimensional Construct in a Cross-Country Study. *International Journal of Science and Math Education* 21, 1841–1864 (2023). https://doi.org/10.1007/s10763-022-10323-0

T.C. Millî Eğitim Bakanlığı. (2024, April). Türkiye Yüzyılı Maarif Modeli [PDF]. https://oltu.meb.gov.tr/meb_iys_dosyalar/2024_06/28183923_turkiyeyuzyilimaarifmodelisunum.pdf

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 639. PME.

TEACHERS' PRACTICES IN SUPPORTING STUDENTS' MATHEMATICS MOTIVATION IN PRIMARY SCHOOL

Iida Tyyskä, Laura Niemi, Anu Laine

University of Helsinki

Motivation plays a significant role in learning mathematics, and identifying instructional practices that improve students' motivation is therefore important (Schukajlow et al., 2023). The purpose of this study is to deepen understanding of how students' motivation in mathematics can be supported in primary education. Specifically, we aim to explore the practices teachers use to foster motivation.

We examine the phenomenom through data-driven content analysis of semi-structured interviews conducted with 15 primary school teachers as part of an international longitudinal MATHMot project. Teachers were selected for the interviews based on the high or increased motivation levels observed in their classrooms in grades 3-5. The theoretical framework is grounded in two motivational theories: SEVT (Eccles & Wigfield, 2020) and SDT (Ryan & Deci, 2020).

According to the interviews, teachers seek to enhance student's motivation in mathematics by supporting learning with differentiated and diverse instructional methods, providing constructive and positive feedback, creating opportunities for success, encouraging positive interactions, connecting mathematical content to everyday life, making learning mathematics enjoyable, and promoting persistence. In the presentation, results will be discussed in detail with examples and connections to motivational theories.

The results show that supporting students' motivation in mathematics is a continuous and multifaceted process. Future research is needed to investigate instructional strategies that foster motivation from the students' perspective.

References

Eccles, J. S., & Wigfield, A. (2020). Form expectancy-value theory to situated expectancy-value theory: A developmental, social cognitive, and sociocultural perspective on motivation. *Contemporary Educational Psychology, 61,* Article 101859.

Ryan, M. R., & Deci, E. L. (2020). Intrinsic and extrinsic motivation from a self-determination theory perspective: Definitions, theory, practices, and future directions. *Contemporary Educational Psychology, 61,* Article 101860.

Schukajlow, S., Rakoczy, K., & Pekrun, R. (2023). Emotions and motivation in mathematics education: Where are we today and where we need to go. *Mathematics Education, 55,* 249-267.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 641. PME.

LEARNING OPPORTUNITIES FOR UNDERSTANDING THE PLACE VALUE SYSTEM IN TEXTBOOKS FOR EARLY MATHEMATICS EDUCATION

Laura Ucke, Moritz Herzog

University of Cologne

Place value understanding is fundamental for early arithmetic and predicts later mathematical performance (Moeller et al., 2011). Jensen et al. (2024) describe regrouping and place value as the two core principles underlying the base-ten system. Regrouping involves combining and decomposing units in tens. The place value principle assigns meaning to digits based on their position and value. Place value instruction is, among other factors, also influenced by mathematics textbooks (Pepin et al., 2001). This study examines place value related learning opportunities in textbooks, focusing on task types, number ranges, and concepts of place value and regrouping. We aim to address the following research questions: (1) To what extent do textbooks offer place value learning opportunities? (2) Which types of learning opportunities address place value concepts? (3) How frequent are core constructs of the place-value system addressed? Five German Grade 1 and 2 textbook series were analyzed using quantitative content analysis (Mayring, 2015). All learning opportunities instructing the place value understanding were coded by type, number range, and principle. Frequencies and percentages were compared across textbooks and grade levels. The Kappa coefficient ranged from .92 to 1. Place value understanding was instructed in 3.74 % (182) of the 4,863 identified learning opportunities. Most of them were exercises (75.27 %) and two-digit numbers (73.63 %). 54.95 % of the learning opportunities focused on place value alone. 37.36 % instructed place value and regrouping combined. The study is limited to a textbook analysis and does not capture classroom implementation.

References

Jensen, S., Gasteiger, H. & Bruns, J. (2024). Place Value and Regrouping as Helpful Constructs to Diagnose Difficulties in Understanding the Place Value System. *Journal für Mathematik-Didaktik, 45(11).*

Mayring, P. (2015). Qualitative Inhaltsanalyse. Grundlagen und Techniken. Beltz.

Moeller, K., Pixner, S., Zuber, J., Kaufmann L. & Nuerk, H.-C. (2011). Early place-value understanding as a precursor for later arithmetic performance – A longitudinal study on numerical development. *Research in Developmental Disabilities, 32(5), 1837-1851.*

Pepin, B., Haggarty, L. & Keynes, M. (2001). Mathematics textbooks and their use in English, French and German classrooms: a way to understand teaching and learning cultures. *Zentralblatt für Didaktik der Mathematik, 33, 158-175.*

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 642. PME.

DOES STATISTICS HAVE A MEANINGFUL STATISTICAL SIGNIFICANCE FOR YOU?: VIEWS ON STATISTICS EDUCATION AND STATISTICAL LITERACY

Emine Gülen Ulusoy, Aslıhan Doğdu, Özge Demirtaş

Marmara University

Increasing data intensity worldwide and the rise of biased statistical interpretations have made it necessary to include statistics in educational curricula. Within this context, statistical literacy has been identified as a necessary skill for becoming active and informed citizens (Franklin et al., 2007). Schoen et al. (2025) likewise emphasized that statistics has been incorporated into mathematics curricula relatively recently and that its distinction from mathematics is still not fully understood. This study investigates, from the views of teachers and pre-service teachers, how statistics education and statistical literacy are defined, and the importance attributed to statistics education. In this context, the research question is "What are the views of mathematics teachers and pre-service mathematics teachers about statistics education and statistical literacy? A qualitative research design was employed, and semi-structured interviews were conducted. The interview questions were developed by researchers and subsequently reviewed by two experts in mathematics education. The participants consisted of nine high school mathematics teachers whose professional experience ranged from 4 to 30 years ($\bar{X} = 18.56, SD = 8.79$), and eight pre-service mathematics teachers, four of whom were in their third year and four in their final year of study. Teachers and pre-service teachers primarily focused on the concepts of analysis, interpretation, and decision-making when describing statistics. Regarding statistics education, participants generally highlighted the teaching of statistical concepts and procedures. While teachers and pre-service teachers shared a common understanding of statistical literacy as the ability to read, analyse, and interpret data, graphs, and tables, one of the most striking findings of the study was the divergence in their views regarding the definition and perception of statistical literacy. While teachers emphasized drawing meaningful inferences from concrete data, pre-service teachers foregrounded reasoning under uncertainty. This divergence suggests that teacher education programs and curriculum designs should explicitly address both views to support a more comprehensive and nuanced understanding of statistical literacy.

References

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) Report: A Pre-K–12 curriculum framework*. American Statistical Association.

Schoen, R. C., Rhoads, C., Perez, A., Jacobbe, T., & Li, L. (2025). Improving the teaching and learning of statistics. *Learning and Instruction*, *95*, 102018.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 643. PME.

ALGORITHM DESIGN IN SOLVING PERMUTATION PROBLEMS: AN ANALYSIS OF PRE-SERVICE TEACHERS' REASONING

Dilara Usta, Mahmut Kertil

Marmara University

Computational Thinking (CT) is increasingly recognized as a key educational competency, involving processes such as abstraction, decomposition, and algorithmic structuring (Shute et al., 2017). Within this perspective, Algorithmic Thinking (AT) refers to organizing complex goals into logically ordered and operationally coherent steps. In mathematics education, this activity has been described as algorithmizing mathematical situations—transforming mathematical relationships into executable structures (Lehmann, 2024). Permutation problems offer a productive context for examining such processes, as they require coordinating constraints, distinguishing cases, and systematically enumerating possibilities which are the areas where students often rely on formula-based reasoning (Lamanna et al., 2022). Addressing the limited research on algorithmic approaches in specific mathematical domains, this qualitative study investigates pre-service teachers' algorithm design processes in solving permutation problems. Sixteen pre-service mathematics teachers enrolled in a Computational Thinking course participated in the study. Data included pseudocode drafts, algorithms developed in Flowgorithm, and semi-structured interviews with five participants. Findings show that participants produced more accurate algorithmic designs when constraints were explicitly stated. Performance declined when they had to identify multiple conditions. Difficulties were especially evident in distinguishing permutation from combination structures and in operationalizing implicit constraints within conditional and iterative constructs. The process of algorithm design provided a context to see participants' conceptual gaps. Therefore, programming-based algorithm design may function primarily as a formative assessment lens, providing insight into students' reasoning. Successful execution of an algorithm design related to a particular mathematical concept have potential to provide students to come up with a clear and robust conceptual understanding.

References

Lamanna, L., Gea, M. M., & Batanero, C. (2022). Do secondary school students' strategies in solving permutation and combination problems change with instruction? *Canadian Journal of Science, Mathematics and Technology Education, 22*(3), 602-616.

Lehmann, T. H. (2024). How current perspectives on algorithmic thinking can be applied to students' engagement in algorithmatizing tasks. *Math. Ed. Research J., 36*(3), 609-643.

Shute, V. J., Sun, C., & Asbell-Clarke, J. (2017). Demystifying computational thinking. *Educational research review*, *22*, 142-158.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 644. PME.

UNLOCKING UNDERSTANDING: DOES FRACTION MAGNITUDE DEVELOPMENT SHAPE STUDENTS' FRACTION SENSE-MAKING?

Jo Van Hoof[1], Jake McMullen[1,2]

[1]Research Institute for Learning Analytics, University of Turku

[2]Department of Teacher Education, University of Turku

Despite the crucial importance of a good fraction magnitude understanding, students' understanding is often marked by misconceptions (e.g., Gabriel et al., 2023; Reinhold et al., 2023). While these misconceptions have been well documented, little is known about their development. Moreover, it is unclear whether development on a frequently used fraction magnitude task (ordering fractions) truly reflects a deeper fraction magnitude conceptual understanding, or if it results from rule-based strategies. This study followed 182 sixth and seventh graders over one school year and found that most retained their fraction magnitude misconceptions, with only a few students progressing toward correct understanding. Next, we tested whether improvement in fraction ordering was related to performance on a fraction sense-making task. Students estimated 12/13 + 7/8 by choosing 1, 2, 19, 21, or "I don't know." With good fraction magnitude understanding, the answer is easily seen as near 2 because both fractions are close to 1. Fraction sense-making task performance did not differ between improvers and non-improvers on the fraction ordering task, suggesting the ordering gains were more procedural than conceptual. Our results confirm previous research showing students' struggle with the conceptual understanding of fractions, which may be partly due to instructional practices that emphasize procedural skills rather than conceptual comprehension. Therefore, fraction instruction should not only focus on teaching how fractions work, but also put emphasis on conceptual understanding of fractions. Next, the methodological implication is that the use of fraction ordering tasks might need to be reconsidered as measure of students' conceptual understanding of the magnitude of fractions.

Acknowledgements

The study is part of the EDUCA RCF Flagship (#358924, #358947).

References

Gabriel, F., Van Hoof, J., Gómez, D. M., & Van Dooren, W. (2023). *Obstacles in the Development of the Understanding of Fractions*. In K. M., Robinson, A. K., Dubé, & D. Kotsopoulos (eds) Mathematical Cognition and Understanding. Springer.

Reinhold, F., Leuders, T., & Loibl, K. (2023). Disentangling magnitude processing, natural number biases, and benchmarking in fraction comparison tasks: A person-centered Bayesian classification approach. *Contemporary Educational Psychology*, *75*, 102224.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 645. PME.

SPATIAL ABILITIES AND MATHEMATICS TEACHER EDUCATION

Luis Carlos Vargas-Zambrano

Pontificia Universidad Católica de Valparaíso

As spatial abilities gain prominence in mathematics and science curricula, the complexity of spatial teaching has become increasingly evident (Bufasi et al., 2024). It is therefore necessary to reexamine the questions originally posed by Bishop (1980) regarding mathematics teachers' spatial abilities: "Should experimental teaching methods in this area take into account the spatial abilities of the teacher? If so, how? […] How much responsibility should mathematics teachers take for the training and teaching of spatial abilities?" (pp. 266–267). To address these questions, a systematic literature review is being conducted using multiple databases, focusing on preservice and in-service mathematics teachers. The search terms include spatial abilities, literacy, thinking, reasoning, visualization, and related constructs. The selected studies are classified according to key perspectives that contribute to understanding visuospatial reasoning *(cognitive science, mathematics education, sociocultural)*; their attention to diagrams, gestures, and embodiment (Sinclair et al., 2016); and their approaches to *training* and *teaching* (Bishop, 1980). Additionally, connections between these perspectives are examined to address the gap in integrating *cognitive science* and *educational research* (Bufasi et al., 2024).

Preliminary findings suggest an increasing interest in mathematics teachers' spatial abilities, with particular emphasis on *cognitive science* research. Studies also indicate connections between *cognitive science* and *mathematics education,* often linked to pedagogical knowledge and pedagogical content knowledge. Moreover, mathematics teachers' responsibility appears to be increasing, driven mainly by policy agendas focused on identifying and training future scientific talent, rather than promoting spatial literacy for citizenship or addressing social inequalities.

References

Bishop, A. J. (1980). Spatial abilities and mathematics education—A review. *Educational Studies in Mathematics, 11,* 257–269. https://doi.org/10.1007/BF00697739

Bufasi, E., Lin, T. J., Benedicic, U., Westerhof, M., Mishra, R., Namsone, D., Dudareva, I., Sorby, S., Gumaelius, L., Klapwijk, R. M., Spandaw, J., Bowe, B., O'Kane, C., Duffy, G., Pagkratidou, M., & Buckley, J. (2024). Addressing the complexity of spatial teaching: A narrative review of barriers and enablers. *Frontiers in Education, 9,* 1–16.

Sinclair, N., Bartolini Bussi, M. G., de Villiers, M., Jones, K., Kortenkamp, U., Leung, A., & Owens, K. (2016). Recent research on geometry education: an ICME-13 survey team report. *ZDM Mathematics Education, 48*(5), 691–719.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions,* p. 646. PME.

INVESTIGATING MATHEMATICAL PROBLEM POSING BY PROSPECTIVE PRIMARY TEACHERS: GAPS AND STRENGTHS

Joëlle Vlassis[1], Isabelle Demonty[2], Ann Derlet[3]

[1]University of Luxembourg, [2]University of Louvain
[3]Haute Ecole Robert Schuman

Recent studies have shown that many prospective primary teachers (PPTs) have difficulties posing a "good" problem. This is generally defined as non-routine (NRP), solvable, with a motivating context and involving a variety of solution strategies (Leavy & Hourigan, 2022). In fact, many PPTs tend to choose closed, routine problems that have one right answer that requires low cognitive demand (Leavy & Hourigan, 2022). However, to understand the quality of the problems posed, it is not enough to examine them in terms of these "general-mathematical elements" ("general") (Ulusoy, 2025). Some problems may be solvable and non-routine, but they may not relate to the mathematical concept in the problem-posing task. We therefore need to examine the problems on the basis of "content-specific elements" ("content") (Ulusoy, 2025). This study's objective is to analyse the characteristics of problems posed by PPTs, considering in the light of both their pedagogical knowledge regarding "good" problems ("general") and their ability to identify the mathematical purpose of those problems ("content").

We administered a multiple-choice questionnaire (MCQ) to 108 PPTs to investigate PPTs' knowledge in these two respects. In addition, 24 PPTs participated in semi-directed interviews. They were asked to pose what they considered to be a "good" problem and to justify their choice. From the "general" point of view, our results showed a more nuanced reality than what is usually described in the research literature. In the MCQ, a large majority of PPTs stated that a "good" problem must be complex, develop reflection in students and involve multiple solving strategies. However, when they posed a problem, half of them posed NRP, but all their problems were solvable. Regarding the "content", many PPTs failed in the MCQ to recognise the mathematical purpose of a Grade 4 problem and in the interviews, half of them were unable to identify the mathematical objective of the problem they themselves posed.

References

Leavy, A., & Hourigan, M. (2022). The framework for posing elementary mathematics problems (F-PosE): Supporting teachers to evaluate and select problems for use in elementary mathematics. *Educational Studies in Mathematics, 111*(1), 147-176.

Ulusoy, F. (2025). Using general-mathematical and content-specific elements to assess the quality of prospective mathematics teachers' problems in a problem-posing context. *Journal of Mathematics Teacher Education*, 1-33.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 647. PME.

HOW COMPUTATIONAL THINKING MAY SUPPORT MATHEMATICS LEARNING IN PRIMARY EDUCATION: ON SUPPORTIVE RELATIONSHIPS IN A LITERATURE REVIEW

Till von Monkiewitsch[1], Lukas Bayer[2], Max Engelhardt[3], Johanna Hedwig Kerres[4], Carina Büscher[1]

[1]University of Cologne, [2]University of Koblenz
[3]Dresden University of Technology, [4]Bielefeld University

Computational thinking (CT) is increasingly discussed in primary mathematics education, where prior reviews highlight its potential to support mathematical competencies and raise questions about how this support is realised (Nordby et al., 2022). To systematically analyse how relationships between CT and mathematics are described in the literature, a systematic literature review following PRISMA guidelines (Page et al., 2021) identified *N = 82* documents in Scopus, of which *n = 32* studies were included. The review reconstructed multiple types of relationships between CT and mathematics. Focusing on supportive relationships, this contribution examines how CT facets are described to support mathematics learning in primary education. Of the 32 included studies, 22 describe CT as supporting mathematics; of these, 20 include supportive effects without reference to specific facets, and in 10 of these no specific facets are identified. Where specified, abstraction and decomposition are among the few recurring facets, illustrating how support is described. Therein, decomposition is described to structure problems into subproblems, while abstraction supports the construction of mathematical representations (e.g., patterns, variables). These descriptions are often situated in programming-based tasks, suggesting that support may emerge through programming as a mediating activity providing context and purpose for applying abstraction and decomposition. This raises the question whether some support attributed to CT is, at least in part, tied to programming as the activity enacted, which itself is described as a supportive CT facet in certain studies. The small number of facet-specific studies and conceptual heterogeneity limit comparability and obscure potential mechanisms. These findings indicate a need for clearer articulation of CT facets and further research on how such supportive relationships are realised across programming and non-programming contexts.

References

Nordby, S. K., Bjerke, A. H., & Mifsud, L. (2022). Computational Thinking in the Primary Mathematics Classroom: A Systematic Review. *Dig. Experiences in Math. Ed.*, *8*, 27–49.

Page, M. J. Moher, D., Bossuyt, P. M., Boutron, I., Hoffmann, T. C., … McKenzie, J. E (2021). PRISMA 2020 explanation and elaboration: Updated guidance and exemplars for reporting systematic reviews. *BMJ*, *372*, Article n160. https://doi.org/10.1136/bmj.n160

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 648. PME.

5-7-YEAR-OLDS DESCRIPTION OF PATTERNS IN A FUNCTIONAL THINKING TASK

Bridget Wadham, Jodie Hunter

Massey University

Functional thinking is a core strand of early algebra and has continued to gain increasing attention within mathematics education research. It well recognised that many young students can express variational patterns and corresponding relationships in more sophisticated ways than previously perceived. However, there is a concern that current learning trajectories may be holding the research field back from recognising the algebraic potential in young children's experiences, due to varying interpretations of what counts as algebra (Walkoe & Levin, 2020).

This oral communication focuses on data drawn from a task-based assessment gathered as part of a larger ongoing project within New Zealand. The task was based on an AAB repeating pattern, and required students to predict further positions, as well as create a table of data using the units of repeat. However, we focus on the open response question “what patterns do you notice” which was the final item in the task. In total, we examined n = 735 responses of 5- 7-year-olds. Using both deductive (Stephens et al., 2017; Warren & Miller, 2010) and inductive approaches, we found that students responses fell under three overarching pathways. First, a variational/correspondence pathway which aligned with the levels of sophistication framework (Stephens et al., 2017). Second, a unit of repeat pathway, consisting of eight unique codes. Third, a foundational pathway, consisting of six unique codes such as “context statement maker” and “number and answer noticer”.

Overall, we identified that whilst young students were able to describe patterns in ways which aligned to functional thinking or unit of repeat pathways, many also described patterns in ways which are not reflected in current algebraic trajectories. In the oral communication we will present examples and further detail on the three over-arching pathways, as well as discuss how codes in the foundational pathway might be harnessed in the development of algebraic thinking.

References

Stephens, A. C., Fonger, N., Strachota, S., Isler, I., Blanton, M., Knuth, E., & Murphy Gardiner, A. (2017). A learning progression for elementary students’ functional thinking. *Mathematical Thinking and Learning*, *19*(3), 143-166.

Walkoe, J., & Levin, M. (2020). Seeds of algebraic thinking. *For the Learning of Mathematics*, *40*(2), 27-31.

Warren, E., & Miller, J. (2010). Exploring Four Year Old Indigenous Students' Ability to Pattern. *International Research in Early Childhood Education*, *1*(2), 42-56.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 649. PME.

TEACHERS' EXPECTATIONS REGARDING THE USE OF LEARNING PROGRESS ASSESSMENT

Jasmin Walter[1], Nils Goldboom[2], Natalie Förster[2], Daniel Sommerhoff[1]

[1]IPN – Leibniz Institute for Science and Mathematics Education
[2]University of Wuppertal

There have been repeated calls for evidence-based teaching and adaptive, formative student support using digital assessment tools. Learning progress assessment (LPA) enables data-driven instruction and holds great potential, as stated by Fuchs et al. (2024), using short, (today mostly) digital, and standardized tests. Based on the provided information, teachers can track students' performance to identify learning difficulties early, respond accordingly, and monitor subsequent changes to assess the effects of the support provided (Nelson et al., 2023). Despite this potential, LPA remains underutilized in German elementary schools. To facilitate the implementation and utilization of LPA and to inform its design, it is imperative to consider elementary school teachers' expectations regarding LPA. These will be crucial for the acceptance of LPA as a tool for data-based teaching and individual support, particularly prior to first-time implementation, when expectations are not yet shaped by experience. These considerations led to the following research questions: *RQ1 What are elementary school teachers' expectations regarding the use of LPA in mathematics lessons? RQ2 Do these expectations differ based on individual and systemic factors?* To answer these questions, this study examines the expectations of 60 elementary school teachers regarding the use of LPA, as well as the influence of individual factors (e.g., assessment skills, ICT literacy) and systemic factors (e.g., technical equipment). Prior to using the intended LPA, data were collected via a questionnaire. The results of the study indicate that teachers primarily have functional expectations, particularly regarding automated, in-depth data interpretation. Expectations regarding easy, time-efficient use of LPA, as well as support and professional development, were mentioned less often. Expectations were largely independent of individual and systemic factors, yet assessment skills were significantly negatively associated with a need for support. Subsequent studies will compare teachers' expectations before and after using LPA to examine how initial expectations change through experience.

References

Fuchs, A., Radkowitsch, A., & Sommerhoff, D. (2024). Using learning progress monitoring to promote academic performance? A meta-analysis of the effectiveness. *Educational Research Review, 46*, 100648.

Nelson, G., Kiss, A. J., Codding, R. S., McKevett, N. M., Schmitt, J. F., Park, S., ... & Hwang, J. (2023). Review of curriculum-based measurement in mathematics: An update and extension of the literature. *Journal of School Psychology*, *97*, 1-42.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 650. PME.

DISPOSITIONAL STRUCTURE OF THINKING SKILLS IN PROPORTIONAL REASONING

Kai-Lin Yang[1], Ting-Ying Wang[1], Ying-Hao Cheng[2], Jian-Cheng Chen[3]
Fang-Jiun Wang[1]

[1]National Taiwan Normal University, [2]University of Taipei
[3]National Taipei University of Education

Proportional reasoning is a foundation for algebraic and advanced mathematical thinking. It requires the coordinated use of thinking skills—critical, creative, reflective, and systems thinking. Critical and creative thinking enable students to question assumptions and generate alternative strategies, whereas reflective and systems thinking support the evaluation of reasoning and the coordination of quantitative relationships and structures for effective problem solving (Wang, 2025; Yang et al., 2026). Together, these skills underpin meaningful engagement in proportional reasoning. Grounded in Bandura's Social Cognitive Theory (SCT) (Bandura, 2001), this study conceptualizes students' dispositions toward these skills through three constructs: behavioral intention, self-efficacy, and value. In SCT, engagement is shaped by students' intention to act, their confidence in their capability, and the value they attribute to the activity. Twelve items were developed by integrating the three dispositional dimensions across the four thinking skills. Two competing hypotheses were tested: H1, that the four skill constructs would better represent students' dispositions; and H2, that the three dispositional constructs would provide a superior representation. Using Partial Least Squares Structural Equation Modeling with a valid sample of 122 seventh graders, the findings supported H2. The three dispositional constructs provided a better structural representation of the 12 items, indicating that, at the dispositional level, the four thinking skills are more coherently organized under broader motivational dimensions than as distinct skill-based constructs.Theoretically, this study extends SCT to the domain of thinking skills in mathematics by clarifying the structural nature of related dispositions. Practically, it suggests that instructional efforts may be more impactful when designed to strengthen these motivational dispositions in ways that generalize across thinking skills.

References

Bandura, A. (2001). Social cognitive theory: An agentic perspective. *Annual review of psychology, 52*(1), 1-26.

Wang, T. Y. (2025). Assessing Systems Thinking Skills in Mathematics: Development and Application of an Operational Framework. *Int. J. Sci. Math. Educ.*, 1-28.

Yang, K. L., Chen, C. H., & Wu, W. R. (2026). Exploring Critical and Reflective Thinking in Mathematical Literacy through Variable-and Person-Centered Approaches. *Think. Ski. Creat.*, 102127.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 651. PME.

STUDENTS' CHALLENGES IN GRAPHING LINEAR AND QUADRATIC FUNCTIONS

Li Wang[1,2]

[1]University of Macau, Macau; [2]ChangJi College

From covariational reasoning perspective, graphing helps students to reason about covariation by treating points as paired values of two quantities and the graph as a trace of how changes in one quantity correspond to changes in the other. Covariational reasoning was defined to be *the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other* (Carlson et al., 2002, p.354). In graphing, students need to sustain covariational reasoning throughout the process, so breakdowns are likely to occur at transitions between adjacent graphing steps. This study investigates into the discontinuity between adjacent graphing steps and examines the grade differences.

Data was collected from 335 students in Grades 7-10 from one school in Xinjiang, China. Eight booklets were developed from three guided graphing tasks related to linear and quadratic functions, each having two versions (Cronbach's α ranged from 0.86 to 0.91), involving. Each student needed to complete one booklet that was randomly assigned. Students' responses were coded using a five-step graphing framework with a three-level scoring 0-1-2: (1) making a table, (2) constructing axes with appropriate labels, (3) labeling scales, (4) plotting points, and (5) constructing the graph. For each adjacent transition (step k to step $k+1$), conditional success rates were calculated to quantify stepwise attrition.

The results revealed that the lowest conditional success rate occurred at the transition from step 1 to step 2, which is around 0.49-0.53 across all participants. This pattern was replicated at each grade level. This indicates that the greatest discontinuity happened at the representational shift from a correspondence display to the construction of a coordinate system that couples two quantities. Meanwhile, the success rates became higher and higher as the grade levels grew.

The results of the current study suggested that the transition from making a table to constructing the coordinate system is not only the starting step of graphing, but also the most possible difficult transition. Suggestions for instructional improvement include the use of dynamic software.

References

Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education*, *33*(5), 352-378. https://doi.org/10.2307/4149958

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 652. PME.

WHAT KNOWLEDGE ARE PROSPECTIVE TEACHERS EXPECTED TO BRING FROM UNIVERSITY? A SURVEY STUDY

Birke-Johanna Weber[1], Daniel Sommerhoff[2], Anika Dreher[3], Aiso Heinze[2]

[1]Kiel University, [2]Leibniz Institute for Science and Mathematics Education
[3]Freiburg University of Education

In many countries, secondary mathematics teacher education includes both academic and practical education. Subject-specific education during the academic phase is often organized in university courses for academic mathematical content knowledge (CK) and pedagogical content knowledge (PCK) with a focus on the former. There is a consensus in recent research in mathematics education that mathematics teachers at secondary schools do not only need CK and PCK but also a profession-specific knowledge about connections between school mathematics and academic mathematics (e.g., Dreher et al., 2018; Wasserman et al., 2023). We refer to this knowledge as *school-related content knowledge* (SRCK). There is an ongoing discussion about the extent to which and how SRCK should be integrated into curricula for teacher education. The aim of our study is to examine the views of teacher educators from the practical phase on this topic, as they – alongside teacher educators from the academic phase – are deeply involved in teacher education and therefore can be seen as experts. We present first results of an ongoing online survey in 14 of 16 federal states in Germany with a sample of 124 teacher educators. Results reveal that a majority of our sample (81.4%) viewed SRCK as requiring targeted learning opportunities beyond those for CK. While 7% of these participants identified the practical phase as the ideal setting for such learning opportunities, 93% favored the academic phase. Subsequently, most participants (74.8%) suggested adjustments in university curricula, specifically calling for a greater emphasis on SRCK at the expense of CK. In sum, our findings support the theoretical argument that (1) SRCK represents a relevant aspect of teachers' professional knowledge, the acquisition of which requires specific learning opportunities that go beyond mere CK training, and (2) that SRCK should be addressed in the academic phase of teacher education. The surveyed views of teacher educators complement existing initiatives that aim to strengthen academic mathematics courses and support SRCK. A next step should be to gain further insight into effective learning opportunities for SRCK and their ideal implementation in teacher education.

References

Dreher, A., Lindmeier, A., Heinze, A. & Niemand, C. (2018). What kind of content knowledge do secondary mathematics teachers need? *Journal für Mathematik-Didaktik, 39*, 319–341.

Wasserman, N. H., Buchbinder, O., & Buchholtz, N. (2023). Making university mathematics matter for secondary teacher preparation. *ZDM – Mathematics Education, 55*, 719–736.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 653. PME.

UNPACKING THE RELATIONSHIP BETWEEN MATHEMATICS SUBJECT MATTER KNOWLEDGE AND TEACHER NOTICING

Yicheng Wei[1], Qiaoping Zhang[2]

[1]The Education University of Hong Kong; [2]The University of Hong Kong

Teacher noticing (TN), which comprises attending, interpreting and responding, is a core competence for understanding students' mathematically thinking and responsive teaching (Jacobs et al., 2010). While scholars hypothesize a shared theoretical space where mathematical knowledge for teaching intersects with professional noticing (Thomas et al., 2017), empirical evidence detailing this exact relationship remains scarce. Specifically, it is unclear how sub-dimensions of subject matter knowledge (SMK) (Ball et al., 2008), namely common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK), predict the specific components of TN. This study investigates the extent to which CCK, SCK, and HCK predict pre-service teachers' abilities to attend, interpret, and respond to real classroom. The study involved 160 mathematics pre-service teachers (PSTs) from China who completed two validated instruments: a written SMK test and a video-based TN assessment. Given the ordinal nature of the dependent variable, three ordinal logistic regression models were constructed to determine the predictive power of SMK dimensions on each TN skill. Results indicate that HCK emerged as the sole positive predictor for attending (β = 0.227, p < .001), suggesting an understanding of the broader mathematical landscape enables teachers to identify critical classroom events. For interpreting, both CCK (β = 0.297, p = .009) and HCK (β = 0.165, p = .013) showed significant positive effects. Finally, only CCK significantly predicted responding (β = 0.361, p = .003). The dominance of CCK in the interpreting and responding phases implies that PSTs rely heavily on basic mathematical correctness rather than leveraging specialized knowledge for highly adaptive instruction. Notably, SCK did not predict any noticing skill, suggesting that for novice teachers, this specialized knowledge may remain inert and not yet functionally connected to practice. These findings empirically validate the intersection of knowledge and noticing, clarifying how specific SMK domains are differentially activated during the different phases of the noticing process.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes It special? *Journal of Teacher Education*, *59*(5), 389–407.

Jacobs, V. R., Lamb, L. L. & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, *2*(41), 169–202.

Thomas J., Jong C., Fisher M. H., Schack E. O. (2017). Noticing and knowledge. Exploring theoretical connections between professional noticing and mathematical knowledge for teaching. *The Mathematics Educator*, 26(2), 3–25.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 654. PME.

MATHEMATICS STUDENTS' CHATBOT ADOPTION

Robert Weinhandl

Center for Open Digital Education, Johannes Kepler University Linz

Large language model (LLM) chatbots can explain concepts and provide step-by-step solutions in mathematics, yet evidence on learning gains is mixed and adoption drivers remain unclear (Cheng et al., 2024). We examined which learning-related factors predict mathematics students' chatbot adoption. We conducted a cross-sectional online survey of upper-secondary mathematics students in Austria (grades 9–13; ages 14–19; N=2,027). Chatbot adoption served as the dependent variable. Predictors included educational-psychology constructs, i.e. student well-being and cognitive engagement, and student characteristics: age, parental education, mathematics performance, and interest in mathematics (school-track proxy). Scale reliabilities were satisfactory (e.g., cognitive engagement, ω=.878; chatbot adoption, ω=.946). We estimated a structural equation model in R/*lavaan* (Rosseel, 2012) using robust maximum likelihood. Model fit was acceptable: CFI = .944, TLI = .936; RMSEA = .043, SRMR = .048. Three direct effects were significant. Lower mathematics performance predicted higher chatbot adoption (β = .132, p = .002). Higher cognitive engagement predicted lower adoption (β = −.116, p = .014). Older age predicted slightly lower adoption (β = −.069, p = .029). Effects of well-being, parental education, and mathematics interest were not significant. Adoption appears driven primarily by performance- and process-related factors rather than affective well-being, socio-economic background, or broad subject interest. Consistent with prior indications that chatbots do not uniformly raise performance (Asare et al., 2023; Cheng et al., 2024), lower-performing students use them more, and students with higher cognitive engagement use them less. Practical implications are twofold: i) Position chatbots as adaptive scaffolds for weaker learners, emphasizing step decomposition, worked-example support, metacognitive prompts, and planned fading of assistance. ii) Implement age-sensitive strategies: more motivational, dialogic support in lower grades and critical evaluation/quality-control practices in higher grades. Limitations include the cross-sectional design, self-reports, and operationalization of *adoption* as frequency rather than quality of interaction.

References

Asare, B., Arthur, Y. D., & Boateng, F. O. (2023). Exploring the impact of ChatGPT on mathematics performance: The influential role of student interest. *Education Science and Management, 1*(3), 158-168. https://doi.org/10.56578/esm010304

Cheng, L., Croteau, E., Baral, S., Heffernan, C., & Heffernan, N. (2024). Facilitating student learning with a chatbot in an online math learning platform. *Journal of Educational Computing Research, 62*(4), 687-717. https://doi.org/10.1177/07356331241226592

Rosseel, Y. (2012). lavaan: An R package for structural equation modeling. *Journal of Statistical Software, 48*, 1-36. https://doi.org/10.18637/jss.v048.i02

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 655. PME.

SENSING MATHEMATICS BEYOND THE CLASSROOM: STUDENTS USING PHOTOGRAPHS TO EXPLORE MATHEMATICS IN EVERYDAY CONTEXT

Sandra Friederike Wieser, Daniela Steflitsch

University of Klagenfurt

Mathematics education research emphasizes that mathematics plays a significant role in societal, political, and environmental contexts and cannot be understood as an isolated school discipline (e.g., Skovsmose, 2023), yet students often struggle to identify such connections in practice. This difficulty is linked to their perception of mathematics, which is shaped by school, peers, and social experiences (De Corte et al., 2002) and is often reduced to procedural calculations and formal algorithms (Reiss & Hammer, 2021). This study employs an autophotography approach to explore what (societal) topics students focus on when photographing aspects of their environment that interest them, and how they describe or apply mathematical concepts in these contexts. A total of 140 students from grades 5 to 12 in Austria submitted photographs accompanied by written explanations, resulting in 303 analyzable contributions.

The findings indicate that only about one-third (110) of all submissions explicitly addressed societal topics. Most of them focused on physics (69), some on biology-related topics (including human body, plants, environment, and sustainability; 19), and finance (8). Students either focused on the photographed objects and mathematized them or used them as a starting point for broader themes and questions, sometimes combining both approaches. The ways in which students used mathematics in their questions varied, with nearly half relying on simple, previously learned procedures such as direct quantification and single-step calculations. The findings align with research showing that students view mathematics as a primary goal and focus on procedural calculation. Even when addressing societal contexts, many mathematized objects and focused on basic procedures. As only one third addressed societal issues, this raises questions about their understanding of "societal" and the limits of the autophotographic approach – two issues we would like to discuss further.

References

De Corte, E., Op't Eynde, P., & Verschaffel, L. (2002). "Knowing What to Believe": The Relevance of Students' Mathematical Beliefs for Mathematics Education. In B. K. Hofer & P. R. Pintrich (Eds.), *Personal Epistemology* (pp. 297–320). Routledge.

Reiss, K., & Hammer, C. (2021). *Grundlagen der Mathematikdidaktik* [*Fundamentals of Mathematics Didactics*]. Springer. https://doi.org/10.1007/978-3-030-65429-0

Skovsmose, O. (2023). *Critical Mathematics Education*. Springer. https://doi.org/10.1007/978-3-031-26242-5

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 656. PME.

NAVIGATING BOUNDARIES AND INCLUSION: A NARRATIVE INQUIRY INTO KOREAN MATHEMATICS-RELATED MAJORS' CAREER TRAJECTORIES

Kyungwon Lee, Min Sik Woo, Oh Nam Kwon

Seoul National University

This study explores the career trajectories of students in mathematics-related majors through narrative inquiry. In Korea, demographic shifts, advances in emerging technologies, and changes in educational policy have reshaped recruitment across education and industry, contributing to increased career uncertainty among these students (e.g., Ministry of Education, 2023). Across national contexts, students in mathematics-related majors increasingly find themselves navigating hybrid professional identities spanning education, industry, and research. This study examines how students narratively construct and interpret their career trajectories. Six undergraduate students majoring in mathematics or mathematics education participated in semi-structured interviews focusing on motivations for choosing their major, experiences in mathematics coursework, and post-graduation plans. The data were analyzed through narrative analysis, informed by Polkinghorne (1995), to understand how career trajectories and their meanings are constructed over time. The narrative analysis yielded two major themes. First, students perceived boundaries across multiple career domains, including mathematics or mathematics education as disciplinary pathways, pursuing graduate studies, entering the teaching profession, and seeking employment in industry, and they experienced tensions while navigating these boundaries. Students moved between certainty and uncertainty regarding particular career paths, which led to internal conflicts. Second, experiences in mathematics coursework were closely associated with students' boundary perceptions. Students selected courses with particular career directions in mind, while in other cases, coursework experiences led them to reconsider and reconstruct their career trajectories. Thus, course taking functioned not merely as an academic requirement but as a site for developing a sense of inclusion and direction within specific career domains. These findings highlight that mathematics coursework plays a mediating role in shaping career trajectories and suggest the need for curriculum design that connects course structures with opportunities for career exploration within mathematics-related higher education contexts.

References

Ministry of Education. (2023). *Mid-to-long-term (2024–2027) teacher supply and demand plan reflecting future educational needs* [Press release].

Polkinghorne, D. E. (1995). Narrative configuration in qualitative analysis. *International Journal of Qualitative Studies in Education, 8*(1), 5–23.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 657. PME.

EARLY CHILDHOOD TEACHERS' BELIEFS ABOUT THE NATURE OF MATHEMATICS AND TEACHING PRACTICE

Bethany Woollacott, Balbir Kaur, Gabriel Lee, Catherine Gripton

University of Nottingham

Beliefs are key determinants in teachers' mathematics practices. This relationship is well-established, with teacher-centred practices commonly associated with more traditional, transmission-oriented beliefs about mathematics and student-centred practice typically associated with more connectionist or discovery-based understandings (e.g., Swan, 2006). In early childhood education, teachers reportedly prefer student-centred practice but evidence of benefit is still contested (Lerkkanen et al., 2012; Lindner & Simpson, 2018). This raises an important question: to what extent are these preferred pedagogic approaches reflected in actual practice, and how do they relate to early childhood teachers' disciplinary beliefs about mathematics? Whilst previous research has explored early childhood educators' beliefs about the importance of teaching mathematics, little research has explored the relationship between their practice and beliefs about the nature of mathematics. Therefore, we surveyed 257 Reception teachers (4-5 years) in England about their mathematical teaching practices and beliefs. This age group is comparatively under-researched yet particularly interesting in England because it is the first year of statutory schooling. We found that Reception teachers reported adopting student-centred teaching approaches more frequently than teacher-centred approaches. This strongly aligned with their beliefs: the vast majority agreed with statements reflecting discovery-based or connectionist conceptions of mathematics, with more disagreement across traditionalist statements. Teachers working in areas with the highest and lowest levels of socio-economic disadvantage tended to espouse more traditionalist practices, and choice of schemes/resources were also associated with variation in practice. Together, these findings raise questions about how school context and perceptions of learners' needs may shape early childhood teachers' practice and the beliefs that underpin them.

References

Lerkkanen, M. K., Kiuru, N., Pakarinen, E., Viljaranta, J., Poikkeus, A. M., Rasku-Puttonen, H., ... & Nurmi, J. E. (2012). The role of teaching practices in the development of children's interest in reading and mathematics in kindergarten. *Contemporary educational psychology*, *37*(4), 266-279.

Lindner, S. M., & Simpson, A. (2018). Towards an understanding of early childhood mathematics education: A systematic review of the literature focusing on practicing and prospective teachers. *Contemporary Issues in Early Childhood*, *19*(3), 274-296.

Swan, M. (2006). Designing and using research instruments to describe the beliefs and practices of mathematics teachers. *Research in Education*, *75*(1), 58-70.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 658. PME.

PROBLEM-POSING PATTERNS ACROSS DATA-BASED TASKS WITHIN A 2×2 FRAMEWORK

Lan-Ting Wu, Feng-Jui Hsieh

National Taiwan Normal University

With the growing prominence of authentic data (OECD, 2023), incorporating real-world datasets into problem-posing has become increasingly important. Yet prior research has emphasized product quality in simplified contexts (Cai et al., 2023), leaving students' framing processes underexplored. This study examines students' problem-posing patterns across two data-based tasks adapted from a released PISA item, focusing on their framing orientations. Participants were 149 Taiwanese 8th graders (randomly selected from four schools). Two open-ended tasks based on the same dataset context were administered: Task 1 (problem posing of questions) required students to construct mathematical questions from a given dataset, and Task 2 (problem posing of justification) required them to generate evidence and explanations for a trend from the same dataset. (score 0-3; inter-rater Kappas > .89). Students were classified as low (0-1) or high (2-3) in each task, forming a 2×2 framework (LL, LH, HL, HH).

The distribution across the 2×2 framework was as follows: LL (38.3%), LH (28.2%), HL (24.2%), and HH (9.4%) revealed that over 52% of students showed inconsistent originality across tasks, supporting its task-dependent nature. Qualitative analysis identified two distinct problem posing patterns: (1) Structurally Oriented Tendency Reasoning (SO-TR), characterized by structural re-representation and systematic reorganization of numerical relations. (2) Trend-Oriented Interpretive Reasoning (TO-IR), characterized by reclassification, holistic interpretation, and trend-oriented reasoning. Notably, even among the high-performing HH group, students demonstrated diverse orientations, with some favoring SO-TR's structural depth while others excelled in TO-IR's contextual inference. Taken together, these findings highlight the importance of examining originality across tasks rather than treating it as a fixed individual trait. Detailed distributions and illustrative examples will be presented during the conference session.

Acknowledgements

The authors gratefully acknowledge the support of the National Science and Technology Council (NSTC), Taiwan (Grant No. MOST 110-2511-H-003-007-MY3).

References

Cai, J., Ran, H., Hwang, S., Ma, Y., Han, J., & Muirhead, F. (2023). Impact of prompts on students' mathematical problem posing. *Journal of Mathematical Behavior, 72*, 101087.

OECD (2023). *PISA 2022 Results (Volume I): The State of Learning and Equity in Education.* OECD Publishing.

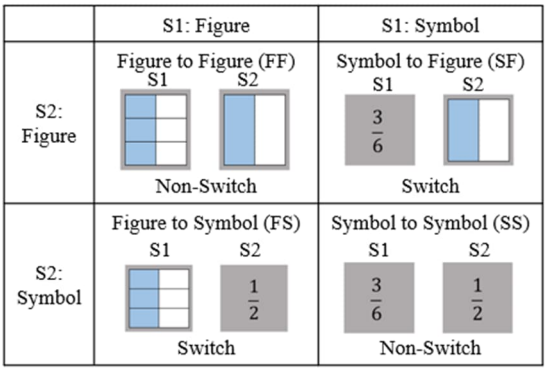
2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 659. PME.

AN ERP STUDY ON COGNITIVE FLEXIBILITY AND UPDATING IN FRACTION EQUIVALENCE ACROSS REPRESENTATIONS

Chen-Yu Yao, Hui-Yu Hsu, Tsu-Jen Ding, Zai-Fu Yao

[1]National Tsing Hua University

Understanding fraction equivalence is fundamental to mathematical learning, requiring learners to flexibly coordinate and update information across different representation systems. This study investigates the behavioral and neural dynamics of fraction equivalence judgments to disentangle the costs of rule switching from representation-based updating. Grounded in Dual Coding Theory (Paivio, 1990), we examine how processing within a single representational system (symbolic-symbolic or figural-figural) versus transitioning across systems (symbolic-figural or figural-symbolic) differentially shapes cognitive load.



Thirty right-handed adults performed a fraction comparison task while EEG data were recorded. The task required participants to judge whether two sequentially presented fractions were equivalent using two formats: symbolic (Arabic numerals) and figural (area models). This generated four transformation conditions: Figure-to-Figure (FF), Symbol-to-Symbol (SS), Figure-to-Symbol (FS), and Symbol-to-Figure (SF). Behavioral performance (accuracy and reaction time) and Event-Related Potentials (N2 and P3 components) were analyzed to evaluate cognitive flexibility and working memory updating demands. Contrary to classical task-switching theories, behavioral results revealed better performance in switch conditions (FS, SF) compared to non-switch conditions (FF, SS). Crucially, processing equivalence within the figural system (FF) imposed the highest cognitive load, resulting in lower accuracy and longer reaction times. Conversely, the symbolic system (SS) was faster and more accurate. Neural data corroborated that SS elicited larger N2 and P3 amplitudes—reflecting efficient cognitive control—poorer FF performance suggests figural representations impose a high cognitive load that may consume a lot of working memory resources. These findings challenge the assumption that visual models universally simplify learning. Cognitive load in fraction equivalence is driven by representational updating rather than rule switching. Evaluating equivalence in area models (FF) requires simultaneously maintaining visual features and actively extracting underlying relational structures. Therefore, identifying a figural fraction and performing equivalence operations upon it are cognitively distinct.

References

Paivio, A. (1990). *Mental representations: A dual coding approach*. Oxford university press.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 660. PME.

IN-SERVICE TEACHERS' LEVELS AND BELIEFS ABOUT STRATEGIC FLEXIBILITY IN NON-ROUTINE PROBLEM SOLVING

Yeliz Yazgan[1], Jon R. Star[2]

[1]Bursa Uludağ University, [2]Harvard Graduate School of Education

This study aims to explore in-service teachers' strategic flexibility level and their underlying beliefs in non-routine problem solving. Strategic flexibility involves generating and selecting appropriate strategies depending on context (Star & Newton, 2009). Non-routine problems provide a key setting for observing this competence (Schoenfeld, 1985). In this context, a qualitative case study design was employed with nine teachers from different school levels (Yin, 2018). Data were collected through semi-structured interviews consisting of two parts. In the first part, teachers solved five non-routine problems and were asked to explain their thinking aloud and produce alternative strategies. In the second part, open-ended questions explored teachers' beliefs about strategic flexibility, including its meaning, classroom use, and instructional conditions. Written solutions and audio recordings were collected for analysis. Strategic flexibility was assessed using a rubric based on multiple strategies and appropriate strategy use. Beliefs were analyzed through content analysis by identifying themes across interview responses.

Results showed most teachers demonstrated high flexibility despite limited prior familiarity. They were strongest in selecting appropriate strategies, while generating multiple strategies within a problem was less common. Teachers emphasized the importance of flexibility for learning, teaching, and real life, but noted students' limited flexibility. Confidence and persistence were identified as important factors. Supportive conditions included safe classroom environments, error tolerance, discussion, and open-ended tasks. However, time and curriculum pressures were key constraints. Teachers consistently linked non-routine problems with flexibility development. These findings highlight the need for professional development focused on evaluating and comparing strategies.

Acknowledgement

Supported by The Scientific and Technological Research Council of Türkiye (TÜBİTAK)

References

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Academic Press.

Star, J. R., & Newton, K. J. (2009). Strategy flexibility. *ZDM, 41*(5), 557–567.

Yin, R. K. (2018). *Case study research and applications* (6th ed.). Sage.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 661. PME.

EXPLORING TRANSITION DYNAMICS BETWEEN ALGORITHMIC THINKING COMPONENTS IN FLOWGORITHM-SUPPORTED MATHEMATICAL TASKS

Merve Yazoğlu Yılmaz, Hatice Akkoç

Marmara University

Algorithmic thinking (AT) encompasses components such as *decomposition* (breaking down a problem into manageable parts), *abstraction* (distinguishing unnecessary information), *algorithm design* (structuring a problem by creating sequential steps), *pattern recognition* (identifying recurring relationships), and *debugging* (identifying and correcting errors) (Lehmann, 2024). Research on how programming tools shape the dynamic relationships among AT components in the context of mathematical tasks is quite limited. Therefore, this study aims to investigate how the components of AT are activated and interconnected as a pre-service mathematics teacher (PMT) engaged in Flowgorithm-supported mathematical tasks. The research question is: What patterns characterize the interconnections among the components of AT when a PMT was engaged in Flowgorithm-supported mathematical tasks?

We designed an embedded single-case study. A task-based interview containing four open-ended mathematical tasks was conducted with a PMT. The interview transcripts were analyzed based on Lehmann's (2024) *AT competency model* adapted to the study context. Interview data were coded based on the AT components and their subskills. The transition orders were determined, and patterns characterizing the interconnections among the components were illustrated using diagrams.

The findings revealed nonlinear, cyclical, and multidirectional transition patterns among AT components. In particular, repetitive transitions were observed between the *algorithm design* and *debugging* components. Although the PMT mostly performed debugging by returning to the *algorithm design* and *decomposition* after detecting the error, consistent bidirectional transitions were observed between the *algorithm design* and *debugging* components. Therefore, although the PMT returned to dividing the algorithm into sub-goals to correct errors, she did not return to *debugging* without first reorganizing the *algorithmic design* or moving to the *abstraction* component. Transition patterns and frequency varied significantly depending on the nature of the mathematical task. Based on the findings regarding the dynamics of AT components, implications for classroom practice regarding the implementation process of Flowgorithm-supported mathematical tasks will be presented.

References

Lehmann, T. H. (2024). Mathematical modelling as a vehicle for eliciting algorithmic thinking. *Educational Studies in Mathematics*, *115*(2), 151-176.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 662. PME.

NEGOTIATING THE LABEL: HIGH SCHOOL MATH TEACHERS' POSITIONING OF EMERGENT BILINGUALS

Jiyeong Yi, Shristi Shrestha, Betsy Araujo Grando

Iowa State University

Supporting emergent bilingual (EB) students in mathematics is critical, as they often face academic challenges rooted in language-based deficit positioning (Yi et al., 2025). This study examines how high school teachers position EBs in mathematics classrooms after participating in collaborative, situated professional development focused on EB learners. Research highlights the importance of engaging EBs in rigorous mathematical tasks while leveraging their cultural and linguistic assets (DiNapoli & Morales, 2021). We implemented a 5-Act Task Framework curriculum designed to support challenging mathematics in culturally and linguistically relevant contexts. Using positioning theory, which examines how roles and responsibilities are constructed through interactions and storylines, we analyze teachers' interactive positioning of EBs in relation to mathematical learning. Few studies examine how teachers' positioning of EBs develops over time; this study addresses that gap. The study took place in an urban district where 21.9% of students are EBs speaking over 100 languages. Over three years, we worked with seven mathematics teachers teaching 100% EB Algebra and Pre-Algebra classes. Data included video recordings of teaching, planning, reflection, and interviews. Three coders analyzed the data in three phases: (1) open coding of positioning acts, (2) development of a coding manual through consensus, and (3) construction of individual and collective storylines. All teaching transcripts were coded and analyzed for themes. This presentation focuses on Phase 2 findings related to teachers' interactive positioning of EBs. Findings show both deficit and asset-based positioning, with asset-based views occurring more frequently but not predominating. Monolingual veteran teachers most often used the "Learner with Difficulties" code, whereas bilingual and early-career teachers more frequently positioned students as "Mathematically Capable." Deficit and asset framings coexisted, with students sometimes positioned simultaneously as capable contributors and as struggling learners. Notably, students were more often positioned as mathematical contributors within collaborative structures. Further implications will be discussed in the presentation.

References

DiNapoli, J., & Morales, H. (2021). Translanguaging to persevere is key for Latinx bilinguals' mathematical success. *Journal of Urban Mathematics Education*, *14*(2), Article 2.

Yi, J., Son, J.-W., & Chang, H. (2025). Teacher positioning impact on students' learning opportunities in multilingual mathematics classrooms. *Journal of Mathematics Teacher Education*. https://doi.org/10.1007/s10857-025-09704-3

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 663. PME.

NOT ALL CORRECT ANSWERS ARE EQUAL: EXAMINING THE ROLE OF STRATEGY IN REPEATING PATTERN ABSTRACTION

Betül Fatma Yıldırım[1], Serkan Özel[2]

[1]Middle East Technical University, [2]Boğaziçi University

Patterns are a central concept in mathematics education and support later achievement and early algebraic thinking (e.g., Rittle-Johnson et al., 2017). Repeating patterns, a foundational pattern type in early childhood, have been widely studied; however, research has largely emphasized accuracy rather than strategy use. This study investigates whether abstraction strategy use predicts performance in Repeating Pattern Abstraction Task (RPAT) beyond the effects of age and Visuospatial Working Memory (WM) in 4- to 6-year-old children. A convergent parallel mixed method design was employed. 56 preschool children participated in one-to-one interviews. The researcher-developed RPAT (12 items; α = .88) and the Corsi span task were administered to assess visuospatial WM. Responses were scored dichotomously, and children's explanations were coded using a hybrid thematic analysis based on Lüken and Sauzet's (2021) strategy codes. A binomial generalized mixed-effects regression tested the effect of strategy use on RPAT accuracy, while a second regression examined the contributions of strategy use, controlling age and visuospatial WM to RPAT accuracy. Binomial generalized mixed-effects analyses indicated that only sequence awareness and unit identification strategies significantly predicted accuracy. Age 6, compared to age 5, was also a significant predictor, whereas no differences were found between 4- and 5-year-olds. Regression analyses further revealed a quadratic relationship between abstraction strategy use and performance. The final model, including age and visuospatial working memory, explained 76% of the variance in abstraction scores. The current study extends the existing literature about repeating patterning by highlighting the importance of children's explanations and strategy use, beyond the effects of age and visuospatial working memory. In the presentation, further results and implications will be discussed in detail.

Acknowledgements: This study was supported by TUBITAK under the Grant Number 124K343 and BIDEB 2210-A National Graduate Scholarship Program.

References

Lüken, M. M., & Sauzet, O. (2021). Patterning strategies in early childhood: a mixed methods study examining 3- to 5-year-old children's patterning competencies. *Mathematical Thinking and Learning, 23*(1), 28–48.

Rittle-Johnson, B., Fyfe, E. R., Hofer, K. G., & Farran, D. C. (2017). Early Math Trajectories: Low-Income Children's Mathematics Knowledge From Ages 4 to 11. *Child Development, 88*(5), 1727–1742.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 664. PME.

MIDDLE SCHOOL STUDENTS' USE OF GENERATIVE ARTIFICIAL INTELLIGENCE IN PROBLEM SOLVING AND PROBLEM POSING

Hongkyung Yun, Kyungwon Lee, Oh Nam Kwon

Seoul National University

This study aims to explore how middle school students use generative artificial intelligence (genAI) in problem solving and problem posing. Problem solving and problem posing provide students with opportunities to experience productive challenges in mathematics learning and to develop their mathematical thinking (Cai, 2022). As genAI has become a tool capable of generating mathematical problems, it has begun to be used within the domain of problem posing (e.g., Walkington et al., 2025). However, there remains a need for a systematic approach to integrating genAI into problem solving and problem posing processes. This study analyzes students' use of genAI in such processes to propose practical directions for its implementation. Seven Korean eighth-grade students individually participated in end-of-semester problem solving and problem posing activities. Given the timing of the study, the activities focused on counting and probability topics included in the curriculum for that grade level. Students used three genAI models (ChatGPT, Gemini, and Copilot) to generate problems and solved one problem from each model. Students then posed their own problems and solved them. Data collected included students' genAI interaction logs, written work, and verbal utterances. Adopting an exploratory approach, Students' use of genAI in problem solving and problem posing was analyzed using an open coding approach. Students engaged with and solved genAI-generated counting or probability problems that varied across models in formatting, contexts, and levels of complexity. After solving these problems, students' posed problems often reflected the situations and contexts of the genAI-generated tasks. Rather than merely modifying numbers or changing conditions, students constructed new problems by drawing on the situations and contexts that structured the original tasks. Students flexibly set conditions when posing problems and solved them using counting or probability concepts. These findings suggest that students can proactively leverage genAI to support both their problem solving and problem posing processes.

References

Cai, J. (2022). What research says about teaching mathematics through problem posing. *Éducation & Didactique*, *16*(3), 31-50.

Walkington, C., Pando, M., Lipsmeyer, L. L., Beauchamp, T., Sager, M., & Milton, S. (2025). Middle school girls using generative AI to engage in mathematical problem-posing. *Mathematical Thinking and Learning*. https://doi.org/10.1080/10986065.2025.2542724

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 665. PME.

ANTICIPATORY TEACHER NOTICING IN THE PRE-ACTIVITY PROCESS OF MATHEMATICAL MODELLING ACTIVITIES WITH GIFTED STUDENTS

Emine Nur Unveren Bilgic

Duzce University

Teacher noticing is defined as the process of paying attention to students' mathematical thinking, interpreting this thinking, and relating it to pedagogical decisions (Jacobs et al., 2010). This process has mostly been examined through classroom interactions; relatively little research has been conducted on how teachers anticipate students' possible ways of thinking prior to implementation. Teachers' prior cognitive constructions play a decisive role, particularly in the planning of open-ended tasks. This study aims to examine the anticipatory noticing structures that emerge in teachers' planning process prior to a modelling activity to be conducted with gifted students. The research was conducted using a qualitative multiple case study design. Two teachers who taught mathematics to gifted students were asked to plan a modelling task, and semi-structured interviews were conducted based on these plans. In the interviews, teachers were asked to anticipate possible solutions, cognitive risks, and intervention strategies. The data were analysed through deductive thematic analysis informed by Jacobs et al.'s (2010) framework of professional teacher noticing—anticipatory attending, anticipatory interpreting, and anticipatory deciding.

The findings indicate that teachers engage in structured mental simulation during the planning process. They anticipate the variables and representations that students may focus on (predictive attention), focusing particularly on the tendency towards rapid abstraction and early generalisation. Assessing that these tendencies may superficialise the modelling cycle (predictive interpretation), they plan interventions that will reopen the neglected stages of the process (predictive decision). The gifted context qualitatively differentiates anticipatory noticing: teachers develop a planning organisation aimed at slowing down the process, preventing early closure, and preserving epistemic depth rather than supportive intervention. These results demonstrate that anticipatory noticing serves a strategic function in modelling pedagogy by preserving epistemic integrity. The study extends teacher noticing beyond the moment of application, making visible the attention–interpretation–decision organisation in the planning process and offering a cognitive contribution to modelling instruction in the context of gifted education.

References

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169–202.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 666. PME.

IMPROVING MATHEMATICAL THINKING BY PLAYING BOARD GAMES IN MATHS LESSONS – A SECOND TAKE

Csilla Zámbó[1], Anna Muzsnay[2], Csaba Szabó[1,3], Sára Szörényi[1]

[1]Eötvös Loránd University, [2]University of Debrecen, [3]Edutus University

Playing board games stimulates the prefrontal and parietal areas essential for mathematical cognition (Newman et al., 2016). This suggests possible advantages to their educational use. While benefits in school settings for students aged 6–13 are increasingly documented (e.g., Vita-Barrull et al., 2024), research in secondary education remains sparse. To address this, our first project phase (Szörényi, Muzsnay, Szabó & Zámbó, 2026) involved 10th graders (N=170) from six schools. For nine weeks, one mathematics lesson per week was replaced by gameplay using non-digital logical or geometrical board games. We examined the effect of the intervention on students' geometrical and logical skills via pre- and post-tests. Results obtained using Anova indicated equivalent performance in logic skills, but significant improvement in geometrical skills in the experimental group (n=79) versus controls (n=91).

In this current proposal, we present the extension of the intervention to a second phase, where the original control group was switched to experimental – playing board games for 1 lesson/week for 9 weeks – and vice versa in one school (N=62 in total), culminating in a post-test measuring logic and geometry skills. The research goal is to examine the medium-term effects on the initial experimental group and within-subject gains, using the input and output test results, supplemented by a qualitative review of test responses for thinking processes. We hypothesize an improvement in the geometry skills of the newly experimental group and maintained performance of the original experimental group. The expected outcomes support integrating board games into mathematics lessons to enhance thinking skills and suggest the need for longitudinal, multi-game replications of the study for broader implementation.

References

Newman, S., Hansen, M., & Gutierrez, A. (2016). An fMRI study of the impact of block building and board games on spatial ability. *Frontiers in Psychology*, 7, 1278. https://doi.org/10.3389/fpsyg.2016.01278

Szörényi, S., Muzsnay, A., Szabó, C., & Zámbó, C. (2026). The impact of commercial board games on secondary school mathematics performance and thinking skills. *International Journal of Educational Research Open*, *10*, 100569.

Vita-Barrull, N., Estrada-Plana, V., March-Llanes, J., Sotoca-Orgaz, P., Guzmán, N., Ayesa, R., & Moya-Higueras, J. (2024). Do you play in class? Board games to promote cognitive and educational development in primary school: A cluster randomized controlled trial. *Learning and Instruction*, *93*, 101946. https://doi.org/10.1016/j.learninstruc.2024.101946

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 667. PME.

RETHINKING MATHEMATICAL KNOWLEDGE AND PRACTICE IN THE AGE OF AI: PERSPECTIVES FROM STEM FACULTY REGARDING UNDERGRADUATE MATHEMATICS

Qi Zhai, Oi-Lam Ng

Department of Curriculum and Instruction, The Chinese University of Hong Kong

The rapid integration of artificial intelligence (AI) across STEM disciplines is reshaping expectations of what it means to be mathematically prepared for undergraduate study and professional practice (Awang et al., 2025). While gaps in tertiary mathematics readiness are well-documented, how these expectations are being reconfigured in the age of AI remains underexplored. Grounded in the Anthropological Theory of the Didactic (ATD) (Chevallard & Bosch, 2020), this study conceptualizes university STEM faculty as the "noosphere"—the sphere of actors influencing the didactic transposition of mathematical knowledge. Accordingly, this study addresses two research questions: (1) What specific mathematical content do STEM academics perceive as essential for STEM undergraduates in the age of AI? and (2) What AI interaction strategies and associated concerns within mathematics learning and problem-solving do they expect from undergraduates? An open-ended qualitative survey was administered to 35 university STEM academics in Hong Kong, a context undergoing rapid AI adoption across industries and education. Data underwent deductive and inductive thematic analysis to uncover faculty's views. The results are organised into three interrelated domains: (1) valued mathematical content, (2) forms of AI use in learning and problem-solving, and (3) concerns surrounding AI integration. The results indicate an expanded and interdisciplinary conception of foundational mathematics (e.g., algebra and calculus), accompanied by an emphasis on data-driven areas (e.g., statistics, probability and mathematical modelling). Furthermore, findings revealed a complex duality in AI use—simultaneously positioned as a tutor to provide individual scaffolding and an assistant to enhance computational efficiency, yet cautioned against as a potential source of over-reliance that may undermine foundational understanding and critical thinking. These insights provide an empirical account of how the academics redefine mathematical knowledge and practice in AI-rich STEM education, ultimately extending the ATD framework to AI-integrated contexts.

References

Awang, L. A., Yusop, F. D., & Danaee, M. (2025). Current practices and future direction of artificial intelligence in mathematics education: A systematic review. *International Electronic Journal of Mathematics Education, 20*(2), em0863.

Chevallard, Y., & Bosch, M. (2020). Didactic transposition in mathematics education. In Lerman, S (Ed), *Encyclopedia of mathematics education* (2nd ed., pp. 214-218). Springer.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 668. PME.

EMERGENT BILINGUAL CHILDREN USING SEMIOTIC RESOURCES TO SUPPORT THEIR MATHEMATICAL ACTIVITY

Tianyi Zhou, John Francisco

University of Massachusetts Amherst

Research on the mathematical learning of bilingual students shows that children draw on multiple communicative resources, not only language, to support their learning (Moschkovich, 2002). For example, research on semiotic resources in mathematics shows that gestures, speech, and inscriptions help make meaning visible (Arzarello et al., 2009). However, research on semiotic resources involving bilingual learners has primarily focused on older students. More needs to be known about how young emergent bilingual children use semiotic resources to support their learning. This qualitative case study examined the following question: How do emergent bilingual kindergarteners use semiotic resources to support their mathematical activity as they work on challenging mathematical tasks? Participants included four five-year-old Chinese-English emergent bilingual children from a pre-K private Chinese immersion school in the northeastern United States. Data included videos of three 30-minute sessions of the students working on two combinatorial tasks. Analysis using a video methodology (Powell et al., 2003) and drawing on semiotic research resulted in six semiotic episodes showing that students used semiotic resources to support their mathematical activity in mainly two ways. First, the children used semiotic resources to represent mathematical ideas, making them visible and concrete to others. For example, they used gestures and manipulatives to display solutions and explain strategies. Second, the children used semiotic resources to support sense-making. For example, they used semiotic resources to interpret what counted as a valid solution, negotiate task premises, and modify tasks. The results show that semiotic resources do not only serve communicative purposes but also shape participation in mathematical activity. The children used semiotic resources to make sense of tasks and decide how to proceed. The study extends semiotic research to early childhood bilingual learners and shows that semiotic resources enhance meaningful participation in learning and should be encouraged in mathematics classrooms.

References

Arzarello, F., Paola, D., Robutti, O., & Sabena, C. (2009). Gestures as semiotic resources in the mathematics classroom. *Educational Studies in Mathematics*, *70*(2), 97–109.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning*, *4*(2–3), 189–212.

Powell, A. B., Francisco, J. M., & Maher, C. A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *The Journal of Mathematical Behavior*, *22*(4), 405-435.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 669. PME.

PREREQUISITES FOR TEACHING MATHEMATICS TO STUDENTS WITH VISUAL IMPAIRMENTS IN INCLUSIVE CLASSROOMS: A SYSTEMATIC REVIEW OF BEST PRACTICE RECOMMENDATIONS

Sophie Zufferey, Laure-Anne Koehli, Valérie Caron, Noémie Lacombe, Myriam Squillaci

Department of Special Education, University of Fribourg

Introduction: The learning process of students with visual impairment (VI) requires specific adaptations, particularly in visually demanding subjects such as mathematics (Hayes & Proulx, 2023). In inclusive settings, teachers need operational and structured guidance to effectively support these learners. However, existing recommendations remains fragmented and unevenly organised.

Aim: This study aimed to identify and systematically structure good practice recommendations for teaching students with VI using an instructional-phase learning.

Method: A systematic review was conducted across major databases covering the period 1990–2025. Sixteen guidelines meeting inclusion criteria were selected and appraised using the AGREE II instrument. Extracted recommendations were analysed and organised according to Hattie's (2017) model: preparation, opening, delivery (learning and feedback), and conclusion.

Results: A total of 192 specific recommendations were identified and showed an uneven distribution across lesson phases. Most recommendations focused on lesson preparation (e.g., adapting the environment and instructional materials), followed by the opening and delivery (learning) phases (e.g., establishing a supportive climate and using multisensory instruction). Only minimal guidance addressed feedback, and no recommendations explicitly targeted the lesson conclusion phase.

Conclusion and implication: The findings confirm the pivotal role of teachers in ensuring accessibility for students with VI and highlighted essential guidance to support instructional decision-making across all phases of a mathematics lesson.

References

Hayes, C., & Proulx, M. J. (2023). Turning a blind eye? Removing barriers to science and mathematics education for students with visual impairments. *British Journal of Visual Impairment, 42*(2), 544–556. https://doi.org/10.1177/02646196221149561

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 670. PME.

SENSING PROPORTION: AN EMBODIED LEARNING DESIGN FOR INCLUSIVE MATHEMATICS EDUCATION

Irma Risdiyanti[1], Zulkardi Zulkardi[1], Ratu Ilma Indra Putri[1]
Rully Charitas Indra Prahmana[2]

[1]Sriwijaya University, [2]Ahmad Dahlan University

In mathematics, students are often able to compare numbers but fail to understand the relationships between quantities. In the context of ratios, they tend to use additive comparisons rather than proportional relationships. This difficulty is more pronounced in students with mild learning difficulties, who process quantities separately and struggle to coordinate them simultaneously. From a cognitive perspective, this relates to limitations in working memory, while embodied cognition emphasizes the coordination of action and perception in building understanding (Abrahamson, 2014). However, how students move from comparing numbers to understanding relationships has rarely been empirically explained. This study aims to explain how students with mild learning disabilities develop proportional reasoning through embodied and contextual learning. Using a design research approach, we developed Local Instructional Theory based on RME-Embodied Learning (Gravemeijer et al., 2006; Zulkardi, Putri & Wijaya, 2020) in the context of designing tour packages. Learning activities included planning travel routes, comparing distances between destinations, selecting modes of transportation, and calculating and comparing tour package costs based on the number of participants. Students were also asked to interpret information from maps, price tables, and itineraries to make efficient decisions. The results showed a consistent reasoning trajectory, from comparing numbers to coordinating quantities, and then establishing multiplicative relationships through representations such as ribbon diagrams and double number lines. These findings confirm that embodied activities are effective when they support relational coordination rather than simply object manipulation.

References

Abrahamson, D. (2014). Embodied design in mathematics learning: A theoretical framework and empirical findings. *Journal of the Learning Sciences, 23*(2), 1–47.

Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 17–51). Routledge.

Zulkardi, Z., Putri, R. I. I., & Wijaya, A. (2020). Two Decades of Realistic Mathematics Education in Indonesia BT – International Reflections on the Netherlands Didactics of Mathematics: Visions on and Experiences with Realistic Mathematics Education. In M. van den Heuvel-Panhuizen (Ed.), International Reflections on the Netherlands Didactics of Mathematics. (pp. 325–340). Springer International Publishing.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 671. PME.

SENSING PROBABILITY: REFLECTIONS OF SIXTH GRADE STUDENTS' EXPERIENCES

Ayşe Tuğba Öner, Zeynep Çiğdem Özcan

İstanbul Medeniyet University

Probability is one of the conceptually most challenging areas of school mathematics for students. Students often struggle to establish the relationship between theoretical and experimental probability and to understand why repeated trials can produce different results each time. These difficulties largely stem from the abstract and conceptually demanding nature of probabilistic reasoning. The embodied learning approach proposes that abstract mathematical concepts can be made meaningful through bodily experience. Embodied learning in mathematics education emphasizes representation through bodily actions (e.g., gesture and movement), which are inherently connected to mathematical concepts and processes. In the context of probability, practices such as taking physical positions and using perception-based designs (Abrahamson, 2014) may support students in constructing abstract concepts through more concrete experiences. This study was conducted to reflect how 6th graders experienced the concept of experimental probability through an embodied learning approach. In this classroom-based study, 24 students participated in three activities. In the first activity, students determined the likelihood of their own verbally constructed statements by positioning themselves on a probability spectrum drawn on the classroom floor and explained why they chose that particular position. In the second activity, students were individually asked to conduct a coin-toss experiment 20 times, represent the outcomes using two different colors, and visualize their results by coloring boxes on a sheet of paper. They were then asked to represent the relative frequencies of heads and tails using their bodies. Finally, the experimental probabilities of heads and tails were discussed based on the total of 480 trials conducted. In the third activity, a spinner divided into four equal sections in different colors was drawn on the classroom floor. Before each spin of the spinner, students predicted which section would occur and, according to predetermined bodily codes based on whether their prediction was correct or not, they observed the distribution of the class. This process created an environment for conceptualizing experimental probability. Observational evidence from the classroom suggested that these activities designed through embodied learning may have supported students' emerging conceptualization of experimental probability.

References

Abrahamson, D. (2014). Building educational activities for understanding: An elaboration on the embodied-design framework and its epistemic grounds. *International Journal of Child-Computer Interaction, 2*(1), 1–16. https://doi.org/10.1016/j.ijcci.2014.07.002

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 672. PME.

VALIDITY AND RELIABILITY STUDY OF COGNITIVE LOAD SCALE TEST FORM: TURKISH VERSION

Zeynep Çiğdem Özcan, Ayşe Tuğba Öner

İstanbul Medeniyet University

Various tools have been developed to assess students' cognitive load (CL) during instruction. One of the most widely used scales is Paas's (1992) single-item scale, which measures the mental effort during learning. In recent years, instead of this single-factor structure, multifactorial structures focusing on the dimensions of CL have been developed and CL has been measured accordingly (Eysink et al., 2009). Furthermore, in addition to measuring students' CL related to the effectiveness of the instruction, it is also important to assess their CL regarding a specific mathematics topic both before and after the implementation. From this point of view, we aimed to develop a Turkish CL scale test form (CLST) that originated from Leppink et al. (2013), which was designed to measure instructional effectiveness and assess three types of CL, whereas CLST aims to assess two types of CL related to a specific mathematics topic, namely algebra. The data were obtained from 7th grade students through a convenient sampling technique. The CLST was presented to 7th graders after an algebra test. Factor analysis was used to assess validity, and reliability was evaluated using Cronbach's alpha coefficients.

The CLST was conducted with the data obtained from 223 seventh-graders to perform EFA. According to EFA, the CLST consisted of two factors: intrinsic load (IL) and extraneous load (EL), which together explained 63.97% of the total variance. CFA was conducted with the data obtained from 354 seventh-graders. The first-order CFA provided a good fit to the data, χ^2 (8) = 36.67, p<.001, RMSEA= .07, NFI=.97, CFI=.98. The reliability statistics for both IL and EL showed acceptable levels of internal consistency that were .85 and .81, respectively.

References

Eysink, T. H. S., de Jong, T., Berthold, K., Kolloffel, B., Opfermann, M., & Wouters, P. (2009). Learner performance in multimedia learning arrangements: An analysis across instructional approaches. *American Educational Research Journal*, *46*(4), 1107–1149. https://doi.org/10.3102/0002831209340235

Leppink, J., Paas, F., Van der Vleuten, C. P. M., Van Gog, T., & Van Merriënboer, J. J. G. (2013). Development of an instrument for measuring different types of cognitive load. *Behavior Research Methods*, *45*(4), 1058–1072.

Paas, F. G. W. C. (1992). Training strategies for attaining transfer of problem-solving skill in statistics: A cognitive-load approach. *Journal of Educational Psychology*, *84*(4), 429–434. https://doi.org/10.1037/0022-0663.84.4.429

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 673. PME.

ABSTRACTING FROM COMPUTATION: ASSESSING PRE-SERVICE MATHEMATICS TEACHERS' ALGEBRAIC THINKING

Begüm Özmusul[1,2], Ali Bozkurt[1]

[1]Gaziantep University, [2]Middle East Technical University

Algebraic thinking extends beyond computing answers to reasoning about structures. For pre-service mathematics teachers (PSMTs), formatively assessing this is challenging. While the Algebraic Habits of Mind framework (Driscoll, 1999) conceptualizes this reasoning, its formative application remains limited. Magiera et al (2017) state that PSMTs need tools to decode reasoning hidden behind correct computations. Addressing this, we operationalized Abstracting from Computation into a diagnostic rubric. The research question is: How can this rubric reveal qualitatively distinct profiles of PSMTs' algebraic thinking masked by procedural correctness? This qualitative study involved 20 third-year PSMTs in an Algebra Teaching course. Data emerged from the "Reversals task," requiring participants to reverse a three-digit number, subtract the smaller, and justify the pattern. This task deliberately distinguishes between empirical trial-and-error and structural abstraction. Written responses were analysed using a three-level rubric (proficient, emerging, no evidence) evaluating sub-habits like generalizing and symbolic justification. Two researchers independently coded all data, resolving discrepancies through discussion to ensure interrater reliability. To illustrate the rubric's diagnostic power within space limits, we contrast two representative cases highlighting the abstraction spectrum among the 20 participants. Both achieved the correct invariant outcome (1089), yet their reasoning differed entirely. PSMT#5 showed no evidence of abstraction; their reasoning was empirical, relying solely on numerical trials (e.g., 541-145=396) to "prove" the pattern. Conversely, PSMT#12 exhibited emerging abstraction. They successfully used symbolic representation (100a+10b+c) to find the structure 99(a-c). However, lacking deductive justification, they reverted to plugging in numbers to verify their own formula. This research highlights that correct answers do not guarantee algebraic thinking. By pinpointing transitional gaps—like PSMT#12's shift from symbolic manipulation to deductive proof—the rubric serves as a crucial formative tool. It offers teacher educators a principled method to move PSMTs from arithmetic verification to structural reasoning.

References

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers grades 6-10*. Heinemann.

Magiera, M. T., Moyer, J. C., & van den Kieboom, L. A. (2017). K-8 pre-service teachers' algebraic thinking: Exploring the habit of mind building rules to represent functions. *Mathematics Teacher Education and Development 19*(2), 25-50.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 674. PME.

THE EMERGENCE OF COMPUTATIONAL THINKING IN PRE-SERVICE TEACHERS' GEOGEBRA-BASED ACTIVITY DESIGN

Özbuğ İlayda Öztürk, Nilüfer Köse

Department of Mathematics Education, Anadolu University

Computational Thinking (CT) enables individuals to tackle various challenges systematically (Wing, 2011) through decomposition, pattern recognition, abstraction, algorithm design, debugging, and evaluation. Since the intellectual processes underlying CT closely parallel mathematical reasoning, supporting pre-service teachers (PSTs) in integrating these skills is crucial for effectively designing technology-enhanced mathematics environments. However, existing research focuses heavily on programming and robotics-based practices, while studies using dynamic software largely target K-12 students. Consequently, how CT emerges beyond coding, particularly utilizing underutilized dynamic tools like GeoGebra's (GGB) spreadsheet view, remains underexplored. Thus, we ask: "How do GGB's dynamic tools—specifically the spreadsheet view, checkboxes, and command writing—support the emergence of CT components during PSTs' activity design processes?"

This qualitative study investigates two PSTs designing an 8th-grade linear relationships activity using GGB. Methodological triangulation was achieved by cross-checking interviews with final design products. Findings report on one representative case (Emir) for an in-depth micro-analysis. Ensuring reliability, two researchers independently conducted thematic analysis based on the CT framework, resolving disagreements through consensus.

Emir demonstrated *decomposition* by breaking the topic into modules explicitly using GGB's checkboxes. For *pattern recognition*, he utilized the spreadsheet view alongside graphical representations to dynamically model the coefficient-slope relationship. His *abstraction* practices involved restricting negative slopes to manage students' cognitive load. He engaged in *algorithm design* through command writing, applying conditional logic ("if-then") and coordinates to link variables. Encountering technical errors, he demonstrated *debugging* by developing pragmatic limitations. Finally, *evaluation* emerged as he critically reviewed the spreadsheet view's pedagogical efficiency.

In conclusion, GGB's dynamic tools directly support the simultaneous emergence of CT components. Findings suggest CT functions not merely as a technical proficiency, but as a multidimensional cognitive tool enhancing PSTs' analytical decision-making and pedagogical practices in non-programming contexts.

References

Wing, J. M. (2011). Research Notebook: Computational thinking—what and why. *The Link Magazine, 6*, 20-23.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: General Contributions*, p. 675. PME.