Quasiweekend III

University of Helsinki

June 2025

Monday

Speaker: Mario Bonk, UCLA

Title: The visual sphere of an expanding Thurston map

Abstract: A Thurston map is a branched covering map on a topological 2-sphere for which the forward orbit of each critical point under iteration is finite. Each such map gives rise to a fractal geometry on its underlying 2-sphere. It is an open problem to determine the conformal dimension of this sphere if the Thurston map is obstructed and not realized as a rational map. In my talk I will report on some recent progress.

Speaker: Susanna Heikkilä, University of Jyväskylä

Title: De Rham algebras of closed quasiregularly elliptic manifolds are Euclidean

Abstract: In this talk, we discuss a result stating that, if a closed manifold admits a non-constant quasiregular map from a Euclidean space, then the de Rham cohomology of the manifold embeds into the Euclidean exterior algebra as a subalgebra. This embedding of algebras yields a topological classification of closed simply connected orientable 4-manifolds admitting a non-constant quasiregular map from the four-dimensional Euclidean space. The talk is based on joint work with Pekka Pankka.

Speaker: Damaris Meier, University of Fribourg

Title: Energy minimizing harmonic 2-spheres in metric spaces

Abstract: In their seminal 1981 article, Sacks-Uhlenbeck famously proved the existence of non-trivial harmonic 2-spheres in every closed Riemannian manifold with non-zero second homotopy group. Their arguments heavily rely on PDE techniques. In this talk, we will explore a novel and conceptually simple metric approach to the existence of harmonic spheres. This allows us to generalize the Sacks-Uhlenbeck result to a large class of compact metric spaces. Based on joint work with Noa Vikman and Stefan Wenger.

Speaker: Panu Lahti, Chinese Academy of Sciences

Title: Functions of bounded variation and nonlocal functionals

Abstract: Starting with the pioneering work of Bourgain, Brezis, and Mironescu in 2001, there has been widespread interest in characterizing Sobolev and BV (bounded variation) functions by means of non-local functionals. In my recent work I have studied, in particular, certain functionals related to the fractional Sobolev seminorms. I will discuss results concerning the convergence of these functionals to the Sobolev or BV seminorms, the concept of Gamma-convergence (with respect to different topologies), and some open problems.

Speaker: Enrico Pasqualetto, University of Jyväskylä

Title: The Isoperimetric Problem in non-smooth context

Abstract: One of the cornerstones of Geometric Analysis in Riemannian manifolds is the remarkable connection between lower Ricci curvature bounds and the Isoperimetric Problem (namely, the study of isoperimetric sets, which are sets that minimize the perimeter under a volume constraint). The aim of this talk is to illustrate how this deep connection persists even in the non-smooth framework of metric measure spaces. In the first part of the presentation, I will discuss topological regularity results for isoperimetric sets under (essentially) minimal assumptions, which cover for example all MCP(K,N) spaces. The latter are metric measure spaces satisfying a rather weak form of lower Ricci bound, the so-called Measure Contraction Property. In the second part, I will showcase much finer results that can be obtained in the more restrictive setting of non-collapsed RCD(K,N) spaces, focusing on sharp concavity properties of the isoperimetric profile and on some of their numerous geometric and analytic consequences. This investigation in RCD spaces gave new insights into the Isoperimetric Problem in non-compact Riemannian manifolds as well.

Speaker: Josh Kline, University of Cincinnati

Title: Self-improvement of fractional Hardy-inequalities in metric measure spaces via hyperbolic fillings

Abstract: In recent years, hyperbolic fillings, as originally due to Gromov, Bonk-Kleiner, and Bourdon-Pajot, have proven to be effective tools with which to study non-local problems in a given metric measure space. In particular, the trace and extension results of Björn-Björn-Shanmugalingam (2022) allow one to transform a problem involving Besov functions on a given doubling metric measure space to a problem involving Newton-Sobolev functions on its uniformized hyperbolic filling. In this talk, we discuss an application of this technique to the study of fractional (θ, p)-Hardy inequalities. By first establishing a new weighted self-improvement result for p-Hardy inequalities and a certain class of regularizable weights, we then use the structure of the hyperbolic filling to obtain self-improvement of the (θ, p)-Hardy inequality in both smoothness parameter θ and exponent p independently. This is joint work with Sylvester Eriksson-Bique.

Tuesday

Speaker: Jeremy Tyson, University of Illinois at Urbana-Champaign

Title: Dimension interpolation and conformal dimension

Abstract: The conformal dimension of a metric space measures its optimal shape from the perspective of quasisymmetric geometry. Dimension interpolation is an emerging program of research in fractal geometry which identifies geometrically natural one-parameter dimension functions interpolating between existing concepts. Two exemplars are the Assouad spectrum, which interpolates between box-counting and Assouad dimension, and the intermediate dimensions, which interpolate between Hausdorff and box-counting dimension. In this talk, I'll discuss mapping-theoretic properties of intermediate dimensions and the Assouad spectrum, with applications to the quasiconformal classification of sets and to the range of conformal Assouad spectrum. The latter results are based on two recent papers, one with Efstathios Chrontsios Garitsis (University of Tennessee) and the other with Jonathan Fraser (University of St. Andrews).

Speaker: Lisa Naples, Fairfield University

Title: Asymptotics of maximum distance minimizers

Abstract: The Maximum Distance Problem asks to find the shortest curve whose r-neighborhood contains a given set. Such curves are called r-maximum distance minimizers. We explore the limiting behavior of r-maximum distance minimizers as well as the asymptotics of their 1-dimensional Hausdorff measures as r tends to zero. Of note, we obtain results involving objects of fractal nature. This talk is based on is joint work with Enrique Alvarado, Louisa Catalano, and Tomás Merchán.

Speaker: Katrin Fässler, University of Jyväskylä

Title: On uniform 1-rectifiability in metric spaces

Abstract: According to a result by Menger, a metric space with at least five elements embeds isometrically into the Euclidean line R if and only if every triple of points in the space embeds isometrically into R. I will explain how an "L^1-quantification" of this result for 1-regular metric spaces – together with work by Hahlomaa and by Schul – can be used to give a new characterization of uniformly 1-rectifiable sets in complete, doubling, and quasiconvex metric spaces. The characterization uses certain "flatness coefficients" that can be interpreted as L^1-based Gromov-Hausdorff beta-numbers, and I hope to shed some light on connections between various versions of these coefficients and the theory of uniform rectifiability.

Speaker: Stefan Wenger, University of Fribourg

Title: The metric fundamental class of a space homeomorphic to a manifold

Abstract: Metric spaces homeomorphic to an oriented closed topological manifold and of finite Hausdorff measure are sometimes called metric manifolds. They frequently appear in the field of analysis on metric spaces, notably in quasiconformal geometry and in parametrization problems for metric spaces. I am interested in finding (minimal) assumptions under which a metric manifold admits a metric notion of fundamental class, defined in terms of metric currents in the sense of Ambrosio-Kirchheim. The existence of such a metric fundamental class has implications for example to the Lipschitz-volume rigidity problem and the question about the existence of Poincaré inequalities on the underlying space.

Speaker: Jani Onninen, Syracuse University

Title: Sobolev Homeomorphic Extensions

Abstract: The classical Jordan–Schönflies theorem states that every topological embedding of the unit circle extends to a homeomorphism of the unit disk. In this talk, I will discuss its Sobolev counterpart. Foundational results such as the Beurling–Ahlfors extension and the Radó–Kneser–Choquet theorem provide key building blocks in the Sobolev setting, but their applicability breaks down for irregular target domains. I will present a complete characterization of circle embeddings that admit Sobolev homeomorphic extensions to the disk, thereby establishing a Sobolev analogue of the Jordan–Schönflies theorem. This result is pivotal for the well-posedness of variational problems in Nonlinear Elasticity and Geometric Function Theory. To illustrate this, I will highlight the model case of the Dirichlet energy, where energy-minimizing mappings, monotone Hopf-harmonics, emerge as natural counterparts to classical harmonic homeomorphisms. This talk is based on joint work with Tadeusz Iwaniec, Aleksis Koski, and Haiqing Xu.

Speaker: Ilmari Kangasniemi, University of Cincinnati

Title: Quasiregular values

Abstract: In this talk, we present the core results of the theory of quasiregular values, which has been developed over the last few years. Given a domain $\Omega \subset \mathbb{R}^n$ and a map $f\colon \Omega \to \mathbb{R}^n$, we say that the map f has a quasiregular value at $y_0 \in \mathbb{R}^n$ if f is locally Sobolev and satisfies $|Df(x)|^n \leq K \det(Df(x)) + \Sigma(x)|f(x) - y_0|^n$ at almost every x, where Σ is a real-valued weight function. A planar precursor to this theory appears in works involving e.g. Vekua, Astala, Päivärinta, Iwaniec, and Martin. Together with Onninen, we developed the n-dimensional version of this theory, which provides single-value versions of many classical results of quasiregular maps under sufficient regularity assumptions

on Σ. The results we present include generalizations of Reshetnyak's theorem, the Liouville theorem, Rickman's Picard theorem, the rescaling principle, metric distortion bounds, results about small values of K, higher integrability of $|Df(x)|$, and Hölder continuity.

Wednesday

Speaker: Matthew Badger, University of Connecticut

Title: Doubling Measures that Charge Rectifiable Surfaces but not Rectifiable Curves

Abstract: I will survey recent work with Raanan Schul on constructions of doubling measures with prescribed lower and upper Hausdorff and packing dimensions in Ahlfors regular metric spaces. Further, we may arrange that the measures are simultaneously countably m-rectifiable and purely $(m-1)$-unrectifiable. To check rectifiability, we introduce a novel "square packing construction" of Lipschitz maps with Euclidean domains and metric targets.

Speaker: Chris Gartland, UC San Diego

Title: Stochastic Embeddings of Metric Spaces and Group Actions on ℓ^1

Abstract: In this talk, we will review the theory of transportation cost Banach spaces and show how an isometric group action on a metric space induces an isometric affine action on the corresponding transportation cost space. Then we define stochastic embeddings between metric spaces and explain how they may be used to study the structure of transportation cost spaces. As an application, we will see that every finitely generated hyperbolic group acts on by uniformly Lipschitz affine transformations with quasi-isometrically embedded orbits.

Speaker: Guy C. David, Ball State University

Title: Coarse tangent fields

Abstract: We will discuss a new notion of "coarse tangent field" for general subsets of Euclidean space. This is work in progress with Sylvester Eriksson-Bique and Raanan Schul.

Thursday

Speaker: Riikka Korte, Aalto University

Title: Conformal transformations of metric measure spaces

Abstract: I will discuss the recent results on conformal deformations of metric measure spaces. Inspired by the stereographic projection and its inverse, the deformations that transform unbounded spaces into bounded ones are called sphericalizations and transformations that transform bounded spaces into unbounded ones are called flattenings. It is possible to construct sphericalizations and flattenings that preserve for example uniformity, doubling property, Poincaré inequality, p-energy and/or Besov energy. These transformations are useful for example in solving a Dirichlet problem on unbounded domains. I will discuss our recent results that are based on joint work with A. Björn, J. Björn, R. Gibara, S. Rogovin, N. Shanmugalingam and T. Takala.

Speaker: Xining Li, Sun Yat-Sen University

Title: Doubling measures and Poincaré inequalities for sphericalizations of metric spaces

Abstract: Building on previous work by Li and Shanmugalingam, who demonstrated that certain large-scale geometric structures supported Poincaré inequalities after a compactification process known as sphericalization, we explore how these inequalities can be preserved under weaker conditions. Specifically, we replace the assumption of annular quasiconvexity with a milder requirement: annular connectedness around a base point for sufficiently large radii. Additionally, we extend the class of admissible measures used in the transformation. Our results show that, even under such relaxed geometric and measure-theoretic conditions, the compactified space continues to support a Poincaré inequality.

Speaker: Marie Snipes, Kenyon College

Title: On the p-Laplace equation in metric measure spaces

Abstract: The p-Laplace equation arises naturally in the study of nonlinear potential theory and variational problems on metric measure spaces that support a doubling measure and a Poincaré inequality. Malý and Shanmugalingam developed a variational approach solving the p-Laplace equation in this context with Neumann boundary data that is bounded and Borel measurable. In this talk, we extend their approach to the natural and functionally relevant condition that the boundary data lines in the dual space to L^p. This is part of a joint work with L. Capogna, J. Kline, R. Korte, and N. Shanmugalingam that aims to develop the theory of non-local (fractional) p-Laplace equations in the metric setting.

Speaker: Kai Rajala, University of Jyväskylä

Title: Quasisymmetric nonparametrization of metric n-spheres

Abstract: Semmes (1996) applied techniques from "wild topology" to construct "good" metric 3-spheres—that is, Ahlfors 3-regular and linearly locally contractible spaces—that nevertheless do not admit quasisymmetric or bi-Lipschitz homeomorphisms to the standard

3-sphere. Heinonen and Wu (2010), and Pankka and Wu (2014), extended these constructions to dimensions four and higher. These results show that the quasisymmetric parametrizations of Ahlfors 2-regular, linearly locally contractible metric 2-spheres constructed by Bonk and Kleiner (2002) do not generalize to higher dimensions. We review these developments and discuss related open problems.

Speaker: Anna Doležalová, The Czech Academy of Sciences

Title: Mission $p < n - 1$: Possible – Nonlinear Elasticity Beyond Conventional Limits

Abstract: In models of Nonlinear Elasticity, the natural physical deformation is a minimizer of an energy functional containing $\int_\Omega |Df(x)|^p dx$ and we search for the minimizer in the Sobolev space $W^{1,p}(\Omega, \mathbb{R}^n)$. In the previous results, one assumes that $p \geq n-1$ to ensure the non-interpenetration of matter. In this talk we prove the lower semicontinuity of an energy functional that allows, for the first time, for $p < n - 1$. Our class of admissible deformations consists of weak limits of Sobolev $W^{1,p}$ homeomorphisms. We also introduce a model that allows for cavitations by studying weak limits of homeomorphisms that can open cavities at some points. In that model we add the measure of the created surface to the energy functional and we again prove lower semicontinuity. This is a joint work in progress with Daniel Campbell and Stanislav Hencl.

Speaker: Jasun Gong, Fordham University

Title: Regularity of elements of certain integrable Teichmüller spaces

Abstract: As introduced by Guo, integrable subspaces of the universal Teichmüller space (or: p-integrable Teichmüller spaces) possess additional geometric properties. For instance, $p = 2$ corresponds to the Weil-Petersson Teichmüller space, and due to work of Takhtajan and Teo it enjoys a complex Hilbert manifold structure. More recently, Wei and Matsuzaki obtained analogous Banach structures for all $p \geq 1$.

The remaining range of $0 < p < 1$ has received less attention in the literature, but quasi-conformal mappings belonging to these p-integrable Teichmüller spaces still possess some interesting properties. In joint work with Melkana Brakalova and Matthew Romney, we show improved regularity of such mappings.

Friday

Speaker: Kari Astala, University of Helsinki

Title: Geometry of the universal Beltrami equation

Abstract: We discuss a specific class of (planar) Beltrami equations which carry surprisingly

strong geometric information. We also explain some of their applications to scaling limits of different random structures on planar domains.

Speaker: Matthew Romney, Stevens Institute of Technology

Title: Conformal dimension of graphs

Abstract: The conformal dimension of a metric space is the infimal Hausdorff dimension of its image over all quasisymmetric mappings onto another metric space. It is a widely studied concept in geometric function theory and related fields. A metric space is minimal for conformal dimension if its conformal dimension is equal to its Hausdorff dimension. We consider the problem of determining whether a metric space is minimal for conformal dimension, with an emphasis on the case of graphs of continuous functions. A recent result of Binder-Hakobyan-Li establishes that many graphs are indeed minimal for conformal dimension, including notably the graph of Brownian motion almost surely. We give a counterpoint to their result by constructing continuous functions whose graphs are not minimal; this construction is robust in a technical sense and gives the most extreme distortion of dimension possible.

Speaker: Gaven Martin, Massey University

Title: From Teichmüller to Shoen–Yau: Extremal mappings between Riemann surfaces

Abstract: There are two now classical descriptions of the moduli space of a Riemann surface via the theory of extremal mappings. The first from Teichmüller in the 1940s (rigorously established by Ahlfors in 1953) and through the existence of extremal quasiconformal mappings. The second is through Schoen-Yau's existence theory for unique harmonic diffeomorphisms in the 1970s, and developed into a theory of moduli by many, including Wolf, Tromba and Wolpert many years later. The important ingredient in both is the existence of a holomorphic quadratic differential, from the Beltrami coefficient of an extremal quasiconformal mapping (Teichmüller) or from the Hopf equation (Harmonic). These quadratic differentials define the cotangent space to the moduli space. Here we show that in fact both of these approaches are manifestations of the same theory (that of existence of diffeomorphic extremal mappings of finite distortion) in limiting regimes. We identify parameterised families of moduli spaces (Beltrami coefficients) interpolating between these two end cases defined by a parametrised family of degenerate elliptic nonlinear PDEs giving holomorphically parameterised homotopy between the extremal quasiconformal mapping [which is not a diffeomorphism] and the harmonic diffeomorphism.