Towards multi-point functions in the Sinh-Gordon quantum field theory in 1+1 dimensions

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Outline

The overall setting of the work

- 1+1 dimensional classical Sinh-Gordon evolution equation

$$\left(\partial_t^2 - \partial_x^2\right)\varphi(\boldsymbol{x}) + \frac{m^2}{g}\sinh[g\varphi(\boldsymbol{x})] = 0 \qquad \boldsymbol{x} = (t, x) \in \mathbb{R}^{1,1}$$

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- Quantisation of wave equations ⇝ QFTs

$$\varphi(\boldsymbol{x}) \hookrightarrow \boldsymbol{\Phi}(\boldsymbol{x}) \in \mathcal{L}(\mathfrak{h}) \, , \qquad o_s(\boldsymbol{x}) \ \left(e.g. \prod_{u=1}^{r} \left\{\partial_t^{\alpha_u}\partial_x^{\beta_u}\varphi(\boldsymbol{x})\right\} \cdot \mathrm{e}^{\gamma\varphi(\boldsymbol{x})}\right) \hookrightarrow \boldsymbol{O}_s(\boldsymbol{x})$$

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- Specific properties $\rightsquigarrow$ universality

$$\big(\boldsymbol{\Omega}, \mathcal{O}(\boldsymbol{x})\,\mathcal{O}^{\dagger}(\boldsymbol{0})\boldsymbol{\Omega}\big) \underset{\|\boldsymbol{x}\|\to 0^+}{\sim} C\,\|\boldsymbol{x}\|^{-\Delta}\big(1 + \mathrm{o}(1)\big)$$

The Bootstrap approach

- Bootstrap construction of 1+1 dim integrable quantum field theories
  '78 Karowski, Weisz , **'80-'84** Smirnov , **'87-'89** Kirillov, Smirnov , **'88** Khamitov
  - Explicit 2-pt fcts (*e.g.* Sinh-G) $\boldsymbol{x} = (t, x) \in \mathbb{R}^{1,1}$, $|t| < |x|$,

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Main result

Closed expressions for truncated multi-pt fcts in Sinh-Gordon IQFT

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$$\mathfrak{h} = \bigoplus_{n=0}^{+\infty} L^2(\mathbb{R}^n_>) \qquad \text{with} \qquad \mathbb{R}^n_> = \Big\{ \boldsymbol{\beta}_n = (\beta_1, \dots, \beta_n) \in \mathbb{R}^n \; : \; \beta_1 > \dots > \beta_n \Big\}$$

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- Vectors $\boldsymbol{f} = (f^{(0)}, \dots, f^{(n)}, \dots) \in \mathfrak{h}$

Basics of the model

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$$S(\beta) = \frac{\tanh[\frac{1}{2}\beta - \mathrm{i}\pi\mathfrak{b}]}{\tanh[\frac{1}{2}\beta + \mathrm{i}\pi\mathfrak{b}]} \qquad \text{with} \qquad \mathfrak{b} = \frac{1}{2}\frac{g^2}{8\pi + g^2} \in \left[\, 0\,;\, \tfrac{1}{2} \,\right]$$

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Translation operator by $\boldsymbol{y} = (y_0, y_1)$

$$\mathrm{U}_{\mathrm{T}_{\boldsymbol{y}}} \cdot \boldsymbol{f} = \Big(\mathrm{U}^{(0)}_{\mathrm{T}_{\boldsymbol{y}}} \cdot f^{(0)}, \dots, \mathrm{U}^{(n)}_{\mathrm{T}_{\boldsymbol{y}}} \cdot f^{(n)}, \dots \Big) \qquad \text{where} \qquad \mathrm{U}^{(n)}_{\mathrm{T}_{\boldsymbol{y}}} \cdot f^{(n)}(\boldsymbol{\beta}_n) = \prod_{a=1}^{n} \mathrm{e}^{\mathrm{i}\boldsymbol{p}(\beta_a) * \boldsymbol{y}} \cdot f^{(n)}(\boldsymbol{\beta}_n)$$

with $\boldsymbol{p}(\beta) = \Big(m\cosh(\beta), m\sinh(\beta) \Big)$ and $\boldsymbol{x} * \boldsymbol{y} = x_0 y_0 - x_1 y_1$

The field operators

- $$\mathfrak{h} = \bigoplus_{n=0}^{+\infty} L^2(\mathbb{R}^n_>) \ni \boldsymbol{f} = (f^{(0)}, \ldots, f^{(n)}, \ldots)$$

- Field operators

$$\mathcal{O}(\boldsymbol{x}) \cdot \boldsymbol{f} = \Big(\big(\mathcal{O}(\boldsymbol{x}) \cdot \boldsymbol{f}\big)^{(0)}, \ldots, \big(\mathcal{O}(\boldsymbol{x}) \cdot \boldsymbol{f}\big)^{(n)}, \ldots \Big)$$

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- Space-time dependence

$$\mathcal{O}(\boldsymbol{x}) = \mathrm{U}_{\mathrm{T}_{\boldsymbol{x}}} \cdot \mathcal{O}(\boldsymbol{0}) \cdot \mathrm{U}^{-1}_{\mathrm{T}_{\boldsymbol{x}}}$$

The explicit construction

Construction of $\mathcal{O}_{nm}(\boldsymbol{x})$ $\rightsquigarrow$ Bootstrap program

'78 Karowski, Weisz , **'84-'86** Smirnov , **'87-'89** Kirillov,Smirnov , **'88** Khamitov

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- 0-particle component $\left(\mathcal{O}(\boldsymbol{x}) \cdot \boldsymbol{f}\right)^{(0)} = \sum_{m \geq 0} \mathcal{O}_{0m}(\boldsymbol{x}) \cdot f^{(m)}$

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Form factor $\mathcal{F}^{(\mathcal{O})}_{m;+} \equiv +$ boundary value of $\mathcal{F}^{(\mathcal{O})}_{m} \in \mathcal{M}\Big(\big\{\boldsymbol{\beta}_m \; : \; 0 < \Im(\beta_a) < \pi \big\}\Big)$

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$\mathcal{M}^{(\mathcal{O})}_{n;m}\big(\boldsymbol{\alpha}_n;\boldsymbol{\beta}_m\big) \equiv$ generalised function

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The form factor axioms

- **Axiom 1** : rapidity exchange

$$\mathcal{F}_n^{(O)}(\beta_1,\ldots,\beta_a,\beta_{a+1},\ldots,\beta_n) = S(\beta_a-\beta_{a+1})\cdot\mathcal{F}_n^{(O)}(\beta_1,\ldots,\beta_{a+1},\beta_a,\ldots,\beta_n)$$

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$$\mathcal{F}_n^{(\mathcal{O})}(\beta_1,\ldots,\beta_a,\beta_{a+1},\ldots,\beta_n) \;=\; \mathsf{S}(\beta_a-\beta_{a+1})\cdot\mathcal{F}_n^{(\mathcal{O})}(\beta_1,\ldots,\beta_{a+1},\beta_a,\ldots,\beta_n)$$

- **Axiom 2** : Monodromy

$$\mathcal{F}_n^{(\mathcal{O})}(\beta_1+2i\pi,\beta_2,\ldots,,\beta_n) \;=\; \mathcal{F}_n^{(\mathcal{O})}(\beta_2,\ldots,\beta_n,\beta_1)$$

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- **Axiom 3** : Kinematic pole

$$-\mathrm{iRes}\Big(\mathcal{F}_{n+2}^{(\mathcal{O})}(\alpha+i\pi,\beta,\boldsymbol{\beta}_n)\cdot \mathrm{d}\alpha\,,\ \alpha=\beta\Big) = \Big\{1-\prod_{a=1}^{n}S(\beta-\beta_a)\Big\}\cdot\mathcal{F}_n^{(\mathcal{O})}(\boldsymbol{\beta}_n)$$

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- **Axiom 4** : Boost

$$\mathcal{F}_n^{(\mathcal{O})}(\boldsymbol{\beta}_n+\Lambda\boldsymbol{e}_n) \;=\; \mathrm{e}^{s_{\mathcal{O}}\Lambda}\cdot\mathcal{F}_n^{(\mathcal{O})}(\boldsymbol{\beta}_n)$$

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All "mild" growth solutions $\hookrightarrow$ operator content

- Long history (**'91-'14**) of devising solutions

Zamolodchikov, Fring, Mussardo, Simonetti, Koubek, Lukyanov, Braznikov, Babujian, Karowski, Zapetal ,Feigin, Lashkievich, Pugai, . . .

The p-function representation

$$\mathcal{F}^{(\mathcal{O})}(\boldsymbol{\beta}_N) \ = \prod_{a<b}^{N} \Big\{ \mathrm{F}(\beta_a - \beta_b) \Big\} \cdot \mathcal{K}_N[p_N^{(\mathcal{O})}](\boldsymbol{\beta}_N)$$

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- Minimal form factor

$$\mathrm{F}(\beta) = \exp\left\{-4\int_0^{+\infty} \mathrm{d}x \, \frac{\sinh(x\mathfrak{b}) \cdot \sinh(x\hat{\mathfrak{b}}) \cdot \sinh(\frac{1}{2}x)}{x\sinh^2(x)} \cos\left(\frac{x}{\pi}(\mathrm{i}\pi - \beta)\right)\right\} \quad \text{for} \quad 0 < \Im(\beta) < 2\pi \, .$$

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$$\mathcal{K}_N[p](\boldsymbol{\beta}_N) = \sum_{\boldsymbol{\ell}_N\in\{0,1\}^N}\prod_{a<b}^{N}\left\{1-\mathrm{i}\frac{(\ell_a-\ell_b)\cdot\sin[2\pi\mathfrak{b}]}{\sinh(\beta_a-\beta_b)}\right\}\cdot\prod_{a=1}^{N}\Big\{(-1)^{\ell_a}\Big\}\cdot p(\boldsymbol{\beta}_N,\boldsymbol{\ell}_N)$$

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- Example ('**98** Braznikov, Lukyanov , '**02** Babujian, Karowski) :

$$p_N^{(\mathrm{e}^{\gamma\boldsymbol{\Phi}})}(\boldsymbol{\beta}_N \mid \boldsymbol{\ell}_N) \; = \; \left\{ -\mathrm{i} \frac{\mathrm{e}^{\frac{1}{2\pi} \int_0^{2\pi\mathfrak{b}} \frac{t\mathrm{d}t}{\sin(t)}}}{\sqrt{\sin[2\pi\mathfrak{b}]}} \right\}^N \cdot \prod_{a=1}^{N} \left\{ \mathrm{e}^{\frac{2\mathrm{i}\pi\mathfrak{b}}{g} \gamma (-1)^{\ell_a}} \right\}$$

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Operator content ⇜⇝ solutions $p_N^{(\mathcal{O})}$

The kernel axioms

⊛ **Axiom 5** : recursive reducibility

$$\mathcal{M}^{(\mathcal{O})}_{n;m}\big(\boldsymbol{\alpha}_n;\boldsymbol{\beta}_m\big) = \mathcal{M}^{(\mathcal{O})}_{n-1;m+1}\big((\alpha_2,\ldots,\alpha_n);(\alpha_1+\mathrm{i}\pi,\boldsymbol{\beta}_m)\big)$$
$$+\sum_{a=1}^{m}2\pi\delta_{\alpha_1;\beta_a}\prod_{k=1}^{a-1}S(\beta_k-\alpha_1)\cdot\mathcal{M}^{(\mathcal{O})}_{n-1;m-1}\big((\alpha_2,\ldots,\alpha_n);(\beta_1,\ldots,\widehat{\beta}_a,\ldots,\beta_m)\big)$$

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Initialisation :

$$\mathcal{M}^{(\mathcal{O})}_{0;m}\big(\emptyset;\boldsymbol{\beta}_m\big) = \mathcal{F}_{m;+}(\boldsymbol{\beta}_m)$$

⊛ **Axiom 5** : recursive reducibility

$$\mathcal{M}^{(O)}_{n;m}\big(\alpha_n;\boldsymbol{\beta}_m\big) = \mathcal{M}^{(O)}_{n-1;m+1}\big((\alpha_2,\dots,\alpha_n);(\alpha_1+\mathrm{i}\pi,\boldsymbol{\beta}_m)\big) + \sum_{a=1}^{m} 2\pi\delta_{\alpha_1;\beta_a}\prod_{k=1}^{a-1} S(\beta_k-\alpha_1)\cdot\mathcal{M}^{(O)}_{n-1;m-1}\big((\alpha_2,\dots,\alpha_n);(\beta_1,\dots,\widehat{\beta}_a,\dots,\beta_m)\big)$$

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⊛ Well-defined distributional induction

$\hookrightarrow$ Combinatorial closed form solution

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⊛ Well-defined distributional induction

↪ Combinatorial closed form solution

Explicit realisation of the fields

Truncated & smeared operators

- Smearing of fields

$$\mathcal{O}[g] = \int_{\mathbb{R}^{1,1}} \mathrm{d}\boldsymbol{x}\, g(\boldsymbol{x})\mathcal{O}(\boldsymbol{x}) , \qquad g \in C_c^\infty(\mathbb{R}^{1,1})$$

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- Projection operators $\mathtt{P}_r$ on $L^2(\mathbb{R}^r_>)$

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- Projection operators P_r on $L^2(\mathbb{R}^r_>)$
- Truncated smeared operators $\mathcal{O}^{(r)}[g] = \mathrm{P}_r\mathcal{O}[g]$
- Truncated regularised correlation functions

$$\left(\boldsymbol{\Omega}, \mathcal{O}_1^{(0)}[g_1]\mathcal{O}_2^{(r_1)}[g_2]\cdots\mathcal{O}_k^{(r_{k-1})}[g_k]\boldsymbol{\Omega}\right), \qquad \boldsymbol{r} = (r_1, \ldots, r_k) \in \mathbb{N}^{k-1}$$

The general formula

Theorem '24, K. Potaux, Simon

Let $g_1, \ldots, g_k \in C_c^\infty(\mathbb{R}^{1,1})$ and $\boldsymbol{r} = (r_1, \ldots, r_{k-1}) \in \mathbb{N}^{k-1}$. Then,

$$\left(\boldsymbol{\Omega}, \mathcal{O}_1^{(0)}[g_1]\mathcal{O}_2^{(r_1)}[g_2]\cdots\mathcal{O}_k^{(r_{k-1})}[g_k]\boldsymbol{\Omega}\right) = \sum_{\boldsymbol{n}\in\mathcal{N}_{\boldsymbol{r}}} \frac{1}{\boldsymbol{n}!} \int_{(\mathbb{R}^{1,1})^k} \prod_{a=1}^{k} \mathrm{d}\boldsymbol{x}_a \mathcal{P}_{\boldsymbol{n}}\left(\{\boldsymbol{x}_a\}, \{\partial_{\boldsymbol{x}_a}\}\right) \cdot \prod_{a=1}^{k} g_a(\boldsymbol{x}_a)$$

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- Summation domain

$$\mathcal{N}_{\boldsymbol{r}} = \left\{\boldsymbol{n} = \left(n_{21}, n_{31}, n_{32}, n_{41}, \ldots, n_{kk-1}\right) : \sum_{u=p+1}^{k}\sum_{s=1}^{p} n_{us} = r_p \quad p = 1, \ldots, k-1\right\} \subset \mathbb{N}^{\frac{k(k-1)}{2}}$$

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- Multi-factorial $\quad \boldsymbol{n}! = \prod_{b>a}^{k} n_{ba}!$

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- Multi-factorial $\quad \boldsymbol{n}! = \prod_{b>a}^{k} n_{ba}!$

- *Explicit* differential polynomial $\mathcal{P}_{\boldsymbol{n}}\left(\{\boldsymbol{x}_a\}, \{\partial_{\boldsymbol{x}_a}\}\right)$

The purely space-like regime

Theorem '24, K. Potaux, Simon

Let $g_1, \dots, g_k \in C_c^\infty(\mathbb{R}^{1,1})$ satisfy for $a < b$

$$(\boldsymbol{x}_b - \boldsymbol{x}_a)^2 < 0 \;\; \& \;\; \boldsymbol{x}_{a;1} > \boldsymbol{x}_{b;1} \quad \text{for any} \quad \boldsymbol{x}_c \in \mathrm{supp}[g_c] \,, \; c \in [\![1 \,;\, k]\!] \,.$$

Then, for $\boldsymbol{r} = (r_1, \dots, r_{k-1}) \in \mathbb{N}^{k-1}$, it holds

$$\Big(\boldsymbol{\Omega}, \mathcal{O}_1^{(0)}[g_1]\mathcal{O}_2^{(r_1)}[g_2] \cdots \mathcal{O}_k^{(r_{k-1})}[g_k]\boldsymbol{\Omega}\Big) = \int\limits_{(\mathbb{R}^{1,1})^k} \prod_{a=1}^{k} \mathrm{d}\boldsymbol{x}_a \cdot \prod_{a=1}^{k} g_a(\boldsymbol{x}_a) \cdot \mathcal{W}_{\boldsymbol{r}}(\boldsymbol{x}_1, \dots, \boldsymbol{x}_k)$$

- Explicit integrand

$$\mathcal{W}_{\boldsymbol{r}}(\boldsymbol{x}_1, \dots, \boldsymbol{x}_k) = \Big(\boldsymbol{\Omega}, \mathcal{O}_1^{(0)}(\boldsymbol{x}_1)\mathcal{O}_2^{(r_1)}(\boldsymbol{x}_2) \cdots \mathcal{O}_k^{(r_{k-1})}(\boldsymbol{x}_k)\boldsymbol{\Omega}\Big)$$

Preliminary notations

- Concatenated vector

$$\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_n) \,, \quad \boldsymbol{\vartheta} = (\vartheta_1, \ldots, \vartheta_m) \qquad \rightsquigarrow \qquad \boldsymbol{\gamma} \cup \boldsymbol{\vartheta} = (\gamma_1, \ldots, \gamma_n, \vartheta_1, \ldots, \vartheta_m)$$

- Reflected vector

$$\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_n) \qquad \rightsquigarrow \qquad \overleftarrow{\boldsymbol{\gamma}} = (\gamma_n, \ldots, \gamma_1)$$

- Vector permutation

$$\mathcal{F}^{(\ell)}(\boldsymbol{\gamma} \cup \boldsymbol{\vartheta}) \; = \; \mathsf{S}(\boldsymbol{\gamma} \cup \boldsymbol{\vartheta} \,|\, \boldsymbol{\vartheta} \cup \boldsymbol{\gamma}) \, \mathcal{F}^{(\ell)}(\boldsymbol{\vartheta} \cup \boldsymbol{\gamma})$$

- Uniform vector $\boldsymbol{e} = (1, \ldots, 1)$
- Macroscopic momentum

$$\overline{\boldsymbol{p}}(\boldsymbol{\gamma}) \; = \; \sum_{a=1}^{n} \boldsymbol{p}(\gamma_a) \,, \qquad \boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_n)$$

The explicit integrand

Explicit density

$$\mathcal{W}_{\boldsymbol{r}}(\boldsymbol{x}_1,\ldots,\boldsymbol{x}_k) = \sum_{\boldsymbol{n}\in\mathcal{N}_{\boldsymbol{r}}} \frac{1}{\boldsymbol{n}!} \prod_{b>a}^{k} \Big\{ \int_{\mathscr{C}_{ba}} \mathrm{d}^{n_{ba}}\gamma^{(ba)} \Big\} \cdot \prod_{b>a}^{k} \Big\{ \mathrm{e}^{\mathrm{i}\overline{\boldsymbol{p}}(\gamma^{(ba)})*\boldsymbol{x}_{ba}} \Big\}$$
$$\times \mathcal{S}(\gamma) \cdot \prod_{p=1}^{k} \mathcal{F}^{(p)}\Big(\overleftarrow{\gamma^{(pp-1)}} \cup \cdots \cup \overleftarrow{\gamma^{(p1)}} + \mathrm{i}\pi\overline{\boldsymbol{e}}, \gamma^{(kp)} \cup \cdots \cup \gamma^{(p+1p)} \Big)$$

- Finite sum of convergent integrals

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$$\times \mathcal{S}(\gamma) \cdot \prod_{p=1}^{k} \mathcal{F}^{(p)}\left(\overleftarrow{\gamma^{(pp-1)}} \cup \cdots \cup \overleftarrow{\gamma^{(p1)}} + \mathrm{i}\pi\overline{\boldsymbol{e}}, \gamma^{(kp)} \cup \cdots \cup \gamma^{(p+1p)} \right)$$

- Finite sum of convergent integrals
- Integration domain $\mathscr{C}_{ba} = \left\{\mathbb{R} + \mathrm{i}\eta^{(ba)}\right\}^{n_{ba}}, \quad 0 < \eta^{(21)} < \eta^{(31)} < \eta^{(32)} < \cdots < \eta^{(kk-1)} \ll 1$

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Explicit density

$$\mathcal{W}_{\boldsymbol{r}}(\boldsymbol{x}_1,\dots,\boldsymbol{x}_k) = \sum_{\boldsymbol{n}\in\mathcal{N}_{\boldsymbol{r}}} \frac{1}{\boldsymbol{n}!}\prod_{b>a}^{k}\Big\{\int_{\mathscr{C}_{ba}} \mathrm{d}^{n_{ba}}\gamma^{(ba)}\Big\}\cdot\prod_{b>a}^{k}\Big\{e^{i\overline{\boldsymbol{p}}(\gamma^{(ba)})*\boldsymbol{x}_{ba}}\Big\}$$

$$\times\mathcal{S}(\gamma)\cdot\prod_{p=1}^{k}\mathcal{F}^{(p)}\Big(\overleftarrow{\gamma^{(pp-1)}}\cup\dots\cup\overleftarrow{\gamma^{(p1)}}+i\pi\overline{\boldsymbol{e}},\gamma^{(kp)}\cup\dots\cup\gamma^{(p+1p)}\Big)$$

- Finite sum of convergent integrals
- Integration domain $\mathscr{C}_{ba} = \{\mathbb{R}+i\eta^{(ba)}\}^{n_{ba}}$, $0<\eta^{(21)}<\eta^{(31)}<\eta^{(32)}<\dots<\eta^{(kk-1)}\ll 1$
- S-matrix factor $$\mathcal{S}(\gamma) = \prod_{\substack{v>p\\p\geq 3}}^{k-1}\prod_{u>s}^{p-1} S\Big(\gamma^{(vu)}\cup\gamma^{(ps)}\mid\gamma^{(ps)}\cup\gamma^{(vu)}\Big)$$

Correlation functions & local commutativity

- Correlation function $\rightsquigarrow$ $\sum_{\boldsymbol{r}\in\mathbb{N}^{k-1}}$ $\hookrightarrow$ totally space-like domains

$$\big(\Omega, \mathcal{O}_1(\boldsymbol{x}_1)\mathcal{O}_2(\boldsymbol{x}_2)\cdots\mathcal{O}_k(\boldsymbol{x}_k)\Omega\big) = \sum_{\boldsymbol{n}\in\mathbb{N}^{\frac{1}{2}k(k-1)}} \frac{1}{\boldsymbol{n}!}\prod_{b>a}^{k}\Big\{\int_{\mathscr{C}_{ba}} \mathrm{d}^{n_{ba}}\gamma^{(ba)}\Big\}\cdot\prod_{b>a}^{k}\Big\{\mathrm{e}^{\mathrm{i}\overline{\boldsymbol{p}}(\gamma^{(ba)})*\boldsymbol{x}_{ba}}\Big\}$$

$$\times\mathcal{S}(\gamma)\cdot\prod_{p=1}^{k}\mathcal{F}^{(p)}\Big(\overleftarrow{\gamma^{(pp-1)}}\cup\cdots\cup\overleftarrow{\gamma^{(p1)}}+\mathrm{i}\pi\overline{\boldsymbol{e}},\gamma^{(kp)}\cup\cdots\cup\gamma^{(p+1p)}\Big)$$

- Correlation function $\rightsquigarrow$ $\sum_{\boldsymbol{r}\in\mathbb{N}^{k-1}}$ $\hookrightarrow$ totally space-like domains

$$(\boldsymbol{\Omega}, \mathcal{O}_1(\boldsymbol{x}_1)\mathcal{O}_2(\boldsymbol{x}_2)\cdots\mathcal{O}_k(\boldsymbol{x}_k)\boldsymbol{\Omega}) = \sum_{\boldsymbol{n}\in\mathbb{N}^{\frac{1}{2}k(k-1)}} \frac{1}{\boldsymbol{n}!}\prod_{b>a}^{k}\Big\{\int_{\mathscr{C}_{ba}} \mathrm{d}^{n_{ba}}\gamma^{(ba)}\Big\} \cdot \prod_{b>a}^{k}\Big\{\mathrm{e}^{\mathrm{i}\overline{\boldsymbol{p}}(\gamma^{(ba)})*\boldsymbol{x}_{ba}}\Big\}$$
$$\times \mathcal{S}(\gamma) \cdot \prod_{p=1}^{k} \mathcal{F}^{(p)}\Big(\overleftarrow{\gamma^{(pp-1)}} \cup \cdots \cup \overleftarrow{\gamma^{(p1)}} + \mathrm{i}\pi\overline{\boldsymbol{e}}, \gamma^{(kp)} \cup \cdots \cup \gamma^{(p+1p)}\Big)$$

- Convergence ?

Correlation functions & local commutativity

⊛ Correlation function ⇝ $\sum_{\boldsymbol{r}\in\mathbb{N}^{k-1}}$ ↪ totally space-like domains

$$\left(\boldsymbol{\Omega}, \mathcal{O}_1(\boldsymbol{x}_1)\mathcal{O}_2(\boldsymbol{x}_2)\cdots\mathcal{O}_k(\boldsymbol{x}_k)\boldsymbol{\Omega}\right) = \sum_{\boldsymbol{n}\in\mathbb{N}^{\frac{1}{2}k(k-1)}} \frac{1}{\boldsymbol{n}!}\prod_{b>a}^{k}\left\{\int_{\mathscr{C}_{ba}} \mathrm{d}^{n_{ba}}\gamma^{(ba)}\right\}\cdot\prod_{b>a}^{k}\left\{\mathrm{e}^{\mathrm{i}\overline{\boldsymbol{p}}(\gamma^{(ba)})*\boldsymbol{x}_{ba}}\right\}$$

$$\times\mathcal{S}(\gamma)\cdot\prod_{p=1}^{k}\mathcal{F}^{(p)}\left(\overleftarrow{\gamma^{(pp-1)}}\cup\cdots\cup\overleftarrow{\gamma^{(p1)}}+\mathrm{i}\pi\overline{\boldsymbol{e}},\gamma^{(kp)}\cup\cdots\cup\gamma^{(p+1p)}\right)$$

⊛ Convergence ?

⊛ If convergent, multi-point fcts have the expected properties

⇝ local commutativity for space-like supports

If $(\boldsymbol{x}_s-\boldsymbol{x}_{s+1})^2 < 0$ for any $\boldsymbol{x}_c\in\mathrm{supp}[g_c]$, $c\in\{s,s+1\}$

$$\left(\boldsymbol{\Omega},\mathcal{O}_1[g_1]\cdots\mathcal{O}_s[g_s]\cdot\mathcal{O}_{s+1}[g_{s+1}]\cdots\mathcal{O}_k[g_k]\boldsymbol{\Omega}\right)=\left(\boldsymbol{\Omega},\mathcal{O}_1[g_1]\cdots\mathcal{O}_{s+1}[g_{s+1}]\cdot\mathcal{O}_s[g_s]\cdots\mathcal{O}_k[g_k]\boldsymbol{\Omega}\right)$$

Review of the results

- ✔ Construction of truncated multi-point functions in 1+1 dimensional Sinh-Gordon IQFT.
- ✔ Local commutativity.

Further study

- ⊛ Convergence of multi-point representations.
- ⊛ UV behaviour.

Happy Birthday Antti