

EXPLAINING QUANTUM MECHANICS TO ANTTI

PATH INTEGRALS AND FRIENDS Helsinki University

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Introduction

Of course, I will not explain anything to Antti.

Since we started collaborating in 1987, we have had many discussions about quantum mechanics.

I think that we more or less agree, but I'll let him say what he thinks in the discussion.

He even very kindly invited me to give a series of lectures in Helsinki on that topic some time ago.

For several years now I realized that I am no longer able to work on the technical problems of mathematical physics, unlike Antti who has brilliantly continued to work in mathematical physics.

So, I have mostly tried to write hopefully pedagogical books and articles on “foundations” or conceptual issues of quantum mechanics and statistical mechanics.

Introduction

Ever since I was student, I have always been puzzled about quantum mechanics. I'll explain why in a minute.

Feynman famously said : "Nobody understands quantum mechanics".

The standard reaction among physicists is : but quantum mechanics works (nobody denies that !); so why bother ?

Or more bluntly : "Shut up and calculate !"

Mathematicians on the other hand marvel at the beauty of algebras of operators acting on infinite dimensional vector spaces.

Mathematical physicists often combine both attitudes (they are the worst !).

Introduction

There are many answers to the “why bother” question :

There is an enormous amount of bad philosophy and pseudo-science in the popular discourses about quantum mechanics.

Physicists dismiss that as “nonsense” but they do not have real answers if quantum mechanics remains “ununderstandable”.

But the main reason for “why bother” is that what we teach to students is unclear and is rarely acknowledged to be so.

And telling them to “shut up and calculate” is the worst possible answer to give to students, from a rational point of view.

Introduction

I will start by explaining the main conceptual problem of quantum mechanics in the traditional way, namely the problem of measurement and of the collapse (of the wave function) and then explain why this is not the most fundamental problem.

I will then give an easy and “obvious” solution, but one that doesn’t work.

Finally I will discuss an even easier solution that does work : the de Broglie-Bohm’s theory.

Why the Problem of Measurement is not the Real Problem

Consider a very simplified measurement process. Let

$$\psi_0 = \left[c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] \varphi_0 ,$$

which is a (tensor) product between the original state of a particle whose spin is going to be measured :

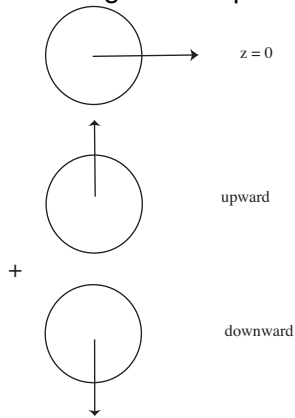
$$c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} ,$$

and the state φ_0 of the measuring device (to simplify matters, we do not include here the wave function of the particle whose spin is being measured, considering only the "spin" part of its quantum state.) .

Why the Problem of Measurement is not the Real Problem

$$\psi_0 = \left[c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] \varphi_0 ,$$

Here φ_0 denotes the initial state of the measuring device, meaning that the pointer is as in the first picture here :

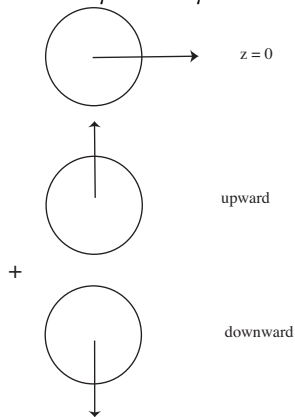


Why the Problem of Measurement is not the Real Problem

The state after the measurement is

$$c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi^\uparrow + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varphi^\downarrow,$$

where φ^\uparrow and φ^\downarrow correspond to the last two pictures here :



Why the Problem of Measurement is not the Real Problem

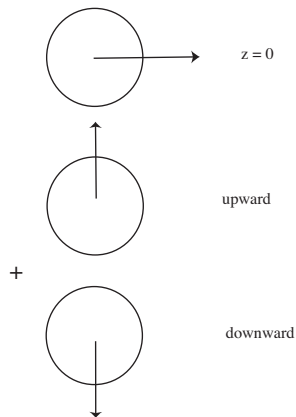
This follows simply from the linearity of Schrödinger's equation :

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi_0 \longrightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi^\uparrow$$
$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \varphi_0 \longrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varphi^\downarrow$$

So,

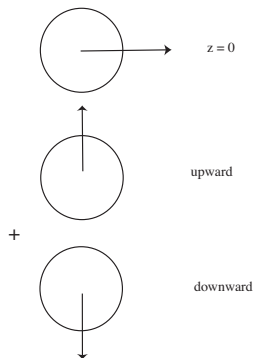
$$\begin{aligned} \psi_0 &= \left[c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] \varphi_0 \\ &\longrightarrow c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi^\uparrow + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varphi^\downarrow, \end{aligned}$$

Why the Problem of Measurement is not the Real Problem



This means that the quantum state of the pointer is a superposition of two macroscopically distinct quantum states : one in which the pointer is pointing upward *and* one in which it is pointing downward.

Why the Problem of Measurement is not the Real Problem

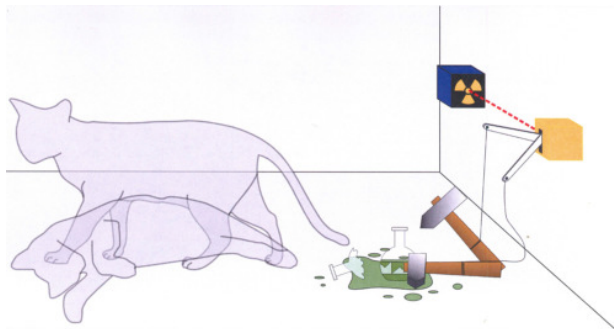


The problem is that we never see the pointer in such a superposed state : we see it *either up or down*, but not both. The ordinary quantum formalism does not correctly predict the state of the measuring device at the end of the experiment, since it unambiguously predicts a superposed state, and this is simply not what is observed.

Why the Problem of Measurement is not the Real Problem

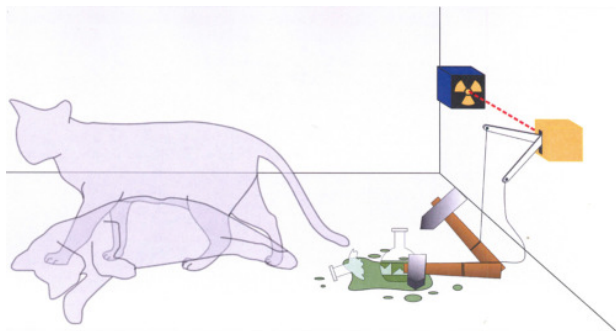
But since the situation is now macroscopic, one may just *look* at the result. If the pointer points upward, we take the state to be $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi^\uparrow$. If the pointer points downward, the state becomes $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \varphi^\downarrow$. One thus reduces the quantum state, which now describes a macroscopic object, just by looking at it.

Why the Problem of Measurement is not the Real Problem



One may also replace the pointer by a cat, as in Schrödinger's dramatic thought experiment : suppose a cat is in a sealed box and there is a purely classical mechanism linking the pointer above to a hammer that will break a bottle containing some deadly poison if the pointer is up, but not if it is down. If the poison is released, it kills the cat.

Why the Problem of Measurement is not the Real Problem



Then, following the same reasoning as above, including now the state of the cat, we get after the measurement :

$$c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \varphi^\uparrow \psi_{\text{cat dead}} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \varphi^\downarrow \psi_{\text{cat alive}} .$$

Why the Problem of Measurement is not the Real Problem

The natural interpretation of the quantum state of the cat is that it is “both alive and dead”. Of course, we never see a cat in such a state. We do not even know what that could mean. But the cat example just dramatizes a problem that occurs already with the pointer, namely the fact that ordinary quantum mechanics predicts macroscopic superpositions that are simply not observed and that are even hard to conceive.

Schrödinger introduced this example as a *reductio ad absurdum* of the usual quantum mechanical view (“Copenhagen”). He called it “quite ridiculous” (burlesque), and he certainly never thought that the cat is “both alive and dead” (contrary to what some people seem to believe).

Why the Problem of Measurement is not the Real Problem

Obviously the state :

$$\frac{1}{\sqrt{2}}(\Psi_{\text{cat alive}} + \Psi_{\text{cat dead}})$$

cannot be a complete description of the cat, which is obviously either alive or dead but not both ! The way out of this problem from the point of view of ordinary quantum mechanics is to introduce again the collapse postulate : when one looks at the cat, one sees whether she is alive or dead and, depending on what one sees, one reduces the wave function of the cat (and of the particle that was measured and is thus coupled to the state of the cat) to either $\Psi_{\text{cat alive}}$ or $\Psi_{\text{cat dead}}$.

Why the Problem of Measurement is not the Real Problem

Since this is a *deus ex machina* from the point of view of the linear Schrödinger evolution, justifying it is often viewed as the main problem in foundations of quantum mechanics.

But there is a deeper problem : neither $\Psi_{\text{cat alive}}$ nor $\Psi_{\text{cat dead}}$ are cats : they are functions defined on a high dimensional space \mathbb{R}^N , N being the number of degrees of freedom describing (classically) the cat (putting aside the “spin” and other such variables) while cats are located in \mathbb{R}^3 . And it is not clear what it means to say that $\Psi_{\text{cat alive}}$ or $\Psi_{\text{cat dead}}$ are descriptions of cats, let alone “complete descriptions” of them.

Like all wave functions, $\Psi_{\text{cat alive}}$ and $\Psi_{\text{cat dead}}$ allow us to predict results of measurements done on the objects that they are attached to, but nothing else.

Why the Problem of Measurement is not the Real Problem

What many people do is to mentally identify cats and wave functions of cats. But that is exactly what we think is illegitimate.

Indeed in orthodox quantum mechanics, one should not think of the cat as made of particles localized in space, since particles have neither position nor velocity in orthodox quantum mechanics, until they are measured.

Note in passing that de Broglie said in his report to the 1927 Solvay Conference : “it seems a little paradoxical to construct a configuration space with the coordinates of points that do not exist”.

He also remarked that, if “the propagation of a wave in space has a clear physical meaning, it is not the same as the propagation of a wave in the abstract configuration space”.

Why the Problem of Measurement is not the Real Problem

But this problem occurs even for one particle : if one knows the quantum state of an electron "out there" in every detail, what does it mean ?

The only honest answer is that, if one brings this electron in a laboratory and one performs some experiment on it, one will get such and such result with such and such probability.

This is very different from what happens in the rest of science !
Example from astronomy.

Why the Problem of Measurement is not the Real Problem

JOHN BELL :

“It would seem that the theory is exclusively concerned about ‘results of measurement’, and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of ‘measurer’ ? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared ? Or did it have to wait a little longer, for some better qualified system . . . with a PhD ?”

Science is verified or confirmed by experiments but is NOT *about experiments* or *about human observations*!

Why the Problem of Measurement is not the Real Problem

JOHN BELL :

"In the beginning natural philosophers tried to understand the world around them. Trying to do that they hit upon the great idea of contriving artificially simple situations in which the number of factors involved is reduced to a minimum. Divide and conquer. Experimental science was born. But experiment is a tool. The aim remains : to understand the world. To restrict quantum mechanics to be exclusively about piddling laboratory operations is to betray the great enterprise. A serious formulation will not exclude the big world outside the laboratory."

Another way to state the fundamental problem of quantum mechanics is that it is not the "problem of measurement" or of the collapse but *the one of the meaning of the wave function outside of laboratories!*

The Naïve Statistical Interpretation

However, there exist a natural interpretation of the quantum formalism that would assuage all these worries.

This interpretation is probably in the mind of most of the “no worry about quantum mechanics” physicists (and was also probably also in the mind of Einstein) : it is the “statistical” one.

According to that interpretation, a state Ψ does not represent an individual system but an *ensemble* of systems.

For any “observable” represented by an operator A , there is, for each individual system, a well-defined value $v(A)$ that a measurement of A would reveal and not create, since it pre-exists to any measurement.

The Naïve Statistical Interpretation

$v(A)$ is an example of what one calls a “hidden variable”, because it is a property of an individual system not included in the quantum state.

Note that the “value” “cat alive” or “cat dead” is also a “hidden variable”, although it is not hidden at all!

The macroscopic world is full of non-hidden “hidden variables”.

The Naïve Statistical Interpretation

In this interpretation, we assume that, if

$$\Psi = \sum_n c_n \Psi_n$$

where the Ψ_n 's are the eigenvectors of A , with eigenvalues λ_n , then the frequency with which $v(A) = \lambda_n$ in an ensemble defined by Ψ is $|c_n|^2$.

So, Born's rule holds.

In that interpretation, there is no problem either with the collapse or reduction of the quantum state : one updates one's probabilities given some new information (compare with coin tossing).

The Naïve Statistical Interpretation

Consider for example the state

$$c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

In the statistical interpretation, if that state is assigned to a large number of particles, it means that a fraction $|c_1|^2$ of them has its spin up and a fraction $|c_2|^2$ of them has its spin down.

The Naïve Statistical Interpretation

But it is a well-known property of quantum mechanics that, if two operators A and B commute ($[A, B] = AB - BA = 0$), then, they are simultaneously measurable, and the results of those measurements have to satisfy

$$v(AB) = v(A)v(B),$$

and

$$v(A + B) = v(A) + v(B).$$

The Naïve Statistical Interpretation

However, the following theorem renders the naïve statistical interpretation untenable.

No hidden variables theorems.

Let \mathcal{A} be the set of self-adjoint operators on a Hilbert space \mathcal{H} with $\dim \mathcal{H}$ at least equal to 4.

Then, there does not exist a map $\nu : \mathcal{A} \rightarrow \mathbb{R}$ such that :

1) $\forall A \in \mathcal{A},$

$\nu(A)$ is an eigenvalue of A .

2) $\forall A, B \in \mathcal{A}$ with $[A, B] = AB - BA = 0,$

$$\nu(AB) = \nu(A)\nu(B),$$

or

$$\nu(A + B) = \nu(A) + \nu(B).$$

The Naïve Statistical Interpretation

These theorems, called the “no hidden variables theorem” are due to Bell and Kochen-Specker, with simplified proofs due to Mermin and Peres.

Their important conclusion is that one cannot have a statistical distribution of maps that do not exist !

Unfortunately, these theorems are very much ignored in the physics community.

I believe that all the talk around the idea that quantum mechanics is only about “information” is implicitly a naïve statistical interpretation, or at least would make sense only if the naïve statistical interpretation was tenable, but it is not.

The Naïve Statistical Interpretation

SUMMARY OF THE PROBLEM POSED BY QUANTUM MECHANICS :

THE STATISTICAL VIEW (WHICH IS THE MOST NATURAL ONE) IS UNTENABLE.

MEASUREMENTS DO NOT SIMPLY "MEASURE". THEY IN SOME SENSE ACT ON THE SYSTEM.

BUT HOW? IN ORDINARY QUANTUM MECHANICS, THEY ARE A DEUS EX MACHINA.

ONLY A MORE DETAILED THEORY CAN EXPLAIN HOW THEY ACT.

The Naïve Statistical Interpretation

We need an “ontology” or “beables” (Bell’s word), namely we need to postulate something that exists outside of laboratories and that is not just the quantum state.

We need an ontology that includes MORE than the measuring devices, but LESS than the values of all the observables.

The de Broglie-Bohm's Theory

*Nature and Nature's Laws lay hid in Copenhagen : God said,
"Let de Broglie-Bohm be !" and all was light.*

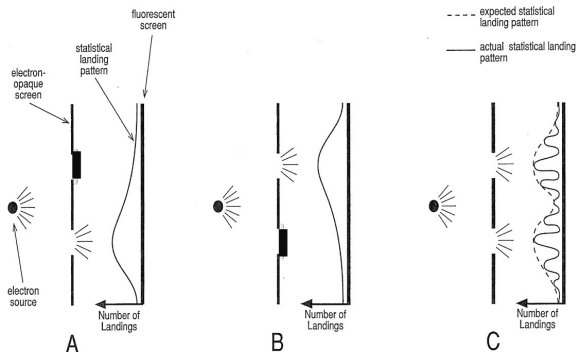
The theory of de Broglie (1927), Bohm (1952), (and Bell, DGZ) :

1. Is a theory of "hidden variables" (although they are not at all hidden),
2. That accounts for all the phenomena predicted by ordinary quantum mechanics,
3. That is not contradicted by the no hidden variables theorems,
4. That explains why measurements do not in general measure pre-existing properties of a system (in other words, it explains why measuring devices have an "active role"),

To achieve that, one has to think outside of the box !

The de Broglie-Bohm's Theory

LET US THINK OF THE DOUBLE SLIT EXPERIMENT



HOW CAN ELECTRONS BE BOTH PARTICLES AND WAVES?

ELEMENTARY MY DEAR BOHR!

THEY ARE PARTICLES *GUIDED* BY WAVES.

The de Broglie-Bohm's Theory

In the de Broglie-Bohm's theory, the state of system is a pair (X, Ψ) , where $X = (X_1, \dots, X_N)$ denotes the actual positions of all the particles in the system under consideration, that exist, whether we measure them or we "look" at them or not.

And $\Psi = \Psi(x_1, \dots, x_N)$ is the usual quantum state, (x_1, \dots, x_N) denoting the arguments of the function Ψ . X are the "hidden variables" in this theory; this is obviously a misnomer, since particle positions are the only things that we ever directly observe (think of the double-slit experiment for example).

The de Broglie-Bohm's Theory

The dynamics of the de Broglie-Bohm's theory is as follows :
both objects Ψ and X evolve in time :

1. SCHRÖDINGER'S EQUATION :

Ψ evolves according to Schrödinger's equation, at all times,
whether one measures something or not :

$$i\partial_t\Psi(x_1, \dots, x_N, t) = (H\Psi)(x_1, \dots, x_N)$$

(with $\hbar = 1$)

where H is the Hamiltonian :

$$H = -\frac{1}{2}\Delta + V,$$

and V is the potential.

THE QUANTUM STATE NEVER COLLAPSES (FOR A
CLOSED SYSTEM).

The de Broglie-Bohm's Theory

2. GUIDING EQUATION :

The evolution of the positions is guided by the quantum state :
writing $\Psi = Re^{iS}$

$$\frac{d}{dt}X_k(t) = \nabla_k S(X_1(t), \dots, X_N(t))$$

for $k = 1, \dots, N$, where $X_1(t), \dots, X_N(t)$ are the actual positions of the particles at time t .

The de Broglie-Bohm's Theory

Double slit experiment : numerical solution in the de Broglie-Bohm theory.

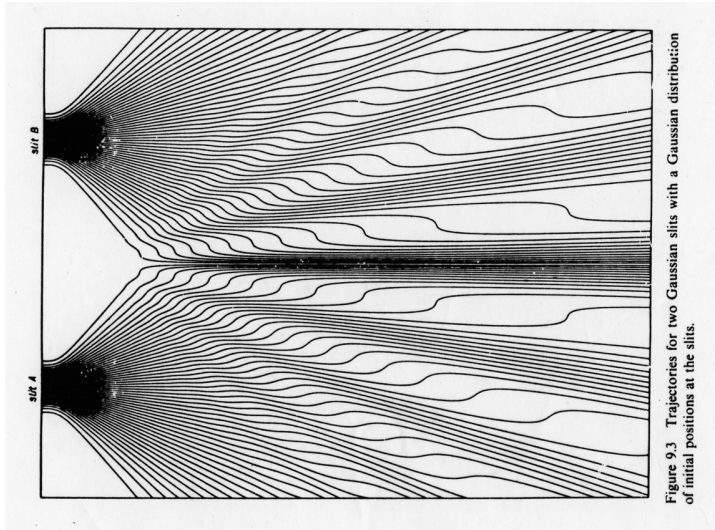


Figure 9.3 Trajectories for two Gaussian slits with a Gaussian distribution of initial positions at the slits.

The de Broglie-Bohm's Theory

It is clear that [the results of the double-slit experiment] can in no way be reconciled with the idea that electrons move in paths. [. . .] In quantum mechanics there is no such concept as the path of a particle.

Lev Landau and Evgeny Lifshitz

The de Broglie-Bohm's Theory

Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given ; that is, a description of it.

Richard Feynman

AND :

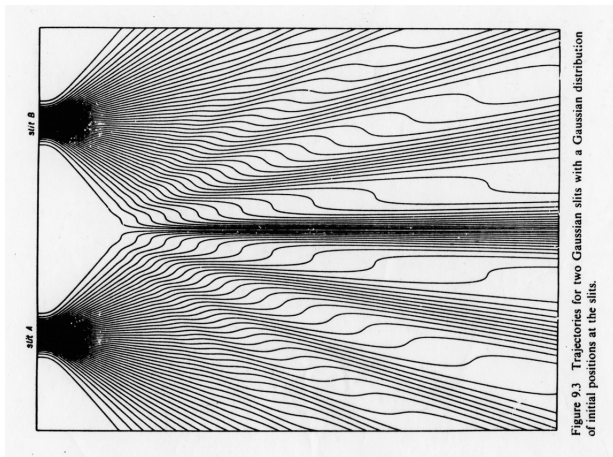
Many ideas have been concocted to try to explain [the interference pattern] in terms of individual electrons going around in complicated ways through the holes. None of them has succeeded.

Richard Feynman

IS IT SO CLEAR?

Think outside of the box !

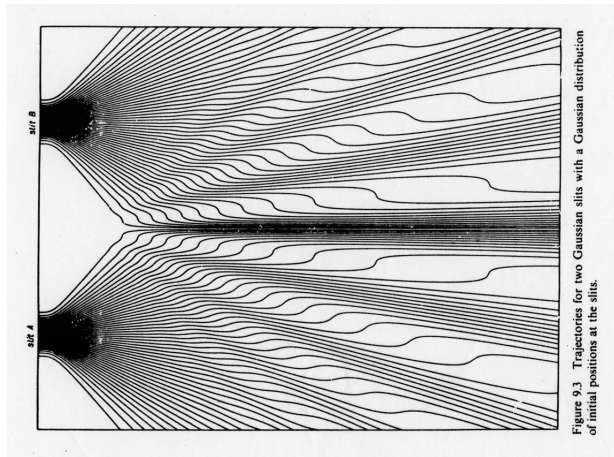
The de Broglie-Bohm's Theory



Motion *in vacuum* highly *non classical*!!

Note that one can determine a posteriori through which hole that particle went!

The de Broglie-Bohm's Theory



Note also the presence of a nodal line : by symmetry of Ψ , the velocity is tangent to the middle line ; thus, particles cannot cross it.

The de Broglie-Bohm's Theory

JOHN BELL :

Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle ? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave ? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in the screen, could be influenced by waves propagating through both holes.

And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored.

The de Broglie-Bohm's Theory

How does the theory of de Broglie-Bohm account for the statistical predictions of quantum mechanics ?

Thanks to equivariance :

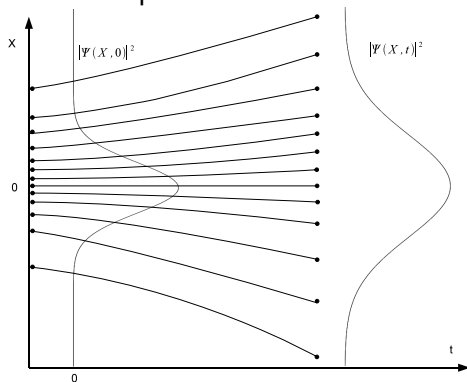
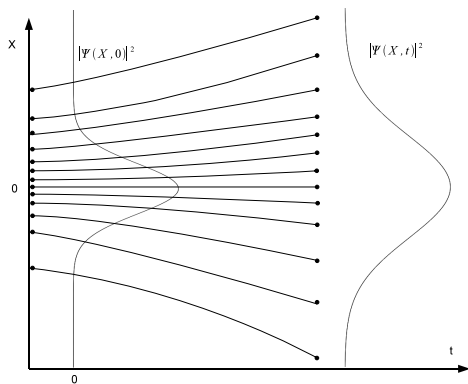


Illustration of the property of equivariance of the $|\Psi(X, t)|^2$ distribution, in one dimension, for a Gaussian Ψ . Each dot represents the position of a particle, both at time 0 and at time t , connected by trajectories.

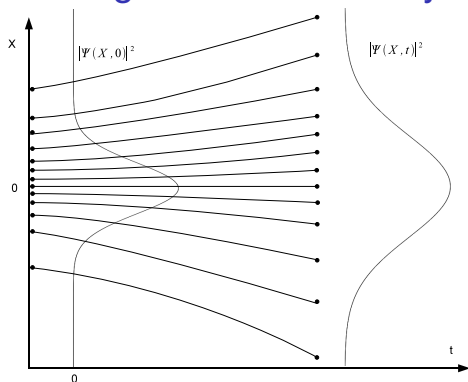
The de Broglie-Bohm's Theory



The initial density of particles ρ_0 is (approximately) given by $\rho_0(X) = |\Psi(X, 0)|^2$, see the left of the picture.

Then, the empirical density of particles at later times ρ_t will satisfy $\rho_t(X) = |\Psi(X, t)|^2$, where $\Psi(X, t)$ is the solution of the Schrödinger equation and ρ_t comes from the guiding equation : $\frac{d}{dt}X_k = \nabla_k S$, with $\Psi = Re^{iS}$.

The de Broglie-Bohm's Theory



So, if we assume that $\rho_0 = |\Psi_0|^2$ at some initial time, $\rho_t(X) = |\Psi(X, t)|^2$ will hold at all times.

The statistical predictions of quantum mechanics are recovered, at least as far as the positions of the particles are concerned.

The assumption that $\rho_0 = |\Psi_0|^2$ is called quantum equilibrium.

The Beauty of de Broglie-Bohm's Theory

But the real beauty of de Broglie-Bohm's theory is that it explains what happens during "measurements".

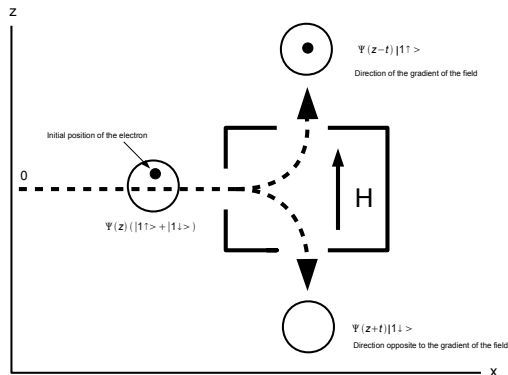
We will consider two examples :

First, a "measurement" of something that does not pre-exist to its "measurement" : the value of the spin.

Then, a "measurement" of something that does pre-exist to its "measurement" but is not measured by that "measurement" : the momentum.

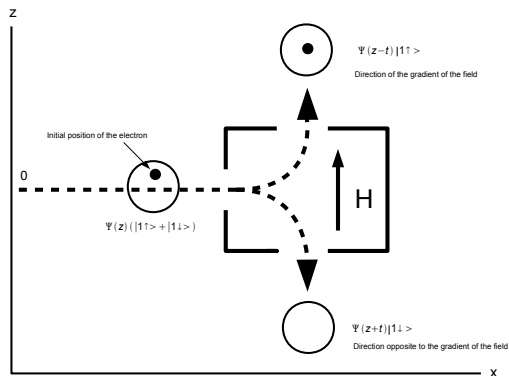
The Beauty of de Broglie-Bohm's Theory

To give an example of a “measurement” of something that does not pre-exist to its “measurement”, consider a Stern-Gerlach apparatus “measuring” the spin. Let H be the magnetic field.



The $|1\uparrow\rangle$ part of the state always goes in the direction of the field, and the $|1\downarrow\rangle$ part always goes in the opposite direction.

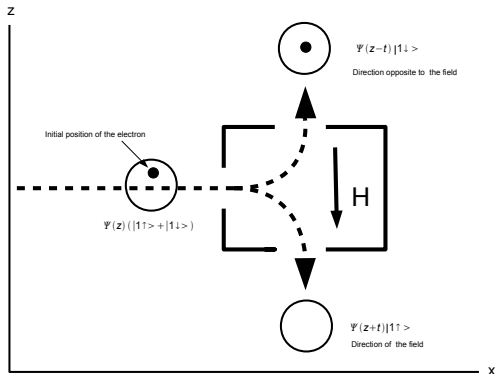
The Beauty of de Broglie-Bohm's Theory



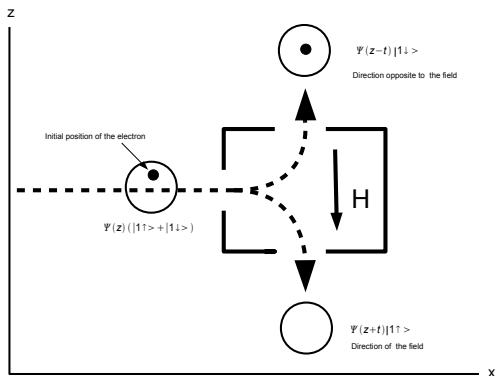
If the particle is initially in the upper part of the support of the wave function (for a symmetric wave function), it will always go upward.

The Beauty of de Broglie-Bohm's Theory

Now, repeat the same experiment, but with the direction of the gradient of the field reversed, and let us assume that the particle starts with *exactly the same wave function and the same position as before*.



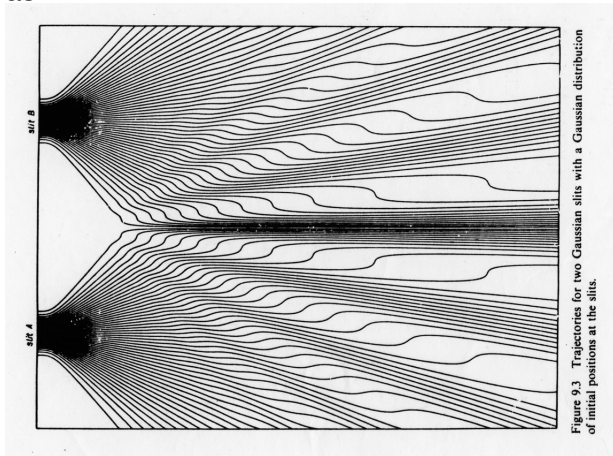
The Beauty of de Broglie-Bohm's Theory



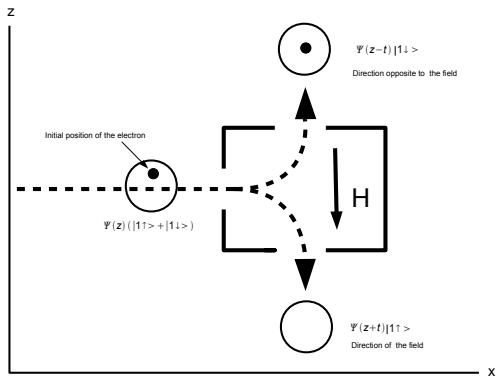
If the particle is initially again in the upper part of the support of the wave function, it will again go upward.

The Beauty of de Broglie-Bohm's Theory

That is because, in that experiment, there is also a nodal line, as in



The Beauty of de Broglie-Bohm's Theory



But going upward means now going in the direction *opposite* to the gradient of the field (since the latter is reversed).

The Beauty of de Broglie-Bohm's Theory

So, the particle whose spin was “up” with the first orientation of the gradient of the field, will “have” its spin “down” with the second orientation of the gradient of the field, although one “measures” exactly the same “operator” (the spin in the vertical direction), with *exactly the same initial conditions (for both the wave function and the position of the particle)*.

So, with two different arrangements of the apparatus measuring the same spin operator, we get different results, for the same initial conditions of the particle.

This is simple illustration of the active role of the measuring device.

Another way to say this : “spin is not real” .

Or, more precisely : quantum “measurements” are interactions that do not measure some property intrinsic to the particle.

The Beauty of de Broglie-Bohm's Theory

To give an example of a “measurement” of something that does pre-exist to its “measurement”, but is not measured by that “measurement”, consider a (spinless) free particle in a box, in its the ground state (for example).

Since its wave function is real (a sine or a cosine), then the particle is at rest. Indeed, write (for one particle) :

$$\psi = Re^{iS}$$

$$\frac{d}{dt}X(t) = \nabla S(X(t)) \quad (1)$$

If $S = 0$, $\frac{d}{dt}X(t) = 0$.

The Beauty of de Broglie-Bohm's Theory

So, we know its velocity : it is zero !

Note that we can also, if we want, measure its position to arbitrary accuracy.

It looks like this violates Heisenberg's inequality !

The Beauty of de Broglie-Bohm's Theory

But deducing the velocity from the theory is not what "measuring the velocity" means in quantum mechanics.

To do that "measurement", one needs to open the box, let the particle move freely ; measure its position at some later time and divide the distance travelled by the elapsed amount of time.

This gives us the "measured velocity", which is not zero and whose statistical distribution, which one can compute, coincides with the quantum prediction and therefore satisfies Heisenberg's inequality.

But that distribution has nothing to do with the true velocity of the particle before the "measurement", which, as we saw, is zero.

The Beauty of de Broglie-Bohm's Theory

All this vindicates the idea that “measurements” are genuine interactions between a system and an apparatus, which was actually an intuition of Bohr :

[. . .] the impossibility of any sharp distinction between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear.

Niels Bohr

But now, this follows from the equations of the theory and not from some more or less philosophical *a priori*.

The Beauty of de Broglie-Bohm's Theory

JOHN BELL :

A . . .charge [against the word measurement] is that the word comes loaded with meaning from everyday life, meaning which is entirely inappropriate in the quantum context. When it is said that something is 'measured' it is difficult not to think of the result as referring to some pre-existing property of the object in question. This is to disregard Bohr's insistence that in quantum phenomena the apparatus as well as the system is essentially involved.

The Beauty of de Broglie-Bohm's Theory

Since “measurements” are interactions that do not reveal pre-existing properties of the system, the de Broglie-Bohm's theory is not refuted by the no hidden variables theorems.

The de Broglie-Bohm's theory is a statistical theory (the distribution of the particle's positions is “random”), but a consistent one, because it does not associate pre-existing values to “observables” other than positions.

One can (rather easily) show that, if one assumes quantum equilibrium for the distribution of the particles' positions, then Born's rule is satisfied for the “measurements” of all “observables”.

Conclusions

In the de Broglie-Bohm's theory, the measurement problem (and the accompanying problem of the collapse) is not solved but dissolved. It is a false problem.

The de Broglie-Bohm theory is not yet another "interpretation" of quantum mechanics.

It is not a different theory than quantum mechanics.

It is simply the (rational) completion of quantum mechanics, which is manifestly incomplete since it does not speak of what happens outside of laboratories.

And ordinary quantum mechanics is simply de Broglie-Bohm's theory applied to what happens in laboratories.

Conclusions

FINALLY, LET ME HOPE THAT I DID EXPLAIN SOMETHING TO ANTTI, WHO HAS EXPLAINED TO ME SO MUCH DURING OUR COLLABORATION!

KIITOS JA HYVÄÄ SYNTYMÄPÄIVÄÄ!