UPDATES

The following updates can be found by searching the program book for the word "UPDATE."

1. John McCarthy's talks in Sessions 3 and 7 have been canceled.
2. Igor Klep's talk in Session 18 has been canceled. The talks of Christian Ikenmeyer and Klemen Šivic in Session 18 have been rescheduled for Tuesday at 2:30 and 3:00, respectively.
3. Session 4: The talks from 2:30 to 4:00 on Thursday take place in Metsätalo, Room 7.
4. Session 7: Connor Evans' talk has been rescheduled for 5:00 on Thursday.
5. Session 13: The talks of Pan Ma and Juha-Matti Huusko have been swapped.
6. Session 15: Joona Oikarinen speaks at 5:30 on Friday.
7. Session 19: Yong Zhang replaces Sayan Das at 5:30 on Thursday.

FURTHER UPDATES ADDED AUG 2:

8. Ken McLaughlin's talk in Session 15 has been canceled and the schedule for Friday has been updated.
8. Session 9: The Thursday talks are in Metsätalo Room 25 instead of Room 8.

Excursion to the Suomenlinna Sea Fortress

Departure from Pier Lyypekinlaituri at 2:30 p.m. and 3:00 p.m. To return from the Suomenlinna Sea Fortress, see the schedule at https://www.frs-finland.fi/suomenlinna-lonna/

Contents

Preface

To be included in the printed copy.

1. Introduction

To be included in the printed copy.

2. Scientific Program

2.1. Schedule.

Monday

8:00-9:00	Registration in Porthania
9:00-9:25	Opening (PI): Virtanen, Helton & Siltanen
9:30-10:25	Simon (PI)
10:30-11:15	Coffee
11:15-12:10	Petermichl (PI)
12:15-12:55	Wick (PI)/Gallardo (PII)
1:00-2:30	Lunch break
2:30-4:30	Sessions
4:30-5:00	Coffee
6:00 p.m.	City Hall Reception

Tuesday

9:00–9:55	Uhlmann (PI) via Zoom
10:00-10:55	Volberg (PI)
11:00-11:30	Coffee
11:30-12:10	Basor (PI)/Jury (PII)
12:15-12:55	Enflo (PI)
1:00-2:30	Lunch break
2:30-4:00	Sessions
4:00-4:30	Coffee
4:30-8:00	Sessions

Wednesday

9:00-9:55	Hytönen (PI)
10:00-10:55	Its (PI)
11:00-11:30	Coffee
11:30-12:10	Aleman (PI)/Wang (PII)
12:15 p.m.	Lunch break and Excursion

Thursday

9:00-9:55	Garcia (PI)
10:00-10:55	Schechtman (PI)
11:00-11:30	Coffee
11:30-12:10	Derksen (PI)/Shargorodsky (PII)
12:15-12:55	Kimsey (PI)/Wojtylak (PII)
1:00-2:30	Lunch break
2:30-4:00	Sessions
4:00-4:30	Coffee
4:30-6:00	Sessions
6:45 p.m.	Concert: Per Enflo, piano (Sipuli)
7:30 p.m.	Conference Dinner (Sipuli)

Friday

9:30–10:10	Zhu (PI)/Hartz (PII)
10:15-10:55	Hassi (PI)/Montes (PII)
11:00-11:30	Coffee
11:30-1:00	Sessions
1:00-2:30	Lunch break
2:30-4:00	Sessions
4:00-4:30	Coffee
4:30-6:00	Sessions
6:00 p.m.	Closing

Locations

PI & PII	Porthania
Sessions	Language Center, Porthania & Metsätalo
Coffee	Porthania Lobby

2.2. **Descriptions of the Special Sessions.**

2.2.1. *Algebraic Structures in Systems and Control Theory.*

Organizers: Mikael Kurula (Åbo Akademi University, Turku, Finland) and Michał Wojtylak (Jagiellonian University, Kraków, Poland)

In systems and control theory several algebraic structures play an important role. Examples are connected with structured matrices (such as Hamiltonian or symplectic matrices), matrix polynomials with even or odd structure, and dynamical systems which are dissipative or port-Hamiltonian (pH). Making extensive use of structure allows for theoretical links between the two research areas and enhanced numerical methods.

Structured matrices and matrix polynomials are a key to the stability and robustness analysis of dynamical systems, and the analysis is to a large extent based on the concepts of indefinite inner product spaces. Structures based on inequalities have also been of great interest recently. PH systems arise from energy-based modelling, and they relax the energy conservation law of Hamiltonian systems in the sense that energy may be exchanged via external ports. The physical properties of pH systems are encoded in the algebraic structure of the coefficient matrices and in geometric structures associated with the flow of the differential equation. Here the structure helps, e.g., for proving existence and uniqueness of solutions, stability and turnpike properties.

The session will cover the following topics from different perspectives: systems and control theory, operator theory, as well as linear algebra and perturbation theory. Talks that stimulate the flow of knowledge between these domains are in particular welcome.

2.2.2. *Harmonic Analysis.*

Organizers: Timo Hänninen (University of Helsinki, Finland) and Emiel Lorist (Delft University of Technology, The Netherlands)

The four-day session covers several aspects of harmonic analysis (and its connections with geometric measure theory, operator theory, and potential theory), with one of the four days focusing particularly on commutator estimates. The speakers of this session range from early career to senior mathematicians in the field. The lectures will be as accessible as possible to early-stage researchers.

2.2.3. *Hilbert Function Spaces in One and Several Variables.*

Organizers: Alberto Dayan (Saarland University, Saarbrücken, Germany) and Michael Hartz (Saarland University, Saarbrücken, Germany)

The topic of this session are Hilbert function spaces, also known as reproducing kernel Hilbert spaces. A particular focus lies on spaces of holomorphic functions in one and several complex variables. We intend to cover developments at the interface of operator theory and complex analysis, such as interpolating sequences, multiplication and composition operators, cyclicity and optimal polynomial approximants, and spaces of Dirichlet series. Moreover, we aim to discuss the recent influx of ideas from non-commutative function theory to classical function spaces.

2.2.4. *Inverse Problems.*

Organizers: Lauri Oksanen (University of Helsinki, Finland) and Tony Liimatainen (University of Helsinki, Finland)

Inverse problems are an active research field of mathematics, where a common goal is to recover information from indirect, incomplete or noisy observations. The study of inverse problems is related to active research in areas of pure mathematics such as harmonic analysis, analysis on manifolds and the study of elliptic partial differential equations. The results and methods of mathematics of IWOTA 2023 workshop find applications in inverse problems.

In Inverse problems session, a wide variety of recent results in the field are presented. The results consider for example inverse problems for various linear and nonlinear partial differential equations, and nonlocal equations. The results also consider spectral theory and operator theory in analytic function spaces.

2.2.5. *Model Spaces, Their Operators, and Applications.*

Organizers: William T. Ross (University of Richmond, Virginia, USA), Javad Mashreghi (Laval University, Quebec, Canada), and Ryan O'Loughlin (University of Leeds, UK)

Model spaces, orthogonal complements of Beurling invariant subspaces for the classical shift on the Hardy space, are ubiquitous in operator theory and function theory. Indeed, in operator theory compressions of the shift to model spaces represent certain classes of Hilbert space contractions. In function theory, model spaces consist of pseudo-continuable functions, which have been shown to have many fascinating properties. Recently, through work initiated by Sarason, model spaces are being explored through the lens of truncated Toeplitz operators.

2.2.6. *Moment Problems and Applications.*

Organizers: Raul E. Curto (University of Iowa, USA)

Over the last two decades, a myriad of new and exciting results have been obtained in the field of moment problems, and significant applications to various areas of pure and applied mathematics have been obtained. The Special Session will aim to bring some of the top contributors to present cutting-edge research and stir renewed interest in these topics.

2.2.7. *Multivariable Operator Theory.*

Organizers: Rongwei Yang (State University of New York at Albany, USA), Joe Ball (Virginia Tech, USA), and Sanne ter Horst (North-West University, South Africa)

Multivariable operator theory (MOT for short) is an active component of functional analysis. It studies joint behaviors of several operators as well as interactions among them. Its long-term objectives can be summarized as follows: 1) Naturally extends operator theory from single operators to several operators; 2) Explore the theory on two fronts: commuting operators and non-commuting operators; 3) Under a multivariable framework, incorporate examples, ideas and methodologies from a wide range of mathematical disciplines; and 4) Build a framework which will facilitate the study of broader multivariate settings in mathematics, science and engineering. MOT is one of the traditional themes at IWOTA meetings. Given the fast progress of MOT in these years, the organizers find it compelling to have this special session to exchange new ideas and discoveries.

2.2.8. *New Trends in Fractional Calculus Operators and Their Applications.*

Organizers: Nelson Vieira (University of Aveiro, Portugal), Milton Ferreira (Polytechnic of Leiria, Portugal), M. Manuela Rodrigues (University of Aveiro, Portugal), and Yuri Luchko (Berliner Hochschule für Technik, Germany)

Within the last few decades, Fractional Calculus enjoyed an increasing interest on the part of the scientific community and became one of the most popular fields of contemporary mathematics. The popularity of Fractional Calculus is due, in part, to its applications in many areas of research and practical activities, including mathematics, physics, engineering, biomedicine, etc. Inside Mathematics, nowadays Fractional Calculus plays an important role in several fields including differential and integro-differential equations, analysis, functional analysis, mathematical physics, computational methods, and the probability theory.

Roughly speaking, Fractional Calculus can be characterized as a study of some generalizations of the definite integrals and integer order derivatives in form of operators of arbitrary order. An important characteristic of these operators is their ability to capture non-linear behaviour and long-term memory effects, say, in biological systems or to enhance the performance and robustness of control systems.

Over the time, development of Fractional Calculus led to some important extensions of several concepts of functional analysis including introduction of new operators, norms, and function spaces that opened new frontiers both in functional analysis and in fractional calculus.

The aim of this Session is to present and to discuss recent advances in fractional calculus from the operator viewpoint. Both theoretical and practical studies in pure and applied mathematics are welcome.

2.2.9. *Non-selfadjoint Operators.*

Organizers: Marco Marletta (Cardiff University, UK) and Christiane Tretter (Bern University, Switzerland)

The interest in non-selfadjoint operators has been steadily growing in the last decades, for at least two reasons. First, their spectral analysis and computation pose interesting challenges, and secondly, there is a wide range of applications e.g. in mathematical physics where non-selfadjoint operators arise. This special session is dedicated to recent developments in this exciting area of operator theory and its applications.

2.2.10. *Noncommutative Geometry.*

Organizers: Tirthankar Bhattacharyya (Indian Institute of Science), Jens Kaad (University of Southern Denmark), David Kyed (University of Southern Denmark), and Walter van Suijlekom (Radboud University Nijmegen, Netherlands)

During the last few years, the mathematical field of noncommutative geometry has witnessed a lot of scientific activity. This includes the intriguing development of unbounded KK-theory and the recent analysis of several different quantum metric spaces, with exciting links to operator algebras. These two main research themes -unbounded KK-theory and quantum metric spaces- will be the focus of the present special session on noncommutative geometry.

2.2.11. *Operator Semigroups: Theory and Applications.*

Organizers: Lassi Paunonen (Tampere University, Finland) and David Seifert (Newcastle University, UK)

The theory of operator semigroups on Banach spaces has a rich history going back to the first half of the 20th century. It is a theory characterised by a beautiful mix of techniques and ideas coming from many different areas of mathematics, including functional analysis, harmonic analysis and complex analysis. Operator semigroups moreover provide the main theoretical framework for the study of various types of evolution equation arising in physics, biology and elsewhere.

The past decade has seen a number of exciting developments in the the- ory of operator semigroups featuring, on the one hand, striking applications to problems arising within mathematics and outside and, on the other hand, deep theoretical contributions, for instance to our understanding of the qualitative and quantitative asymptotic behaviour of operator semigroups. One particu- larly notable area of progress has been the study of energy decay for damped waves.

This special session aims to bring together a number of leading researchers in the field of operator semigroups. There will be speakers from various differ- ent mathematical backgrounds, and the hope is to generate a fruitful exchange of ideas which will help to prepare the ground for many further years of exciting work on the theory and applications of operator semigroups.

2.2.12. *Operator Theory in Elliptic Partial Differential Equations.*

Organizers: Giuseppe Cardone (University of Naples Federico II, Naples, Italy) and Jari Taskinen (University of Helsinki, Finland)

The topics of the session include aspects of spectral theory, asymptotic methods, homogenization theory, equations in waveguides and related fields.

2.2.13. *Operator Theory on Analytic Function Spaces.*

Organizers: Santeri Miihkinen (University of Reading, UK), Antti Perälä (Umeå University, Sweden), Nikolai Vasilevski (CINVESTAV, Mexico City, Mexico), and Kehe Zhu (State University of New York at Albany, USA)

The interaction between analytic function spaces and operator theory is a fruitful area of modern analysis. Many problems in abstract operator theory become more approachable once the operators are unitarily transformed to analytic function spaces. A classical example is Beurling's theorem on invariant subspaces of the unilateral shift operator. The focus of this special session is on analytic function spaces and operators acting on them. Spaces that are frequently used in operator theory include the Hardy space, the Bergman space, the Dirichlet space, and the Fock space. Operators on analytic function spaces that are extensively studied in recent years include Toeplitz operators, Hankel operators, and composition operators.

2.2.14. *Quantum Harmonic Analysis.*

Organizers: Raffael Hagger (Kiel University, Germany) and Robert Fulsche (University of Hannover, Germany)

In the broad sense, Quantum Harmonic Analysis (QHA) is dedicated to the transfer of harmonic analysis tools to quantum mechanics, and as such is a field as old as quantum mechanics itself. More specifically, one may also understand QHA as a formalism developed in R. Werner's celebrated 1984-paper "Quantum Harmonic Analysis on phase space." This formalism, originating from considerations in theoretical physics, did not get much traction in mathematical analysis until recently, but can now be viewed as a prime example of a mathematical application of quantum physics (rather than the other way around, which seems more common). Notably, a nice collection of applications to (Toeplitz) operator theory and time-frequency analysis has been found in recent years. Current fields of interest in QHA include: further development of the mathematical foundations of the theory, reformulating and extending known results in this formalism in order to get a better understanding of the interplay between the different fields, and the exploration of even more applications both ways. Besides pushing QHA itself forward by bringing the experts together, the aim of this special session is also to introduce it to a broader audience who wants to get in touch with this beautiful subject. Talks are welcome from a wide range of topics including Toeplitz operators, time-frequency analysis and quantization.

2.2.15. *Random Matrix Theory and Mathematical Physics.*

Organizers: Christian Webb (University of Helsinki, Finland) and Roozbeh Gharakhloo (University of California, Santa Cruz, USA)

The session will cover a wide range of topics unified by their connection to random matrix theory and mathematical physics, with a broad interpretation. We expect to attract speakers with a diverse range of expertise, including probabilistic and analytical approaches to random matrix theory, statistical and quantum physics, integrable partial differential equations, interacting particle systems, random growth models, and determinantal point processes. Both senior and young speakers will be represented in the session.

2.2.16. *Spectral Inequalities and Null-Controllability.*

Organizers: Michela Egidi (University of Rostock, Germany), Albrecht Seelmann (Technical University Dortmund, Germany), and Matthias Täufer (University of Hagen, Germany)

Intuitively, *Spectral inequalities* can be considered as a replacement for the futile attempt to invert a multiplication with an indicator function of a proper subset in an L^p space. Clearly, this can be an operator with a large kernel and invertibility can fail dramatically. However, it can be made invertible if restricted to a sequence of suitably chosen, nested (spectral) subspaces. *Spectral inequalities* are bounds on the growing norms of the inverses as these subspaces exhaust the L^p space, and thus quantify the loss of invertibility.

This is particularly relevant for controllability of parabolic problems such as the heat equation, and one of the first proofs of null-controllability of the heat equation by Lebeau and Robbiano 1995 relied on this very technique. In the recent years, spectral inequalities have gained attention due to several parallel developments:

(1) The classic Lebeau-Robbiano method for null-controllability has been generalized to a more abstract operator-theoretic framework.
(2) Inputs from other fields such as Harmonic Analysis, Mathematics Physics, Inverse Problems and Unique Continuation Principles have led to new spectral inequalities.

Our special session will unite experts working on spectral inequalities and operator theoretic aspects of null-controllability of parabolic problems.

2.2.17. *Spectral Theory and Partial Differential Operators.*

Organizers: Yuri Latushkin (University of Missouri, USA) and Selim Sukhtaiev (Auburn University, Alabama, USA)

We intend to bring together experts working on applications of abstract methods in spectral theory of unbounded operators in Hilbert spaces to the theory of boundary value problems for partial differential equations and quantum graphs. Special attention will be given to applications of abstract boundary triplets. Various index theorems for partial differential operators will be discussed, in particular, we plan to involve people working on the Malsov index, a modern geometric tool in spectral theory.

2.2.18. *Symmetries, Positivity and Representations.*

Organizers: Jurij Volcic (Drexel University, USA) and Visu Makam (Radix Trading Europe B. V., Netherlands)

Several recent developments in operator theory and its applications call for a more algebraic approach. This session aims to explore the interplay between symmetries, positivity, and representations in operator theory, present new advances on this topic, and establish connections between experts from the associated fields. The discussed themes include real algebraic geometry, computational complexity, tensors, invariants, representation theory, quantum information and noncommutative function theory.

2.2.19. *Operator Theory and Applications.*

The session consists of contributed talks related to various aspects of operator theory and its applications.

2.3. **Organizers and Contact Email.**

IWOTA 2023 is organized by

- Jani Virtanen (chair), University of Helsinki and University of Reading
- Hans-Olav Tylli, University of Helsinki
- Kari Astala, University of Helsinki

in collaboration with the IWOTA executive steering committee members

- J. William Helton (chair of the IWOTA steering committee), UC San Diego
- Igor Klep, University of Ljubljana, Slovenia
- Hugo J. Woerdeman, Drexel University

Conference email: `iwota2023@helsinki.fi`

3. Social Program

The IWOTA 2023 social program consists of the City Hall Reception on Monday the 31st, Excursion to Suomenlinna on Wednesday the 2nd, and Concert by Per Enflo followed by the Conference Dinner on Thursday the 3rd.

3.1. **City Hall Reception.** The City of Helsinki holds a reception on Monday the 31st on the occasion of IWOTA 2023. Notice that confirmation of the preregistration is required at the entrance.

Date and Time: Monday, July 31, 2023, 6:00 p.m. to 7:30 p.m.

Venue: Helsinki City Hall, Banquet Hall

Address: Pohjoisesplanadi 11-13, Helsinki

"The Helsinki City Hall was designed by C.L. Engel as a hotel in 1833 and the building has been the City Hall since the 1930s. The Helsinki coat-of-arms can be seen on the tympanum. With the exception of the facade, the building was completely rebuilt in 1967-1970 designed by the architect Aarno Ruusuvuori." www.myhelsinki.fi/see-and-do/sights/helsinki-city-hall

3.2. **Excursion to Suomenlinna.** A visit to the Suomenlinna Sea Fortress, one of the seven World Heritage Sites in Finland, takes place on the afternoon of Wednesday the 2nd. A ferry ride to the island takes about 15 minutes.

Date: Wednesday, August 2, 2023 (precise time to be announced)

Departure: Market Square, Helsinki

"Situated on a group of islands off Helsinki, Suomenlinna was built during the Swedish era as a maritime fortress and a base for the Archipelago Fleet. Work on the fortress began in 1748." https://www.suomenlinna.fi/en/fortress/

3.3. **Concert and Conference Dinner.** Prior to the conference dinner, Per Enflo performs a concert at Restaurant Sipuli.

Date and Time: Thursday, August 3, 2023, at 6:45 p.m.

Venue: Restaurant Sipuli, Winter Garden

Address: Kanavaranta 7, Helsinki

Program

Per Enflo, piano

L.v. Beethoven (1770-1827)

Sonata in C major, op.53 (Waldstein)

Allegro con brio

Introduzione, Adagio molto

Rondo, Allegretto moderato

Prestissimo

F. Schubert (1797-1828)

Impromptu in G flat major, op.90 no.3

R. Schumann (1810-1856)

Vogel als Prophet, from Waldszenen, op. 82

Conference Dinner.

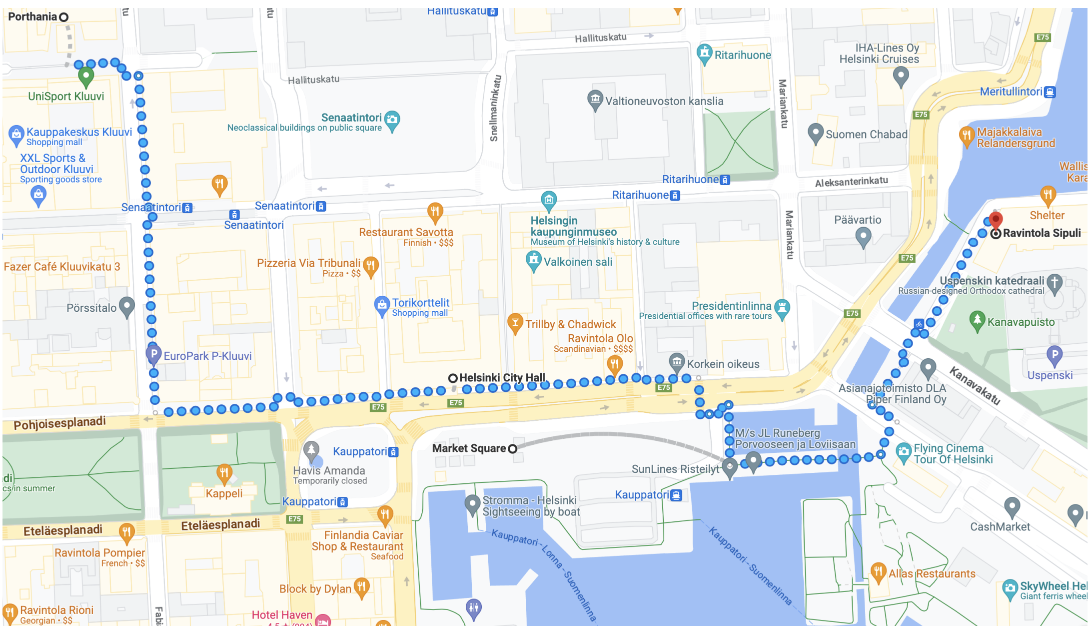
Date and Time: Thursday, August 3, 2023, at 7:30 p.m.

Venue: Restaurant Sipuli, Winter Garden

Address: Kanavaranta 7, Helsinki

Restaurant Sipuli is located in Katajanokka, an island district known for elegant art nouveau buildings and the gold-domed, red-brick Uspenski Cathedral, only a short walk from the Market Square and Porthania.

3.4. **Map.** The dotted path in the following map connects the conference site (Porthania) with the Helsinki City Hall, Market Square (for ferries to the Suomenlinna Sea Fortress), and Restaurant Sipuli.



4. Schedule and Abstracts of the Invited Talks

Monday in Porthania I and II

9:00-9:25	Opening: Jani Virtanen (Main Organizer) J. William Helton (Chair of the IWOTA Steering Committee) Samuli Siltanen (Vice Dean of the Faculty of Science)	
9:30-10:25	Barry Simon *A tale of three coauthors: comparison of Ising models*	chair: Antti Kupiainen
11:15-12:10	Stefanie Petermichl *The dyadic and the continuous Hilbert transforms with values in Banach spaces*	chair: Tuomas Hytönen
12:15-12:55	Brett Wick *Singular Integral Operators on the Fock Space*	chair: Kari Astala
12:15-12:55	Eva Gallardo Gutierrez *Compact perturbations of normal operators: spectral idempotents, decomposability and functional models*	chair: Hans-Olav Tylli

Tuesday in Porthania I and II

9:00-9:55	Gunther Uhlmann *The Dirichlet-to-Neumann map and inverse problems*	chair: Matti Lassas
10:00-10:55	Alexander Volberg *Some applications of harmonic analysis on Hamming cube*	chair: Kari Astala
11:30-12:10	Estelle Basor *Some applications of Fredholm theory*	chair: Jani Virtanen
11:30-12:10	Michael Jury *Hankel forms over a free monoid*	chair: Raul Curto
12:15-12:55	Per Enflo *On the Invariant Subspace Problem in Hilbert space*	chair: Haakan Hedenmalm

Wednesday in Porthania I and II

9:00-9:55	Tuomas Hytönen *Matrix-weighted function spaces*	chair: Christiane Tretter
10:00-10:55	Alexander Its *Toeplitz and Hankel determinants.* *A Riemann-Hilbert point of view*	chair: Jani Virtanen
11:30-12:10	Alexandru Aleman *Cyclicity in weighted Besov spaces*	chair: Eero Saksman
11:30-12:10	Yi Wang *Trace formulas and quantization*	chair: Antti Perälä

Thursday in Porthania I and II

9:00-9:55	Stephan Ramon Garcia *What can chicken nuggets tell us about symmetric functions, positive polynomials, random norms, and AF algebras?*	chair: Hugo Woerdeman
10:00-10:55	Gideon Schechtman *The number of closed ideals in the algebra of bounded operators on Lebesgue spaces*	chair: Hans-Olav Tylli
11:30-12:10	Harm Derksen *Similarity of matrix tuples*	chair: Jurij Volcic
11:30-12:10	Eugene Shargorodsky *Variations on Liouville's theorem*	chair: Roland Duduchava
12:15-12:55	David Kimsey *Moment indeterminateness: The Marcel Riesz principle*	chair: Sanne ter Horst
12:15-12:55	Michał Wojtylak *On localisation of spectra of matrix polynomials with certain definiteness conditions on coefficients*	chair: André Ran

Friday in Porthania I and II

9:30-10:10	Kehe Zhu *The Bargmann transform*	chair: William Ross
9:30-10:10	Michael Hartz *Boundary values in the Drury-Arveson space on the ball*	chair: Tirthankar Bhattacharyya
10:15-10:55	Seppo Hassi *Some classes of generalized boundary triplets, Weyl functions, and applications*	chair: Joseph Ball
10:15-10:55	Alfonso Montes Rodríguez *The even Gauss operator in the Klein-Gordon equation and hyperbolic Fourier series*	chair: Nicholas Young

4.1. **Abstracts of the Plenary Talks.**

Stephan Ramon Garcia, Pomona College, USA Th 9:00, PI

What can chicken nuggets tell us about symmetric functions, positive polynomials, random norms, and AF algebras?

Abstract. Numerical semigroups are combinatorial objects that lead to deep and subtle questions. With tools from complex, harmonic, and functional analysis, probability theory, algebraic combinatorics, and computer-aided design, we answer virtually all asymptotic questions about factorization lengths in numerical semigroups. Our results yield uncannily accurate predictions, along with unexpected results about symmetric functions, trace polynomials, and the statistical properties of certain AF C*-algebras.

Research partially supported by NSF Grants DMS-1800123 and DMS-2054002. Joint work (in various combinations) with K. Aguilar, A. Böttcher, Á. Chávez, L. Fukshansky, M. Omar, C. O'Neill, J. Volčič and undergraduate students J. Hurley, G. Udell, T. Wesley, S. Yih.

Tuomas Hytönen, University of Helsinki, Finland W 9:00, PI

Matrix-weighted function spaces

Abstract. The notion of the vector-valued functions that are square-integrable with respect to a matrix-valued weight goes back to Wiener and Masani (1950's) in their development on the prediction theory for multivariate stochastic processes. The key identification of the weights for which classical operators like the Hilbert transform act boundedly on these spaces was made by Treil and Volberg in the late 1990's. Extensions to p-integrable functions with other exponents followed shortly after, and the study of Besov spaces with matrix weights was started by Frazier and Roudenko in the early 2000's.

In this talk, I will survey a recent systematic theory of function spaces with a matrix weight. The results include characterizations of these spaces in terms of related sequence spaces, "almost diagonal" conditions that imply the boundedness of weakly defined operators on these spaces, and consequences for classical operators like singular integrals. Many of these results find their sharp form when stated in terms of the "dimension" of the weight, which seems to be a new concept even for scalar-valued weights. These recent results are joint work with F. Bu, D. Yang and W. Yuan.

Alexander Its, IUPUI, Indianapolis, USA W 10:00 PI

Toeplitz and Hankel determinants. A Riemann-Hilbert point of view

Abstract. We review some of the old and new results concerning the asymptotic analysis of Toeplitz and Hankel determinants. The main focus will be in the "Riemann-Hilbert" side of the story. We will also discuss the interplay of the Riemann-Hilbert scheme and the more traditional, operator theory based techniques.

Stefanie Petermichl, University of Würzburg, Germany M 11:15, PI

The dyadic and the continuous Hilbert transforms with values in Banach spaces

Abstract. The classical Haar shift was invented twenty years ago by the speaker to solve a question phrased by Pisier on Hankel operators with matrix symbol. It has ever since played a role in harmonic analysis and operator theory. We modify the definition of this naive original Haar shift in a natural way to obtain the so-called dyadic Hilbert transform to resemble much more closely the actual Hilbert transform.

We show that the Hilbert transform with values in a Banach space is Lp bounded if and only if the dyadic Hilbert transform is, with a linear relation of the norms.

The same question for the standard dyadic Haar multiplier T (hence the UMD constant of Banach spaces) and the Hilbert transform H is a famous elusive conjecture. Bourgain showed a quadratic dependence of 'H bounded implies T bounded' and Burkholder showed a quadratic dependence for 'T bounded implies H bounded'. Both have resisted improvement since the 80s.

Gideon Schechtman, The Weizmann Institute, Israel Th 10:00, PI

The number of closed ideals in the algebra of bounded operators on Lebesgue spaces

Abstract. I intend to review what is known about the closed ideals in the Banach algebras $L(L_p(0,1))$ of bounded linear operators on $L_p(0,1)$. Then concentrate mainly on two results: The first with Bill Johnson and Gilles Pisier is the existence of a continuum of closed ideals in $L(L_1(0,1))$. The second, with Bill Johnson, shows that for $1 < p \neq 2 < \infty$ there are exactly $2^{2^{\aleph_0}}$ different closed ideals in $L(L_p(0,1))$.

Barry Simon, Caltech, USA M 9:30, PI

A tale of three coauthors: comparison of Ising models

Abstract. On Friday, Jan 14, 2022, I had a draft of a single author paper intended for the Lieb Festschrift. Six days later, the paper had three authors. This talk will explain the interesting story, expose some underlying machinery and sketch the proof of a lovely inequality on certain finite sums.

Gunther Uhlmann, University of Washington, USA Tu 9:00, PI

The Dirichlet-to-Neumann map and inverse problems

Abstract. The Dirichlet-to-Neumann (DN) map maps the Dirichlet data to Neumann data of solutions to partial differential equations. In this talk we consider several inverse problems associated to the DN map including Calderon's inverse problem, inverse problems for nonlinear elliptic equations and nonlocal operators.

Alexander Volberg, Michigan State University, USA Tu 10:00, PI

Some applications of harmonic analysis on Hamming cube

Abstract. We will mention applications to Banach space theory (the solution of Enflo's problem) and to learning theory: how to learn large matrices by a few queries by using the dimension free Bohnenblust—Hille and Remez inequalities.

4.2. **Abstracts of the Semi-plenary Talks.**

Alexandru Aleman, University of Lund, Sweden W 11:30, PI

Cyclicity in weighted Besov spaces

Abstract. If H is a reproducing kernel Hilbert space and $Mult(H)$ denotes the space of pointwise multipliers of that space, we say that $f \in H$ is *cyclic* if $Mult(H)$ is dense in H. The first important example of cyclic functions are the so called outer functions which emerge from Beurling's famous theorem about invariant subspaces of the unilateral shift operator on the Hardy space H^2. Later, the work of Korenblum, Brown and Shields revealed that in smaller spaces of analytic functions in the disc, cyclicity of an outer function also depends on the size of its zero-set on the boundary. In general, a complete characterization of such functions is lacking, but the area is rich with deep results.

Much less is known in the setting of spaces of functions on the unit ball in several complex variables and as is to expect, the situation is quite complicated. For example, there are polynomials without zeros in the ball which are not cyclic in the standard Drury-Arveson space. The purpose of the talk is to present some recent results in this direction and the material is based on joint work with K.M. Perfekt, S. Richter, C. Sundberg and J. Sunkes.

Estelle Basor, American Institute of Mathematics, USA Tu 11:30, PI

Some applications of Fredholm theory

Abstract. This talk will show how the theory of Fredholm determinants ties three different topics together. These are asymptotics of structured matrices, averages over classical groups, and factorizations of certain polynomials. The averages and factorizations arise in some number theory problems. The key to all three topics is finding exact identities for determinants of classes of structured matrices.

Harm Derksen, Northeastern University, USA Th 11:30, PI

Similarity of matrix tuples

Abstract. An invertible $n \times n$ matrix acts on an m-tuple of $A = (A_1, A_2, \ldots, A_m)$ of $n \times n$ matrices by simultaneous conjugation. The study of simultaneous similarity plays a crucial role in many areas such as operator theory, invariant theory, complexity theory and the representation theory of algebras. In this talk I will discuss several new results that exemplify the connections between these areas, including a joint work with Igor Klep, Visu Makam and Jurij Volčič that resolves a 2003 conjecture of Hadwin and Larson.

Per Enflo, Kent State University, USA Tu 12:15, PI

On the Invariant Subspace Problem in Hilbert space

Abstract. A method to construct invariant subspaces for a general operator T on Hilbert space is presented. It represents a new direction of my "method of extremal vectors", first presented in 1998 in Ansari-Enflo [1].

The Main Construction of the method gives a non-cyclic vector of T by gradual approximation by "almost non-cyclic" vectors. There are reasons, why the Main Construction cannot work for some weighted shifts. But when the Main Construction fails, one gets, by using the information obtained, invariant subspaces of T, similar to those of weighted shifts.

The sequence $y(n)$ of "almost non-cyclic" vectors follows the formula $y(n+1) = y(n)+r(T)y(n)$, which will allow for efficient use of elementary Fourier Analysis. The more general formula $y(n+1) = y(n) + z$ would lead to difficult problems concerning the relation between T and T^*, problems which may be of interest in themselves.

[1] S.Ansari, P.Enflo, "Extremal vectors and invariant subspaces", Transactions of Am. Math. Soc. Vol. 350 no.2, 1998, pp. 539-558

Eva Gallardo Gutierrez, Complutense University of Madrid, Spain M 12:15, PII

Compact perturbations of normal operators: spectral idempotents, decomposability and functional models

Abstract. Decomposable operators were introduced by Foias in the sixties and many operators in Hilbert spaces as unitary operators, self-adjoint operators or more generally normal operators are decomposable. In a broad sense, decomposable operators have the most general kind of spectral decomposition possible. In this talk we will discuss the decomposability of compact perturbations of normal operators and show that a large class of finite-rank perturbations of diagonalizable normal operators on a separable, infinite-dimensional complex Hilbert space are decomposable. Consequently, every operator T in such a class has a rich spectral structure and plenty of non-trivial closed hyperinvariant subspaces, extending in particular recent results of Foias, Jung, Ko and Pearcy. If time permits, we will discuss functional models that arise naturally in this context. (Based on joint works with F. J. González-Doña).

Michael Hartz, Saarland University, Germany F 9:30, PII

Boundary values in the Drury-Arveson space on the ball

Abstract. The classical Hardy space consists of holomorphic functions on the open unit disc. A theorem of Fatou shows that every function in the Hardy space has a radial limit at almost every point on the unit circle.

I will talk about an extension of Fatou's theorem to the Drury-Arveson space, which is a space of holomorphic functions on the open unit ball that in many ways plays the role of the Hardy space in several variables. This is joint work with Nikolaos Chalmoukis.

Seppo Hassi, University of Vaasa, Finland F 10:15, PI

Some classes of generalized boundary triplets, Weyl functions, and applications

Abstract. In this talk various classes of generalized boundary triplets and associated Weyl functions are introduced together with some applications of this technique to ordinary and partial differential operators. Geometric and characteristic properties associated with different types of boundary triplets are expressed in analytic terms via the corresponding Weyl functions. These functions play a fundamental role, for instance, in the study of spectral theoretic properties of differential operators generated by the underlying boundary conditions. In the application part of the talk we discuss Laplacian operators on bounded domains with smooth, Lipschitz, and even rough boundary and, for instance, momentum, Schrödinger, and Dirac operators with local point interactions on discrete sets.

The talk is based on joint work and some recent papers with Vladimir Derkach, Mark Malamud, and Henk de Snoo.

Michael Jury, University of Florida, USA Tu 11:30, PII

Hankel forms over a free monoid

Abstract. We discuss some analytic aspects (boundedness, compactness etc.) of Hankel-like operators defined over a free monoid, from the point of view of noncommutative function theory. In particular we investigate the success and failure of various ways of generalizing the Nehari theorem in this setting, and indicate some open problems. The results and problems are motivated by applications to weighted finite automota (WFA), approximation by noncommutative rational functions, and by connections with function theory in the Drury-Arveson space.

David Kimsey, Newcastle University, England Th 12:15, PI

Moment indeterminateness: The Marcel Riesz principle

Abstract. In general, the discrete data encoded in the power moments of a positive measure μ on $\mathbb{R}^d$, with

$$\int_{\mathbb{R}^d} |x^\gamma| d\mu(x) := \int \cdots \int_{\mathbb{R}^d} |\prod_{j=1}^{d} x_j^{\gamma_j}| d\mu(x_1, \ldots, x_d) < \infty$$

for all $\gamma := (\gamma_1, \ldots, \gamma_d) \in \mathbb{N}_0^d$, is incomplete for recovery, leading to the concept of moment indeterminateness. On the other hand, classical integral transforms (Fourier-Laplace, Fantappiè and Poisson) of such measures are complete and are often invertible with an effective inverse operation. The gap between the two non-uniqueness/uniqueness phenomena is manifest in the dual picture, when trying to extend the corresponding positive linear functional from the polynomial algebra to the full space of continuous functions. This point of view was advocated by Marcel Riesz a century ago in the single real variable setting.

In this talk, we shall revisit, in the context of several real variables, M. Riesz' variational principle. The result is an array of necessary and sufficient moment indeterminateness criteria, some raising real algebra questions, others involving intriguing analytic problems. We shall see that all indeterminateness criteria gravitate around the concept of moment separating function.

This talk is based on a joint work with Mihai Putinar.

Alfonso Montes Rodríguez, University of Seville, Spain F 10:15, PII

The even Gauss operator in the Klein-Gordon equation and hyperbolic Fourier series

Abstract. A pair (Γ, Λ), where $\Gamma \subset \mathbb{R}^2$ is a locally rectifiable curve and $\Lambda \subset \mathbb{R}^2$ is a *Heisenberg uniqueness pair* if an absolutely continuous finite complex-valued Borel measure supported on Γ whose Fourier transform vanishes on Λ necessarily is the zero measure. Here, absolute continuity is with respect to arc length measure. If Γ is the hyperbola $x_1x_2 = M^2/(4\pi^2)$, where $M > 0$ is the mass, and Λ is the lattice-cross $(\alpha\mathbb{Z} \times \{0\}) \cup (\{0\} \times \beta\mathbb{Z})$, where α, β are positive reals, then (Γ, Λ) is a Heisenberg uniqueness pair if and only if $\alpha\beta M^2 \leq 4\pi^2$. The Fourier transform of a measure supported on a hyperbola solves the one-dimensional Klein-Gordon equation, so the theorem supplies discrete uniqueness sets for a class of solutions to this equation. By rescaling, we may assume that the mass equals $M = 2\pi$, and then the above-mentioned theorem is equivalent to the following assertion: *the functions*

$$e^{i\pi\alpha mt}, \quad e^{-i\pi\beta n/t}, \qquad m, n \in \mathbb{Z},$$

span a weak-star dense subspace of $L^\infty(\mathbb{R})$ if and only if $0 < \alpha\beta \leq 1$. The proof involved ideas from Ergodic Theory. To be more specific, in the critical regime $\alpha\beta = 1$, the crucial fact was that the Gauss-type map $t \mapsto -1/t$ modulo $2\mathbb{Z}$ on $[-1, 1]$ has an ergodic absolutely continuous invariant measure with infinite total mass.

As for the holomorphic counterpart, *it can be shown that the functions*

$$e^{i\pi\alpha mt}, \quad e^{-i\pi\beta n/t}, \qquad m, n \in \mathbb{Z}_+ \cup \{0\},$$

span a weak-star dense subspace of $H^\infty_+(\mathbb{R})$ *if and only if* $0 < \alpha\beta \leq 1$. Here, $H^\infty_+(\mathbb{R})$ is the subspace of $L^\infty(\mathbb{R})$ which consists of those functions whose Poisson extensions to the upper half-plane are holomorphic. In the critical regime $\alpha\beta = 1$, the proof relies on the nonexistence of a certain invariant distribution in the predual of real H^∞ for the above mentioned Gauss-type map on the interval $[1, 1]$, which is a new result of dynamical flavor. To attain it, we need a subtle analysis of the iterates of the even Gauss operator

$$(\mathbf{P}f)(x) = \sum_{k \in \mathbb{Z}\setminus\{0\}} \frac{1}{(x + 2k)^2} f\left(\frac{-1}{x + 2k}\right)$$

We have to handle in detail series of powers of the even Gauss operator, a rather intractable problem where even the recent advances by Melbourne and Terhesiu do not apply. More specifically, our approach – which is obtained by combining ideas from Ergodic Theory with ideas from Harmonic Analysis – involves a splitting of the Hilbert kernel, as induced by the transfer operator. The careful analysis of this splitting involves handling the Hurwitz zeta function as well as to the theory of totally positive matrices.

The previous results have been developed with H. Hedenmalm.

Finally, with the aim to delete points of the lattice cross Λ, very recently, with A. Bakan, H. Hedenmalm,, D. Rachenko and M. Viazovska, we have developed a theory on Hyperbolic Fourier series in which certain classes of complex functions f defined on $\mathbb{R}$ can be represented in terms of hyperbolic series

$$f(x) = \sum_{n\in\mathbb{Z}\setminus\{0\}} a_n e^{i\pi nt} + b_n e^{-i\pi nt}$$

where a_n and b_n are complex numbers.

Eugene Shargorodsky, King's College London, England Th 11:30, PII

Variations on Liouville's theorem

Abstract. A classical theorem of Liouville states that a function that is analytic and bounded on the entire complex plane is in fact constant. The same conclusion is true for a function that is harmonic and bounded on $\mathbb{R}^n$.

The talk discusses generalisations of Liouville's theorem to nonlocal translation-invariant operators. It is based on a joint work with D. Berger and R.L. Schilling, and a further joint work in progress with the same co-authors and T. Sharia. We consider operators with continuous but not necessarily smooth symbols. It follows from our results that if $\{\eta \in \mathbb{R}^n \mid m(\eta) = 0\} \subseteq \{0\}$, then, under suitable conditions, every polynomially bounded weak solution f of the equation $m(D)f = 0$ is in fact a polynomial, while sub-exponentially growing solutions admit analytic continuation to entire functions on $\mathbb{C}^n$.

Yi Wang, Chongqing University, China W 11:30, PII

Trace formulas and quantization

Abstract. We give several trace formulas for Toeplitz operators on analytic function spaces on the unit ball. We also give some asymptotic trace formulas with respect to the Toeplitz quantization. In particular, an explicit trace formula for semicommutators of Toeplitz operators is obtained. The Helton-Howe trace formula is shown to be invariant about the quantization. And an asymptotic formula for the Connes-Chern character is also obtained.

Brett D. Wick, Washington University in Saint Louis M 12:15, PI

Singular Integral Operators on the Fock Space

Abstract. In this talk we will discuss the recent solution of a question raised by K. Zhu about characterizing a class of singular integral operators on the Fock space. We show that for an entire function φ belonging to the Fock space $\mathcal{F}^2(\mathbb{C}^n)$ on the complex Euclidean space $\mathbb{C}^n$, the integral operator

$$S_\varphi F(z) = \int_{\mathbb{C}^n} F(w) e^{z\cdot\bar{w}} \varphi(z - \bar{w})\, d\lambda(w), \quad z \in \mathbb{C}^n,$$

is bounded on $\mathcal{F}^2(\mathbb{C}^n)$ if and only if there exists a function $m \in L^\infty(\mathbb{R}^n)$ such that

$$\varphi(z) = \int_{\mathbb{R}^n} m(x) e^{-2\left(x - \frac{i}{2}z\right)^2} dx, \quad z \in \mathbb{C}^n.$$

Here $d\lambda(w) = \pi^{-n}e^{-|w|^2}dw$ is the Gaussian measure on $\mathbb{C}^n$.

With this characterization we are able to obtain some fundamental results of the operator S_φ, including the normality, the C^* algebraic properties, the spectrum and its compactness. Moreover, we obtain the reducing subspaces of S_φ.

In particular, in the case $n = 1$, this gives a complete solution to the question proposed by K. Zhu for the Fock space $\mathcal{F}^2(\mathbb{C})$ on the complex plane $\mathbb{C}$ (Integr. Equ. Oper. Theory **81** (2015), 451–454). This talk is based on joint work with Guangfu Cao, Ji Li, Minxing Shen, and Lixin Yan.

Michał Wojtylak, Jagiellonian University, Kraków, Poland Th 12:15, PII

On localisation of spectra of matrix polynomials with certain definiteness conditions on coefficients

Abstract. Localising spectra of matrix polynomials is an important task, e.g., in view of umerical methods. We will concentrate on matrix polynomials related to port-Hamiltonian systems. The plot of the talk wll be led by the following results on a regular matrix polynomial

$$P(\lambda) = \sum_{j=0}^{d} \lambda^j A_j.$$

Thm.1. If all A_i's are positive semidefinite, then the eigenvalues are located in the angle arg $z \leq \pi/d$.

Proof technique: Numerical range of a matrix polynomial.

Source: C. Mehl, V. Merhmann, M.W., Matrix Pencils with Coefficients that have Positive Semidefinite Hermitian Parts, *SIMAX*, 2022

Thm.2. If $d \leq 2$ and all A_0, A_2 and the Hermitian part of A_1 are positive semidefinite, then the eigenvalues are in the closed left-half plane.

Proof technique: Linearisation.

Source: C. Mehl, V. Merhmann, M.W., Linear algebra properties of dissipative Hamiltonian descriptor systems,*SIMAX*, 2018

Thm.3. If $d \leq 3$, A_3, A_2, A_1 and the skew-Hermitian part of A_0 are positive semidefinite, then then the eigenvalues are located in the angle arg $z \leq \pi/3$.

Proof technique: Multivariate matrix polynomials, polarisation.

Source: O. Szymański, M.W., Stability of matrix polynomials in one and several variables, *LAA*, 2023

We will also discuss the multiplicity of eigenvalues on the imaginary axis.

Kehe Zhu, SUNY at Albany, USA F 9:30, PI

The Bargmann transform

Abstract. The Bargmann transform is a unitary operator from L^2 of the real line to the Fock space F^2 of the complex plane. Under the Bargmann transform, many classical operators on L^2 take fascinating new forms on the Fock space. The talk will focus on several examples of such operators, including the Fourier transform and the Hilbert transform. Motivated by the Hilbert transform, we also introduce several classes of "singular" integral operators on the Fock space and discuss their boundedness. As an application, we construct a C^*-algebra of entire functions that is contained in the Fock space.

5. Schedules and abstracts of the special sessions

1. Algebraic Structures in Systems and Control Theory
   Organized by Mikael Kurula and Michał Wojtylak
2. Harmonic Analysis
   Organized by Timo Hänninen and Emiel Lorist
3. Hilbert Function Spaces in One and Several Variables
   Organized by Alberto Dayan and Michael Hartz
4. Inverse Problems
   Organized by Lauri Oksanen and Tony Liimatainen
5. Model Spaces, Their Operators, and Applications
   Organized by William Ross, Javad Mashreghi, and Ryan O'Loughlin
6. Moment Problems and Applications
   Organized by Raul E. Curto
7. Multivariable Operator Theory
   Organized by Rongwei Yang, Joe Ball, and Sanne ter Horst
8. New Trends in Fractional Calculus Operators and Their Applications
   Organized by N. Vieira, M. Ferreira, M. M. Rodrigues, and Yu. Luchko
9. Non-selfadjoint Operators
   Organized by Marco Marletta and Christiane Tretter
10. Noncommutative Geometry
    Organized by T. Bhattacharyya, J. Kaad, D. Kyed, and W. van Suijlekom
11. Operator Semigroups: Theory and Applications
    Organized by Lassi Paunonen and David Seifert
12. Operator Theory in Elliptic Partial Differential Equations
    Organized by Jari Taskinen and Giuseppe Cardone
13. Operator Theory on Analytic Function Spaces
    Organized by N. Vasilevski, K. Zhu, A. Perälä, and S. Miihkinen
14. Quantum Harmonic Analysis
    Organized by Raffael Hagger and Robert Fulsche
15. Random Matrix Theory and Mathematical Physics
    Organized by Christian Webb and Roozbeh Gharakhloo
16. Spectral Inequalities and Null-Controllability
    Organized by Michela Egidi, Albrecht Seelmann, and Matthias Täufer
17. Spectral Theory and Partial Differential Operators
    Organized by Yuri Latushkin and Selim Sukhtaiev
18. Symmetries, Positivity and Representations
    Organized by Jurij Volcic and Visu Makam
19. Operator Theory and Applications
    Contributed talks

5.1. **Algebraic Structures in Systems and Control Theory.**

Tuesday 2:30-4:00 in Metsätalo, Hall 2 chair: Michał Wojtylak

2:30-2:55	André Ran *Distributed state estimation with communication of observations*
3:00-3:25	Riccardo Morandin *A Galerkin approach for general port-Hamiltonian descriptor systems*
3:30-3:55	Nathanael Skrepek *A port-Hamiltonian approach to coupled cable-field problems*

Tuesday 4:30-6:00 in Metsätalo, Hall 2 chair: Michał Wojtylak

4:30-4:55	Dorothea Hinsen *On discrete-time port-Hamiltonian (descriptor) systems*
5:00-5:25	Alexander A. Wierzba *On BIBO stability of infinite-dimensional linear state-space systems*
5:30-5:55	Babhrubahan Bose *Birkhoff-James orthgonality and its local symmetry in some sequence spaces*

Thursday 2:30-4:00 in Metsätalo, Hall 2 chair: Mikael Kurula

2:30-2:55	Joseph Ball *Structured multidimensional linear systems and function-theoretic operator theory on the commutative and freely noncommutative unit ball*
3:00-3:25	Sanne ter Horst *The Infinite-Dimensional Bounded Real Lemma in Continuous Time*
3:30-3:55	Bertin Zinsou *Asymptotics of the eigenvalues of self-adjoint fourth order problems*

Thursday 4:30-6:00 in Metsätalo, Hall 2 chair: Mikael Kurula

4:30-4:55	Zinaida Lykova *Function theory of the tetrablock*
5:00-5:25	Olof Staffans *Tall-Inner/Bi-Inner Factorizations in State/Signal Optimal Control*

5.1.1. *Abstracts.*

Joseph Ball, Virginia Tech

Structured multidimensional linear systems and function-theoretic operator theory on the commutative and freely noncommutative unit ball

Abstract. The synthesis of energy-preserving discrete-time-invariant linear systems and function-theoretic operator theory (Hardy spaes over the unit disk, Sz.-Nagy-Foias and de Branges-Rpovnyak dilation and model theory for a contraction operator) already acheived a fairly definitive form in the 1970s. A sample result from this theory is: *if the system matrix* $\mathbf{U} = \left[\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right] : \left[\begin{smallmatrix} \mathcal{X} \\ \mathcal{U} \end{smallmatrix}\right] \to \left[\begin{smallmatrix} \mathcal{X} \\ \mathcal{Y} \end{smallmatrix}\right]$ (here $\mathcal{X}$ is the *state space*, $\mathcal{U}$ is the *input space*, and $\mathcal{Y}$ is the *output space*) *is unitary with state operator A strongly stable* (i.e., $\|A^n x\| \to 0$ as $n \to \infty$ for each $x \in \mathcal{X}$), *then the observability operator $\mathcal{O}_{C,A} \colon x \mapsto C(I - \lambda A)^{-1}x$ is isometric from $\mathcal{X}$ into the vectorial Hardy space $H^2_{\mathcal{Y}}$ and the transfer function $\Theta_{\mathbf{U}}(\lambda) = D + \lambda C(I_{\mathcal{X}} - \lambda A)^{-1}B$ is inner* (i.e., the multiplication operator $M_{\Theta_{\mathbf{U}}} \colon f(\lambda) \mapsto \Theta_{\mathbf{U}}(\lambda) \cdot f(\lambda)$ is isometric from $H^2_{\mathcal{U}}$ into $H^2_{\mathcal{Y}}$). The talk will review recent extensions of these ideas to multivariable settings, both commutative (where H^2 on the unit disk is replaced by a (possibly weighted) Drury-Arveson/Bergman space on the ball $\mathbb{B}^d \subset \mathbb{C}^d$ and where the system evolution is along the nonnegative integer lattice $\mathbb{Z}^d_+$), and freely noncommutative (where H^2 on the disk is replaced by a (possibly weighted) Hardy-Fock space over a free noncommutative monoid $\mathbb{F}^d_+$ consisting of what are now called noncommutative functions, and with system evolution along the finitely generated free monoid $\mathbb{F}^d_+$). Many

results for the two settings are completely parallel to each other, but there are instances where the system-theory/function-theory fit is much better for the freely noncommutative setting than for the commutative setting. Recent references for the noncommutative setting are listed below. This is joint work with Vladimir Bolotnikov of William & Mary (Williamsburg, VA USA).

[1] J.A. Ball and V. Bolotnikov, *Noncommitative Function-theoretic Opertor Theory and Applications*, Cambridge Tracts in Mathematics **225**, Cambridge University Press, 2022.

[2] D.S. Kaliuzhnyi-Verbovetskyi and V. Vinnikov, *Foundations of Free Noncommutative Function Theory*, Mathematical Surveys and Monographs **199**, Amer. Math. Soc., Providence, RI, 2014.

Babhrubahan Bose, Department of Mathematics, Indian Institute of Science, Bengaluru

Birkhoff-James orthgonality and its local symmetry in some sequence spaces

Abstract. We study Birkhoff-James orthogonality and its local symmetry in some sequence spaces namely ℓ_p, for $1 \leq p \leq \infty$, $p \neq 2$ and c, c_0 and c_{00}. Using the characterization of the local symmetry of Birkhoff-James orthogonality, we characterize isometries of each of these spaces onto itself and obtain the Banach-Lamperti theorem for onto operators on the sequence spaces.

This is a joint work with Saikat Roy and Debmalya Sain.

Dorothea Hinsen, TU Berlin

On discrete-time port-Hamiltonian (descriptor) systems

Abstract. Port-Hamiltonian (pH) systems have been studied extensively for linear continuous-time dynamical systems. In this talk a discrete-time pH descriptor formulation is presented for linear, completely causal, scattering passive dynamical systems that is purely based on the system coefficients. The relation of this formulation to positive and bounded real systems and the characterization via positive semidefinite solutions of Kalman-Yakubovich-Popv inequalities is also studied.

This talk is based on the paper [1].

[1] Cherifi, K., H. Gernandt, D. Hinsen und V. Mehrmann (2023). “On discrete-time port-Hamiltonian (descriptor) systems”. doi: 10.48550/ARXIV.2301.06731

Zinaida Lykova, Newcastle University, UK

Function theory of the tetrablock

Abstract. The set

$$\overline{\mathbb{E}} = \{x \in \mathbb{C}^3 : \quad 1 - x_1 z - x_2 w + x_3 zw \neq 0 \text{ whenever } |z| < 1, |w| < 1\}$$

is called the tetrablock and has intriguing complex-geometric properties. It is a polynomially convex, nonconvex set and is starlike about 0. This set is associated with the μ_{Diag}-synthesis interpolation problem. For the tetrablock $\overline{\mathbb{E}}$, we describe a rich structure of interconnections between four objects: the set $\mathrm{Hol}(\mathbb{D}, \overline{\mathbb{E}})$ of analytic functions from the disc into $\overline{\mathbb{E}}$, the 2×2 matricial Schur class $\mathcal{S}^{2\times 2}$, the Schur class $\mathcal{S}_2$ of the bidisc, and the set $\mathcal{R}$ of pairs of positive kernels on the bidisc subject to a boundedness condition. The rich structure related to the construction of analytic matrix functions can be summarised diagrammatically as

$$\begin{array}{ccc} \mathcal{S}^{2\times 2} & \longleftrightarrow & \mathcal{R} \\ \updownarrow & & \updownarrow \\ \mathrm{Hol}(\mathbb{D}, \overline{\mathbb{E}}) & \longleftrightarrow & \mathcal{S}_2. \end{array}$$

This rich structure combines with the classical realization formula and Hilbert space models in the sense of Agler to give a theoretical method for the construction of the required interpolating functions. The distinguished boundary of the tetrablock is homeomorphic to the solid torus $\bar{\mathbb{D}} \times \mathbb{T}$. We exploit the geometry of $\overline{\mathbb{E}}$ to develop an explicit and detailed structure theory for the rational $\overline{\mathbb{E}}$-inner functions. These are the analytic functions from the unit disc $\mathbb{D}$ to $\overline{\mathbb{E}}$ that map $\mathbb{T}$ to the distinguished boundary $b\overline{\mathbb{E}}$ of $\overline{\mathbb{E}}$. Note that any analytic functions from the disc into $\overline{\mathbb{E}}$ can be approximated uniformly on compact subsets by rational $\overline{\mathbb{E}}$-inner functions.

The talk is based on the papers

J. Agler, Z. A. Lykova and N. J. Young, A case of mu-synthesis as a quadratic semidefinite progam, *SIAM J. Control and Optimization*, **51 (3)** (2013) 2472-2508.

D. C. Brown, Z. A. Lykova and N. J. Young, A rich structure related to the construction of holomorphic matrix functions, *Journal of Functional Analysis*, **272**(4) (2017), 1704–1754.

O. M. O. Alsalhi and Z.A. Lykova, Rational tetra-inner functions and the special variety of the tetrablock, *Journal of Mathematical Analysis and Applications*, **506** (2022), the article number 125534, 52 pages.

D. Alpay, T. Bhattacharyya, A. Jindal and P. Kumar, A dilation theoretic approach to approximation by inner functions, arXiv:2203.10936 [math.CV], 9 Jan. 2023, 15 pages.

Riccardo Morandin, Technische Universität Berlin

A Galerkin approach for general port-Hamiltonian descriptor systems

Abstract. In many applications, port-Hamiltonian (pH) systems of partial differential equations are used to model individual components of a complex network, that are then interconnected imposing power-preserving constraints on the corresponding boundary values. This produces hybrid systems that include algebraic constraints, i.e., differential-algebraic equations, that are still pH. This appears e.g. in the modeling of gas networks and power networks. In this talk we introduce a general formulation for pH descriptor systems, that allows the mixture of finite and infinite-dimensional components, and algebraic constraints. The main idea consists in defining a bilinear form over the space of flow-effort pairs, that does not necessarily correspond to the standard inner product of some Hilbert space. Such systems satisfy the usual power balance equation and dissipation inequality, and allow for structure-preserving interconnection. Furthermore, we show that Galerkin schemes can be applied to systems of this form in a structure-preserving fashion, providing a paradigm for the development of dedicated space-time discretization and model order reduction schemes.

André Ran, Department of Mathematics, Vrije Universiteit Amsterdam and Research Focus: Pure and Applied Analytics, North West University, South Africa

Distributed state estimation with communication of observations

Abstract. Consider the two-input, two-output linear time-invariant stochastic system in discrete time:

$$\begin{aligned} x(t+1) &= A\ x(t) + B_1 u_1(t) + B_2 u_2(t) + Mv(t), \\ y_1(t) &= C_1 x(t) + D_1 u_1(t) + N_1 v(t), \\ y_2(t) &= C_2 x(t) + D_2 u_2(t) + N_2 v(t). \end{aligned}$$

Here, $x(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^{m_i}$, $y_i(t) \in \mathbb{R}^{p_i}$ ($i = 1, 2$). Finally, $v(t) \in \mathbb{R}^{m_v}$ is standard white noise. The noise on state and outputs is uncorrelated, i.e.,

$$MN_1^T = 0, \qquad MN_2^T = 0, \qquad N_2 N_1^T = 0.$$

Furthermore, N_1 and N_2 are full rank, $N_1 N_1^T$ is invertible and $N_2 N_2^T$ is invertible.

The goal of the talk is to discuss state estimation, where only $y_1(t)$ and a linear function of $y_2(t)$ are available.

That is, for purpose of state estimation we have

$$y(t) = \begin{bmatrix} C_1 \\ LC_2 \end{bmatrix} x(t) + \begin{bmatrix} D_1 & 0 \\ 0 & LD_2 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} + \begin{bmatrix} N_1 \\ LN_2 \end{bmatrix} v(t),$$

for some $L \in \mathbb{R}^{p \times p_2}$.

We shall assume that $p \leq p_2$, and that L is chosen to be full rank.

We consider two cases:
Case 1. rank L free,
Case 2. rank L fixed.

It turns out that the first case is fully solvable, while the second case is actually not a well-posed problem. For the second case an alternative will be proposed.

The talk is based on joint work with Jan van Schuppen.

Nathanael Skrepek, TU Freiberg

A port-Hamiltonian approach to coupled cable-field problems

Abstract. We investigate the electromagnetic interactions of cable harnesses. We model every single cable as a 1-D transmission line, which is embedded in a 3-D electromagnetic field. The cables interact with each other via this 3-D field, which is modeled by Maxwell's equations. Hence, every single component of our model fits into the framework of port-Hamiltonian systems. However, the difficulty that arises in the coupling is the mismatch of dimensions. We will overcome this issue by lifting the dynamics of the 1-D transmission line on a 2-D surface, which fits the boundary ports of Maxwell's equations. Moreover, we go beyond the description of the model and show well-posedness of the coupled system.

Olof Staffans, Åbo Akademi University

Tall-Inner/Bi-Inner Factorizations in State/Signal Optimal Control

Abstract. As is well known, every externally stabilizable linear stationary discrete time i/s/o system Σ can be externally stabilized in an ℓ^2-optimal way, i.e., to each initial state there corresponds a unique optimal input which minimizes the future cost, given by the ℓ^2-norm of the combined input/output signal of the system. The optimal control minimizes the norm from an input disturbance to the combined input/output signal, and it is of state feedback type. The map $\mathfrak{F}_0$ from the disturbance to the optimal input/output signal has a flat frequency response $\widehat{\mathfrak{F}}_0$, which can be renormalized to be inner. It turns out that $\widehat{\mathfrak{F}}_0$ is weakly left invertible, and from $\widehat{\mathfrak{F}}_0$ it is possible to get a normalized weakly right coprime fractional H^∞ representation of the transfer function $\widehat{\mathfrak{D}}$ of Σ. This weakly right coprime representation uniuqe, up to a unitary similarity transformation.

If we instead of the ℓ^2-optimal state feedback use another state feedback to externally stabilize Σ, then we get a different fractional H^∞ representation of $\widehat{\mathfrak{D}}$, which we can normalize to be inner after factoring out an invertible spectral factor. It turns out that the new map $\mathfrak{F}$ from the disturbance to the i/o signal always contains $\mathfrak{F}_0$ as a left factor, and that the corresponding right factor in the factorization $\mathfrak{F} = \mathfrak{F}_0\mathfrak{F}_1$ is bi-inner. These two factors are essentially unique, and they are characterized by the fact that $\mathfrak{F}_0$ is the minimal factor and $\mathfrak{F}_1$ is the maximal factor among all possible tall-inner/bi-inner factorizations of $\mathfrak{F}$.

This talk is based on joint work with Mark Opmeer.

Sanne ter Horst, North-West University

The Infinite-Dimensional Bounded Real Lemma in Continuous Time

Abstract. The bounded real lemma (BRL) is a classical result in systems theory, which provides a linear matrix inequality criterium for dissipativity, via the Kalman-Yakubovich-Popov (KYP) inequality. Extensions to infinite dimensional systems, although already present in the work of Yakubovich, have only been studied systematically in the last few decades. In this context various notions of stability, observability and controllability exist, and depending on the hypothesis one may have to allow the KYP-inequality to have unbounded solutions which forces one to consider the KYP-inequality in a spatial form. In the present paper we consider the BRL for continuous time, infinite dimensional, linear well-posed systems via an adaptation of Willems' storage function approach. We avoid making use of the Cayley transform and work only in continuous time, since the Cayley transform does not preserve exponential stability, which is an important condition in the strict version of the BRL.

The talk is based on joint work with Joe Ball and Mikael Kurula.

Alexander A. Wierzba, University of Twente

On BIBO stability of infinite-dimensional linear state-space systems

Abstract. In this contribution we consider bounded-input-bounded-output (BIBO) stability of systems described by infinite-dimensional linear state-space representations. We use tools from operator theory (semigroups, system nodes) to fill the so far unattended gap of a formal definition and characterization of BIBO stability in this general case.

Furthermore, we provide several sufficient conditions guaranteeing BIBO stability and discuss to which extent this property is preserved under additive and multiplicative perturbations of the associated semigroup generator.

This contribution is based upon joint work with Felix L. Schwenninger and Hans Zwart. The preprint is available at https://arxiv.org/abs/2303.18148.

Bertin Zinsou, University of the Witwatersrand

Asymptotics of the eigenvalues of self-adjoint fourth order problems

Abstract. A regular fourth order differential equation which depends quadratically on the eigenvalue parameter λ is considered with classes of separable boundary conditions, where exactly one of the boundary conditions depends on λ linearly. This problem is described by a quadratic operator polynomial with self-adjoint operators. The location of the eigenvalues is investigated and the first four terms of the eigenvalues are provided.

References

[1] B. Zinsou, Asymptotics of the eigenvalues of self-adjoint fourth order problems, *Differ. Equ. Dyn. Syst.* (2021) https://doi.org/10.1007/s12591-021-00567-7.
[2] M. Möller, B. Zinsou, Asymptotics of the eigenvalues of self-adjoint fourth order differential operators with separated eigenvalue parameter dependent boundary conditions, *Rocky Mountain. J. Math.* **47(6)** (2017), 2013–2042. DOI: 10.1216/RMJ-2017-47-6-2013.
[3] Möller, M, Zinsou, B.: Self-adjoint Fourth Order Differential Operators With Eigenvalue Parameter Dependent Boundary Conditions. Quaestiones Math., **34**, 393–406 (2011). doi: 10.2989/16073606.2011.622913.

5.2. **Harmonic Analysis.**

Monday 2:30-4:30 in Porthania, Suomen Laki Hall

2:30-2:55	Igor Verbitsky *Nonlinear potential theory for equations of p-Laplace type with sub-natural growth terms*
3:00-3:25	Carlos Pérez *Extensions of Sobolev inequalities for linear and non-linear operators thru Harmonic Analysis*
3:30-3:55	Maria Carmen Reguera *Quadratic sparse domination beyond the integral realm*
4:00-4:25	Alexander Volberg *Harmonic analysis on boolean cube and beyond and some application to learning*

Tuesday 2:30-4:00 in Porthania, Suomen Laki Hall

2:30-2:55	Andrei Lerner *A boundedness criterion for the maximal operator on variable Lebesgue spaces*
3:00-3:25	Guillermo Rey *Greedy approximation algorithms for sparse collections*
3:30-3:55	Zoe Nieraeth *Extrapolation in quasi-Banach function spaces*

Tuesday 4:30-6:30 in Porthania, Suomen Laki Hall

4:30-4:55	Olli Saari *Quantitative characterization of reverse Hölder weights through Carleson conditions*
5:00-5:25	Antti Vähäkangas *Muckenhoupt distance functions and weakly porous sets*
5:30-5:55	Emma-Karoliina Kurki *Weak reverse Hölder inequalities on metric measure spaces*
6:00-6:25	Kim Myyryläinen *Parabolic Muckenhoupt weights*

Thursday 2:30-4:00 in Porthania, Suomen Laki Hall

2:30-2:55	Ji Li *Schatten class of commutator in the two weight setting and applications*
3:00-3:25	Jaakko Sinko *Off-diagonal two-weight boundedness and compactness of commutators*
3:30-3:55	Tuomas Oikari *On the boundedness and compactness of commutators along monomial curves*

Thursday 4:30-5:30 in Porthania, Suomen Laki Hall

4:30-4:55	Emil Vuorinen *Multiresolution Analysis and Zygmund Dilations*
5:00-5:25	Henri Martikainen *Entangled dilations: commutator estimates and multilinear aspects*
5:30-5:55	Tuomas Hytönen *Some remarks on convex body domination*

Friday 11:30-1:00 in Porthania, Suomen Laki Hall

11:30-11:55	Błażej Wróbel *On a dimension-free control of maximal Riesz transforms in terms of the corresponding Riesz transforms*
12:00-12:25	Lenka Slavíková *Local bounds for singular Brascamp-Lieb forms with cubical structure*
12:30-1:00	Tuomas Orponen *Progress on the Furstenberg set problem*

Friday 2:30-4:00 in Porthania, Suomen Laki Hall

2:30-2:55	David Seifert *Rates of decay in Tauberian theorems*
3:00-3:25	Valentina Casarino *Functional calculus in a general Gaussian setting*
3:30-3:55	Lars Niedorf *Restriction type estimates and spectral multipliers on two-step stratified Lie groups*

5.2.1. *Abstracts.*

Valentina Casarino, Università degli Studi di Padova

Functional calculus in a general Gaussian setting

Abstract. We consider the operator $m(\mathcal{L})$, where m is a function of Laplace transform type defined in the right half-plane, and $\mathcal{L}$ is an Ornstein–Uhlenbeck operator in $\mathbb{R}^n$, with drift given by a real matrix B whose eigenvalues have negative real parts. We discuss the mapping properties of $m(\mathcal{L})$ between Lebesgue spaces, with respect to the invariant measure in $\mathbb{R}^n$. This is a joint work with Paolo Ciatti and Peter Sjögren.

[1] V. Casarino, P. Ciatti and P. Sjögren, Spectral multipliers in a general Gaussian setting, `arXiv:2202.01547`.

Tuomas Hytönen, University of Helsinki, Finland

Some remarks on convex body domination

Abstract. Convex body domination is an important elaboration of the technique of sparse domination that has seen significant development and applications over the past ten years. In this talk, I will present an abstract framework for convex body domination, which also applies to Banach space -valued functions, and yields matrix-weighted norm inequalities in this setting. We explore applications to "generalised commutators", obtaining new examples of bounded operators among linear combinations of compositions of pointwise multipliers and a singular integral operator. The talk is based on an arXiv preprint with the same title.

Emma-Karoliina Kurki, Aalto University

Weak reverse Hölder inequalities on metric measure spaces

Abstract. It is well known that in Euclidean spaces the reverse Hölder inequality (RHI) implies the Muckenhoupt A_p condition. This statement remains true in reasonably general metric measure spaces, but the necessary condition appears to be unknown. If we weaken the RHI by increasing the support on the right-hand side and hereby give up on the doubling property, the problem ceases to exist. I discuss the theory of weak reverse Hölder inequalities (WRHI) and related Muckenhoupt weights in metric measure spaces of homogeneous type. In particular, we present characterizations of the WRHI, which is the nondoubling analogy of the RHI with an increasing support on the right-hand side. The talk is based on joint work with Juha Kinnunen and Carlos Mudarra.

Andrei Lerner, Bar-Ilan University

A boundedness criterion for the maximal operator on variable Lebesgue spaces

Abstract. Let $\mathcal{P}$ denote the class of all exponents $p(\cdot) : \mathbb{R}^n \to [1, \infty)$ such that the Hardy–Littlewood maximal operator M is bounded on the variable Lebesgue space $L^{p(\cdot)}$. In 2004, L. Diening found the first non-trivial sufficient condition on $p(\cdot)$ for which $p(\cdot) \in \mathcal{P}$. Since then the class $\mathcal{P}$ has been intensively studied in a number of works. However, the problem of finding a full characterization of $\mathcal{P}$ remained open. In this talk we will discuss a recent solution to this problem.

Ji Li, Macquarie University

Schatten class of commutator in the two weight setting and applications

Abstract. It is well-known that S. Bloom first established the characterization for boundedness of commutator of Hilbert transform in the two weight setting in 1985. We further characterized the compactness (joint work with Michael Lacey), and then provided a first step towards the theory on singular value estimate (joint work with Michael Lacey and Brett Wick). Comparing to Janson–Wolff and Rochberg–Semmes, our technique bypasses the use of Fourier. Applications to other settings will also be discussed, where Fourier is not available.

Henri Martikainen, Washington University in St. Louis

Entangled dilations: commutator estimates and multilinear aspects

Abstract. “Entangled” systems of dilations refer to dilations of the general type

$$(x_1, \ldots, x_m) \mapsto (\delta_1^{\lambda_{11}} \cdots \delta_k^{\lambda_{1k}} x_1, \ldots, \delta_1^{\lambda_{m1}} \cdots \delta_k^{\lambda_{mk}} x_m), \qquad \delta_1, \ldots, \delta_k > 0,$$

where $\mathbb{R}^d$ is viewed as the m-parameter *product space* $\mathbb{R}^d = \prod_{i=1}^m \mathbb{R}^{d_i}$. Invariances under such dilations appear naturally throughout analysis – for example, in $\mathbb{R}^3$ the *Zygmund dilations* $(x_1, x_2, x_3) \mapsto (\delta_1 x_1, \delta_2 x_2, \delta_1 \delta_2 x_3)$ are compatible with the group law $(x_1, x_2, x_3) \odot (y_1, y_2, y_3) = \big(x_1 + y_1, x_2 + y_2, x_3 + y_3 + \alpha(x_1 y_2 - y_1 x_2)\big)$ of the Heisenberg group.

The so-called Zygmund weights satisfy the usual A_p condition but with the supremum running over the Zygmund rectangles

$$\mathcal{R}_Z := \{R = I \times J \times K \subset \mathbb{R}^3 : \ell(K) = \ell(I)\ell(J)\}.$$

Our recent examples show that weighted estimates for Zygmund invariant singular integrals with the optimal class of Zygmund weights require that the underlying kernels satisfy a fast decay rate from the Zygmund manifold $|x_1 x_2| = |x_3|$, and do not otherwise necessarily hold. This e.g. limits the use of the so-called Cauchy integral trick for proving commutator estimates, since it relies on weighted estimates.

In this talk we discuss how our Zygmund multiresolution methods allow us to prove estimates for commutators $[b, T]f = bTf - T(bf)$ even in the regime where the singular integral T does not satisfy weighted estimates with Zygmund weights. We also discuss bilinear versions of the recent dyadic multiresolution methods for Zygmund dilations and the related $T1$ type results, and explain the state of the weighted estimates in the multilinear setting.

Kim Myyryläinen, Aalto University, Finland

Parabolic Muckenhoupt weights

Abstract. We discuss parabolic Muckenhoupt weights related to a doubly nonlinear parabolic partial differential equation (PDE). In the natural geometry of the PDE, the time variable scales to the power in the structural conditions for the PDE. Consequently, the Euclidean balls and cubes are replaced by parabolic rectangles respecting this scaling in all estimates. The main challenge is that in the definition of parabolic Muckenhoupt weights one of the integral averages

is in the past and the other one in the future with a time lag between the averages. The main theorems include a characterization of weak and strong type weighted norm inequalities for parabolic forward in time maximal operators. In addition, we give a Jones type factorization and a Coifman–Rochberg type characterization result for the parabolic Muckenhoupt weights. If time permits, we will discuss the parabolic Muckenhoupt A_∞ theory and related open problems.

Lars Niedorf, Kiel University

Restriction type estimates and spectral multipliers on two-step stratified Lie groups

Abstract. Let G be a two-step stratified Lie group, and let L be a homogeneous sub-Laplacian on G. We consider the operators $F(L)$ defined via functional calculus. Due to a celebrated theorem of Christ and Mauceri/Meda, which is an extension of the classical Mikhlin–Hörmander theorem, the operator $F(L)$ is of weak type $(1,1)$ and bounded on L^p for $1 < p < \infty$ whenever F satisfies a scale-invariant smoothness condition of order $s > Q/2$, where Q denotes the homogeneous dimension of the Lie group G. Müller/Stein and Hebisch discovered that for Heisenberg (-type) groups, this threshold can be pushed down to $s > d/2$, with d being the topological dimension of G. This result has since then been extended to various other settings. In this talk, I present one such spectral multiplier theorem in the setting of Métivier groups, which only requires $s > d\,(1/p - 1/2)$ as a regularity condition for boundedness on L^p. The proof is based on a new restriction type estimate, which holds even for arbitrary two-step stratified Lie groups.

Zoe Nieraeth, BCAM

Extrapolation in quasi-Banach function spaces

Abstract. Rubio de Francia's extrapolation theorem allows one to show that an operator that is bounded on weighted Lebesgue spaces for a single exponent and with respect to all weights in the associated Muckenhoupt class has to also be bounded for every exponent. As a matter of fact, in the previous years it has been shown that the operator has to be bounded on a much larger class of weighted spaces, including weighted Lorentz, variable Lebesgue, and Morrey spaces. In this talk I will discuss a unification and extension to a limited range setting of some of these results by presenting an extrapolation theorem for general quasi-Banach function spaces. Time permitting, I will also discuss an application to compact extrapolation that is part of a joint work with Emiel Lorist.

Tuomas Oikari, University of Jyväskylä

On the boundedness and compactness of commutators along monomial curves

Abstract. I discuss the boundedness and compactness of the commutator

$$bH_\gamma - H_\gamma b =: [b, H_\gamma] : L^p(\mathbb{R}^n) \to L^q(\mathbb{R}^n), \qquad p, q \in (1, \infty),$$

where H_γ is the Hilbert transform along a monomial curve $\gamma : \mathbb{R} \to \mathbb{R}^n$.

Tuomas Orponen, University of Jyväskylä

Progress on the Furstenberg set problem

Abstract. An s-Furstenberg set is a set $F \subset \mathbb{R}^2$ with the following property: for every $e \in S^1$, there is a line $l_e \subset \mathbb{R}^2$ parallel to e such that $\dim(F \cap l_e) \geq s$. Wolff conjectured in the 90s that every s-Furstenberg set $F \subset \mathbb{R}^2$ satisfies $\dim_{\mathrm{H}} F \geq (3s + 1)/2$. This remains open, but I will discuss recent partial progress, obtained in collaboration with Pablo Shmerkin.

Carlos Perez, University of the Basque Country and BCAM

Extensions of Sobolev inequalities for linear and non-linear operators thru Harmonic Analysis

Abstract. In this talk I will present some new extensions of the global classical Sobolev type inequalities for linear and non-linear operators. It is a joint collaboration with Cong Hoang and Kabe Moen.

Maria Carmen Reguera, Universidad de Malaga

Quadratic sparse domination beyond the integral realm

Abstract. We discuss quadratic sparse domination. A form of sparse domination that is suited for square functions. In this talk we will focus on building the theory for non-integral square functions. That includes square functions associated to non-elliptic PDEs. This talk is based on joint work with Gianmarco Brocchi and Julian Bailey.

Guillermo Rey, UAM

Greedy approximation algorithms for sparse collections

Abstract. I'll describe a greedy algorithm that approximates the Carleson constant of a collection of general sets. The approximation has a logarithmic loss in a general setting, but is optimal up to a constant with only mild geometric assumptions. The constructive nature of the algorithm gives additional information about the almost-disjoint structure of sparse collections. Some of the applications of the algorithm will be explored.

Olli Saari, Universitat Politècnica de Catalunya

Quantitative characterization of reverse Hölder weights through Carleson conditions

Abstract. Weights (non-negative locally integrable functions) satisfying a reverse Hölder condition are important in the study of harmonic measure and boundary value problems for elliptic partial differential equations. In this talk, I will explain a quantitative version of their characterization through a Carleson condition, originally observed by Fefferman, Kenig and Pipher without focus on estimation of constants. The quantitative estimate of this talk applies to almost flat weights and has direct consequences for elliptic measures on the boundary of the Euclidean upper half space for a certain class of differential operators with rough coefficients. This is based on joint work with Simon Bortz and Moritz Egert.

David Seifert, Newcastle University

Rates of decay in Tauberian theorems

Abstract. Let X be a Banach space and let $f \in \mathrm{BUC}(\mathbb{R}_+, X)$. Suppose there exists a function $F \in L^1_{\mathrm{loc}}(\mathbb{R}, X)$ such that

$$\lim_{\alpha\to 0+} \int_{\mathbb{R}} (\mathcal{L}f)(\alpha + is)\psi(s)\,\mathrm{d}s = \int_{\mathbb{R}} F(s)\psi(s)\,\mathrm{d}s$$

for all test functions $\psi \in C_c^\infty(\mathbb{R})$, where $\mathcal{L}$ denotes the Laplace transform. In other words, we assume that the distributional Fourier transform of f (when extended by zero to the real line) is a *regular* distribution. Then $\|f(t)\|_X \to 0$ as $t \to \infty$, by a famous Tauberian theorem due to Ingham and Karamata. This result and its variants have been used to give short proofs of the prime number theorem, but they also play an important role in the modern asymptotic theory of C_0-semigroups. In this talk I will give a simple proof of the Ingham–Karamata theorem and provide an overview of results from the last 15 years on decay rates in Tauberian theorems, with an emphasis on results for operator semigroups.

Jaakko Sinko, University of Helsinki

Off-diagonal two-weight boundedness and compactness of commutators

Abstract. By a commutator, we mean a commutator of a singular integral and a pointwise multiplier. They have important applications in the theory of partial differential equations. In this talk, a commutator maps between weighted Lebesgue spaces. Characterisations of boundedness/compactness of commutators have been studied in recent years. Most of these have been in the on-diagonal and/or one-weight setting, the former meaning that the domain and codomain Lebesgue spaces are associated with the same exponent. We complement these results by providing the missing two-weight off-diagonal cases, while allowing the singular integral operator to be any Calderón-Zygmund operator. The talk is based on joint work with T. Hytönen, T. Hänninen, E. Lorist and T. Oikari.

Lenka Slavíková, Charles University

Local bounds for singular Brascamp-Lieb forms with cubical structure

Abstract. In this talk, we discuss boundedness properties of certain multilinear forms that involve a Calderón-Zygmund kernel and possess a cubical structure. Special instances of these forms have found applications in enumerative combinatorics and ergodic theory. Passing through local and sparse bounds, we prove a range of L^p bounds for these forms, extending thus an earlier result which only allowed for one particular tuple of exponents. New in this context is the use of a modified strong maximal function. This is a joint work with P. Durcik and C. Thiele.

Antti Vähäkangas, University of Jyväskylä

Muckenhoupt distance functions and weakly porous sets

Abstract. In this talk we consider the following question. Assume that $E \subset \mathbb{R}^n$. When does $w(x) = \operatorname{dist}(x, E)^{-\alpha}$ belong to a Muckenhoupt class A_p? We report recent qualitative results that connect this question to geometry, by means of a weak porosity condition on E. Quantitative results are given in terms of a Muckenhoupt exponent of the set, assuming a priori its weak porosity. The results are based on a joint work with Theresa C. Anderson, Juha Lehrbäck and Carlos Mudarra.

Igor Verbitsky, University of Missouri, Columbia, Missouri, USA

Nonlinear potential theory for equations of p-Laplace type with sub-natural growth terms

Abstract. We intend to discuss recent results on weighted norm inequalities and nonlinear potential estimates associated with quasilinear equations of the type

$$-\Delta_p u = \sigma u^q + \mu, \quad u \geq 0 \quad \text{in } \mathbb{R}^n, \qquad \liminf_{x \to \infty} u(x) = 0,$$

where $\Delta_p u = \operatorname{div}(\nabla u \, |\nabla u|^{p-2})$ is the p-Laplace operator, $0 < q < p-1$, and $\mu, \sigma \geq 0$ are locally finite Borel measures on $\mathbb{R}^n$. Bilateral pointwise estimates of solutions, along with existence and uniqueness results, will be presented. More general quasilinear equations with $\mathcal{A}$-Laplace operators $\operatorname{div}\mathcal{A}(x, \nabla u)$ in place of Δ_p will be covered as well.

Alexander Volberg, Michigan State University, USA

Harmonic analysis on boolean cube and beyond and some application to learning

Abstract. Classical harmonic analysis (Poincaré inequalities, singular operators) being moved to discrete setting reveals many surprises.

It also proves to be rather useful in providing the solutions to old problems of Banach space theory, graph theory and theoretical computer science.

I will present examples of the successful applications of harmonic analysis on hypercube (and beyond) to solve some problems in these areas.

The emphasis will be made on non-commutative Bohnenblust—Hille inequality, dimension free discrete Remez inequality, and their application to learning big matrices by small number of queries.

Emil Vuorinen, University of Helsinki

Multiresolution Analysis and Zygmund Dilations

Abstract. Zygmund dilations are defined by $(x_1, x_2, x_3) \mapsto (sx_1, tx_2, stx_3)$, where $(x_1, x_2, x_3) \in \mathbb{R}^3$ and $s, t > 0$. The talk concerns multi-parameter singular integral operators which are adapted to these dilations. We will discuss a representation theorem and weighted estimates of such operators.

Błażej Wróbel, Institute of Mathematics, Polish Academy of Sciences and Institute of Mathematics, University of Wrocław

On a dimension-free control of maximal Riesz transforms in terms of the corresponding Riesz transforms

Abstract. We prove a dimension-free $L^p(\mathbb{R}^d)$, $1 < p < \infty$, estimate for (arbitrary) higher order maximal truncated Riesz transforms in terms of the corresponding Riesz transforms. Similar estimates are also obtained for vectors of maximal Riesz transforms. Our results are a dimension-free extension of the work of J. Mateu, J. Orobitg, C. Pérez, and J. Verdera.

Based on joint work with Maciej Kucharski and Jacek Zienkiewicz (Wrocław).

5.3. **Hilbert Function Spaces in One and Several Variables.**

Monday 2:30-4:30 in Porthania, PI chair: Alberto Dayan

2:30-2:55	William Ross *Sharp estimates of the solution to Bézout's identity*
3:00-3:25	Ryan O'Loughlin *Symmetric Tensor Products: An Operator Theory Approach*
3:30-3:55	Sebastian Toth *The generalized disk algebra*
4:00-4:30	Athanasios Kouroupis *Beurling integer systems with RH and Bohr's theorem*

Tuesday 2:30-4:00 in Porthania, PI chair: Ryan O'Loughlin

2:30-2:55	Javad Mashreghi *Hol*($\mathbb{D}$) *is gigantic!*
3:00-3:25	Nikolaos Chalmoukis *Holomorphic semigroups and Sarason's characterization of vanishing mean oscillation*
3:30-3:55	Christopher Felder *Forward Operator Monoids*

Tuesday 4:30-6:00 in Porthania, PI chair: Chris Felder

4:30-4:55	Catherine Bénéteau *Open problems related to optimal polynomial approximants in several variables*
5:00-5:25	Myrto Manolaki *Behaviour of optimal polynomial approximants on the unit circle*
5:30-5:55	Alessandro Monguzzi *Analysis on worm domains*

Thursday 2:30-4:00 in Porthania, PI chair: Nikos Chalmoukis

2:30-2:55	Andreas Hartmann *Carleson's formula in weighted Dirichlet spaces*
3:00-3:25	Georgios Tsikalas *Interpolating sequences for pairs of spaces*
3:30-3:55	Giuseppe Lamberti *Random interpolating sequences in the Nevanlinna class*

Thursday 4:30-6:00 in Porthania, PI chair: Myrto Manolaki

4:30-4:55	Soumitra Ghara *A local Douglas formula for higher order weighted Dirichlet-type integrals*
5:00-5:25	Santu Bera *Dirichlet-type spaces of the bidisc, Gleason's problem and Toral* 2*-isometries*
5:30-5:55	Nicola Arcozzi *Carleson measures for the Dirichlet space on the bidisc*

Friday 11:30-1:00 in Porthania, PI chair: George Tsikalas

11:30-11:55	Marco M. Peloso *Paley–Wiener and Bernstein spaces in several complex variables*
12:00-12:25	Subhankar Mahapatra *Vector valued de Branges spaces of entire functions based on pairs of Fredholm operator valued functions*
12:30-12:55	Cody B. Stockdale *On the* $T1$ *theorem for compactness of Calderón-Zygmund operators*

Friday 2:30-4:00 in Porthania, PI chair: Cody Stockdale

2:30-2:55	Nathan Wagner *Riesz-Kolmogorov Type Compactness Criteria in Function Spaces with Applications*
3:00-3:25	Poornendu Kumar *Distinguished varieties*
3:30-3:55	Jeet Sampat *Isomorphism problem of homogeneous subvarieties of the nc unit polydisk $\mathfrak{D}_d$*

Friday 4:30-5:30 in Porthania, PI chair: Michael Hartz

4:30-4:55	Jens de Vries *A unified approach to von Neumann's inequality and Crouzeix's conjecture*
~~5:00-5:25~~	~~John McCarthy~~ UPDATE *Inner composition operators on model spaces*

5.3.1. *Abstracts.*

Nicola Arcozzi, University of Bologna

Carleson measures for the Dirichlet space on the bidisc

Abstract. Carleson measures for the Dirichlet space on the bidisc are characterized in terms of a potential theoretic condition. This requires developing basic tools of potential theory in two parameters. The main difficulty to overcome is the failure of the maximum principle. Work in collaboration with P. Mozolyako, K.M. Perfekt, G. Sarfatti, A. Volberg, and others.

Catherine Bénéteau, University of South Florida

Open problems related to optimal polynomial approximants in several variables

Abstract. In this talk, I will discuss optimal polynomial approximants (opas) in certain Hilbert spaces of analytic functions of several variables. Opas are polynomials that indirectly approximate inverses of functions in the space. I will examine some stubborn open problems and discuss questions related to inner functions in these spaces.

Santu Bera, Indian Institute of Technology Kanpur, India.

Dirichlet-type spaces of the bidisc, Gleason's problem and Toral 2-isometries

Abstract. We introduce and study Dirichlet-type spaces $\mathcal{D}(\mu_1, \mu_2)$ of the unit bidisc $\mathbb{D}^2$, where μ_1, μ_2 are finite positive Borel measures concentrated on the unit circle. In general, these spaces differ from the Hilbert tensor product of Dirichlet-type spaces $\mathcal{D}(\mu_1)$ and $\mathcal{D}(\mu_2)$. We show that the coordinate functions z_1 and z_2 are multipliers for $\mathcal{D}(\mu_1, \mu_2)$ and the complex polynomials are dense in $\mathcal{D}(\mu_1, \mu_2)$. Further, we obtain the division property and solve Gleason's problem for $\mathcal{D}(\mu_1, \mu_2)$. In particular, we show that the commuting pair M_z of the multiplication operators M_{z_1} and M_{z_2} on $\mathcal{D}(\mu_1, \mu_2)$ defines a cyclic toral 2-isometry and M_z^* belongs to the Cowen-Douglas class $\mathbf{B}_1(\mathbb{D}^2)$. Moreover, we formulate a notion of the wandering subspace suitable to the multivariable operator theory and use it to show that an analytic toral 2-isometric pair T can be modeled as the multiplication pair M_z on $\mathcal{D}(\mu_1, \mu_2)$ if and only if $\ker(T^*)$ is a cyclic, wandering subspace for T. This is a joint work with Sameer Chavan and Soumitra Ghara.

Nikolaos Chalmoukis, University of Milano - Bicocca

Holomorphic semigroups and Sarason's characterization of vanishing mean oscillation

Abstract. It is a classical theorem of Sarason that an analytic function of bounded mean oscillation (BMOA), is of vanishing mean oscillation if and only if its rotations converge in norm to the original function as the angle of the rotation tends to zero. In a series of two papers

Blasco et al. have raised the problem of characterizing all semigroups of holomorphic functions that can replace the semigroup of rotations in Sarason's Theorem. In this talk we will give a complete answer to this question, in terms of a logarithmic vanishing oscillation condition on the infinitesimal generator of the semigroup. In particular we confirm the conjecture of Blasco et al. that all such semigroups are elliptic. We also investigate the analogous question for the Bloch and the little Bloch space and surprisingly enough we find that the semigroups for which the Bloch version of Sarason's Theorem holds are exactly the same as in the BMOA case. This is a joint work with Vassilis Daskalogiannis.

Jens de Vries, University of Twente

A unified approach to von Neumann's inequality and Crouzeix's conjecture

Abstract. We prove bounds for a class of algebra homomorphisms arising in the study of spectral sets by involving extremal functions and vectors. These bounds, which refine a recent result by Crouzeix–Greenbaum, are used to recover three celebrated results on spectral sets by Crouzeix–Palencia, Okubo–Ando and von Neumann in a unified way. We also discuss further applications.

This talk is based on joint work with Felix Schwenninger, arXiv:2302.05389.

Christopher Felder, Indiana University, Bloomington

Forward Operator Monoids

Abstract. This talk will introduce a *forward operator monoid*– a collection of bounded linear operators acting on a separable Hilbert space, which contains the identity, and has a strict monoid structure.

We will then discuss generalized inner and cyclic vectors for these monoids, and connect this to some interesting open problems in analysis, including some completeness and approximation problems in Hilbert function spaces.

Soumitra Ghara, Indian Institute of Technology Kanpur

A local Douglas formula for higher order weighted Dirichlet-type integrals

Abstract. We prove a local Douglas formula for higher order weighted Dirichlet-type integrals. With the help of this formula, we discuss the multiplier algebra of the associated higher order weighted Dirichlet-type spaces $\mathcal{H}_\mu$, induced by an m-tuple $\mu = (\mu_1, \ldots, \mu_m)$, $m \geq 1$, of finite non-negative Borel measures on the unit circle. In particular, it is shown that any weighted Dirichlet-type space of order m, for $m \geq 3$, forms an algebra under the pointwise product. We also show that every non-zero closed M_z-invariant subspace of $\mathcal{H}_\mu$, has codimension 1 property if $m \geq 3$ or μ_2 is finitely supported. This talk is based on a joint work with Rajeev Gupta and Ramiz Reza.

Andreas Hartmann, Université De Bordeaux

Carleson's formula in weighted Dirichlet spaces

Abstract. Outer functions are completely determined by the modulus of their boundary values in the Hardy space, and their norms are determined by $|f|$. When considering the related question in the Dirichlet space, things become more complicated since the norm in this space depends on the derivative of the function and oscillation has to be taken into account. More than 90 years ago, Jesse Douglas obtained a formula allowing to compute the Dirichlet integral – which says exactly when a holomorphic function is in the Dirichlet space – in terms of difference quotients of f on the unit circle $\mathbb{T}$. In the late 50s, work by Devinatz and Hirschman showed that the result generalizes to Dirichlet spaces $\mathcal{D}_\alpha$ weighted with $\omega_\alpha(z) = (1-|z|)^\alpha$. These results require the knowledge of f on $\mathbb{T}$ and not only its modulus $|f|$. It was Carleson, who managed

shortly afterwards to reveal a description of the Dirichlet integral (in the classical Dirichlet space) based only on the modulus of an outer function on $\mathbb{T}$. The aim of this work is to discuss such a result in the weighted Dirichlet space $\mathcal{D}_\alpha$.

This is joint work with Brahim Bouya

Athanasios Kouroupis, Norwegian University of Science and Technology (NTNU)

Beurling integer systems with RH and Bohr's theorem

Abstract. Given an arbitrary increasing sequence $q = \{q_n\}_{n\geq 1}$, $1 < q_n \to \infty$, such that $\{\log q_n\}_{n\geq 1}$ is linearly independent over $\mathbb{Q}$, we will denote by $\mathbb{N}_q = \{\nu_n\}_{n\geq 1}$ the set of numbers that can be written (uniquely) as finite products with factors from q, ordered in an increasing manner. The numbers q_n are known as Beurling primes, and the numbers ν_n are Beurling integers. The corresponding generalized Dirichlet series are of the form

$$f(s) = \sum_{n\geq 1} a_n \nu_n^{-s}.$$

The space $\mathcal{H}_q^2$ of generalized Dirichlet series with square summable coefficients is defined as

$$\mathcal{H}_q^2 = \left\{ f(s) = \sum_{n\geq 1} \frac{a_n}{\nu_n^s} : \|f\|^2_{\mathcal{H}_q^2} = \sum_{n\geq 1} |a_n|^2 < +\infty \right\}.$$

In the classical case, where $\nu_n = n$, Bohr's theorem holds: if f converges somewhere and has an analytic extension which is bounded in a half-plane $\{\Re s > \theta\}$, then it actually converges uniformly in every half-plane $\{\Re s > \theta + \varepsilon\}$, $\varepsilon > 0$. We prove, under very mild conditions, that given a sequence of Beurling primes, a small perturbation yields another sequence of primes such that the corresponding Beurling integers satisfy Bohr's condition, and therefore the theorem. Applying our technique in conjunction with a probabilistic method, we find a system of Beurling primes for which both Bohr's theorem and the Riemann hypothesis are valid. This provides a counterexample to a conjecture of H. Helson concerning outer functions in Hardy spaces of generalized Dirichlet series. The function $f \in \mathcal{H}_q^2$ is said to be outer if $\{fg : g \in \mathcal{H}_q^\infty\}$ is dense in $\mathcal{H}_q^2$.

Conjecture: If $\mathbb{N}_q$ is a Beurling system that satisfies Bohr's condition and f is outer in $\mathcal{H}_q^2$, then f never has any zeros in its half-plane of convergence.

This is joint work with Frederik Broucke and Karl–Mikael Perfekt.

Poornendu Kumar, Indian Institute of Science, Bangalore

Distinguished varieties

Abstract. A distinguished variety in $\mathbb{C}^2$ has been the focus of much research in recent years because of good reasons. One of the most important results in operator theory is Ando's inequality which states that for any pair of commuting contractions (T_1, T_2) and two variables polynomial p, the operator norm of of the operator $p(T_1, T_2)$ does not exceed the sup norm of p over the bidisc, i.e.,

$$\|p(T_1, T_2)\| \leq \sup_{(z_1,z_2)\in\mathbb{D}^2} |p(z_1, z_2)|.$$

A quest for an improvement of Ando's inequality led to the study of distinguished varieties. Since then, distinguished varieties are a fertile venue for function theoretic operator theory and connection to algebraic geometry.

In this talk, we shall see a new description of distinguished varieties with respect to the bidisc. It is in terms of the joint eigenvalue of a pair of commuting linear pencils. There is a characerization known of $\mathbb{D}^2$ due to a seminal work of Agler–McCarthy. We shall see how the Agler–McCarthy

characterization can be obtained from the new one and vice versa. Using the new characterization of distinguished varieties, we improved the known description by Pal–Shalit of distinguished varieties over the symmetrized bidisc:

$$\mathbb{G} = \{(z_1 + z_2, z_1 z_2) \in \mathbb{C}^2 : (z_1, z_2) \in \mathbb{D}^2\}.$$

Moreover, we will see complete algebraic and geometric characterizations of distinguished varieties with respect to $\mathbb{G}$.

This talk is based on joint works with Prof. Tirthankar Bhattacharyya and Prof. Haripada Sau.

Giuseppe Lamberti, University of Bordeaux

Random interpolating sequences in the Nevanlinna class

Abstract. The study of interpolating sequences for analytic functions in one or more complex variables is one of the main research areas in complex analysis. For many spaces, like Hardy spaces, these sequences are well understood while for others, like Dirichlet spaces, there exists a characterization which is not very easy to verify. In other circumstances, a characterisation does even not exist. In this scenario it is useful to consider a random setting, which can help us to understand when interpolation is “generic”. In particular in this talk we are going to recall deterministic interpolation in the Nevanlinna class, to then consider a random setting, more specifically a radial model (where points’ radii are fixed, while the arguments are uniformly distributed).

Subhankar Mahapatra, Indian Institute of Technology Ropar

Vector valued de Branges spaces of entire functions based on pairs of Fredholm operator valued functions

Abstract. We consider vector valued reproducing kernel Hilbert spaces (RKHS) $\mathcal{H}$ of entire functions associated with operator valued kernel functions. In Particular, we intend to observe the transition of the theory of de Branges spaces based on matrix valued reproducing kernels to the operator valued reproducing kernels. For this purpose, we construct the de Branges operators $\mathfrak{E} = (E_-, E_+)$ analogous to de Branges matrices with the help of pairs of Fredholm operator valued entire functions on $\mathfrak{X}$, where $\mathfrak{X}$ is a complex seperable Hilbert space. In this direction, we discuss a few explicit examples of these de Branges operators. We also highlight the intimate connection of the multiplication operator by the independent variable with the de Branges spaces. Finally, we present a characterization of the newly defined RKHS $\mathcal{B}(\mathfrak{E})$ based on the de Branges operator $\mathfrak{E} = (E_-, E_+)$ under some special restrictions.
This presentation is part of the joint work “Vector valued de Branges spaces of entire functions based on pairs of Fredholm operator valued functions and functional model,”arXiv:2302.06297v1 [math.FA], with my PhD supervisor Dr. Santanu Sarkar.

Myrto Manolaki, University College Dublin

Behaviour of optimal polynomial approximants on the unit circle

Abstract. The notion of *optimal polynomial approximants* was introduced to investigate the phenomenon of cyclicity in certain Hilbert spaces of analytic functions on the unit disc, including the classical Hardy, Bergman and Dirichlet spaces. In this talk, we will focus on the limiting behaviour of sequences of optimal polynomial approximants on subsets E of the unit circle. In particular, we will see that if E has zero arclength measure, then, for most functions in the Hardy space, this behaviour is extremely chaotic. We will also discuss properties of such functions and related open questions. (Based on joint work with Catherine Bénéteau, Oleg Ivrii and Daniel Seco.)

Javad Mashreghi, Laval University, Quebec, Canada

Hol($\mathbb{D}$) is gigantic!

Abstract. There are several ways to justify that Hol($\mathbb{D}$), the space of all holomorphic functions on the open unit disc $\mathbb{D}$, is huge. For example, several analytic functions with pathological boundary behaviors have been constructed/found for specific purposes since early twentieth century, e.g., by Wolff (1921), Littlewood (1930), Frostman (1942), Rudin (1954), Bagemihl–Seidel (1954), Lohwater–Piranian (1957), MacLane (1962), Hayman (1964), Ahern (1979), Aleman–Richter-Ross (1998), etc. In this short talk we look at the problem from a different point of view: Every Banach space has a copy inside Hol($\mathbb{D}$). More precisely, if Y is any separable, infinite-dimensional, complex Banach space, and $(\alpha_n)_{n\geq 0}$ is any positive sequence such that $\lim_{n\to\infty} \alpha_n^{1/n} = 1$, then there exists a Banach holomorphic function space $X \subset$ Hol($\mathbb{D}$) such that

(i) X is isometrically isomorphic to Y,
(ii) X contains all functions holomorphic in a neighborhood of $\overline{\mathbb{D}}$,
(iii) polynomials are dense in X,
(iv) $\|z^n\|_X = \alpha_n$ for all $n \geq 0$.

This result has been extended by José Bonet Solves to Fréchet spaces.

The second part of the talk is based on joint work with Thomas Ransfrod.

John McCarthy, Washington University

Inner composition operators on model spaces

Abstract. In 2013 Mashreghi and Shabankhah described when composition with an inner function ϕ mapped a model space into itself. When ϕ is not elliptic, their analysis depends on when the equation $\Theta \circ \phi = \tau\Theta$ has a solution with an inner function Θ and a unimodular number τ. We will discuss when this equation can be solved.

This is joint work with Isabelle Chalendar and Pavel Gumenyuk.

Alessandro Monguzzi, University of Bergamo

Analysis on worm domains

Abstract. In this talk I will discuss some aspect of the analysis on worm domains with emphasis on the regularity of the Bergman and Szegő projections on such domains.

Ryan O'Loughlin, University of Leeds

Symmetric Tensor Products: An Operator Theory Approach

Abstract. Although tensor products and their symmetrisation have appeared in mathematical literature since at least the mid nineteenth century, they rarely appear in the function-theoretic operator theory literature. In this talk I will introduce the symmetric and antisymmetric tensor products from an operator theoretic point of view. I will present results concerning some of the most fundamental operator-theoretic questions in this area, such as finding the norm and spectrum of the symmetric tensor products of operators. I will then work through some examples of symmetric tensor products of familiar operators, such as the unilateral shift, the adjoint of the shift and diagonal operators.

Marco M. Peloso, Università degli Studi di Milano

Paley–Wiener and Bernstein spaces in several complex variables

Abstract. In these lectures I will present some recent results on Paley–Wiener and Bernstein spaces in one and several variables. I will first recall some basic properties of the classical Bernstein spaces in one variable. Then I will illustrate the new results and some open problems. The first result is the characterisation of the dual of $\mathcal{B}^1_\kappa$ in one variable. Then I will switch to several variables and introduce a new class of Bernstein spaces and describe several functional properties of theses spaces. These spaces can be described as the space of entire functions of exponential type whose restriction to a Siegel CR manifold is L^p integrable with respect to a natural measure. These lectures are based on works in collaborations with C. Bellavita, with A. Monguzzi and M. Salvatori, and with M. Calzi.

William Ross, University of Richmond

Sharp estimates of the solution to Bézout's identity

Abstract. A well-known theorem of Étienne Bézout (1730-1783) says that if $A, B \in \mathbb{C}[z]$ with no common roots, then there are $R, S \in \mathbb{C}[z]$ with $\deg R \leq \deg B - 1$ and $\deg S \leq \deg A - 1$ such that $AR + BS \equiv 1$. In this joint work with E. Fricain, A. Hartmann, and D. Timotin, we obtain sharp estimates of the coefficients of R and S that is reminiscent of estimates in the corona theorem for H^∞. As an application of these estimates, we give a corona theorem for certain de Branges–Rovnyak spaces.

Jeet Sampat, Technion - Israel Institute of Technology

Isomorphism problem of homogeneous subvarieties of the nc unit polydisk $\mathfrak{D}_d$

Abstract. For a subvariety $\mathfrak{V} \subset \mathfrak{D}_d$, we consider $A(\mathfrak{V}) = \overline{\mathbb{C}\langle z_1, \ldots, z_d\rangle|_{\mathfrak{V}}}^{\|\cdot\|_\infty}$, i.e. the sup-norm closure of free polynomials in $H^\infty(\mathfrak{V})$ – the algebra of bounded nc functions on $\mathfrak{V}$. We show that when $\mathfrak{V}$ is homogeneous, $A(\mathfrak{V})$ is completely isometrically isomorphic to $A(\mathfrak{D}_d)/I(\mathfrak{V})$, where $I(\mathfrak{V})$ is the ideal of functions in $A(\mathfrak{D}_d)$ that vanish on $\mathfrak{V}$. Using this identification, we investigate the problem of when $A(\mathfrak{V})$ and $A(\mathfrak{W})$ are completely isometrically isomorphic to one another for two homogeneous subvarieties $\mathfrak{V}$ and $\mathfrak{W}$. If time permits, we shall extend some of our results to a very general class of domains, namely *nc analytic polyhedra* with minor additional constraints.

On our way, we also showcase a dramatic departure from the rich function theory that exists for the nc unit row-ball $\mathfrak{B}_d$. For instance, $H^\infty(\mathfrak{B}_d)$ has a natural faithful representation on the nc Drury-Arveson space $\mathcal{H}^2_d$ as its multiplier algebra, however we can show that $H^\infty(\mathfrak{D}_d)$ can never be represented as the multiplier algebra of a nc RKHS over $\mathfrak{D}_d$ where the monomials are orthogonal to one another!

This research is a part of on-going joint work with Orr Shalit.

Cody B. Stockdale, Clemson University

On the $T1$ theorem for compactness of Calderón-Zygmund operators

Abstract. We give a new formulation of the $T1$ theorem for compactness of Calderón-Zygmund singular integral operators. In particular, we prove that a Calderón-Zygmund operator T is compact on $L^2(\mathbb{R}^n)$ if and only if $T1, T^*1 \in \mathrm{CMO}(\mathbb{R}^n)$ and T is weakly compact. Compared to existing compactness criteria, our characterization more closely resembles the classical $T1$ theorem for boundedness and avoids technical conditions involving the Calderón-Zygmund kernel. Our proof follows a simple, self-contained argument and relies on a characterization of a general class of almost-diagonalized operators.

Sebastian Toth, Saarland University

The generalized disk algebra

Abstract. The algebra $A(\mathbb{D})$ of continuous functions on the closed unit disk, which are analytic on the interior part, the so called disk algebra, is often a useful tool in function theory and functional analysis. The algebra can be also seen as the closure of the polynomials in H^∞, which is the multiplier algebra of the Hardy space H^2. Given a unitarily invariant reproducing kernel Hilbert space H, for example a radially weighted Besov space, I will talk about a generalized concept, namely the closure of the polynomials $A(H)$ in the multiplier algebra $M(H)$ with respect to the multiplier norm.

Georgios Tsikalas, Washington University in St. Louis

Interpolating sequences for pairs of spaces

Abstract. In 2019, Aleman, Hartz, McCarthy and Richter characterized interpolating sequences for multiplier algebras of Hilbert function spaces with a complete Pick kernel. We discuss an extension of their result to pairs of spaces $(\mathcal{H}_s, \mathcal{H}_\ell)$, where s, ℓ are reproducing kernels on a set X, s is a complete Pick kernel and ℓ/s is also a kernel. Specifically, it turns out that a sequence is interpolating for

$$\mathrm{Mult}(\mathcal{H}_s, \mathcal{H}_\ell) = \{\phi : X \to \mathbb{C} \mid \phi \cdot f \in \mathcal{H}_\ell,\ \forall f \in \mathcal{H}_s\}$$

in this setting if and only if it generates a Carleson measure for $\mathcal{H}_s$ and is n-weakly separated by ℓ for every $n \geq 2$, the latter condition being slightly stronger than weak separation by ℓ. We also exhibit examples to show that, unlike the case of a single complete Pick kernel $s = \ell$, n-weak separation cannot, in general, be replaced by weak separation by ℓ.

Nathan Wagner, Brown University

Riesz-Kolmogorov Type Compactness Criteria in Function Spaces with Applications.

Abstract. The classical Riesz-Kolmogorov theorem gives a characterization of the precompact subsets of L^p in terms of a uniform spatial decay condition and a uniform continuity condition. In this talk, we will introduce new variants of this result relevant to complex analysis and operator theory. These precompactness characterizations include statements in the abstract setting of Hilbert spaces with continuous Parseval frames, as well as a variety of Hilbert and Banach function spaces, including the Paley-Wiener space and Besov-Sobolev spaces of analytic functions on the unit ball (a scale of spaces which includes the Hardy, Bergman, and Dirichlet spaces). We will also briefly mention some applications of these results to Toeplitz and Hankel operators. This talk is based on joint work with Mishko Mitkovski, Cody Stockdale, and Brett Wick.

5.4. **Inverse Problems.**

Monday 2:30-4:30 in Language Center, Room 206

2:30-2:55	Matti Lassas *Mapping properties of neural networks, neural operators, and inverse problems*
3:00-3:25	Leo Tzou *A geometric user interface for analytic WF calculus for FIO*
3:30-3:55	Yi-Hsuan Lin *Inverse source problems of local, nonlocal and nonlinear equations*
4:00-4:25	Teemu Tyni *Stability of an inverse problem for a nonlinear wave equation*

Tuesday 2:30-4:00 in Metsätalo, Hall 4

2:30-2:55	Samuli Siltanen *TBA*
3:00-3:25	Mikko Salo *Instability in inverse problems*
3:30-3:55	Catalin Carstea *An inverse problem for the porous medium equation*

Tuesday 4:30-6:30 in Porthania, P673

4:30-4:55	Luisa Faella *Monotonicity Principle for Nonlinear Electrical Conductivity Tomography*
5:00-5:25	Pavel Dubovski *Recovery of the integral kernel in the kinetic fragmentation equation*
5:30-5:55	Lady Estefania Murcia Lozano *Sinc method for spectrum completion and inverse Sturm-Liouville problems*

Thursday 2:30-4:00 in Metsätalo, Room 7

2:30-2:55	Lassi Päivärinta *The Hilbert transform on finite curves and Schiffer's inverse scattering problem in 2D*
3:00-3:25	Medet Nursultanov *Disjoint data inverse problem on manifolds with quantum chaos bounds*
3:30-4:00	Giovanni Covi *Reducing the fractional Calderón problem to the classical case via the Caffarelli-Silvestre extension*

5.4.1. *Abstracts.*

Catalin Carstea, National Yang Ming Chiao Tung University, Hsinchu, Taiwan

An inverse problem for the porous medium equation

Abstract. The porous medium equation is a degenerate parabolic type quasilinear equation that models, for example, the flow of a gas through a porous medium. In this talk I will present recent results on uniqueness in the inverse boundary value problem for this equation. Inverse boundary value problems are the problems of reconstructing the unknown coefficients of a partial differential equation from the knowledge of the set of all possible Cauchy data on the boundary. The talk is based on work with T. Ghosh & G. Nakamura and T. Ghosh & G. Uhlmann.

Giovanni Covi, University of Bonn

Reducing the fractional Calderón problem to the classical case via the Caffarelli-Silvestre extension

Abstract. We show that any uniqueness result for the classical (possibly anisotropic) Calderón problem implies a uniqueness result for the related fractional Calderón problem. In particular, the partial DN map for the fractional problem for $n \geq 3$ determines the full DN map for the local problem. Our method involves the Caffarelli-Silvestre extension. We also highlight the natural obstructions in the reversal of this procedure. This is a joint work with Tuhin Ghosh, Angkana Rüland and Gunther Uhlmann.

Pavel Dubovski, Stevens Institute of Technology

Recovery of the integral kernel in the kinetic fragmentation equation

Abstract. We address the recovery kernel problem for the equation

$$\frac{\partial c(x,t)}{\partial t} = -c(x,t)\int_0^x F(y,x-y)dy + 2\int_x^\infty c(y,t)F(x,y-x)dy.$$

Kernel F is supposed to be close to a constant. If the unknown kernel F is either above or below that constant, then we prove the uniqueness of the solution. The method is based on the transformation of the problem to the inverse problem for the right-hand side of the equation with a modified linear operator. We form a variational functional, its minimization uses the system of direct, adjoint and control equations. Then working backwards we recover the kernel in the original problem. We assume that the only available information are partial observations of the solution at two time instants. This is a joint research with O. Alomari.

Luisa Faella, University of Cassino and Southern Lazio, Italy

Monotonicity Principle for Nonlinear Electrical Conductivity Tomography

Abstract. We treat an inverse electrical conductivity problem which deals with the reconstruction of nonlinear electrical conductivity starting from boundary measurements in steady currents operations. In this framework, a key role is played by the monotonicity principle, which establishes a monotonic relation connecting the unknown material property to the (measured) Dirichlet-to-Neumann operator (DtN). Monotonicity Principles are the foundation for a class of noniterative and real-time imaging methods and algorithms.We prove that the monotonicity principle for the Dirichlet energy in nonlinear problems holds under mild assumptions. Then, we show that apart from linear and p-Laplacian cases, it is impossible to transfer this monotonicity result from the Dirichlet energy to the DtN operator. To overcome this issue, we introduce a new boundary operator, identified as the average DtN operator.

Authors: A. Corbo Esposito, L. Faella, G. Piscitelli, R. Prakash, A. Tamburrino

Matti Lassas, University of Helsinki, Finland

Mapping properties of neural networks, neural operators, and inverse problems

Abstract. We will consider mapping properties of neural networks and neural operators which are infinite dimensional generalizations of neural networks. In particular, we consider the injectivity of neural networks and universal approximation property of injective neural networks. In addition, we study approximation of probability measures using neural networks that are compositions of invertible flow networks and injective layers and present applications in inverse problems. The results have been done in collaboration with M. de Hoop, I. Dokmanic, T. Furuya, P. Pankka and M. Puthawala.

Yi-Hsuan Lin, National Yang Ming Chiao Tung University, Hsinchu, Taiwan

Inverse source problems of local, nonlocal and nonlinear equations

Abstract. In this talk, we perform inverse source problems for local, nonlocal and nonlinear equations. Unlike linear differential equations, which always has gauge invariance. We investigate the gauge symmetry could be broken for several nonlinear and nonlocal equations, which leads unique determination results for certain equations. The talk is based on several joint works with Yavar Kian, Tony Liimatainen and Hongyu Liu.

Lady Estefania Murcia Lozano (Center for Research and Advanced Studies of the National Poly- technic Institute (CINVESTAV), Queretaro, Mexico)

Sinc method for spectrum completion and inverse Sturm-Liouville problems

Abstract. A new representation for solutions of the Sturm-Liouville equation $-y''+q(x)y = \lambda y$, $x \in (0, L)$ with $q(x) \in W_2^{-1}(0, L)$ is obtained. The solutions are represented as cardinal series. For this purpose, we apply the Whittaker–Shannon–
Kotelnikov sampling theorem for functions from the Paley–Wiener space. Several papers use the sinc method to solve Sturm-Liouville problems, e.g., in [1], [2]. In the present work we develop its application to inverse Sturm-Liouville problems. Additionally, we complete the spectrum, that is, compute more eigenvalues without any additional information on the potential and with the aid of the new cardinal series representations for solutions of the Sturm-Liouville equation. Based on these results, we propose a method for numerical solution of the inverse Sturm-Liouville problem of recovering the potential from two spectra. The approach is analogous to that from [3] where the Neumann series of Bessel functions representations for solutions of Sturm-Liouville equations were used.

The talk is based on a joint work with Vladislav V. Kravchenko.

[1] M. H. Annaby and R. M. Asharabi, On sinc-based method in computing eigenvalues of boundary-value problems, SIAM J. Num. Anal., 64, 671–690, 2008.

[2] A. Boumenir and B. Chanane, Eigenvalues of Sturm-Liouville systems using sampling theory, Appl. Anal., 62, 323-334, 1996.

[3] V. V. Kravchenko, Spectrum completion and inverse Sturm–Liouville problems, Math Meth Appl Sci., 46, issue 5, 5821-5835, 2023.

Medet Nursultanov, University of Helsinki

Disjoint data inverse problem on manifolds with quantum chaos bounds

Abstract. We consider the inverse problem to determine a smooth compact Riemannian manifold (M, g) from a restriction of the source-to-solution operator, $\Lambda_{\mathcal{S},\mathcal{R}}$, for the wave equation on the manifold. Here, $\mathcal{S}$ and $\mathcal{R}$ are open sets on M, and $\Lambda_{\mathcal{S},\mathcal{R}}$ represents the measurements of waves produced by smooth sources supported on $\mathcal{S}$ and observed on $\mathcal{R}$. We demonstrate that $\Lambda_{\mathcal{S},\mathcal{R}}$ determines the manifold (M, g) uniquely under the following spectral bound condition for the set $\mathcal{S}$: There exists a constant $C > 0$ such that any normalized eigenfunction ϕ of the Laplace-Beltrami operator on (M, g) satisfies

$$1 \leq C \|\phi|_{\mathcal{S}}\|_{L^2(\mathcal{S})}.$$

(Joint work with Matti Lassas, Lauri Oksanen, Lauri Ylinen)

Lassi Päivärinta, Tallinn University of Technology, Estonia

The Hilbert transform on finite curves and Schiffer's inverse scattering problem in 2D

Abstract. In the talk we discuss the Hilbert transform defined in function spaces on finite smooth curves in the complex plane. Especially we are interested in the inverse transform when it exists. This question is classically (Carleman, Tricomi) well understood if the curve is an interval (a piece of a line).

In addition, we study the inverse obstacle scattering problem in R^n called also the Schiffer's problem and explain the connection to the Hilbert transform in the case $n = 2$.

This is a joint work with Emilia Blåsten, LUT University, Petri Ola, University of Helsinki and Sadia Sadique, Taltech University

Mikko Salo, University of Jyväskylä

Instability in inverse problems

Abstract. Many inverse problems, such as the Calderón problem related to electrical imaging, are known to be highly sensitive to noise. Such problems are called ill-posed or unstable, as opposed to being well-posed (a notion introduced by J. Hadamard in 1902). Instability is a crucial feature in the mathematical theory of these problems and it affects the performance and design of computational methods for solving them.

We discuss a general framework for studying instability in inverse problems based on smoothing/compression properties of the forward map, together with estimates for entropy and capacity numbers in relevant function spaces. The methods apply to various linear and nonlinear inverse problems as well as unique continuation problems, and they are valid for general geometries and low regularity coefficients. We will also discuss recent results explaining from this point of view the phenomenon of stability increasing with frequency in certain inverse scattering problems.

This talk is based on joint works with Herbert Koch and Angkana Rüland (Bonn) and with Pu-Zhao Kow (Jyväskylä) and Sen Zou (Fudan).

Samuli Siltanen, University of Helsinki, Finland

To be announced

Teemu Tyni, University of Oulu

Stability of an inverse problem for a nonlinear wave equation

Abstract. An important aspect of inverse problems is the stability of the recovery of the quantity of interest. In practise one would like an estimate on the size of error one makes in the reconstruction given the size of error in the measurement. I will discuss the stability of reconstruction of a potential function in a nonlinear wave equation on a Lorentzian manifold. We show that by using the nonlinearity of the equation, the potential function can be recovered in a Hölder stable way from the Dirichlet-to-Neumann map. The proof is constructive and the main tools we employ are the higher order linearization method and careful use of Gaussian beam solutions to the wave equation.

This talk is based on a joint work with Matti Lassas, Tony Liimatainen and Leyter Potenciano-Machado.

Leo Tzou, University of Amsterdam

A geometric user interface for analytic WF calculus for FIO

Abstract. We find a geometric description for the propagation of regularity for a special class of elliptic FIOs in the analytic category. This can be seen as the initial step towards a calculus for analytic FIOs in the double fibration setting as in the smooth case. We describe various transforms in integral geometry that can fit into this setting.

5.5. Model Spaces, Their Operators, and Applications.

Monday 2:30-4:30 in Porthania, P673 chair: Javad Mashreghi

2:30-2:55	Dmitry Khavinson *Some open problems in star-shift invariant subspaces*
3:00-3:25	Christopher Felder *Inner functions and Optimal Polynomial Appoximants*
3:30-3:55	Catherine Bénéteau *Distribution of zeros of optimal polynomial approximants*
4:00-4:25	Dragan Vukotić *Domination (maximum) principle for weighted Bergman spaces*

Tuesday 2:30-4:00 in Porthania, P673 chair: William Ross

2:30-2:55	Mishko Mitkovski *Quantitative Estimates for Riesz Sequence Constants in Model Spaces*
3:00-3:25	Bartosz Malman *Boundary interpolation in model spaces*
3:30-3:55	Alberto Dayan *Interpolating Sequences for Complete Pick Spaces*

Tuesday 4:30-6:00 in Metsätalo, Hall 4 chair: Ryan O'Loughlin

4:30-4:55	Adem Limani *M_z-invariant subspaces in the Bloch space and Model spaces*
5:00-5:25	Robert T.W. Martin *Lebesgue decomposition via reproducing kernels*
5:30-5:55	Gabriel Prajitura *Some questions in Linear Dynamics*

Thursday 2:30-4:00 in Porthania, P673 chair: Javad Mashreghi

2:30-2:55	Marek Ptak *General (X, Y)–operator invariance and applications*
3:00-3:25	Marcu-Antone Orsoni *Separation of singularities in spaces of analytic functions*
3:30-3:55	Alexander Kheifets *Automorphic Carathéodory—Julia Theorem and related boundary interpolation*

Friday 11:30-1:00 in Porthania, P673 chair: Ryan O'Loughlin

11:30-11:55	Victor Bailey *Frames via Unilateral Iterations of Bounded Operators*
12:00-12:25	Frej Dahlin *Generalizations of de Branges-Rovnyak spaces*
12:30-12:55	Apoorva Singh *Multiplication by a finite Blaschke factor on generalized Hardy spaces*

Friday 2:30-4:00 in Porthania, P673 chair: William Ross

2:30-2:55	Muyan Jiang *Numerical Range of Reciprocal Matrices*
3:00-3:25	Luis Manuel Tovar *Bicomplex Bergman and Bloch Spaces*
3:30-3:55	Hyun Kwon *Similarity of Cowen-Douglas Operator Tuples*

5.5.1. *Abstracts.*

Victor Bailey, Georgia Institute of Technology

Frames via Unilateral Iterations of Bounded Operators

Abstract. Dynamical Sampling is, in a sense, a hypernym classifying the set of inverse problems arising from considering samples of a signal and its future states under the action of a bounded linear operator. Recent works in this area consider questions such as when can a given frame for a separable Hilbert Space, $\{f_k\}_{k \in I} \subset H$, be represented by iterations of an operator on a single vector and what are necessary and sufficient conditions for a system, $\{T^n \varphi\}_{n=0}^{\infty} \subset H$, to be a frame? In this talk, we will discuss the connection between frames given by iterations of a bounded operator and the theory of model spaces in the Hardy-Hilbert Space as well as necessary and sufficient conditions for a system generated by the orbit of a pair of commuting bounded operators to be a frame. This is joint work with Carlos Cabrelli.

Catherine Bénéteau, University of South Florida

Distribution of zeros of optimal polynomial approximants

Abstract. In this talk, I will examine the limiting zeros of optimal polynomial approximants (opas) in certain Hilbert spaces of analytic functions of the disk. Opas are polynomials that indirectly approximate inverses of functions in the space. These polynomials have zeros which made them interesting to engineers in the 1970s in the context of digital filters in signal processing. I will talk about the analogue of the theorem of Jentzsch from 1914 for opas and discuss open problems related to their zero distributions.

Frej Dahlin, Lund University

Generalizations of de Branges-Rovnyak spaces

Abstract. Some natural generalizations of sub-Hardy (de Branges-Rovnyak) spaces are Hilbert spaces of analytic functions in the disc, where the backward shift acts as a contraction. The sub-Bergman spaces introduced by K. Zhu are a different generalization which is interesting in its own right. These are essentially a particular case of Hilbert spaces of analytic functions in the disc, where the forward shift satisfies a famous hereditary inequality of S. Shimorin. The basic observation used in the talk is that such spaces are reproducing kernel Hilbert spaces whose kernel is obtained by dividing a given kernel (like the Szegö or Bergman kernel) by a normalized complete Nevanlinna-Pick kernel. The aim is to deduce some general properties of these objects. The talk will be centered around some approximation results in these spaces. Joint work with A. Aleman.

Alberto Dayan, Saarland University

Interpolating Sequences for Complete Pick Spaces

Abstract. In this talk we will review some notions of separated and interpolating sequences for reproducing kernel Hilbert spaces and their multiplier algebras. In particular, we will focus on the case in which such Hilbert spaces enjoy the complete Pick property. Many well known and well studied Hilbert spaces of analytic functions have such property: some main examples are the Dirichlet and the Hardy space on the unit disc, or the Drury-Arveson space on the unit ball. Interpolating sequences for the Hardy space are very well understood since the celebrated work of Carleson and Shapiro&Shields, and many efforts have been spent in order to extend such results for the Dirichlet space and the Drury-Arveson space. For instance, Bishop showed that any strongly separated sequence for the Dirichlet space is onto interpolating, while Marhsall&Sundberg constructed a sequence in the unit disc that is strongly separated for the Dirichlet space but that doesn't generate a Carleson measure for the Dirichlet space. We show that the positive result of Bishop holds true in any complete Pick reproducing kernel Hilbert

space: in any such space, all strongly separated sequences are onto interpolating (that is, every uniformly minimal sequence of normalized kernel functions enjoys a lower Riesz bound). Moreover, we show that for the Drury-Arveson space on the two dimensional unit ball one can construct a sequences that is strongly separated but that doesn't even generate a finite measure.

This is a joint work with Nikolaos Chalmoukis and Michael Hartz.

Christopher Felder, Indiana University, Bloomington

Inner functions and Optimal Polynomial Appoximants

Abstract. Consider $\mathcal{H}$, a reasonably nice Hilbert space of analytic functions, and let $f \in \mathcal{H}$. The nth optimal polynomial approximant to $1/f$ is the polynomial that solves the minimization problem

$$\min_{p \in \mathcal{P}_n} \|pf - 1\|_{\mathcal{H}},$$

where $\mathcal{P}_n$ is the set of complex polynomials of degree less than or equal to n. These approximants were devised in an attempt understand cyclic vectors for the forward shift. However, Bénéteau et al. uncovered an interesting relationship between inner functions and their optimal approximants– the approximants are all constant. It turns out this actually characterizes inner functions.

In this talk, we will discuss the interplay between inner functions and optimal polynomial approximants, not only in Hilbert spaces, but also in Banach spaces (based partly on joint work w/ R. Centner & R. Cheng).

Dmitry Khavinson, University of South Florida

Some open problems in star-shift invariant subspaces

Abstract. We shall discuss some open problems concerning existence of functions with singular inner factors in model spaces generated by infinite Blaschke products. The problem stems from questions in optimal control, some results were obtained two decades ago in the Akeroyd - Khavinson - Shapiro paper , yet many open questions remain. We shall also discuss the problem of existence of smooth functions in star-shift invariant subspaces.

Alexander Kheifets, University of Massachusetts Lowell, USA

Automorphic Carathéodory - Julia Theorem and Related Boundary Interpolation.

Abstract. Let w be an analytic function on the unit disk, $|w(\zeta)| \le 1$. Let t_0 be a point on the unit circle, $|t_0| = 1$. The classical Carathéodory - Julia Theorem states in particular that if w and w' have nontangential boundary values w_0, $|w_0| = 1$ and w_0', respectively, at this point t_0, then

$$t_0 \frac{w_0'}{w_0} \ge 0. \tag{1}$$

Moreover, the theorem states that $\dfrac{w(\zeta) - w_0}{\zeta - t_0}$ belongs to the Hardy class H^2.

Conversely, for every numbers w_0, $|w_0| = 1$ and w_0' such that (1) holds there exists an analytic function w on the unit disk, $|w(\zeta)| \le 1$ with nontangential boundary values of w and w' at t_0 equal w_0 and w_0', respectively.

Let Γ be a Fuchsian group acting on the unit disk. Let β be a unitary character of this group. Let w be an analytic function on the unit disk, $|w(\zeta)| \le 1$ which is β-automorphic, that is

$$w(\gamma(\zeta)) = \beta(\gamma) w(\zeta)$$

for every $\gamma \in \Gamma$. The goal of the work is to establish an analogue of (1) in this case. In general the right hand side is positive and depends on β.

To have a meaningful construction one needs a condition on group Γ that guarantees existence for every character α of an α-automorfic function h such that $\frac{h(\zeta)-1}{\zeta-t_0} \in H^2$. Recall that existence for every character α of an α-automorfic function $h \in H^2$ is equivalent to the famous Widom condition (given in terms of the Green function of group Γ) and necessary and sufficient condition of existence for every character α of an α-automorfic function h such that $\frac{h(\zeta)}{\zeta-t_0} \in H^2$ was established in a recent joint work with Peter Yuditskii (given in terms of the Martin function of Γ with singularity at t_0).

Hyun Kwon, University at Albany, SUNY

Similarity of Cowen-Douglas Operator Tuples

Abstract. The definition of a Cowen-Douglas operator can be extended to a commuting Cowen-Douglas operator tuple and the existence of an associated holomorphic eigenvector bundle over the point spectrum follows as well. It was previously shown that the similarity between Cowen-Douglas operators is determined by the trace of the curvatures of the eigenvector bundles. We show an analogous result in the operator tuples setting. The talk is based on joint work with K. Ji, S. Ji, and J. Xu.

Adem Limani, Universitat Autònoma de Barcelona, Barcelona.

M_z-invariant subspaces in the Bloch space and Model spaces

Abstract. This talk is devoted to the investigation of M_z-invariant subspaces generated by inner functions in the classical Bloch space, and its connection to Model spaces. Here $M_z f(z) = zf(z)$ denotes the linear operator of multiplication by the independent variable z. The Bloch space has a rich history, originating from the work around the 1920's of Bloch, Landau and Valiron, on right inverses of holomorphic maps in the unit disc $\mathbb{D}$. The space of Bloch functions is also very intimately related to the theory of conformal mappings. In the end of the 1980's, Makarov established a deep connection between Bloch functions and dyadic martingales with uniformly bounded jumps, which further enhanced the development of probabilistic tools for analyzing boundary behaviors of conformal maps. As the Bloch space in the setting of Bergman spaces plays the analogue of $BMOA$ in the classical Hardy spaces, many natural problems in operator and function theory, such as describing interpolating sequences and invariant subspaces of various operators, may also be phrased therein. In multiple occasions, the intuitions and corresponding results for analytic function spaces determined by growth conditions, seem to largely deviate from the phenomenons occurring in the Bloch space. Our purpose is to illustrate this point for certain M_z-invariant subspaces and cyclic vectors in the Bloch space, answering a few problems left open around the 1990's by Anderson, Brown, Fernandez and Shields. This talk is based on joint work with Artur Nicolau.

Bartosz Malman, Mälardalen University

Boundary interpolation in model spaces

Abstract. It is well-known that any model space K_θ contains plenty of functions which are also members of the disk algebra $\mathcal{A}$. I will discuss some boundary interpolation properties in this class. For which closed sets E on the circle $\mathbb{T}$ can we always extend a continuous function $f \in C(E)$ to a member of $K_\theta \cap \mathcal{A}$? For any inner function θ, there are plenty of sets E for which this kind of interpolation is possible, and plenty for which it is not. Similar questions can be asked in the context of de Branges-Rovnyak spaces.

The talk is based on work (in progress) with Linus Bergqvist from Stockholm University.

Robert T.W. Martin, University of Manitoba

Lebesgue decomposition via reproducing kernels

Abstract. Given any positive, finite and regular Borel measure, μ, on the complex unit circle, one can construct a reproducing kernel Hilbert space of analytic functions in the disk by equipping the vector space of '$\mu-$Cauchy transforms of analytic polynomials' with the $L^2(\mu)-$inner product of the polynomials. (The images of such spaces under certain isometric multipliers are the de Branges–Rovnyak subspaces of H^2 of the disk, including the model spaces – if the measure is singular with respect to Lebesgue measure.) This allows one to translate properties of pairs of such measures, in particular, domination, absolute continuity and mutual singularity, into corresponding properties of their spaces of Cauchy transforms. We will show that one can use this to develop a new approach to Lebesgue decomposition and the Radon–Nikodym theorem using functional analysis and reproducing kernel theory.

This is joint work with Mr. J. Bal and Mr. F. Naderi of the University of Manitoba.

Mishko Mitkovski, Clemson University

Quantitative Estimates for Riesz Sequence Constants in Model Spaces

Abstract. I will present some preliminary results concerning the classical interpolation problem in model spaces. It is well-known that in a model space a sequence of points is interpolating if and only if the corresponding sequence of normalized reproducing kernels forms a Riesz sequence. The lower constant in the Riesz sequence inequality measures the quality of the interpolation. I will present results towards quantification of this constant.

Muyan Jiang, UC Berkeley

Numerical Range of Reciprocal Matrices

Abstract. By definition, reciprocal matrices are tridiagonal n-by-n matrices A with constant main diagonal and such that $a_{i,i+1}a_{i+1,i} = 1$ for $i = 1, \ldots, n-1$. For a reciprocal matrix with any dimension, we derived an exact formula of its Kippenhahn polynomial (numerical range generating polynomial) in terms of the coefficients. We quantifiably described criteria under which the Kippenhahn curves of such matrices consist of elliptical components only, along with conjectures of some properties. As an example, we illustrate the process of deriving such criteria by the case of 7×7 reciprocal matrices.

Marcu-Antone Orsoni, University of Toronto

Separation of singularities in spaces of analytic functions.

Abstract. Let Ω_1 and Ω_2 be two open subsets of the complex plane with non empty intersection. The separation of singularities problem can be stated as follows: if f is holomorphic on $\Omega_1 \cap \Omega_2$, can we find two functions f_1 and f_2 holomorphic on Ω_1 and Ω_2 respectively, such that $f = f_1 + f_2$? This problem has been solved in 1935 by N. Aronszajn who proved that the answer is positive, regardless of the open sets Ω_1 and Ω_2. The same question can be asked in a space X of holomorphic functions: if $f \in X(\Omega_1 \cap \Omega_2)$, can we find two functions $f_1 \in X(\Omega_1)$ and $f_2 \in X(\Omega_2)$ such that $f = f_1 + f_2$? In this talk, I will give an overview of the known results on different spaces of analytic functions and present some new results on the Bergman space. Based on a joint work with Andreas Hartmann.

Gabriel Prajitura, SUNY Brockport

Some questions in Linear Dynamics

Abstract. We will discuss some questions in Linear Dynamics. The choice is rather personal and not of the popular kind. However, we believe them to be relevant and important for the future of the field.

This is joint work with Gabriela Ileana Sebe from Politehnica University of Bucharest, Department of Applied Sciences & Gheorghe Mihoc - Caius Iacob Institute of Mathematical Statistics and Applied Mathematics of the Romanian Academy, Bucharest Romania

Marek Ptak, University of Agriculture in Kraków, Poland

General (X, Y)–operator invariance and applications

Abstract. Motivated by the near invariance of model spaces for the backward shift, a general notion of (X, Y)–invariant operators is introduced. The relations between this class of operators and the near invariance properties of their kernels are studied. This general approach can be applied to a wide class of operators defined as compressions of multiplication operators, in particular to Toeplitz operators and truncated Toeplitz operators, to study the invariance properties of their kernels (general Toeplitz kernels).

Joint work with M. Cristina Câmara and K. Kliś–Garlicka.

Apoorva Singh, Shiv Nadar Institution of Eminence (Deemed to be University), India

Multiplication by a finite Blaschke factor on generalized Hardy spaces

Abstract. The classical Beurling-Helson-Lowdenslager theorem characterizes all shift-invariant subspaces of the Hardy space H^2 over the unit disk and the Lebesgue space L^2 on the unit circle. The shift-invariant subspace theorem has also been extended to the L^p spaces, $1 \leq p \leq \infty$. Recently, a broader class of Hardy spaces (H^α) and Lebesgue spaces (L^α) have been introduced on the unit circle by Chen in two cases: First, when α is a continuous $\|\cdot\|_1$-dominating normalized gauge norms, and second when α is a continuous rotationally symmetric norm. The space L^α is the α-closure of L^∞ and H^α is the α-closure of H^∞. The classical H^p and L^p spaces ($1 \leq p < \infty$) are special cases of H^α and L^α spaces. A Beurling-Helson-Lowdenslager type result for the operator of multiplication by the coordinate function z on L^α as well as H^α has also been proved.

In this work, we study the invariance in a general class of Hardy spaces H^α associated with continuous $\|.\|_1$-dominating normalized gauge norms. We characterize the closed subspaces of H^α invariant under the operator of multiplication by a finite Blaschke factor $B(z)$. We also derive the common invariant subspaces of H^α under multiplication by all the natural powers of $B(z)$ except the first power which is equivalent to characterizing invariance under the algebra $H_1^\infty(B) = \{f(B(z)) : f \in H^\infty \text{ and } f'(0) = 0\}$. The sharp descriptions of the invariant and common invariant subspaces have been obtained for a special class of continuous rotationally symmetric norms. For this case, the underlying Blaschke factor is the monomial $B(z) = z^n$. The invariance result leads to a general inner-outer factorization for all functions in the space H^α equipped with a continuous rotationally symmetric norm.

Luis Manuel Tovar, Instituto Politecnico Nacional, MEXICO

Bicomplex Bergman and Bloch Spaces

Abstract. Using several operators, in this paper we define bicomplex weighted Bergman spaces in the bidisc and their associated weighted Bergman projections, where the respective Bergman kernels are determined.

Dragan Vukotić, Universidad Autónoma de Madrid, Spain

Domination (maximum) principle for weighted Bergman spaces

Abstract. The domination (maximum) principle, formulated by Korenblum in the late 1980s for Bergman spaces A^p, states that there exists a radius $c \in (0,1)$ such that for any two functions f, $g \in A^p$ that satisfy $|f(z)| \le |g(z)|$ for $c \le |z| < 1$, we must have $\|f\|_p \le \|g\|_p$. It is known that this principle holds for $1 \le p < \infty$ and fails for $0 < p < 1$, thanks to the work of a number of authors starting with Hayman and Hinkkanen in 1999.

In this joint work with Iason Efraimidis and Adrián Llinares, we discuss a generalization of this principle to weighted Bergman spaces with radial weights and comment on the properties of the optimal radius.

5.6. **Moment Problems and Applications.**

Monday 2:30-4:30 in Language Center, Room 203 chair: Raul Curto

2:30-2:55	Sarah Hess *A Cone Filtration in the Dual Setting to the Moment Problem*
3:00-3:25	Lawrence Fialkow *The core set and some dual cones on matrix algebras*
3:30-3:55	Paweł Pietrzycki *Noncommutative approximation in operator theory*
4:00-4:25	Raúl E. Curto *Geometrically regular weighted shifts*

Tuesday 2:30-4:00 in Language Center, Room 203 chair: Sarah-Tanja Hess

2:30-2:55	Chafiq Benhida *MID and subnormal safe quotients for geometrically regular weighted shifts*
3:00-3:25	Rajae Ben Taher *The moment problem for the recursive sequence with periodic coefficients*
3:30-3:55	Philipp di Dio *Derivatives of Moments and Moment Functionals*

Tuesday 4:30-6:00 in Language Center, Room 203 chair: Sarah-Tanja Hess

4:30-4:55	Seonguk Yoo *On a construction method of new moment sequences*
5:00-5:25	Michał Buchała *Backward extensions of truncated moment sequences: an application to completion problems for weighted shifts on directed trees*
5:30-5:55	Aljaž Zalar *Bounds on the size of moment matrix extensions to solve the truncated moment problem on some rational curves*

Thursday 2:30-4:00 in Language Center, Room 203 chair: Aljaž Zalar

2:30-2:55	George R. Exner *The role of signed representing measures in subnormality and related properties of weighted shifts*
3:00-3:25	El Hassan Zerouali *Recursive approach to the matrix moment problem*
3:30-3:55	Shubham Jain *Hardy-Hilbert spaces of the generalized Hartogs triangle and joint subnormality*

Thursday 4:30-5:00 in Language Center, Room 203 chair: Aljaž Zalar

4:30-4:55	Franciszek Hugon Szafraniec *Composition in reproducing kernel Hilbert spaces*

Friday 11:30-1:00 in Language Center, Room 203 chair: Raul Curto

11:30-11:55	Greg Blekherman *Nonnegative Polynomials and Truncated Moment Problem on Curves*
12:00-12:25	Piotr Pikul *A bridge between graphs and operators*
12:30-12:55	David P Kimsey *Matrix-valued moment problems*

5.6.1. *Abstracts.*

Rajae Ben Taher, Department of Mathematics, University Moulay Ismail, Meknes, Morocco; E-mail: bentaher89@hotmail.fr

The moment problem for the recursive sequence with periodic coefficients

Abstract. In the present talk, we are interested in studying the moment problem for the recursive sequence with periodic coefficients. A linearization process allows us to generate subsequences satisfying a linear recurrence relation, with constant coefficients. A relevant question is whether there is link between the moment problem for a recursive sequence of periodic coefficients of period $p \geq 1$, and the moment problem of its related p recursive subsequences of constant coefficients. To answer this question, a detailed study is conducted in the case when the period $p = 2$. New results related to their moment problem are established and some examples are provided. Furthermore, some open questions have been raised.

Chafiq Benhida, Université des Sciences et Technologies de Lille, France

MID and subnormal safe quotients for geometrically regular weighted shifts

Abstract. In joint work with Raúl E. Curto (University of Iowa, USA) and George R. Exner (Bucknell University, USA) we consider geometrically regular weighted shifts (GRWS), namely those with weights $\alpha_n = \sqrt{\frac{p^n+N}{p^n+D}}$, where $p > 1$ and N and D are parameters so that $(N, D) \in (-1, 1) \times (-1, 1)$. We study the zone of pairs (M, P) for which the weight $\frac{\alpha(N,D)}{\alpha(M,P)}$ gives rise to a moment infinitely divisible ($\mathcal{MID}$) or a subnormal weighted shift, and deduce immediately the analogous results for product weights $\alpha(N, D)\alpha(M, P)$ instead of quotients. (While this talk is self-contained, it is the second in a sequence of connected talks by the authors.)

Greg Blekherman, Georgia Tech

Nonnegative Polynomials and Truncated Moment Problem on Curves

Abstract. I will present some recent results on the structure of the cone of nonnegative polynomials on curves, and their applications to the truncated moment problem. The picture will be most complete for genus 1 curves; the most explicit example of these are smooth cubic planar curves. Joint work with Lorenzo Baldi and Rainer Sinn.

Michał Buchała, Jagiellonian University, Kraków, Poland

Backward extensions of truncated moment sequences: an application to completion problems for weighted shifts on directed trees

Abstract. For a given directed tree and weights attached to its subtree the completion problem is to determine whether these weights may be completed in a way to obtain a bounded weighted shift on the whole tree, which further satisfies additional conditions. In [Ex1] Exner et.al. posed a subnormal completion problem for weighted shifts on directed trees with one branching point and in [Ex2] the same authors solved this problem in very particular case of 2-generation completion on trees with trunk of length 1. During the talk we present a general approach to p-generation subnormal and completely hyperexpansive completion problems for weighted shifts on directed trees with one branching point. We give a convenient characterization of k-step backward extensions of truncated moment sequences on $(0, \infty)$ and $(0, 1]$, which is a key tool used in solving these problems; our results extends the theory developed by Krein and Nudel'man in [KN1]. The talk is based on [Bu1].

[Bu1] M. Buchała, *Subnormal and completely hyperexpansive completion problem for weighted shifts on directed trees*, arXiv:2207.01593 (2022).

[Ex1] G.R. Exner, I.B. Jung, J. Stochel, H.Y. Yun, *A Subnormal Completion Problem for Weighted Shifts on Directed Trees*, Integr. Equ. Oper. Theory 90, 72 (2018).

[Ex2] G. Exner, I.B. Jung, J. Stochel, H.Y. Yun, *A subnormal completion problem for weighted shifts on directed trees II*, Integr. Equ. Oper. Theory 92, 8 (2020).

[KN1] M.G. Krein, A.A. Nudel'man, *The Markov Moment Problem and Extremal Problems*, Transl. Math. Monographs, vol. 50, American Math. Soc., Providence, 1977.

Raúl E. Curto, University of Iowa, USA

Geometrically regular weighted shifts

Abstract. In joint work with Chafiq Benhida (Université des Sciences et Technologies de Lille, France) and George R. Exner (Bucknell University, USA), we study a general class of weighted shifts whose weights α are given by $\alpha_n = \sqrt{\frac{p^n+N}{p^n+D}}$, where $p > 1$ and N and D are parameters so that $(N, D) \in (-1, 1) \times (-1, 1)$. A few examples of these shifts have appeared previously, usually as examples in connection with some property related to subnormality.

In sectors nicely arranged in the unit square of the parameter space for (N, D), we show that these geometrically regular weighted shifts (GRWS) exhibit a wide variety of properties: moment infinitely divisible, subnormal, $k-$ but not $(k + 1)$–hyponormal, or completely hyperexpansive, and with a variety of well-known functions (such as Bernstein functions) interpolating their weights squared or their moment sequences. The GRWS provide subshifts of the Bergman shift with geometric, not linear, spacing in the weights which are moment infinitely divisible.

This new family of weighted shifts provides a useful addition to the library of shifts with which to explore new definitions and properties.

Philipp di Dio, University of Konstanz

Derivatives of Moments and Moment Functionals

Abstract. We present the concept of derivatives of moments. Moments and derivatives of functions have been used together before and several reconstruction results were gained. We unify and extend these concepts by introducing the concept of derivatives of moments and moments functionals. Simple proofs of previous results and extensions are gained. We especially emphasize the application to Gaussian mixtures. This talk is based on [P. di Dio, *The multidimensional truncated moment problem: Gaussian mixture reconstruction from derivatives of moments*, JMAA **517** (2023), 126592].

El Hassan Zerouali, Mohammed V University in Rabat - Morocco.

Recursive approach to the matrix moment problem

Abstract. We investigate in this paper the classical matrix moment problems of Hamburger, Stieltjes, and Hausdorff type by the mean of the description of non-negative matrix polynomials and by the use of linear recurrence relations. We extend several known results in the scalar moment problem to the matrix moment setting.

George R. Exner, Bucknell University USA

The role of signed representing measures in subnormality and related properties of weighted shifts

Abstract. In joint work with Chafiq Benhida (Université des Sciences et Technologies de Lille, France) and Raúl E. Curto (University of Iowa, USA), consideration of the geometrically regular weighted shifts (cf. the talk by Curto) displayed the importance of signed representing measures (henceforth, Berger charges) in conditions such as subnormality, moment indivisibility, k-hyponormality, and complete hyperexpansiveness of weighted shifts, and certain "Schur

quotients" of such shifts (cf. the talk by Benhida). We highlight the role of Berger charges in that setting and present general results for weighted shifts, including a Berger charge for certain of the completely hyperexpansive weighted shifts and its connection to the Levy-Khinchin measure, and show that for a weighted shift having a Berger charge of the form $\sum_{n=0}^{\infty} a_n r^n$ for some $0 < r < 1$, a certain asymptotic k-hyponormality forces $a_n \geq 0$, $0 \leq n \leq k$. (While this talk is self-contained, it is the third in a sequence of connected talks by the authors.)

Lawrence Fialkow, SUNY New Paltz

The core set and some dual cones on matrix algebras

Abstract. Let S denote a truncated n-dimensional real multisequence. In [J. Operator Theory, 2020], G. Blekherman and the author proved that S has a (finitely atomic) representing measure supported in $\mathbb{R}^n$ if and only if the *core variety* associated to S (or to its functional L_S) is nonempty. In a continuation of this joint work, we present an abstract version of this result that describes an iterative procedure for determining whether an element of a finite dimensional vector space V belongs to a prescribed convex cone in V. With a view toward determining whether or not a given linear map T on the matrix algebra $M_n(\mathbb{R})$ is *positive*, we study certain dual cones related to computing the *core set* of T relative to the cone of all positive maps on $M_n(\mathbb{R})$.

Sarah Hess, University of Konstanz

A Cone Filtration in the Dual Setting to the Moment Problem

Abstract. The classical n-dimensional moment problem asks if for a fixed linear functional $L\colon \mathbb{R}[X] \to \mathbb{R}$, there exists a non-negative Radon measure μ such that $L(p) = \int p\, d\mu$ for any $p \in \mathbb{R}[X]$. The answer is affirmative if and only if L is non-negative on the cone $\mathcal{P}$ of polynomials in $\mathbb{R}[X]$ that are non-negative on $\mathbb{R}^n$ [1, 3]. Thus, the cone of linear functionals with a measure as above is dual to $\mathcal{P}$.

The cone $\mathcal{P}$ is strictly greater than the subcone Σ of polynomials with a sums-of-squares respresentation. Indeed, let $\mathcal{P}_{n,2d}$ and $\Sigma_{n,2d}$ be the subcones of homogeneous polynomials of degree $2d$ in $\mathcal{P}$ and Σ, respectively, then $\mathcal{P}_{n,2d} = \Sigma_{n,2d}$ if and only if $n = 2$, $2d = 2$ or $(n, 2d) = (3, 4)$ [2]. This result is known as Hilbert's 1888 Theorem and shall be generalized in the inequality cases during this talk.

To this aim, we induce a filtration of intermediate cones between $\mathcal{P}_{n,2d}$ and $\Sigma_{n,2d}$ along projective varieties via a Gram matrix approach and investigate it for strict inclusions. Important tools therein are considerations of varieties of minimal degree and permutations of non-negative homogeneous polynomials without sums-of-square representations. This is a joint work with C. Goel and S. Kuhlmann.

[1] E. K. Haviland, *On the moment problem for distribution functions in more than one dimension II*, Amer. J. Math., 58:164–168, 1936.

[2] D. Hilbert, *Über die Darstellung definiter Formen als Summe von Formenquadraten*, Math. Ann., 32:342–350, 1888; Ges. Abh. 2:154–161, Springer, Berlin, reprinted by Chelsea, New York, 1981.

[3] M. Riesz, *Sur le problème de moments: Troisième note*, volume 17, Arkiv för matematik, Astronomi och Fysik, 1923.

Shubham Jain, Indian Institute of Technology Kanpur

Title of Talk Hardy-Hilbert spaces of the generalized Hartogs triangle and joint subnormality

Abstract. We introduce a family of Hardy-Hilbert spaces of the generalized Hartogs triangle and discuss the notion of subnormality in the context of the multiplication n-tuples $\mathcal{M}_z$ on these

spaces. We also discuss the construction of a Hardy space on the n-dimensional Hartogs triangle generalizing a result of A. Monguzzi. This talk is based on a joint work with S. Chavan and P. Pramanick.

David P Kimsey, Newcastle University

Matrix-valued moment problems

Abstract. Given a matrix-valued truncated or full sequence $(S_j)_{j=0}^m$, the matrix-valued moment problems entails determining whether or not the given truncated or full sequence has a matrix-valued representing measure, i.e., determining whether or not there exists a matrix-valued measure σ such that

$$S_j = \int_{\mathbb{R}} x^j \, d\sigma(x) \qquad \text{for} \quad j = 0, 1, ldots, m.$$

In this talk, we shall sketch a solution in the univariate setting, with the aim of highlighting differences from the scalar-valued setting. Passing to the multidimensional setting, we will present a solution which makes a concrete connection with zeros of certain matrix-valued polynomials. Time permitting, we will also touch on analogues of the aforementioned moment problems when matrices are replaced with Clifford algebras possessing a certain signature.

Paweł Pietrzycki, Jagiellonian University in Kraków

Noncommutative approximation in operator theory

Abstract. Motivated both by the fundamental role of the classical Choquet boundary in classical approximation theory, and by the importance of approximation in the contemporary theory of operator algebras, Arveson introduced hyperrigidity as a form of approximation that captures many important operator-algebraic phenomena. I will discuss new results on the notion of hyperrigidity such as the characterization of spectral measures, the intertwining theorem, the relationship between the convergence of subnormal operators in weak and strong operator topologies, and new examples of sets of generators that are hyperrigid. The talk is based on joint works with Jan Stochel.

Piotr Pikul, Jagiellonian University in Kraków

A bridge between graphs and operators

Abstract. A decade ago there was introduced [1] a generalisation of well known weighted shifts on ℓ^2, namely the weighted shifts on directed trees, where the linear order of coordinates in ℓ^2 is replaced by a more involved graph structure. Next generalisation, in form of shifts on directed *forests*, appears to show even tighter bound between graph structure and the operator-theoretic properties.

Directed forest can be defined in the following lightweight form:

A *directed forest* is a pair $\mathcal{T} = (V, \mathsf{p})$, where p is a self-map of the set V satisfying:

$$\text{if } \mathsf{p}^n(v) = v \text{ for some } v \in V \text{ and } n \geq 1, \text{ then } \mathsf{p}(v) = v.$$

Roots are then "vertices with loops" i.e. $v \in \mathsf{root}(\mathcal{T})$ if and only if $\mathsf{p}(v) = v$. The weighted shift on $\mathcal{T}$ with complex weights $\boldsymbol{\lambda} = \{\lambda_v\}_{v \in V}$ is defined like a weighted composition operator in $\ell^2(V)$, by the formula $S_{\boldsymbol{\lambda}} f(v) = \lambda_v f(\mathsf{p}(v))$ with additional assumption that $\lambda_\omega = 0$ whenever ω is a root. If roots are the only vertices with zero weights, we say that the weighted shift is *proper*.

There are following advantages of the directed forest approach (cf. [2]):

- The class of weighted shifts on directed forests is closed on taking orthogonal sums or positive integer powers. What is important, k-th power of a bounded weighted shift is defined over the k-th power of the original forest.

- Many results (and essentially proofs) regarding weighted shifts on directed trees remain valid. For example characterisation of dense definiteness, boundedness, hyponormality or description of polar decomposition.
- Every weighted shift on a directed forest is actually proper, if we consider appropriate underlying forest (i.e. if we "remove edges with zero weights"). Together with known fact about shifts on directed trees, it implies that without loss of generality we can assume that all weights (except zeros for the roots) are positive real numbers.

The main result (not yet published) regarding weighted shifts on directed forests is full characterisation of all the directed forests on which every hyponormal weighted shift is power hyponormal. It is a known phenomenon for classical weighted shifts on the sequence space ℓ^2.

Theorem. For a directed forest $\mathcal{T} = (V, \mathsf{p})$ the following are equivalent:

(1) every bounded hyponormal weighted shift on $\mathcal{T}$ is power hyponormal,
(2) if $S_{\boldsymbol{\lambda}}$ is a bounded hyponormal weighted shift on $\mathcal{T}$, then $S_{\boldsymbol{\lambda}}^2$ is hyponormal,
(3) the leafless support of $\mathcal{T}$ is forkless.

The *leafless support* is the thickest leafless directed forest thinner than the given one (where thickness is simply the inclusion of the traditional sets of graph edges). A forest is called *forkless* if every vertex which is not a root has precisely one child.

[1] Z. Jabłoński, I. B. Jung, J. Stochel, *Weighted shifts on directed trees*, Mem. Amer. Math. Soc. **216** (2012), no. 1017.

[2] P. Pikul, *Backward extensions of weighted shifts on directed trees*, Integr. Equ. Oper. Theory **94** (2022), article 26.

Franciszek Hugon Szafraniec, Uniwersytet Jagielloński, Kraków

Composition in reproducing kernel Hilbert spaces
à rebours

Abstract. This is based on a missing talk scheduled for IWOTA2022, section 21.

The talk will be an expanded (or in other words updated) version of FH Szafraniec, Transporting positive definiteness, *Acta Sci. Math.* **88**(2022), 505–514.

Seonguk Yoo, Gyeongsang National University

On a construction method of new moment sequences

Abstract. In this talk we provide a way to construct new moment sequences from a given moment sequence. An operator based on multivariate positive polynomials is applied to get the new moment sequences. A class of new sequences is corresponding to a unique symmetric polynomial; if this polynomial is positive, then the new sequence becomes again a moment sequence. We will see for instance that a new sequence generated from minors of a Hankel matrix of a Stieltjes moment sequence is also a Stieltjes moment sequence. (This talk is based on joint work with Seunghwan Baek and Hayoung Choi.)

Aljaž Zalar, University of Ljubljana

Bounds on the size of moment matrix extensions to solve the truncated moment problem on some rational curves

Abstract. Given a finite d–dimensional real multisequence $\beta^{(2n)} = \{\beta_i\}_{|i|\leq 2n}$ and a closed set K of $\mathbb{R}^d$, the truncated moment problem (TMP) on K asks to characterize the existence of a positive Borel measure μ on $\mathbb{R}^d$ with support on K such that the elements of $\beta^{(2n)}$ are the moments of μ. Let $\mathcal{Q} = \{g_0 = 1, g_1, \ldots, g_m\} \subseteq \mathbb{R}[\underline{x}]$ be a subset of d-variate polynomials and

$K_{\mathcal{Q}} = \{x \in \mathbb{R}^d \colon g_1(x) \geq 0, \ldots, g_m \geq 0\}$ the corresponding semialgebraic set. $K_{\mathcal{Q}}$ can have one of the following properties:

$(P_{n,m})$ Every $p \in \mathbb{R}[\underline{x}]$ with $\deg p \leq 2n$ and $p|_{K_{\mathcal{Q}}} > 0$ is of the form

$$p = \sum_{j=0}^{m} g_j \sum_k f_{jk}^2, \text{ where } f_{jk} \in \mathbb{R}[\underline{x}], \deg(g_j f_{jk}^2) \leq 2m.$$

$(S_{n,k})$ $\beta^{(2n)}$ has a representing measure supported on $K_{\mathcal{Q}}$ if and only if the moment matrix $M(n)$ admits a positive extension such that all localizing moment matrices $M_{g_i}(n+k)$ are positive for $i = 1, \ldots, m$.

By results of Curto and Fialkow [1], the implications $(P_{n+1,n+k}) \Rightarrow (S_{n,k})$ and $(S_{n,k}) \Rightarrow (P_{n,n+k})$ hold true. In [2], Fialkow solved the TMP on plane rational curves $y = q(x)$ or $yq(x) = 1$ where q is a univariate polynomial, by establishing the property $(P_{n+1,n+k})$ where k is a quadratic function in n and $\deg(q)$. In the talk we will see how the TMP on these curves can be solved by proving the property $(S_{n,\deg q+1})$ directly. This also leads to improved degree bounds in the positivity certificates, i.e., $(P_{n,\deg q+1})$ holds. The same approach establishes the property $(S_{n,j+1})$ for irreducible curves $x^i y^j = 1$, $i, j \in \mathbb{N}$, $i < j$ and shows that irreducible curves $y^i = x^j$ with $i, j \geq 2$, do not have the property $(S_{n,k})$ for arbitrary k. Moreover, an optimal bound on the number of atoms in a minimal representing measure for all the above curves is obtained.

[1] R. Curto, L. Fialkow, *An analogue of the Riesz-Haviland theorem for the truncated moment problem.* J. Funct. Anal. 225 (2008), 2709–2731.

[2] L. Fialkow, Solution of the truncated moment problem with variety $y = x^3$, Trans. Amer. Math. Soc. 363 (2011), 3133–3165.

[3] A. Zalar, The truncated moment problem on curves $y = q(x)$ and $yx^\ell = 1$, to appear in Linear and Multilinear Algebra.

5.7. **Multivariable Operator Theory.**

Monday 2:30-4:30 in Porthania, PII chair: Joe Ball

2:30-2:55	Robert T.W. Martin	
	Toeplitz factorization in the full Fock space	
3:00-3:25	Amit Maji	
	Commuting isometries and characterization of invariant subspaces in the polydisc	
3:30-3:55	Baruch Solel	
	Function Theory and W^*-Categories	
4:00-4:25	Ran Kiri	
	Subhomogeneous Operator Systems and Classification of Operator Systems Generated by Λ-Commuting Unitaries	

Tuesday 2:30-4:00 in Porthania, PII chair: Sanne ter Horst

2:30-2:55	Joseph Ball
	Pairs of commuting Hilbert-space contraction operators: dilation theory and functional models
3:00-3:25	Haripada Sau
	A Constrained Ando Dilation Problem
3:30-3:55	David Kimsey
	Multidimensional spectral theorem for a commuting tuple of normal operators on a Clifford module

Tuesday 4:30-7:30 in Porthania, PII chair: Sanne ter Horst

4:30-4:55	Hui Dan
	Kozlov's claims and the Beurling-Wintner problem for characteristic functions
5:00-5:25	Michael Stessin
	Spectral analysis near a point of regular reducibility
5:30-5:55	Abhay Jindal
	Complete Nevanlinna-Pick kernels and the characteristic function
6:00-6:25	Kate Howell
	The Characteristic Polynomial of Projections
6:30-6:55	Paramita Pramanick
	Multiplication tuples homogeneous under the unitary group
7:00-7:25	Kui Ji
	Geometry of holomorphic vector bundles and similarity of commuting tuples of operators

Thursday 2:30-4:00 in Porthania, PII chair: Haripada Sau

2:30-2:55	Hugo J. Woerdeman
	Completing an Operator Matrix and the Free Joint Numerical Radius
3:00-3:25	Slaviša Djordjević
	Spectra of a commuting pair of operators, a linear pencil and the spherical Aluthge transform
3:30-3:55	Zeljko Cuckovic
	Essential norms of Toeplitz operators on domains in $\mathbb{C}^n$ and the Berezin transform

Thursday 4:30-6:00 in Porthania, PII chair: Haripada Sau

4:30-4:55	Joshua Isralowitz *Toeplitz products with matrix symbols on the Fock space*
5:00-5:25	Connor Evans UPDATE *A model formula for bounded holomorphic functions on the symmetrized rectangle*
5:30-5:55	Penghui Wang *Essential normality of quotient modules in weighted Bergman modules on the polydiscs*

Friday 11:30-1:00 in Porthania, PII chair: Robert Martin

11:30-11:55	H. Turgay Kaptanoğlu *Uncertainty principles in holomorphic function spaces on the unit disc and ball*
12:00-12:25	Kai Wang *Law of Large Numbers in Determinantal Point Processes*
12:30-12:55	Nicholas Young *Herglotz and Nevanlinna representations in several variables*

Friday 2:30-4:00 in Porthania, PII chair: Joe Ball

2:30-2:55	Michio Seto *An indefinite Schwarz–Pick inequality on the bidisk*
3:00-3:25	
3:30-3:55	Eli Shamovich *Grassmannians of complete Pick spaces*

Friday 4:30-6:00 in Porthania, PII chair: Sanne ter Horst

4:30-4:55	Raúl E. Curto *Norm and numerical radius of single operators through tools and techniques from multivariable operator theory*
5:00-5:25	Gerald Wanjala *The Numerical Range and Stability of Index for Multivalued Linear Operators*
5:30-5:55	Jiayang Yu *L^2 methods in infinite dimensional spaces*

5.7.1. *Abstracts.*

Joseph Ball, Virginia Tech

Pairs of commuting Hilbert-space contraction operators: dilation theory and functional models

Abstract. A seminal result in operator theory is the Sz.-Nagy dilation theorem: any Hilbert space contraction operator T on $\mathcal{H}$ can be dilated to a unitary operator $\mathcal{U}$ on $\widetilde{\mathcal{K}} \supset \mathcal{H}$, or equivalently, any such T can be lifted to an isometry V on $\mathcal{K} \supset \mathcal{H}$ (i.e., $\mathcal{H}$ is invariant for V^* and $V^*|_{\mathcal{H}} = T^*$). Furthermore the minimal isometric lift of T is unique up to a unitary change of coordinates in $\mathcal{K}$ and correspondingly in $\mathcal{H}$. The Sz.-Nagy–Foias functional-model identifies the change of coordinates which leads to a functional-model representation for V on a functional-model Hilbert space $\mathcal{K}_\Theta$ and for T on $\mathcal{H}_\Theta \subset \mathcal{K}_\Theta$ defined solely in terms of the Sz.-Nagy–Foias characteristic function $\Theta = \Theta_T$ of T. This, combined with spectral theory for the unitary part of T if T has a unitary part, reduces the study of a general contraction operator T to the study of a contractive analytic function Θ on the unit disk, in principle a much simpler object than T (at least in the case when Θ is matrix-valued). Here we obtain the analogue of these results for the case of a commuting contractive pair (T_1, T_2) in place of a single contraction operator T.

The analogue of the first step of this program has already appeared in the 1963 result of Andô: *any commuting pair* (T_1, T_2) *of Hilbert-space contraction operators on* $\mathcal{H}$ *can be lifted to a commuting pair* (V_1, V_2) *of isometries on* $\mathcal{K} \supset \mathcal{H}$. However it is well known that, unlike the situation for the univariate setting, the minimal Andô lift for the commutative contractive-pair case lacks uniqueness (with respect to a unitary change of coordinates). Here we present a new, more constructive proof of Andô's result leading to a Sz.-Nagy-Foias-type functional model for the lift which in addition identifies additional independent lift-invariants (called *Andô tuples*) which serve to classify the distinct unitary-equivalence classes of minimal Andô lifts for the given contractive pair (T_1, T_2).

This lack of uniqueness limits the utility of such minimal Andô lifts for the construction of a functional model for a commuting contractive-pair (T_1, T_2) itself. There is an intermediate type of lift, called *pseudo-commuting contractive lift* $(\mathbb{W}_1, \mathbb{W}_2, \mathbb{W})$ of the operator triple $(T_1, T_2, T = T_1T_2)$ associated with the commuting contractive pair (T_1, T_2), obtained by compressing any choice of minimal Andô lift (V_1, V_2, V_1V_2) to the copy of the minimal Sz.-Nagy–Foias isometric lift of the product contraction $T = T_1T_2$ embedded in the functional-model of the Andô lift (V_1, V_2). The operators $\mathbb{W}_1, \mathbb{W}_2, \mathbb{W}$ in this lift are no longer commuting isometries, but are characterized by a slight weakening of the condition that $\mathbb{W}_1$, $\mathbb{W}_2$ be commuting isometries with product equal to $\mathbb{W}$, namely the property of being what we call a *pseudo-commuting contractive triple*, which still guarantees that $\mathbb{W}_1$, $\mathbb{W}_2$ are multiplication operators of a simple form and $\mathbb{W}$ is equal to the Sz.-Nagy–Foias model minimal isometric lift of the product operator $T = T_1T_2$. In the Sz.-Nagy–Foias-like model form, the characteristic function Θ_T is augmented by the *Fundamental-Operator pair* (G_1, G_2) of (T_1^*, T_2^*) (determined uniquely from (T_1^*, T_2^*) as the unique solution of a certain system of operator equations), together with a commuting pair of unitary operators $(W_{\sharp 1}, W_{\sharp 2})$, defined on the L^2-component of the Sz.-Nagy–Foias functional-model space for the isometric lift of T which are canonically and uniquely determined by (T_1, T_2), so that the combined collection $((G_1, G_2), (W_{\sharp 1}, W_{\sharp 2}), \Theta_{T_1T_2})$ (called the *characteristic triple* for (T_1, T_2)) is a complete unitary invariant for the commuting contractive pair (T_1, T_2) analogous to the characteristic function Θ_T alone for the case of a single contraction operator T. There is also a notion of *admissible triple* $\Xi := ((G_1, G_2), (W_1, W_2), \Theta)$ as the substitute for a *purely contractive analytic function* Θ in the Sz.-Nagy–Foias theory, from which one can construct a functional-model commuting contractive operator-pair $(T_{\Xi,1}, T_{\Xi,2})$ having its characteristic triple "coinciding" with the original admissible triple in the appropriate precise sense.

This is joint work with Haripada Sau of the Indian Institute of Science Education and Research, Maharashta.

Zeljko Cuckovic, University of Toledo, Ohio

Essential norms of Toeplitz operators on domains in $\mathbb{C}^n$ and the Berezin transform

Abstract. In light of the Axler-Zheng theorem for the Toeplitz operators on the unit disk, and subsequent extensions, includiong our version for smoothly bounded pseudoconvex domains Ω in $\mathbb{C}^n$, we know that the Berezin transform plays a crucial role in characterizing compactness of Toeplitz operators. This suggests that the measure of non-compactness of an operator T in a natural Toeplitz subalgebra on Ω, should be related to the properties of its Berezin transform on the boundary. In fact we show that the essential norm of T is related to the norm of its Berezin transform, not on the full boundary, but on the set of strongly pseudoconvex points. This leads us to the question of the regularity of the Berezin transform of operators and we show that the boundary geometry plays an important role. (Joint work with Sonmez Sahutoglu)

Raúl E. Curto, The University of Iowa

Norm and numerical radius of single operators through tools and techniques from multivariable operator theory

Abstract. We employ tools and techniques from multivariable operator theory to obtain new proofs and extensions of well known inequalities regarding the norm and the numerical radius of elementary operators defined on the C^*–algebra of all bounded operators on Hilbert space, or on the $*$–ideal of Hilbert-Schmidt operators. In the process, we provide new insights on the study of Heinz-type inequalities related to the arithmetic-geometric mean inequality, and generalize results of several authors, including R. Bhatia, G. Corach, C. Davis, F. Kittaneh, and M.S. Moslehian.

To estimate the norm, our approach exploits, in particular, the Spectral Mapping Theorem for the Taylor spectrum, and Ky Fan's Dominance Theorem. For the numerical radius, we use S. Hildebrandt's description of the numerical range of an operator in terms of the norm of its translates.

The talk is based on recent research with Sang Hoon Lee (Chungnam National University, Republic of Korea) and Jasang Yoon (The University of Texas Rio Grande Valley, USA).

Keywords: Numerical radius, Taylor spectrum, Ky Fan's Dominance Theorem.

Hui Dan, Sichuan University

Kozlov's claims and the Beurling-Wintner problem for characteristic functions

Abstract. There is a long-standing problem raised by Beurling and Wintner on completeness of the dilation system $\{\varphi(kx) : k = 1, 2, \cdots\}$ generated by the odd periodic extension on $\mathbb{R}$ of any $\varphi \in L^2(0, 1)$. In this talk, we will introduce Kozlov's astonishing claims in 1950s and exhibit some results on the Beurling-Wintner problem for characteristic functions, especially the Kozlov completeness problem.

Slaviša Djordjević, Benemérita Universidad Autónoma de Puebla, México

Spectra of a commuting pair of operators, a linear pencil and the spherical Aluthge transform

Abstract. In this talk, we review and investigate some properties of a linear pencil in the frame of the point spectrum and an isolated point in its spectrum. We next give relationships among the spectrum of the linear pencil, the Taylor spectrums of a commuting pair and its spherical Aluthge transform through projection maps and the spectral mapping theorem. Moreover, using the generalized spherical Aluthge transforms for commuting pairs of operators, we study connection of a nontrivial joint invariant (resp. hyperinvariant) subspaces for the original commuting pair and the generalized spherical Aluthge transform.

Kate Howell, State University of New York at Albany, USA

The Characteristic Polynomial of Projections

Abstract. The characteristic polynomial is an important topic in linear algebra and matrix theory, and it has a wide range of applications in mathematics, science, and engineering. A multivariable version of the characteristic polynomial, on the other hand, has been ostensibly missing until recently. Given matrices $A_1, ..., A_n$, we may define their characteristic polynomial as $Q_A = \det(z_0 I + z_1 A_1 + \cdots + z_n A_n)$. Recent studies have shown that Q_A offers an efficient tool for us to describe the interactions as well as the joint behavior of the involved matrices. In the past, it has been difficult to classify pairs of projections. However, by turning to the Tits representation of Coxeter groups and writing the Tits representation ρ as $\rho = 2p - I$, we find that the Tits representation may be written as a projection p. Furthermore, by connecting projections with the characteristic polynomial Coxeter groups with respect to the Tits representation, we find that two n-tuples of projections onto hyperplanes are unitarily equivalent if

their characteristic polynomials coincide. Moreover, two tuples of any projections (p_1, p_2) and (q_1, q_2) are unitarily equivalent if and only if their characteristic polynomials coincide.

Joshua Isralowitz, University at Albany, SUNY

Toeplitz products with matrix symbols on the Fock space

Abstract. In a 2014 paper, H.R. Choe, J. Park, and K. Zhu showed that the Toeplitz product $T_f T_{\bar{g}}$ with nonzero symbols f, g in the standard Fock space F^p for $0 < p < \infty$ is bounded precisely in the "trivial" setting $f = e^q$ and $g = ce^{-q}$. Here, q is a complex linear polynomial and $c \in \mathbb{C}$ (in which case $T_f T_{\bar{g}}$ is a constant multiple of a unitary operator, and thus invertible).

In this talk, we discuss the boundedness and invertibility of the product $T_F T_{G^*}$ of Toeplitz operators with matrix symbols F and G having entries in F^p. In particular, we focus on what algebraic structure F and G is forced to have if the product $T_F T_{G^*}$ is bounded and invertible.

Kui Ji, Hebei Normal University

Geometry of holomorphic vector bundles and similarity of commuting tuples of operators

Abstract. In this talk, a new criterion for the similarity of commuting tuples of operators on Hilbert spaces is introduced. As an application, we obtain a geometric similarity invariant of tuples in the Cowen-Douglas class which gives a partial answer to a question raised by R.G. Douglas. Moreover, we also discuss some applications of the main theorem on weakly homogeneous operators and so on. This is joint work with Yingli Hou, Shanshan Ji and Jing Xu.

Jiayang Yu, Sichuan University, Chengdu, China

L^2 methods in infinite dimensional spaces

Abstract. The classical L^2 estimate for the $\overline{\partial}$ operators is a basic tool in complex analysis of several variables. Naturally, it is expected to extend this estimate to infinite dimensional complex analysis, but this is a longstanding unsolved problem, due to the essential difficulty that there exists no nontrivial translation invariance measure in the setting of infinite dimensions. The main purpose in this series of work is to give an affirmative solution to the above problem, and apply the estimates to the solvability of the infinite dimensional $\overline{\partial}$ equations. In this part, we consider the most difficult case, i.e., the underlying space is a general pseudo-convex domain. In order to solve this longstanding open problem, we introduce several new concepts and techniques, which have independent interest and pave the way for research that investigates some other issues in infinite-dimensional analysis.

Abhay Jindal, Indian Institute of Science, Bengaluru, India

Complete Nevanlinna-Pick kernels and the characteristic function

Abstract. We shall extend the classical theory of Sz.-Nagy and Foias about the characteristic function of a contraction to a commuting tuple $(T_1, \ldots, T_d)$ of bounded operators satisfying the natural positivity condition of $1/k$-contractivity for an irreducible unitarily invariant complete Nevanlinna-Pick kernel. The characteristic function is a multiplier from $H_k \otimes E$ to $H_k \otimes F$, *factoring* a certain positive operator, for suitable Hilbert spaces E and F depending on the tuple $(T_1, \ldots, T_d)$. There is a converse, which roughly says that if a kernel k *admits* a characteristic function, then it has to be an irreducible unitarily invariant complete Nevanlinna-Pick kernel. The characterization explains, among other things, why in the literature an analogue of the characteristic function for a Bergman contraction ($1/k$-contraction where k is the Bergman kernel), when viewed as a multiplier between two vector valued reproducing kernel Hilbert spaces, requires a different (vector valued) reproducing kernel Hilbert space as the domain. This is a joint work with Tirthankar Bhattacharyya.

H. Turgay Kaptanoğlu, Bilkent University

Uncertainty Principles in Holomorphic Function Spaces on the Unit Disc and Ball

Abstract. On a large family of function spaces on the unit disc that include the Bergman-Besov Hilbert spaces, we find self-adjoint weighted shift operators that are fractional differential operators of half order whose commutators are the identity thereby obtaining uncertainty relations in these spaces. We identify the functions that yield equality in the uncertainty inequalities. We also obtain joint average uncertainty relations for pairs of commuting tuples of operators on the same spaces defined on the unit ball.

David Kimsey

Multidimensional spectral theorem for a commuting tuple of normal operators on a Clifford module

Abstract. In this talk, we will consider the problem of finding a spectral representation for a commuting tuple of normal operators on a Clifford module based on the S-spectrum. We shall see that such a representation involves a new notion of multidimensional spectrum, which we will call the joint S-spectrum. Applications to a certain noncommutative multidimensional moment problem will be considered.

Ran Kiri, Technion - Israel Institute of Technology

Subhomogeneous Operator Systems and Classification of Operator Systems Generated by Λ-Commuting Unitaries

Abstract. A unital C^*-algebra is called N-subhomogeneous if its irreducible representations are finite dimensional with dimension at most N. We extend this notion to operator systems, replacing irreducible representations by boundary representations. This is done by considering the matrix state space associated with an operator system, identifying the boundary representations as absolute matrix extreme points. We show that two N-subhomogeneous operator systems are completely order equivalent if and only if they are N-order equivalent. Moreover, we show that a unital N-positive map into a finite dimensional N-subhomogeneous operator system is completely positive. We apply these tools, to classify pairs of q-commuting unitaries up to $*$-isomorphism. Similar results are obtained for operator systems related to higher dimensional non-commutative tori.

Amit Maji, Indian Institute of Technology Roorkee, India

Commuting isometries and characterization of invariant subspaces in the polydisc

Abstract. In this talk, we present an explicit version of Berger, Coburn and Lebow's classification result for pure pairs of commuting isometries in the sense of an explicit recipe for constructing pairs of commuting isometric multipliers with precise coefficients. Secondly, we give a complete characterization of invariant subspaces for $(M_{z_1}, \dots, M_{z_n})$ on the Hardy space $H^2(\mathbb{D}^n)$ over the unit polydisc $\mathbb{D}^n$ in $\mathbb{C}^n$, $n > 1$. In particular, this yields a complete set of unitary invariants for invariant subspaces for $(M_{z_1}, \dots, M_{z_n})$ on $H^2(\mathbb{D}^n)$. As a consequence, we classify a large class of n-tuples of commuting isometries. All of our results hold for vector-valued Hardy spaces over $\mathbb{D}^n$, $n > 1$.

Robert T.W. Martin, University of Manitoba

Toeplitz factorization in the full Fock space

Abstract. A Toeplitz operator on the Hardy space, H^2, of square–summable Taylor series in the complex unit disk is defined as the compression of a multiplication operator on L^2 of the circle to H^2. A positive semi-definite Toeplitz operator, T, is *factorizable* if $T = M_h^* M_h$, for some bounded, analytic (and outer) h. It follows easily from the Riesz exponential integral

formula for outer functions that a positive Toeplitz operator is factorizable if and only if it has a log–integrable symbol. On the other hand, Szegö's theorem can be applied to show that a positive semi-definite Toeplitz operator is factorizable if and only if it has a positive and factorizable minorant.

We study the factorization of positive 'multi–Toeplitz' operators in the non-commutative, multivariate setting of the *full Fock space* or *free Hardy space* of square–summable power series in several non-commuting variables. In this more abstract setting we no longer have access to the concrete Riesz formula for outer functions, boundary values, or measure–theoretic notions such as 'log–integrability', and so new techniques seem to be required. Several new characterizations equivalent to the existence of a positive factorizable Toeplitz minorant will be presented.

This is joint work with Prof. M.T. Jury (U. Florida).

John McCarthy, Washington University in St. Louis

Common range of co-analytic Toeplitz operators on the Drury-Arveson space

Abstract. We characterize the common range of the adjoints of cyclic multiplication operators on the Drury–Arveson space. We show that a function belongs to this common range if and only if its Taylor coefficients satisfy a simple decay condition, which can be viewed as a smoothness condition.

This is joint work with A. Aleman, M. Hartz and S. Richter.

Paramita Pramanick, Indian Institute of Technology Kanpur

Multiplication tuples homogeneous under the unitary group

Abstract. Let $\mathcal{U}(d)$ be the group of $d \times d$ unitary matrices. We find conditions to ensure that a $\mathcal{U}(d)$-homogeneous d-tuple $\boldsymbol{T}$ is unitarily equivalent to multiplication by the coordinate functions on some reproducing kernel Hilbert space $\mathcal{H}_K(\mathbb{B}_d, \mathbb{C}^n) \subseteq \mathrm{Hol}(\mathbb{B}_d, \mathbb{C}^n)$, $n = \dim \cap_{j=1}^{d} \ker T_j^*$. We describe this class of $\mathcal{U}(d)$-homogeneous operators, equivalently, non-negative kernels K quasi-invariant under the action of $\mathcal{U}(d)$. We classify quasi-invariant kernels K transforming under $\mathcal{U}(d)$ with two specific choice of multipliers. A crucial ingredient of the proof is that the group $SU(d)$ has exactly two inequivalent irreducible unitary representations of dimension d and none in dimensions $2, \ldots, d-1$, $d \geq 3$. We obtain explicit criterion for boundedness, reducibility and mutual unitary equivalence among these operators. (This is a joint work with S. Ghara, S. Kumar and G. Misra.)

Haripada Sau

A Constrained Ando Dilation Problem

Abstract. The Andô's Inequality and its remarkable improvements first for certain matrices by Agler and McCarthy (Acta Math., 2005) and then for certain operators by Das and Sarkar (J. Funct. Anal., 2017) motivate us to ask a question which can be seen as a Constrained Andô Dilation problem. The statement involves a class of two-variable polynomials with a geometric condition on its zero sets, called the toral polynomials. If a pair of commuting operators is annihilated by a toral polynomial, then the pair is called a toral pair.

Does every toral pair of commuting contractions lift to a toral pair of commuting isometries?

In this talk, we shall see how exactly the results cited above inspire the question and why one may want to find an answer to the problem.

Michio Seto, National Defense Academy of Japan

An indefinite Schwarz–Pick inequality on the bidisk

Abstract. In this talk, we introduce a Schwarz–Pick type inequality on the bidisk. Our inequality is indefinite in a certain sense and obtained by an application of the Hilbert module structure of the Hardy space over the bidisk. Moreover, we discuss an interpolation problem for rational maps in the bidisk.

Eli Shamovich, Ben-Gurion University of the Negev

Grassmannians of complete Pick spaces

Abstract. Recently, Ofek, Pandey, and Shalit have introduced a number of Banach-Mazur type distances on equivalence classes of complete Pick spaces of fixed finite dimension. In this talk, I will describe another possible distance which comes from considering the corresponding projections on the Drury-Arveson space. I will explain why that the new metric induces the same topology as the metrics of Ofek, Pandey, and Shalit. However, this metric is not equivalent. I will describe a possible application for obtaining invariants of operator algebras corresponding to algebraic sub varieties of the unit ball. This talk is based on joint work in progress with Prahllad Deb and Jonathan Nureliyan.

Baruch Solel, Technion

Function Theory and W^*-Categories

Abstract. Free nc function theory is an extension of the theory of holomorphic functions of several complex variables to the theory of functions on matrix tuples $Z = (Z_1, \cdots, Z_d)$ where $Z_i \in M_n(\mathbb{C})$ and n is allowed to vary.

An nc function is a function defined on such tuples Z and takes values in $\cup_{n\in\mathbb{N}} M_n(\mathbb{C})$ which is graded and respects direct sums and similarity (equivalently, respects intertwiners).

The classical correspondence between positive kernels and Hilbert spaces of functions has been recently extended by Ball, Marx and Vinnikov to nc completely positive kernels and Hilbert spaces of nc functions. In a previous, unpulished work, we have developed a similar theory for matricial functions where $\mathbb{C}$ is replaced by a von Neumann algebra M, $\cup_{n\in\mathbb{N}} M_n(\mathbb{C})$ is replaced by a suitable disjoint union of correspondences over M and the "index set" $\mathbb{N}$ is replaced by the set of representations of M.

Thus, in both situations we deal with structures that are fibred. In each settings there are situations where we need to move among fibres. This led us to consider actions of categories on fibred sets and we study here functions and kernels that are invariant under certain actions of categories.

This is a joint work (in progress) with Paul Muhly.

Michael Stessin, State University of New York, University at Albany

Spectral analysis near a point of regular reducibility

Abstract. For a tuple of $n \times n$ matrices $A_1, \ldots, A_k$ the *determinantal hypersurface (determinantal manifold) of the pencil* $x_1A_1 + \cdots + x_nA_n$ is defined as

$$\sigma(A_1, ..., A_n) = \Big\{[x_1 : \cdots : x_n] \in \mathbb{CP}^{n-1} : \ det\big(x_1A_1 + \cdots + x_nA_n\big) = 0\Big\}.$$

Determinantal hypersurfaces of matrix pencils have been under investigation for more than a century, mostly in the framework of algebraic geometry, with the main focus on the question when a hypersurface in the projective space admits a determinantal representation.

Infinite dimensional analog of determinantal manifold for a tuple of operators acting on a Hilbert space was introduced by R.Yang in 2009 and was called *the projective joint spectrum of a*

tuple. If the dimension of the Hilbert space is finite, the projective joint spectrum coincides with the corresponding determinantal hypersurface. Projective joint spectra have been intensly investigated in the last decade in the framework of operator theory. The main focus of this investigation shifted to the question of the connection between the geometry of the projective joint spectrum (determinantal manifold) and the relations between operators (matrices) in the tuple.

In the finite dimensional case a determinantal hypersurface is an algebraic manifold in the projective space. In the infinite dimensional case the joint spectrum is an analytic set near every isolated spectral point of finite multiplicity of an operator in the tuple.

Local spectral analysis near a regular point of a determinantal manifold (projective joint spectrum) provides a substantial information about the mutual behavior of the elements in the tuple (see Stessin, Tchernev 2019, Peebles, Stessin 2020).

In this talk we consider results related to local spectral analysis near a point in the singular locus of the joint spectrum which is a regular point of reducibility. As an application we obtain a spectral rigidity result for representations of Coxeter group

Kai Wang, School of Mathematical Sciences, Fudan University

Law of Large Numbers in Determinantal Point Processes

Abstract. In the talk we will show a weak law of large numbers for the determinantal point processes from zeros of Gaussian analytic functions on the unit disc. This is a joint work with prof. Yanqi Qiu.

Penghui Wang

Essential normality of quotient modules in weighted Bergman modules on the polydiscs

Abstract. In this talk, I will introduce the recent development on the essential normality of quotient modules of analytic Hilbert modules over the polydisc. It will be seen that the essential normality of the quotient modules is closed related to the distinguished varieties, which were introduced by Agler and McCarthy. Some application on the Hilbert-Schmidtness of submodules in Hardy modules over the polydisc will be considered. The talk is based on the joint work with K.Guo and C.Zhao.

Gerald Wanjala, Sultan Qaboos University, Oman

The Numerical Range and Stability of Index for Multivalued Linear Operators

Abstract. For multivalued linear operators $\mathcal{A}$ and $\mathcal{B}$ on a Hilbert space H, we define the numerical range of $\mathcal{A}$ relative to $\mathcal{B}$ to be the subset $\Theta_{\mathcal{B}}(\mathcal{A})$ of $\mathbb{C}$ defined by

$$\Theta_{\mathcal{B}}(\mathcal{A}) = \{\langle v, w\rangle : v \in \mathcal{A}(u), w \in \mathcal{B}(u) \text{ with } \|w\| = 1 \ \&\ u \in D(\mathcal{A}) \cap D(\mathcal{B})\}.$$

If $\mathcal{B}$ is the identity relation $\mathcal{I}$ on H, then $\Theta_{\mathcal{I}}(\mathcal{A})$ is simply the classical numerical range $\Theta(\mathcal{A})$ of $\mathcal{A}$ given by

$$\Theta(\mathcal{A}) = \{\langle v, u\rangle : v \in \mathcal{A}(u),\ u \in D(\mathcal{A}) \text{ with } \|u\| = 1\}.$$

In this talk, we use the numerical range $\Theta_{\mathcal{B}}(\mathcal{A})$ of $\mathcal{A}$ relative to $\mathcal{B}$ to study the stability of the index $\kappa(\mathcal{A} - \xi\mathcal{B})$, where ξ is a constant. In particular, we show that $\kappa(\mathcal{A} - \xi\mathcal{B})$ is constant for ξ outside the closure of the numerical range $\Theta_{\mathcal{B}}(\mathcal{A})$ of $\mathcal{A}$ relative to $\mathcal{B}$.

Recall that the index $\kappa(\mathcal{A})$ of a multivalued linear operator $\mathcal{A}$ is defined by $\kappa(A) := \alpha(A) - \beta(A)$, where $\alpha(A) := \dim N(A)$ and $\beta(A) := \dim H/R(A)$.

Hugo J. Woerdeman, Drexel University

Completing an Operator Matrix and the Free Joint Numerical Radius

Abstract. Ando's classical characterization of the unit ball in the numerical radius norm was generalized by Farenick, Kavruk, and Paulsen using the free joint numerical radius of a tuple of Hilbert space operators $(X_1, \ldots, X_m)$. In particular, the characterization leads to a positive definite completion problem. In this paper, we study various aspects of Ando's result in this generalized setting. Among other things, this leads to the study of finding a positive definite solution L to the equation

$$L = I + \sum_{j=1}^{m} \left[\left(L^{\frac{1}{2}} X_j^* L X_j L^{\frac{1}{2}} + \frac{1}{4} I \right)^{\frac{1}{2}} + \left(L^{\frac{1}{2}} X_j L X_j^* L^{\frac{1}{2}} + \frac{1}{4} I \right)^{\frac{1}{2}} \right],$$

which may be viewed as a fixed point equation. Once such a fixed point is identified, the desired positive definite completion is easily obtained. Along the way we derive other related results including basic properties of the free joint numerical radius and an easy way to determine the free joint numerical radius of a tuple of generalized permutations. This talk is based on joint work with Kennett L. Dela Rosa.

Nicholas Young, Newcastle University

Herglotz and Nevanlinna representations in several variables

Abstract. I will present some classical results of Herglotz and Nevanlinna which give representation formulae for certain holomorphic functions of a single variable. I will then introduce several-variable analogues of the Herglotz and Nevanlinna classes of functions. I will describe four types of representations of functions in these two classes, will sketch the proof of their existence, and will analyse the function-theoretic properties (for example, growth at infinity) of functions having the different types of representation.

The talk is based on joint work with Jim Agler, J. E. McCarthy and Ryan Tully-Doyle.

[1] J. Agler, J. E. McCarthy and N. J. Young, *Operator analysis: Hilbert Space Methods in Complex Analysis*, Cambridge Tracts in Mathematics **219**, Cambridge University Press, 2020, 389pp.

[2] J. Agler, R. Tully-Doyle and N. J. Young, 'Nevanlinna representations in several variables', *J. Functional Analysis*, **270(8)** (2016) 3000-3046.

Jiayang Yu

L^2 methods in infinite dimensional spaces

Abstract. The classical L^2 estimate for the $\overline{\partial}$ operators is a basic tool in complex analysis of several variables. Naturally, it is expected to extend this estimate to infinite dimensional complex analysis, but this is a longstanding unsolved problem, due to the essential difficulty that there exists no nontrivial translation invariant measure in the setting of infinite dimensions. The main purpose in this series of work is to give an affirmative solution to the above problem, and apply the estimates to the solvability of the infinite dimensional $\overline{\partial}$ equations. In this first part, we focus on the simplest case, i.e., L^2 estimates and existence theorems for the $\overline{\partial}$ equations on the whole space of ℓ^p for $p \in [1, \infty)$. The key of our approach is to introduce a suitable working space, i.e., a Hilbert space for (s,t)-forms on ℓ^p (for each nonnegative integers s and t), and via which we define the $\overline{\partial}$ operator from (s,t)-forms to $(s,t+1)$-forms and establish the exactness of these operators, and therefore in this case we solve a problem which has been open for nearly forty years.

5.8. New Trends in Fractional Calculus Operators and Their Applications.

Tuesday 2:30-4:00 in Language Center, Room 403 chair: Nelson Vieira

2:30-2:55	Tillmann Kleiner *Convolution on Distribution Spaces Characterized by Regularization*
3:00-3:25	Arran Fernandez *Parametrised families of fractional calculus operators: an algebraic viewpoint*
3:30-3:55	Milton Ferreira *Hypercomplex operator calculus for the fractional Helmholtz equation*

Tuesday 4:30-6:00 in Language Center, Room 403 chair: Arran Fernandez

4:30-4:55	Hüseyin Budak *Simpson-type inequalities for ψ-Hilfer fractional integrals*
5:00-5:25	M. Manuela Rodrigues *Fractional problems with ψ-Hilfer derivative*
5:30-5:55	Nelson Vieira *Time-fractional telegraph equation with ψ-Hilfer fractional derivative in higher dimensions*

Thursday 2:30-4:00 in Language Center, Room 403 chair: M. Manuela Rodrigues

2:30-2:55	Kai Diethelm *Terminal Value Problems for Fractional Ordinary Differential Equations: Analytical Properties of Their Solutions and an Efficient Numerical Algorithm*
3:00-3:25	Natalia Dilna *General Exact Solvability Conditions for the Initial Value Problems for Linear Fractional Functional Differential Equations*
3:30-3:55	Hanna Britt Soots *A numerical method for time-fractional sub-diffusion problems*

Thursday 4:30-6:00 in Language Center, Room 403 chair: Milton Ferreira

4:30-4:55	Pavel Dubovski *Quasi-Bessel and Legendre fractional operators*
5:00-5:25	Fatma Al-Musalhi *Analytical solutions to non-homogeneous fractional differential equations*
5:30-5:55	Hasan Kara *Generalizations in conformable fractional integral inequalities with parameters*

5.8.1. *Abstracts.*

Fatma Al-Musalhi, Sultan Qaboos University, Oman

Analytical solutions to non-homogeneous fractional differential equations

Abstract. Non-homogeneous fractional differential equations containing variable coefficients and fractional derivatives of different types are considered. Analytical solutions to these equations are obtained using the successive approximation method. The obtained solutions are expressed in integral forms. Example solutions with particular choices of the non-homogeneous term are presented. Some of the existing results in the literature appear as special cases of the general solutions obtained in this work. The inverse source problem of a fractional diffusion equation is presented as an application. A solution to this problem is constructed based on appropriate eigenfunction expansions and results on existence are established.

Hüseyin Budak, Düzce University, Turkey

Simpson-type inequalities for ψ-Hilfer fractional integrals

Abstract. The concept of ψ-Hilfer fractional integrals generalizes the some well-known fractional integrals such as Riemann-Liouville fractional integrals, Hadamard fractional integrals, etc. In this study, we first prove two identities involving ψ-Hilfer fractional integrals for differentiable functions. By utilizing these equalities, we establish several Simpson-type inequalities for ψ-Hilfer fractional integrals. For this purpose, we consider the concept of convexity and well-known Hölder inequality. Moreover, we discuss the connections between our main findings and previous studies.

[1] H. Budak, F. Hezenci, and H. Kara, *On parameterized inequalities of Ostrowski and Simpson type for convex functions via generalized fractional integrals*, Math. Meth. Appl. Sci. **44**-No.17 (2021), 12522–12536.

[2] J. Chen and X. Huang, *Some new inequalities of Simpson's type for s-convex functions via fractional integrals*, Filomat **31**-No.15 (2017), 4989–4997.

[3] M. Iqbal, S. Qaisar, and S. Hussain, *On Simpson's type inequalities utilizing fractional integrals*, J. Comput. Anal. Appl **23**-No.6 (2017) 1137–1145.

[4] S. Kermausuor, *Simpson's type inequalities via the Katugampola fractional integrals for s-convex functions*, Kragujev. J. Math. **45**-No.5 (2021), 709–720.

[5] C. Luo and T. Du, *Generalized Simpson type inequalities involving Riemann-Liouville fractional integrals and their applications*, Filomat, **34**(3), (2020), 751-760.

[6] M.Z. Sarikaya, E. Set, and M.E. Özdemir, *On new inequalities of Simpson's type for functions whose second derivatives absolute values are convex*, J. Appl. Math. Stat. Inform. **9**-No.1 (2013), 37–45.

[7] M. Z. Sarikaya, E. Set, and M.E. Ozdemir, *On new inequalities of Simpson's type for s-convex functions*, Comput. Math. Appl. **60**-No.8 (2010), 2191–2199.

Kai Diethelm, Technische Hochschule Würzburg-Schweinfurt, Germany

Terminal Value Problems for Fractional Ordinary Differential Equations: Analytical Properties of Their Solutions and an Efficient Numerical Algorithm

Abstract. When fractional ordinary differential equations are combined with terminal conditions, one obtains a problem whose nature differs significantly from the classical and deeply investigated initial value problems. In this talk, we briefly discuss sufficient conditions under which such terminal value problems have unique continuous solutions. Then we present results showing how these solutions depend on the given terminal condition. Finally, we exploit these analytical observations to design rapidly converging numerical methods for solving problems of this kind.

The talk is based on joint work with H. T. Tuan (Vietnamese Academy of Science and Technology) and F. Uhlig (Auburn University).

Natalia Dilna, Institute of Mathematics of the Slovak Academy of Sciences, Slovakia

General Exact Solvability Conditions for the Initial Value Problems for Linear Fractional Functional Differential Equations

Abstract. The primary interest of our investigation in [1] is to find precise requirements sufficient for the existence of the unique solution of the initial-value problem for fractional functional differential equations (FFDEs), in contrast with the more general conditions. The novelty of the main result is given by the fact that it is not necessary to calculate fractional derivatives here. Therefore, such a condition is much more practical to use in variable FFDEs. The main focus of the investigations is on obtaining the new method practiced with exact results for linear FFDEs that could be useful for various applications. Also, the discrete memory effect model and the model of the pantograph type from electrodynamics are studied.

This research was supported by the Slovak Grant Agency VEGA-SAV, Grant No. 2/0127/20

[1] N. Dilna, *General exact solvability conditions for the initial value problems for linear fractional functional differential equations*, Arch. Math.-Brno **59**-No.1 (2023), 11–19.

Pavel Dubovski
Stevens Institute of Technology, USA

Quasi-Bessel and Legendre fractional operators

Abstract. We introduce fractional quasi-Bessel operators

$$\sum_{i=1}^{m} d_i x^{\xi_i} D^{\alpha_i} u(x) + (x^{\beta} - \nu^2) u(x)$$

and construct the existence theory for the corresponding equations in the class of fractional series solutions. In order to find the parameters of the series, we derive the characteristic equation, which is surprisingly independent of the terms with non-matching parameters $\xi_i \neq \alpha_i$. As a direct corollary, the method allows to analyze quasi-Euler, constant-coefficient, and Legendre equations and is applicable to the existence result for elliptic-like PDEs with fractional Cauchy-Euler operator. The theoretical findings are justified computationally.

The results are obtained jointly with L. Boyadjiev (City University of New York, USA) and J.A. Slepoi (Stevens Institute of Technology, USA).

[1] P.B. Dubovski and J.A. Slepoi, *Construction and analysis of series solutions for fractional quasi-Bessel equations*, Fract. Calc. Appl. Anal. **25**-No.3 (2022), 1229–1249.

[2] L. Boyadjiev, P.B. Dubovski, and J.A. Slepoi, *Existence for partial differential equations with fractional Cauchy-Euler operator*, J. Math. Sci. **266**-No.2 (2022), 285–294.

[3] P.B. Dubovski and J.A. Slepoi, *Analysis of solutions of some multi-term fractional Bessel equations*, Fract. Calc. Appl. Anal. **24**-No.5 (2021), 1380–1408.

Arran Fernandez, Eastern Mediterranean University, Northern Cyprus

Parametrised families of fractional calculus operators: an algebraic viewpoint

Abstract. One of the new trends in fractional calculus operators is the creation of many different operators defined by convolution integrals combined with differentiation. Several broad classes of operators have been defined by considering certain general types of kernel functions, such as analytic kernels and Sonine kernels. But none of the broad classes so far have enabled a general version of the semigroup property, an important fact about some types of fractional calculus (such as Prabhakar) which enables many further properties to be neatly obtained. Some of the general kernels (such as Sonine kernels) do not even have an explicit fractional order parameter, while others (such as analytic kernels) do not even have elegant inversion properties in general. Here, we introduce a new algebraic perspective on generalised fractional calculus, inspired by ideas from operational calculus, which enables parametrised families of fractional calculus operators to be constructed in a way that preserves various desired properties while allowing high generality.

Milton Ferreira, Polytechnic of Leiria, Portugal

Hypercomplex operator calculus for the fractional Helmholtz equation

Abstract. In this talk, we construct analytical solutions for the fractional homogeneous and inhomogeneous Helmholtz equation where fractional derivatives are in the sense of Caputo or Riemann-Liouville. With the fundamental solution at hand, we construct an operator calculus in the framework of hypercomplex function theory to deal with boundary value problems. We show an interesting triple-duality relation between left and right derivatives, Caputo and Riemann-Liouville derivatives, and eigensolutions of antipodal eigenvalues in terms of a generalized Borel-Pompeiu formula. Some examples are presented to illustrate our results.

Hasan Kara, Düzce University, Turkey

Generalizations in conformable fractional integral inequalities with parameters

Abstract. This current study presents a novel equality for differentiable convex functions involving conformable fractional integrals. Using this equality, we establish new parameterized inequalities via conformable fractional integrals. These inequalities are significant due to their utilization of convexity, which enhances their applicability in various mathematical applications. Additionally, we use special cases of the obtained theorems to derive both previously achieved and new results. Overall, this research offers valuable insights into the use of conformable fractional integrals in solving complex mathematical problems involving convex functions.

This is a joint work with Hüseyin Budak and Fatih Hezenci.

[1] H. Budak, F. Hezenci, and H. Kara, *On generalized Ostrowski, Simpson and Trapezoidal type inequalities for co-ordinated convex functions via generalized fractional integrals*, Adv. Differ. Equ. **2021** (2021), Article No.312 (32pp.).

Tillmann Kleiner, ICP, Universität Stuttgart, Germany

Convolution on Distribution Spaces Characterized by Regularization

Abstract. Motivated by interpreting fractional calculus operators as convolution operators, locally convex convolutor spaces are studied which consist of those distributions that define a continuous convolution operator mapping from the space of test functions into a given locally convex lattice of measures [1]. The convolutor spaces are endowed with the topology of uniform convergence on bounded sets. Their locally convex structure is characterized via regularization and function-valued seminorms under mild structural assumptions on the space of measures. Many recent generalizations of classical distribution spaces turn out to be special cases of the general convolutor spaces introduced here. Recent topological characterizations of convolutor spaces via regularization are extended and improved. In applications to fractional calculus the convolutor spaces guarantee that convolution of distributions inherits continuity properties from those of bilinear convolution mappings between the locally convex lattices of measures.

This is a joint work with R. Hilfer (ICP, Universität Stuttgart).

[1] T. Kleiner and R. Hilfer, *Convolution and distribution spaces characterized by regularization*, Math. Nachr. **296**-No.5 (2023), 1938–1963.

M. Manuela Rodrigues, CIDMA & Department of Mathematics, University of Aveiro, Portugal

Fractional problems with ψ-Hilfer derivative

Abstract. In this work, we consider the multidimensional time-fractional diffusion equation with the ψ-Hilfer derivative. This fractional derivative enables the interpolation between Riemann–Liouville and Caputo fractional derivatives and its kernel depends on an arbitrary positive monotone increasing function ψ, thus encompassing several fractional derivatives in the literature. This allows us to obtain general results for different families of problems that depend on the function ψ selected. We obtain a solution representation in terms of convolutions involving Fox H-functions for the Cauchy problem associated with our equation. Series representations of the first fundamental solution are explicitly obtained for any dimension.

Finally, some plots of the fundamental solution are presented for particular choices of the function ψ and the order of differentiation.

[1] N. Vieira, M.M. Rodrigues, and M. Ferreira, *Time-fractional diffusion equation with ψ-Hilfer derivative*, Comput. Appl. Math. **41**-No.6 (2022), Article No. 230 (26pp.),

Hanna Britt Soots, Institute of Mathematics and Statistics of University of Tartu, Estonia

A numerical method for time-fractional sub-diffusion problems

Abstract. The time-fractional diffusion equation has been found in many real-life processes, where anomalous diffusion occurs. Due to the fact that the analytical solution of the fractional diffusion equation is not usually known, effective numerical methods are vital when solving real world problems. In this presentation, we will explore how combining two techniques—the method of lines and collocation method—can yield a higher degree of accuracy than a two-way discretization approach.

To demonstrate our findings, we investigate the initial value problem

$$D_t^\alpha u(x,t) - p\frac{\partial^2 u(x,t)}{\partial x^2} + \nu(x)u(x,t) = f(x,t) \tag{2}$$

for $(x,t) \in Q = (0,L) \times (0,b]$ with

$$u(0,t) = \psi_0(t), \quad t \in (0,b], \tag{3}$$

$$u(L,t) = \psi_L(t), \quad t \in (0,b], \tag{4}$$

$$u(x,0) = \phi(x), \quad x \in [0,L]. \tag{5}$$

where $0 < \alpha < 1$, p is a positive constant (also called in literature the general diffusion coefficient), $\nu \geq 0$ and $\nu, f, \psi_0, \psi_L, \phi$ are continuous in their respective fields.

The idea is first use the method of lines: create a uniform mesh on the space-interval $[0,L]$ and approximate the space derivative in (2) by the second order finite difference. We get a system of equations for the $y_i(t) \approx u(x_i,t)$, where x_i is the gridpoint in $[0,L]$ and t belongs to $[0,b]$.

We denote $z_i = D^\alpha y_i$ and use the collocation-based approach to approximate z_{iN} using the graded grid on the time-interval $[0,b]$ and Lagrange fundamental polynomials.

In the talk we describe the existence, uniqueness and regularity properties of the solution to problem, study the convergence and convergence order of our method and present some numerical examples.

Nelson Vieira, CIDMA & Department of Mathematics, University of Aveiro, Portugal

Time-fractional telegraph equation with ψ-Hilfer fractional derivative in higher dimensions

Abstract. In this work, we consider the multidimensional time-fractional diffusion equation with the ψ-Hilfer derivative. This fractional derivative depends on an arbitrary positive monotone increasing function ψ and encompass several fractional derivatives in the literature. This allows us to obtain general results for different families of problems that depend on the function ψ selected. We obtain a solution representation in terms of convolutions involving Fox H-functions for the Cauchy problem associated with our equation. Series representations of the first fundamental solution are explicitly obtained for any dimension as well as the fractional moments of arbitrary positive order. For the one-dimensional case, the fundamental solution corresponds to a probability density function for any admissible ψ. Some plots of the fundamental solution are presented for particular choices of the function ψ and the order of differentiation.

[1] N. Vieira, M. Ferreira, and M.M. Rodrigues, *Time-fractional telegraph equation with ψ-Hilfer derivatives*, Chaos Solitons Fractals **162** (2022), Article No. 112276.

5.9. Non-selfadjoint Operators.

Monday 2:30-4:30 in Language Center, Room 204 chair: Marco Marletta

2:30-2:55	Jonathan Ben-Artzi *Can we always approximate spectra?*
3:00-3:25	Christiane Tretter *Analysis and eigenvalue computation for Klein-Gordon equations*
3:30-3:55	Petr Blaschke *Asymptotic eigenvalue distribution of complex non-self-adjoint Jacobi matrices*
4:00-4:25	Raffael Hagger *Double-layer potentials on dilation-invariant domains*

Thursday 2:30-4:00 in Metsätalo, Room 25 chair: Jonathan Ben-Artzi

2:30-2:55	Oscar F. Bandtlow *Lower bounds for regularised determinants and an application to estimating spectral variation*
3:00-3:25	Annemarie Luger *Quasi-Herglotz functions*
3:30-3:55	Christian Emmel *Realizations of meromorphic functions of bounded type*

Thursday 4:30-6:00 in Metsätalo, Room 25 chair: Annemarie Luger

4:30-4:55	Francesco Ferraresso *Spectral properties of dissipative Maxwell systems in inhomogeneous materials*
5:00-5:25	Ramesh Golla *Absolutely norm attaining operators*
5:30-5:55	Yuri Tomilov *Instability for semilinear evolution equations via operator theory*

5.9.1. *Abstracts.*

Oscar F. Bandtlow, Queen Mary University of London

Lower bounds for regularised determinants and an application to estimating spectral variation

Abstract. Let p be a positive integer. Given an operator A belonging to the Schatten class S_p over an infinite-dimensional separable Hilbert space, consider the corresponding regularised determinant of order p, that is,

$$\det_p(1-A) = \prod_{n=1}^{\infty}(1-\lambda_n)\exp\left(\sum_{k=1}^{p-1}\frac{\lambda_n^k}{k}\right),$$

where $(\lambda_n)_{n\in\mathbb{N}}$ denotes the sequence of eigenvalues of A. The regularised determinant plays an important role in perturbation theory, as $z \mapsto \det_p(1-zA)$ is an entire function of finite order with its zeros coinciding with the reciprocals of the eigenvalues of A.

In this talk I will report on new lower bounds of the form

$$|\det_p(1-z^{-1}A)| \geq \exp\left(-c_p\frac{\|A\|_p^p}{\operatorname{dist}(z,\sigma(A))^p}\right) \quad (\forall z \in \mathbb{C}\setminus\sigma(A))$$

where $\|A\|_p$ denotes the Schatten norm of A, $\operatorname{dist}(z,\sigma(A))$ the distance of z to the spectrum $\sigma(A)$ of A and the constant c_p does not depend on A.

I will also explain how this lower bound can be used to obtain explicit upper bounds for the operator norm $\|(zI-A)^{-1}\|$ of the resolvent of a Schatten class operator A expressible in terms of the Schatten norm and the distance of z to $\sigma(A)$ only. These bounds in turn yield explicit upper bounds for the spectral variation of two Schatten class operators, that is, the Hausdorff distance of their spectra, in terms of their Schatten norms and the operator norm of their difference.

The resulting resolvent and spectral variation bounds improve existing bounds of this form obtained by different means (see [1,2,3]). Moreover, unlike the methods used in [1,2,3], the approach described above easily generalises to the Banach space case.

References:

[1] O. F. Bandtlow: Estimates for norms of resolvents and an application to the perturbation of spectra, *Math. Nachr.*, 267 (2004), 3–11.
[2] M. I. Gil': *Operator Functions and Localization of Spectra*, Berlin, Springer, 2003.
[3] A. Pokrzywa: On continuity of spectra in norm ideals, *Linear Algebra Appl.* 69 (1985), 121–130.

Jonathan Ben-Artzi, Cardiff University

Can we always approximate spectra?

Abstract. The purpose of this talk is to present recent results providing algorithms that can approximate resonances (both 'quantum' resonances of Schrödinger operators and 'classical' resonances generated by obstacles in $\mathbb{R}^n$). However, this will be preceded by a discussion of the so-called Solvability Complexity Index Hierarchy, introduced by A. Hansen, which is an abstract theory for the classification of the complexity of a variety of computations. This will demonstrate that the existence of algorithms such as the resonance algorithms is not at all obvious: for instance, there does not exist an algorithm for computing the spectra of bounded infinite matrices! This is based on various joint works, most recently with M. Marletta and F. Rösler.

Petr Blaschke, Silesian University in Opava, Czech Republic

Asymptotic eigenvalue distribution of complex non-self-adjoint Jacobi matrices

Abstract. We investigate the asymptotic eigenvalue distribution of tri-diagonal symmetric $n \times n$ matrices $J_n(a,b)$:

$$J_n(a,b)_{i,i} := b\left(\frac{i}{n}\right), \qquad J_n(a,b)_{i+1,i} := a\left(\frac{i}{n}\right),$$

where the diagonals are determined by sampling values of complex functions $a, b : I \to \mathbb{C}$ from the unit interval $I := [0,1]$.

It has been numerically observed that eigenvalues of J_n cluster on certain curves in $\mathbb{C}$ as $n \to \infty$. The problem of determining these curves leads (for specific functions a, b) to a problem of finding an asymptotic expansion of a Laplace type contour integral with multiple saddle points.

Christian Emmel, Stockholm University

Realizations of meromorphic functions of bounded type

Abstract. A realization of an analytic function is a way of writing the function in terms of the resolvent of an self-adjoint relation acting on a Krein space (an indefinite inner product space). In this talk, we show that every meromorphic function of bounded type has a realization. Moreover, we discuss when this realization can be constructed in a minimal way.

Francesco Ferraresso, Università di Sassari

Spectral properties of dissipative Maxwell systems in inhomogeneous materials

Abstract. Electromagnetic waves propagating through conductive media tend to lose part of their energy; from a mathematical point of view, conductivity makes the underlying Maxwell operator non-selfadjoint. I will discuss some recent results about the spectrum of the dissipative Maxwell system and the lossy Drude-Lorentz model in (possibly) unbounded domains of the three-dimensional Euclidean space. The essential spectrum is characterised as the union of two

parts, one related to the behaviour of divergence-free vector fields at 'infinity'; the other to the loss of ellipticity of a suitably defined $\operatorname{div} p(\omega)\nabla$ operator. Spectral approximation via domain truncation is shown to be reliable outside an explicit, 'small' set of spectral pollution. Based on joint work with S. Bögli, M. Marletta, and C. Tretter.

Ramesh Golla, Indian Institute of Technology Hyderabad

Absolutely norm attaining operators

Abstract. A bounded linear operator T defined on a Hilbert space H is called *norm attaining* if there exist $x \in H$ with unit norm such that $\|Tx\| = \|T\|$. If for every non-zero closed subspace $M \subseteq H$, the operator $T|_M : M \to H$ is norm attaining, then T is called an *absolutely norm attaining* operator or $\mathcal{AN}$-operator.

In this talk, we discuss basic characterizations, properties and spectral representations of this class of operators.

Raffael Hagger, Kiel University

Double-layer potentials on dilation-invariant domains

Abstract. Consider the Dirichlet problem

$$\begin{aligned} \Delta u &= 0 \quad \text{on } \Omega, \\ u &= f \quad \text{on } \partial\Omega \end{aligned}$$

on a bounded Lipschitz domain $\Omega \subset \mathbb{R}^d$. With the help of a fundamental solution, in this case given by the Newton potential Φ, this problem can be translated into an integral equation involving the double layer (or Neumann-Poincaré) operator

$$D : L^2(\partial\Omega) \to L^2(\partial\Omega), \quad Dg(x) := \int_{\partial\Omega} \frac{\partial\Phi(x-y)}{\partial n(y)} g(y)\,\mathrm{d}s(y).$$

The spectral radius of this (non-selfadjoint) operator is conjectured to be less than or equal to $\frac{1}{2}$. This has been verified for several special cases including C^1-domains and polygons, but the general case has been open for decades. In this talk we will study this conjecture on domains that are locally dilation invariant, which means that the boundary can be locally described by either a C^1-graph or by the graph of a Lipschitz continuous function f satisfying $f(\alpha y) = \alpha f(y)$ for a fixed $\alpha \in (0,1)$. The study of such domains is motivated by a recent paper of Chandler-Wilde and Spence [1], where it was shown that for these domains the (essential) numerical range can get arbitrarily large.

Joint work with S.N. Chandler-Wilde, K.-M. Perfekt and J.A. Virtanen [2].

[1] S.N. Chandler-Wilde, E. Spence: *Coercivity, essential norms, and the Galerkin method for second-kind integral equations on polyhedral and Lipschitz domains*, Numer. Math. 150, 299-371 (2022).

[2] S.N. Chandler-Wilde, R. Hagger, K.-M. Perfekt, J.A. Virtanen: *On the spectrum of the double layer operator on locally-dilation-invariant Lipschitz domains*, Numer. Math. 153, 653-699 (2023).

Annemarie Luger, Stockholm University, Sweden

Quasi-Herglotz functions

Abstract. With (resolvents of) self-adjoint operators in Hilbert spaces there are associated Herglotz-Nevanlinna functions, i.e., analytic functions that map the complex upper half plane into itself.

In this talk we are going to discuss an extension of this class, namely Quasi-Herglotz-functions, which by definition are linear combinations of the last named. This class is surprisingly rich, has interesting analytic properties, and is naturally related to non-selfadjoint operators.

This talk is based on joint work with Mitja Nedic.

Yuri Tomilov, IM PAN, Warsaw

Instability for semilinear evolution equations via operator theory

Abstract. I will present a new approach to the study of growth and instability for solutions to semilinear evolution equations with compact nonlinearities in Banach spaces. This approach shows, in particular, that compact nonlinear perturbations of linear evolution equations can be treated as linear ones as far as the growth of their solutions is concerned. I will explain how appropriate resolvent bounds can be used to describe the norm growth of solutions for a dense set of initial data. Several applications of the new approach, including asymptotic properties of solutions to semilinear damped wave equations, will also be discussed.

This is joint work with V. Müller and R. Schnaubelt.

Christiane Tretter, University of Bern, CH

Analysis and eigenvalue computation for Klein-Gordon equations

Abstract. In this talk we will study the spectral properties of Klein-Gordon problems. Special attention will be paid to analytic estimates and enclosures for both real and non-real eigenvalues. We will also touch upon the computational complexity of the Klein-Gordon spectral problem in the framework of the Solvability Complexity Index (SCI) Hierarchy.

5.10. Noncommutative Geometry.

Monday 2:30-4:30 in Language Center, Room 405 chair: Tirthankar Bhattacharyya

2:30-2:55	Konrad Aguilar *The strongly Leibniz property and the Gromov–Hausdorff propinquity*
3:00-3:25	Are Austad *Detecting ideals in reduced crossed product C^*-algebras of topological dynamical systems*
3:30-3:55	Jens Kaad *Spectral metrics on quantum projective spaces*
4:00-4:25	Carla Farsi *Metric spectral triples, Gromov-Hausdorff limits, and inductive limits*

Tuesday 2:30-4:00 in Language Center, Room 405 chair: Giovanni Landi

2:30-2:55	Sophie Emma Zegers *Split extensions and KK-equivalences for quantum flag manifolds*
3:00-3:25	Yufan Ge *Subproduct systems from $SU(2)$-representations*
3:30-3:55	Debashish Goswami *Quantum Isometry Groups of compact metric space*

Tuesday 4:30-6:00 in Language Center, Room 405 chair: Carla Farsi

4:30-4:55	Paolo Piazza *Primary and Secondary Invariants of Dirac operators on G-proper manifolds*
5:00-5:25	Chi-Keung Ng *Reconstructing von Neumann algebras from metric spaces*
5:30-5:55	Walter van Suijlekom *Tolerance relations and operator systems*

Thursday 2:30-4:00 in Language Center, Room 405 chair: Walter van Suijlekom

2:30-2:55	Bram Mesland *The Levi-Civita connection on modules of non commutative differential forms*
3:00-3:25	Joakim Arnlind *Levi-Civita connections for a class of noncommutative minimal surfaces*
3:30-3:55	Giovanni Landi *Hopf algebroids, Atiyah sequences and noncommutative gauge theories*

Thursday 4:30-6:00 in Language Center, Room 405 chair: Joakim Arnlind

4:30-4:55	Priyanga Ganesan *A quantum isometry game for quantum metric spaces*
5:00-5:25	Kanat Tulenov *On Fourier multipliers on quantum tori and applications*
5:30-5:55	Franz Luef *Metaplectic transformations for Heisenberg modules*

Friday 11:30-1:00 in Language Center, Room 405 chair: Jens Kaad

11:30-11:55	Sushil Singla *Sequence of operator algebras converging to odd spheres in the quantum Gromov-Hausdorff distance*
12:00-12:25	Malte Leimbach *On spectral truncations of tori*
12:30-12:55	Jacopo Zanchettin *Morita equivalence for the Erhesmann-Schauenburg Hopf algebroid*

Friday 2:30-4:00 in Language Center, Room 405 chair: Jens Kaad

2:30-2:55 Koen van den Dungen
Generalised Dirac-Schrödinger operators

5.10.1. *Abstracts.*

Konrad Aguilar, Pomona College

The strongly Leibniz property and the Gromov–Hausdorff propinquity

Abstract. We construct a new version of the dual Gromov–Hausdorff propinquity that is sensitive to the strongly Leibniz property. In particular, this new distance is complete on the class of strongly Leibniz quantum compact metric spaces. Then, given an inductive limit of C*-algebras for which each C*-algebra of the inductive limit is equipped with a strongly Leibniz L-seminorm, we provide sufficient conditions for placing a strongly Leibniz L-seminorm on an inductive limit such that the inductive sequence converges to the inductive limit in this new Gromov–Hausdorff propinquity. As an application, we place new strongly Leibniz L-seminorms on AF-algebras using Frobenius–Rieffel norms, for which we have convergence of the Effros–Shen algebras in the Gromov–Hausdorff propinquity with respect to their irrational parameter. (This is joint work with Stephan Ramon Garcia, Elena Kim, and Frédéric Latrémolière, arXiv: 2301.05692).

Joakim Arnlind

Levi-Civita connections for a class of noncommutative minimal surfaces

Abstract. We study connections on hermitian modules, and show that metric connections exist on regular hermitian modules, i.e finitely generated projective modules together with a non-singular hermitian form. In addition, we develop an index calculus for such modules, and provide a characterization in terms of the existence of a pseudo-inverse of the matrix representing the hermitian form with respect to a set of generators. The framework is applied to a class of noncommutative minimal surfaces, for which there is a natural concept of torsion, and show that there exist metric and torsion-free connections for every minimal surface in this class.

Are Austad, University of Southern Denmark, Odense

Detecting ideals in reduced crossed product C^*-algebras of topological dynamical systems

Abstract. Given a discrete twisted C^*-dynamical system (A,Γ,α,σ), we would like to understand the ideal structure of the reduced crossed product $C^*_r(\Gamma,A;\alpha,\sigma)$. One way to go about this is to attempt to detect the ideals using subalgebras of $C^*_r(\Gamma,A;\alpha,\sigma)$. We say (A,Γ,α,σ) has the *ℓ^1-ideal intersection property* if every non-zero ideal of $C^*_r(\Gamma,A;\alpha,\sigma)$ has non-zero intersection with the ℓ^1-crossed product $\ell^1(\Gamma,A;\alpha,\sigma)$. We will present classes of groups Γ for which $(C(X),\Gamma,\alpha,\sigma)$ has the ℓ^1-ideal intersection property for all choices of $C(X)$, action α and twist σ. As a by-product, we find the first new class of discrete groups for which $\ell^1(\Gamma)$ has a unique C^*-norm since the '70s.

This is joint work with Sven Raum.

Carla Farsi, University of Colorado/Boulder

Metric spectral triples, Gromov-Hausdorff limits, and inductive limits

Abstract. Metric spectral triples are spectral triples in which the semi-norm associated to the commutator of the Dirac operator is a norm inducing the weak* topology on the state space of the spectral triple unital C*-algebra. Latrémolière recently introduced a metric on this class [5], the spectral propinquity, which is zero exactly when the spectral triples are unitary equivalent. This metric dominates the propinquity for quantum compact metric spaces. After introducing the main definitions, we will review some results that connect (isometry groups of)

metric spectral triples to inductive limits. Based on joint work with: T. Landry, N. Larsen, F. Latremoliere, and J. Packer:

[1] J. Bassi, R. Conti, C. Farsi, and F. Latrémolière, *Isometry groups of inductive limits of metric spectral triples and Gromov-Hausdorff convergence*, submitted, arXiv:2302.09117.

[2] R. Conti and C. Farsi, *Isometries of Kellendonk-Savinien spectral triples and Connes metrics*, Internat. J. Math. 33 (2022), no. 13, Paper No. 2250084, 26 pp.

[3] C. Farsi, T. Basa Landry, N. S. Larsen, and J. Packer, *Spectral triples for noncommutative solenoids and a Wiener's lemma*, J. Noncommutative Geom, to appear (2022), ArXiv: 2212.07470.

[4] C. Farsi, F. Latrémolière, and J. Packer, *Convergence of inductive sequences of spectral triples for the spectral propinquity*, arXiv:2301.00274.

[5]F. Latrémolière, *The Gromov-Hausdorff propinquity for metric spectral triples*, Adv. Math. **404** (2022), Paper No. 108393, 56pp.

Priyanga Ganesan, University of California San Diego

A quantum isometry game for quantum metric spaces

Abstract. In recent years, nonlocal games have received significant attention in operator algebras and resulted in highly fruitful interactions, including the recent resolution of the Connes Embedding Problem. A nonlocal game involves two non-communicating players who cooperatively play to win against a referee. In this talk, I will introduce a nonlocal game that captures an isometry game for quantum metric spaces in the sense of Kuperberg and Weaver. This nonlocal game involves quantum inputs and quantum outputs and generalises the metric isometry game introduced by K. Eifler. I will discuss the game C*-algebra in this case using a recent result of Brannan-Harris-Todorov-Turwoska (2023).

Yufan Ge, Leiden University

Subproduct systems from $SU(2)$-representations

Abstract. Subproduct systems, as introduced by Shalit and Solel in 2009, are families of C^*-correspondences endowed with suitable structure maps. We follow up on recent work by F. Arici and J. Kaad on $SU(2)$-equivariant subproduct systems arising from irreducible representations of the Lie group $SU(2)$ and extend their results to reducible representations. In particular, we obtain commutation relations for the Toeplitz algebras and Cuntz–Pimsner algebras. Moreover we show that the Toeplitz algebra is equivariantly KK-equivalent to the algebra of complex numbers $\mathbb{C}$. In this way, we compute K-theory groups of the corresponding Cuntz–Pimsner algebra. This talk is based on the joint work with F. Arici.

Debashish Goswami

Quantum Isometry Groups of compact metric space

Abstract. We give a brief overview of existence and some explicit examples of the so-called quantum isometry group for a compact metric space satisfying certain mild conditions. This is a generalization of the group of isometries of the metric space to the framework of compact quantum groups. The talk will be a combination of some slightly old results of the speaker and more recent results obtained by the speaker in a joint work with A. Chirvasitu.

Jens Kaad, University of Southern Denmark

Spectral metrics on quantum projective spaces

Abstract. A spectral metric space is a unital spectral triple satisfying that the coordinate algebra becomes a compact quantum metric space via the seminorm which measures the size of first order derivatives. In this talk we investigate the spectral metric properties of quantum projective spaces. The geometric framework for this investigation is provided by the unital spectral triples introduced by D'Andrea and Dabrowski in their CMP paper from 2010. We shall see that these unital spectral triples are in fact spectral metric spaces and, if time permits, indicate how this can be proved. This result makes it possible to investigate the spectral metric continuity properties of quantum projective spaces under variations of the deformation parameter q. It can moreover be viewed as a first step towards an understanding of the higher Vaksman-Soibelman spheres from the point of view of spectral metric spaces. The talk is based on work in progress with Max Holst Mikkelsen.

Giovanni Landi

Hopf algebroids, Atiyah sequences and noncommutative gauge theories

Abstract. We consider noncommutative principal bundles which are equivariant under a triangular Hopf algebra and analyze an associated (noncommutative) gauge groupoid as well as an Atiyah sequence of braided infinite dimensional Lie algebras which are related to gauge transformations acting on connections. From this sequence we derive a Chern-Weil homomorphism and braided Chern-Simons terms. We present explicit examples over noncommutative spheres.

Malte Leimbach, Radboud Universiteit Nijmegen

On spectral truncations of tori

Abstract. We discuss the matter of Gromov-Hausdorff convergence of the metric spaces arising from spectral truncations of the d-dimensional torus, as introduced by Connes and van Suijlekom. Such a convergence result holds if the $*$-algebra of smooth functions (on the torus) and the operator systems arising from its spectral truncations are C^1-*approximately order isomorphic.* We present a candidate for such an approximate isomorphism and explore connections to the rich theory of Fourier and Schur multipliers, which ultimately allow us to prove convergence (at least in low dimensions). This talk is based on joint work with Walter van Suijlekom.

Franz Luef, Norwegian University of Science and Technology

Metaplectic transformations for Heisenberg modules

Abstract. We discuss the structure of Heisenberg modules over isomorphic noncommutative tori and explain that the metaplectic representation, aka Weil representation, allows one to relate the respective module structures over two isomorphic noncommutative tori. Furthermore, are we using results and notions from linear symplectic geometry to develop a geometric approach towards basic results in the theory of Morita equivalence such as the one by Rieffel-Schwarz. This is joint work with Michael Gjertsen (NTNU Trondheim).

Bram Mesland, Leiden University, the Netherlands

The Levi-Civita connection on modules of non commutative differential forms

Abstract. In this talk I will present a new, operator theoretic construction of the Levi-Civita connection on a Riemannian manifold. The construction allows us to deduce the mild technical assumptions needed for the construction of the Levi-Civita connection on the module of non commutative differential forms associated to a spectral triple. This is joint work with Adam Rennie.

Chi-Keung Ng

Reconstructing von Neumann algebras from metric spaces

Abstract. Given a metric space $(X, \mathbf{d})$ with diameter $D < +\infty$. We introduce, in terms of maximal D-separated subsets of X, the notion of cyclic representations of X. Through cyclic representations, "bounded universal quantum functions" will be defined. Denote by $L^{\mathfrak{q}}_{\infty}(X)$ the JW^*-subalgebra of the von Neumann algebra of bounded universal quantum functions on X generated by the canonical image of X.

Let M be a von Neumann algebra with no type I_2 summand, and $\mathfrak{S}(M)$ be the set of its normal states, equipped with the metric $\mathbf{d}$ induced by the norm $\|\cdot\|_{M_*}$. It is shown that there is a Jordan *-isomorphism from M onto $L^{\mathfrak{q}}_{\infty}(\mathfrak{S}(M))$. In this way, we reconstruct the JW-algebra M_{sa} from the metric space $\mathfrak{S}(M)$.

Furthermore, M induces a structure on equivalence classes of cyclic representations of $\mathfrak{S}(M)$ that allows us to recover the von Neumann algebra structure of M completely.

Paolo Piazza, Sapienza Università di Roma

Primary and Secondary Invariants of Dirac operators on G-proper manifolds

Abstract. Abstract. In this talk I shall explain how cyclic cohomology and K-theory can be used in order to define and investigate primary and secondary invariants of G-equivariant Dirac operators on a cocompact G-proper manifold, with G a connected real reductive Lie group. I will first treat, rapidly, the case of cyclic cocycles associated to elements in the differentiable cohomology of G; I will then go on to delocalized cyclic cocycles, the main topic of this talk. I will explain the challenges in defining the delocalized eta invariant associated to the orbital integral defined by a semisimple element g in G and in showing that such an invariant enters in an Atiyah-Patodi-Singer index theorem for cocompact G-proper manifolds. I will then move to a higher version of these results, using the higher delocalized cyclic cocycles defined by Song and Tang. This talk is based on work with Hessel Posthuma and two recent articles with Hessel Posthuma, Yanli Song and Xiang Tang, the second one still work in progress but close to be finished.

Sushil Singla, University of Primorska, Slovenia

Sequence of operator algebras converging to odd spheres in the quantum Gromov-Hausdorff distance

Abstract. Marc Rieffel had introduced the notion of quantum Gromov-Hausdorff distance on compact quantum metric spaces and found a sequence of matrix algebras that converges to the space of continuous functions of two sphere in this distance, that one finds in many scattered places in the theoretical physics literature. The compact quantum metric spaces and convergence in the quantum Gromov-Hausdorff distance has been explored by a lot of mathematicians in the last two decades. We will define compact quantum metric space structure on the sequence of Toeplitz algebras on generalized Bergman space and prove that it converges to the space of continuous function on odd spheres in the quantum Gromov-Hausdorff distance. This is a joint work with Prof. Tirthankar Bhattacharyya.

Kanat Tulenov, Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan, and PDE and Analysis centre, Ghent University, Ghent, Belgium. E-mail: kanat.tulenov@ugent.be

On Fourier multipliers on quantum tori and applications

Abstract. In this work, we study L_p-L_q Fourier multiplier theorem on quantum tori. We obtain an analogue of Nikolskii's inequality on quantum tori. As applications we establish embedding theorems between Besov spaces on quantum tori, as well as embeddings between Besov and Wiener and Beurling spaces.

Koen van den Dungen, University of Bonn

Generalised Dirac-Schrödinger operators

Abstract. A Dirac-Schrödinger operator is a (self-adjoint) Dirac-type operator on a noncompact complete Riemannian manifold with a suitable skew-adjoint potential (which is invertible near infinity). In the classical setting, this potential is typically given by a matrix-valued function on the manifold. More generally, we will allow the potential to consist of unbounded operators acting on an infinite-dimensional Hilbert space (or even a Hilbert module over some C^*-algebra). We will present a generalised Callias Theorem, which shows that (under suitable assumptions) the Fredholm index of a generalised Dirac-Schrödinger operator on a noncompact manifold can be computed by restricting to a compact hypersurface.

Walter van Suijlekom, Radboud University Nijmegen

Tolerance relations and operator systems

Abstract. We extend the scope of noncommutative geometry by generalizing the construction of the noncommutative algebra of a quotient space to situations in which one is no longer dealing with an equivalence relation. For these so-called tolerance relations, passing to the associated equivalence relation looses crucial information as is clear from the examples such as the relation $d(x, y) < \epsilon$ on a metric space. Fortunately, thanks to the formalism of operator systems such an extension is possible and provides new invariants, such as the C*-envelope and the propagation number. After a thorough investigation of the structure of the (non-unital) operator systems associated to tolerance relations, we analyze the corresponding state spaces. In particular, we determine the pure state space associated to the operator system for the relation $d(x, y) < \epsilon$ on a path metric measure space. (joint work with Alain Connes)

Jacopo Zanchettin, SISSA

Morita equivalence for the Erhesmann-Schauenburg Hopf algebroid

Abstract. In this talk, I will recall the notion of principal bibundle for commutative Hopf algebroids introduced by El Kaoutit and Kowalzig and adapt it to Schauenburg's Hopf algebroids. Eventually, I will show that any such Hopf algebroid admitting a principal bibundle with a Hopf algebra is isomorphic to the Ehresmann-Schauenburg Hopf algebroid associated with a Hopf-Galois extension. This reproduces the classical result that every Lie groupoid is Morita equivalent to a Lie group if and only if it is the gauge groupoid of a principal bundle. In the last part, I will discuss how to get the same result starting from a monoidal equivalence.

Sophie Emma Zegers, Charles University, Prague

Split extensions and KK-equivalences for quantum flag manifolds

Abstract. In the study of noncommutative geometry, various classical spaces have been given a quantum analogue. Examples include Drinfeld-Jimbo quantum flag manifolds for which the C^*-completions have recently been described as graph C^*-algebras by Brzeziński, Krähmer, Ó Buachalla and Strung. One example of a quantum flag manifold is the quantum complex projective space $C(\mathbb{C}P_q^n)$ which is known to be a graph C^*-algebra due to Hong and Szymański.

In this talk, I will first present the explicit KK-equivalence between $C(\mathbb{C}P_q^n)$ and the commutative algebra $\mathbb{C}^{n+1}$ constructed in collaboration with Francesca Arici. The KK-equivalence is constructed by finding an explicit splitting for the short exact sequence of C^*-algebras $\mathcal{K} \to C(\mathbb{C}P_q^n) \to C(\mathbb{C}P_q^{n-1})$. In the construction of a splitting it is crucial that $C(\mathbb{C}P_q^n)$ can be described as a graph C^*-algebra. Secondly, I will present how this approach can be used to construct KK-equivalences in the more general framework of quantum flag manifolds which is based on ongoing work with Réamonn Ó Buachalla and Karen Strung.

5.11. **Operator Semigroups: Theory and Applications.**

Monday 2:30-4:30 in Porthania, PIII

2:30-2:55	Floris Roodenburg *The Dirichlet Laplacian on weighted Sobolev spaces with non-A_p weights*
3:00-3:25	Jarosław Sarnowski *Extrapolation of boundedness of periodic Fourier multipliers*
3:30-3:55	Sebastian Król *The well-posedness of periodic integro-differential equations via harmonic analysis methods*
4:00-4:25	Emiel Lorist *An interpolation approach to boundary value problems*

Tuesday 2:30-4:00 in Porthania, PIII

2:30-2:55	Sascha Trostorff *M-Accretive Realisations of Skew-Symmetric Operators*
3:00-3:25	Yuri Tomilov *Rational Approximations of Semigroups Revisited*
3:30-3:55	Lutz Weis *The absolute functional calculus for sectorial operators and regularity estimates for evolution equations*

Tuesday 4:30-6:00 in Language Center, Room 204

4:30-4:55	Paolo Ciatti *On the variation operator for the Ornstein–Uhlenbeck semigroup in dimension one*
5:00-5:25	Julian Hölz *Uniform convergence of an irreducible stochastic semigroup modelling gene expression dynamics*
5:30-5:55	Jan van Neerven *Thermal time as an unsharp observable*

Thursday 2:30-4:00 in Porthania, PIII

2:30-2:55	Charles Batty *A Variant of the Katznelson-Tzafriri Theorem*
3:00-3:25	Andrew Pritchard *Semi-Uniform Stability of Semigroups and Their Cogenerators*
3:30-3:55	Lassi Paunonen *Non-uniform stability of damped contraction semigroups*

Thursday 4:30-6:00 in Porthania, PIII

4:30-4:55	Nicolas Vanspranghe *Non-uniform Datko-Pazy criteria and asymptotics of contraction semigroups*
5:00-5:25	Perry Kleinhenz *Energy decay for the damped wave equation with unbounded damping*
5:30-5:55	Nathanael Skrepek *Semi-uniform stability of Maxwell's equations via boundary feedback*

Friday 11:30-1:00 in Porthania, PIII

11:30-11:55	Christian Budde *Non-autonomous Desch–Schappacher perturbations*
12:00-12:25	Christian Seifert *Perturbation theory for of non-autonomous second-order abstract Cauchy problems*
12:30-1:00	Andreas Buchinger, *Infinite-Dimensional Control Systems as Evolutionary Equations*

Friday 2:30-4:00 in Porthania, PIII

2:30-2:55	Roland Schnaubelt *Error analysis of the Lie splitting for semilinear wave equations at H^1-regularity*
3:00-3:25	Katharina Klioba *Temporal approximation of stochastic evolution equations*
3:30-3:55	Shubham Rastogi *Factorization of the right-shift-semigroup and a BCL type theorem*

Friday 4:30-6:00 in Porthania, PIII

4:30-4:55	Alejandro Mahillo *Characterization of generalized Holder spaces through semigroups associated with the discrete Laplacian*

5.11.1. *Abstracts.*

Charles Batty, University of Oxford, UK

A Variant of the Katznelson-Tzafriri Theorem

Abstract. In 1986, Katznelson and Tzafriri proved that, if T is a power-bounded operator on a Banach space X, and the spectrum of T meets the unit circle only at 1, then $\|T^n(I-T)\| \to 0$ as $n \to \infty$. Actually they went further and proved that $\|T^n f(T)\| \to 0$ if T and f satisfy certain conditions. Soon afterwards, analogous results were obtained for bounded C_0-semigroups $(T(t))_{t\geq 0}$. I will speak about a recent result obtained by David Seifert and myself.

Andreas Buchinger, TU Bergakademie Freiberg, Germany

Infinite-Dimensional Control Systems as Evolutionary Equations

Abstract. The theory of evolutionary equations respectively evolutionary well-posedness of PDEs - that means unique solvability as well as continuous and causal dependence on given data - is a concept afforded by Rainer Picard et al. In this talk, a possible evolutionary approach to controllability and observability of PDEs and to control theory for PDEs in general is presented.

Christian Budde, University of the Free State, South Africa

Non-autonomous Desch–Schappacher perturbations

Abstract. For many processes in sciences, the coefficients of the partial differential equation describing a dynamical system as well as the boundary conditions of it may vary with time. In such cases one speaks of *non-autonomous* (or time-varying) evolution equations. From an operator theoretical point of view one considers families of Banach space operators which depend on the time parameter and studies the associated non-autonomous abstract Cauchy problem. In particular, for fixed $T > 0$ and a family of linear (and typically unbounded) operators $(A(t), \mathrm{D}(A(t)))_{t\in[0,T]}$ on a Banach space X one considers the non-autonomous abstract Cauchy problem given by

$$\text{(nACP)} \qquad \begin{aligned} \dot{u}(t) &= A(t)u(t), \quad T \geq t \geq s \geq 0, \\ u(s) &= x \in X. \end{aligned}$$

While in the autonomous case operator semigroups yield fundamental solutions and thus provide an appropriate solution concept, for the non-autonomous case one needs to make use of so-called evolution families $(U(t,s))_{t\geq s}$, which give rise to a notion of well-posedness [3, Sect. 3.2]. Perturbation theory is a powerful tool in order to study existence, uniqueness and other qualitative properties and allows for a more general and abstract view on non-autonomous abstract Cauchy problems. More precisely, additionally given a family $(B(t), \mathrm{D}(B(t)))_{t\in[0,T]}$ of linear operators in X one studies the perturbed Cauchy problem

$$\begin{aligned} \dot{u}(t) &= (A(t)+B(t))u(t), \quad T \geq t \geq s \geq 0, \\ u(s) &= x \in X, \end{aligned}$$

trying to make use of information of the unperturbed Cauchy problem (nACP). Time-dependent perturbations of evolution equations have attracted a lot of interest in the past, see for example [2, 4]. In this talk, we consider time-dependent Desch–Schappacher perturbations of non-autonomous abstract Cauchy problems and apply our result to non-autonomous uniformly strongly elliptic differential operators on L^p-spaces. This is joint work with C. Seifert [1].

[1] Budde, C., Seifert, C.: Non-autononomous Desch–Schappacher perturbations. Accepted to IWOTA Lancaster Proceedings. https://arxiv.org/abs/2109.00225

[2] Evans, D.E.: Time dependent perturbations and scattering of strongly continuous groups on Banach spaces. Math. Ann. **221**(3), 275–290 (1976). https://doi.org/10.1007/BF01596393

[3] Nickel, G.: On evolution semigroups and wellposedness of nonautonomous Cauchy problems. Ph.D. thesis, Eberhard-Karls-Universität Tübingen (1996)

[4] Rhandi, A.: Extrapolation methods to solve non-autonomous retarded partial differential equations. Studia Math. **126**(3), 219–233 (1997).
https://doi.org/10.4064/sm-126-3-219-233

Paolo Ciatti, Università di Padova, Italy

On the variation operator for the Ornstein–Uhlenbeck semigroup in dimension one

Abstract. We consider the variation seminorm of the Ornstein–Uhlenbeck semigroup in dimension one, taken with respect to t. We show that this seminorm defines an operator of weak type $(1,1)$ for the relevant Gaussian measure. The analogous L^p estimates for $1 < p < \infty$ were already known.

This is a joint work done in collaboration with Valentina Casarino (Università di Padova) and Peter Sjögren (University of Gothenburg and Chalmers University of Technology).

Julian Hölz, University of Wuppertal, Germany

Uniform convergence of an irreducible stochastic semigroup modelling gene expression dynamics

Abstract. In a 2018 paper of Kurasov, Lück, Mugnolo and Wolf a coupled system of stochastic partial differential equation was proposed to model gene expression dynamics. This is modeled on L^1 over the disjoint union of two intervals. Later, the authors have proved existence and strong convergence of the solutions of the model to equilibrium.

We improve upon these results by showing the uniform convergence to equilibrium. To this end we employ a recent convergence theorem for stochastic, irreducible semigroups that yields uniform convergence by showing that the semigroups contain partial integral operators and that 0 is a pole of the generator's resolvent.

This is joint work with Alexander Dobrick from the Christian-Albrechts-University of Kiel.

Perry Kleinhenz, Michigan State University, USA

Energy decay for the damped wave equation with unbounded damping

Abstract. The damped wave equation describes the evolution of a vibrating system exposed to a damping force. When the damping is bounded, the decay of energy has been thoroughly studied using semigroup decay results. I will discuss some polynomial energy decay results when the damping is unbounded, which requires adaptation in the semigroup approach. I will also discuss a backwards uniqueness result, which begins to clarify a surprising finite time extinction result from the literature.

Katharina Klioba, Hamburg University of Technology, Germany

Temporal approximation of stochastic evolution equations

Abstract. In this talk, I will present optimal bounds for the pathwise uniform strong error arising from temporal discretisation of semi-linear stochastic evolution equations. We assume a certain convergence rate for the approximation of the semigroup as well as global Lipschitz continuity and linear growth of nonlinearity and noise. Under these assumptions, we obtain the same convergence rate for the pathwise uniform approximation error of the semi-linear SPDE up to a logarithmic correction factor. This extends and improves previous results from splitting to general time discretisation schemes and from the group to the semigroup case.

We illustrate how novel maximal inequalities for stochastic convolutions were used to obtain these results, which are applicable to a large class of hyperbolic equations. As an example, we discuss the convergence rates of implicit Euler and splitting for the nonlinear Schrödinger equation with multiplicative noise.

This talk is based on joint work with Mark Veraar (TU Delft).

Sebastian Król, Adam Mickiewicz University in Poznań, Poland

The well-posedness of periodic integro-differential equations via harmonic analysis methods

Abstract. I will discuss the well-posedness and regularity properties of solutions to the following abstract periodic problem

$$(AEE) \qquad\qquad \partial P \partial u + B\partial u + Au + c * u = f,$$

which is initially stated in the space of X-valued periodic distributions $\mathcal{D}(\mathbb{T}; X)$, where X is a given Banach space, P, B, A denote linear operators on X, and $c \in \mathcal{D}(\mathbb{T}; X)$.

Under suitable assumptions imposed on the geometry of X, I will characterize the conditions on the operators P, B, A, and the convolutor c, which ensure the well-posedness of (AEE) for a large class of function spaces.

More precisely, I will propose a new type of joint multiplier condition (involving the convolutor c and the multiplier symbol of a solution operator of (AEE)), which allows handling different questions on the solvability of many particular forms of (AEE) in a unified manner. In particular, I will show how the structure of (AEE) affects the relation between the regularities of the symbols of Fourier multipliers, which are involved in such studies. In connection with some recent multiplier results [1,2], it allows us to extend the known theory for such periodic problems from L^p-setting to a much wider context of general Banach function spaces. In particular, I will clarify the phenomenon of extrapolation of the L^p-maximal regularity for several periodic evolution equations modelled by (AEE).

[1] S. Król, J. Sarnowski, *The invariant subspaces of periodic Fourier multipliers with application to abstract evolution equations*, arXiv:2301.12451

[2] S. Król, *The maximal regularity property of abstract integro-differential equations*, J. Evol. Equ. 23 (2023), doi.org/10.1007/s00028-023-00878-y

Emiel Lorist, TU Delft, The Netherlands

An interpolation approach to boundary value problems

Abstract. Let X be a Banach space, let $-A$ be the generator of a C_0-semigroup and consider the abstract Cauchy problem

$$\begin{cases} u_t + Au &= f, \\ u(0) &= u_0. \end{cases}$$

We say that A has maximal L^p-regularity if for any $f \in L^p((0,T);X)$ the (mild) solution to the abstract Cauchy problem with $u_0 = 0$ satisfies $Au, u_t \in L^p((0,T);X)$. It is a classical result that one can then allow for initial data u_0 in the real interpolation space $(X, D(A))_{1-\frac{1}{p},p}$.

Now assume X is a space of functions on some domain $\mathcal{O} \subseteq \mathbb{R}^d$ and the restriction $u|_{(0,T)\times\partial\mathcal{O}}$ makes sense for solutions to the abstract Cauchy problem. Then one may ask for which $g\colon (0,T) \times \partial\mathcal{O}$ one can show maximal regularity for the boundary value problem

$$\begin{cases} u_t + Au &= f, \\ u|_{(0,T)\times\partial\mathcal{O}} &= g, \\ u(0) &= u_0. \end{cases}$$

In this talk I will discuss this problem from an interpolation theory point of view, which is based on joint work with Nick Lindemulder.

Alejandro Mahillo, Universidad de Zaragoza

Characterization of generalized Holder spaces through semigroups associated with the discrete Laplacian

Abstract. Our work builds on the seminal papers by Stein and Taibleson from the late 1960s, where they established integral estimates of the Gauss and Poisson semigroups, $\{e^{t\Delta_d}\}_{t>0}$ and $\{e^{-y\sqrt{-\Delta_d}}\}_{y>0}$, to characterize Lipschitz (or Hölder) and Besov functions. This type of results are beneficial since they allow us to obtain regularity results on the semigroup in a much more direct way, without going through the computation of pointwise expression. This makes us question what happens with other types of "laplacians", as for example in the discrete case of these spaces.

In the context of discrete Hölder spaces, recently Abadías and De León-Contreras provide a characterization using the heat and Poisson semigroups associated with the discrete Laplacian. It then naturally arises to wonder whether the results obtained by Stein and Taibleson for Besov spaces are also verified for the discrete case.

Our paper provides a positive answer to this question by characterizing discrete Besov spaces using the heat and Poisson semigroups associated with the discrete Laplacian. To achieve this, we compute certain bounds on the discrete heat kernel that we believe have intrinsic relevance.

Additionally, we demonstrate the practical significance of our results by computing fractional powers of some finite difference operators.

Lassi Paunonen, Tampere University, Finland

Non-uniform stability of damped contraction semigroups

Abstract. In this presentation we study the stability properties of strongly continuous semigroups generated by operators of the form $A - BB^*$, where A generates a unitary group or a contraction semigroup, and B is a possibly unbounded operator. Such semigroups are encountered in the study of hyperbolic partial differential equations with damping on the boundary or inside the spatial domain. In the case of multi-dimensional wave equations with viscous damping, the associated semigroup is often not uniformly exponentially stable, but instead only

"polynomially" or "non-uniformly stable". Motivated by such situations, we present general sufficient conditions for polynomial and non-uniform stability of the semigroup generated by $A - BB^*$ in terms of generalised observability-type conditions of the pair (B^*, A). The proofs in particular involve derivation of resolvent estimates for $A - BB^*$ on the imaginary axis. In addition, we apply the results in studying the stability of hyperbolic PDEs with partial or weak dampings.

The research is joint work with R. Chill, D. Seifert, R. Stahn and Y. Tomilov (https://arxiv.org/abs/1911.04804)

Andrew Pritchard, Newcastle University, UK

Semi-Uniform Stability of Semigroups and Their Cogenerators

Abstract. The cogenerator of a bounded C_0-semigroup $(T(t))_{t\geq 0}$ with generator A is defined as $V(A) = (A+I)(A-I)^{-1}$, and the corresponding discrete semigroup $(V(A)^n)_{n=1}^{\infty}$ is the Crank-Nicolson discretisation scheme. For both C_0-semigroups and discrete semigroups, there are notions of semi-uniform stability: for C_0-semigroups, one studies the decay of $\|T(t)(I-A)^{-1}\|$ as $t \to \infty$; for a discrete semigroup $(V^n)_{n=1}^{\infty}$, the quantified Katznelson-Tzafriri theorem allows one to study the decay of $\|V^n(I-V)\|$ as $n \to \infty$.

In this talk, we will discuss results which the relate the rate of decay of $\|T(t)(I-A)^{-1}\|$ to the rate of decay of $\|V(A)^n(I-V(A))\|$. We consider separately the cases of semigroups on Banach spaces and Hilbert spaces. These results build on work published by M. Wakaiki in 2021.

This is joint work with David Seifert.

Shubham Rastogi, Indian Institute of Science, Bangalore

Factorization of the right-shift-semigroup and a BCL type theorem

Abstract. The famous Wold decomposition gives us the structure of an isometry on a Hilbert space. Berger, Coburn, and Lebow (BCL) obtained the structure for a tuple of commuting isometries acting on a Hilbert space. In this talk, we shall discuss the structure of a pair of commuting C_0-semigroups of isometries and obtain a BCL type result.

The right-shift-semigroup $S^{\mathcal{E}} = (S_t^{\mathcal{E}})_{t\geq 0}$ on $L^2(\mathbb{R}_+, \mathcal{E})$ for any Hilbert space $\mathcal{E}$ is defined as

$$(S_t^{\mathcal{E}} f)x = \begin{cases} f(x-t) & \text{if } x \geq t, \\ 0 & \text{otherwise,} \end{cases}$$

for $f \in L^2(\mathbb{R}_+, \mathcal{E})$. Cooper showed that the role of the unilateral shift in the Wold decomposition of an isometry is played by the right-shift-semigroup for a C_0-semigroup of isometries. The factorizations of the unilateral shift have been explored by BCL, we are interested in examining the factorizations of the right-shift-semigroup. Firstly, we shall discuss the contractive C_0-semigroups which commute with the right-shift-semigroup. Then, we give a complete description of the pairs (V_1, V_2) of commuting C_0-semigroups of contractions which satisfy $S^{\mathcal{E}} = V_1 V_2$, (such a pair is called as a *factorization* of $S^{\mathcal{E}}$), when $\mathcal{E}$ is a finite dimensional Hilbert space.

This talk is based on the joint work with Prof. T. Bhattacharyya, Prof. K. B. Sinha and Dr. Vijaya Kumar U.

Floris Roodenburg, TU Delft, The Netherlands

The Dirichlet Laplacian on weighted Sobolev spaces with non-A_p weights

Abstract. We consider the Laplace operator with Dirichlet boundary conditions on the half-space and we study its properties on weighted Sobolev spaces with power weights which fall outside the class of Muckenhoupt A_p weights. Whereas the Dirichlet heat semigroup is bounded on Sobolev spaces without weights, it turns out that the semigroup grows polynomially on

Sobolev spaces with power weights. Moreover, for the shifted Dirichlet Laplacian it is found that it admits a bounded H^∞-calculus on these weighted Sobolev spaces. Furthermore, the non-A_p power weights are important to study (stochastic) partial differential equations on bounded domains. This is joint work with Emiel Lorist and Mark Veraar.

Jarosław Sarnowski, Nicolaus Copernicus University in Toruń, Poland

Extrapolation of boundedness of periodic Fourier multipliers

Abstract. During the talk I will show how methods of harmonic analysis let us extrapolate L^p-boundedness of a periodic Fourier multiplier (which symbol satisfies Marcinkiewicz type conditions) to boundedness with respect to a large class of Banach function spaces. The strategy of the proof is based on devoloping transference techniques which allow us to obtain crucial periodic ingredients (of our proof) from the Euclidean approach. In particular, the procedure of extending a symbol from periodic to non-periodic case (preserving regularity and boundedness of corresponding multiplier operators) will be shown. The classical Jodeit theorem and de Leeuw theorem will be revisited in a generalized form. The presented multiplier theorems imply extrapolation of maximal regularity from L^p space to general Banach function spaces Φ for many abstract evolution equations (where generators of analytic semigroups appear). Sufficient conditions for boundedness of a Fourier multiplier with respect to Besov and Triebel-Lizorkin spaces defined on the basis of Φ will also be discussed.

[1] S. Król, J.Sarnowski *The invariant subspaces of periodic Fourier multipliers with application to abstract evolution equations*, arXiv:2301.12451

Roland Schnaubelt, Karlsruhe Institute of Technology, Germany

Error analysis of the Lie splitting for semilinear wave equations at H^1-regularity

Abstract. We study the semilinear wave equation

$$\partial_t^2 u = \Delta u + \mu |u|^{\alpha-1} u, \qquad t \geq 0, \quad x \in \mathbb{R}^3, \tag{1}$$

for data $(u(0), \partial_t u(0))$ in the energy space $H^1(\mathbb{R}^3) \times L^2(\mathbb{R}^3)$, where $\alpha \in [3,5]$ and $\mu \in \{-1,1\}$. For $\alpha = 5$, equation (1) is energy-critical, and this quintic nonlinearity is the borderline case for local wellposedness. Time integration schemes for (1) have been studied before only at higher regularity levels. The local wellposedness theory suggests to exploit the dispersive nature of the system by using Strichartz estimates. Motivated by related work on the nonlinear Schrödinger equation, we first show discrete-in-time Strichartz estimates for the linear wave equation on $\mathbb{R}^3$. These require frequency cut-offs of the solution because of the discrete setting. We then analyze a Lie splitting scheme with frequency cut-offs for (1) and data in the energy space. Our main result yields first-order convergence in $L^2(\mathbb{R}^3) \times \dot{H}^{-1}(\mathbb{R}^3)$ of the time-discrete approximation to the solution $(u, \partial_t u)$ of (1). This is joint work with Maximilian Ruff (Karlsruhe).

Christian Seifert, Hamburg University of Technology, Germany

Perturbation theory for of non-autonomous second-order abstract Cauchy problems

Abstract. In this talk, we focus on non-autonomous second order abstract Cauchy problems in Banach spaces, which model non-autonomous wave equations. First, we will study fundamental solutions and their existence by means of the corresponding non-autonomous first-order system. We then turn to a perturbation result where the perturbation consists of a suitably bounded family of operators. As an application, we study perturbations of wave equations.

The talk is based on joint work with Christian Budde (University of the Free State, Bloemfontein, South Africa).

Nathanael Skrepek, TU Freiberg, Germany

Semi-uniform stability of Maxwell's equations via boundary feedback

Abstract. We regard the undamped Maxwell's equations as a linear port-Hamiltonian boundary control and observation system on a bounded Lipschitz domain. We apply a feedback control law that stabilizes the system in a semi-uniform manner without any further geometrical assumption on the domain. This will be achieved by separating the equilibriums from the system and show that the remaining system is described by an operator with compact resolvent. Furthermore, we will apply a unique continuation principle on the resolvent equation to show that there are no eigenvalues on the imaginary axis.

Yuri Tomilov, IM PAN, Warsaw, Poland

Rational Approximations of Semigroups Revisited

Abstract. In this talk, I will present a new, functional calculus approach to the study of rational approximations for operator semigroups on Hilbert spaces. The approach is based on the so-called $\mathcal{B}$-calculus for semigroup generators introduced and developed recently by C. Batty, A. Gomilko and myself. I will explain how functional calculi ideas can be used to obtain sharp stability estimates for rational approximations, and to equip the approximations with optimal convergence rates.

This is joint work with A. Gomilko.

Sascha Trostorff, CAU Kiel, Germany

M-Accretive Realisations of Skew-Symmetric Operators

Abstract. Skew-symmetric operators naturally arise in the study of many partial differential equations in mathematical physics, e.g. transport equation, wave equation, Maxwell's equations, etc. In order to solve the corresponding Cauchy problem, one is interested in m-accretive realisations of these operator, since these (or the negative of it) are precisely the generators of contraction semigroups (linear and non-linear ones).
It is the purpose of this talk to present a characterisation result of all m-accretive realisations (linear and non-linear) of skew-symmetric operators in terms of suitable associated 'deficiency spaces' and mappings between them. Moreover, we illustrate how the abstract results can be used to identify the 'right' boundary conditions for differential operators to generate contraction semigroups.
This talk is based on a joint work with Rainer Picard (TU Dresden, Germany).

Jan van Neerven, TU Delft, The Netherlands

Thermal time as an unsharp observable

Abstract. Using the theory of C_0-groups, we show that the Connes-Rovelli thermal time associated with the quantum harmonic oscillator can be described as an (unsharp) observable, that is, as a positive operator valued measure. We furthermore present extensions of this result to the free massless relativistic particle in one dimension and to a hypothetical physical system whose equilibrium state is given by the noncommutative integral. This is joint work with Pierre Portal (ANU, Canberra).

Nicolas Vanspranghe, Tampere University, Finland

Non-uniform Datko-Pazy criteria and asymptotics of contraction semigroups

Abstract. We present sufficient conditions for polynomial stability of operator semigroups on Banach spaces in terms of integrability of certain orbits (or weak orbits in the Hilbert space setting). As an application, we extend and simplify earlier stability results for "damped" contraction semigroups in relation to relaxed observability-type conditions in the time domain. Examples coming from partial differential equations will illustrate the talk. *Joint work with Lassi Paunonen and David Seifert.*

Lutz Weis, KIT, University of Karlsruhe, Germany

The absolute functional calculus for sectorial operators and regularity estimates for evolution equations

Abstract. The absolute functional calculus for a sectorial operator, introduced by N. Kalton and T. Kucherenko, is stronger than the classical H_∞ functional calculus, in particular it allows for an operator-valued functional calculus and sum theorems without additional R-boundedness conditions. Since it is closely linked with the real interpolation method it can be applied in spaces without the UMD property. In this talk we give some new examples of operators with an absolute functional calculus and apply it to maximal regularity estimates for deterministic and stochastic evolution equations with a analytic semigroup generator. In particular, one obtains a unified approach to some classical results of Da Prato and Grisvard.

5.12. **Operator Theory in Elliptic Partial Differential Equations.**

Tuesday 2:30-4:00 in Language Center, Room 204 chair: Jari Taskinen

2:30-2:55	Pavel Exner *Geometrically Induced Discrete Spectrum of Soft Quantum Waveguides*
3:00-3:25	Massimo Lanza de Cristoforis *A singularly perturbed problem for the Helmholtz equation. A functional analytic approach*
3:30-3:55	Maria-Eugenia Pérez-Martínez *Homogenization for alternating nonlinear Robin-Winkler boundary conditions*

Tuesday 4:30-8:00 in Porthania, PIII chair: Massimo Lanza de Cristoforis

4:30-4:55	Andrii Khrabustovskyi *Operator estimates for Neumann sieve problem*
5:00-5:25	Giuseppe Cardone *Elliptic operators with Steklov condition perturbed by Dirichlet condition on a small part of boundary*
5:30-5:55	Marzieh Baradaran *Spectral properties of periodic quantum graphs with time-reversal non-invariant vertex coupling*
6:00-6:25	Jari Taskinen *Spectral Laplace problem and heat equation in periodic domains*
6:30-6:55	Jonathan Ben-Artzi *Convergence rates in dynamical systems lacking a spectral gap*
7:00-7:25	George Tephnadze *Bi-Laplace-Beltrami equation on hypersurfaces and Γ-convergence*
7:30-7:55	Christine Pfeuffer *Invariance of the Fredholm Index of Non-Smooth Pseudodifferential Operators*

Thursday 2:30-4:00 in Language Center, Room 204 chair: Giuseppe Cardone

2:30-2:55	Roland Duduchava *Laplace-Beltrami equation on Lipschitz hypersurface in the generic Bessel potential spaces*
3:00-3:25	Pier Domenico Lamberti *Spectral analysis of Steklov problems on varying domains*
3:30-3:55	Roman Šimon Hilscher *Singular Sturmian comparison theorems*

Thursday 4:30-6:00 in Language Center, Room 204 chair: Maria-Eugenia Pérez-Martínez

4:30-4:55	Carmen Perugia *The unfolding operator for the homogenization of a non-Newtonian fluid in a thin domain with oscillating boundary*
5:00-5:25	Delfina Gómez *Asymptotics for some spectral problems in thin domains*
5:30-5:55	Medea Tsaava *Mixed boundary value problems for the Helmholtz equation in a model 2D double angular domain*

Friday 11:30-1:00 in Language Center, Room 204 chair: Pavel Exner

11:30-11:55	Margarita Tutberidze *Dirichlet and Neumann boundary value problems for the Helmholtz equation in a double angle*
12:00-12:25	Larysa Khilkova *Asymptotic behavior of stable and unstable structures made of thin beams*
1:00-1:25	Zurab Vashakidze *Wave Propagation in a Triangular Lattice with Discrete Sources Placed on Line Segments*

Friday 2:30-4:00 in Language Center, Room 204 chair: Roland Duduchava

2:30-2:55	Josh Isralowitz *Agmon Theory and decay estimates for the fundamental solution of an elliptic system with matrix potential*
3:00-3:25	Zeinab Ashtab *The Neumann problem on a torus*
3:30-4:00	Rakesh Kumar *Operator Analysis of non-self-adjoint bent waveguide problem*

5.12.1. *Abstracts.*

Zeinab Ashtab, Cinvestav-IPN

The Neumann problem on a torus

Abstract. We consider the Dirichlet-to-Neumann mapping and the Neumann problem for the Laplace operator on a torus, given in toroidal coordinates. The Dirichlet-to-Neumann mapping is expressed with respect to series expansions in toroidal harmonics and thereby reduced to algebraic manipulations on the coefficients. A simple method for computing the numerical solutions of the corresponding Neumann problem is presented, and numerical illustrations are provided. We combine the results for interior and exterior domains to solve the Neumann problem for a toroidal shell.

Marzieh Baradaran, University of Hradec Králové

Spectral properties of periodic quantum graphs with time-reversal non-invariant vertex coupling

Abstract. Motivated by the application of quantum graphs to model the anomalous Hall effect, we discuss spectral properties of periodic quantum graphs assuming that the vertex coupling is manifestly non-invariant with respect to the time reversal. Special attention is paid to the asymptotic behavior of the spectral bands in the high-energy regime; we see that the Band-Berkolaiko universality holds as long as the graph edge lengths are incommensurate. Moreover, we see that the transport properties of the graphs depend substantially on the network topology, in particular, on the parity of the vertices involved.

This is a joint work with Pavel Exner and Jiří Lipovský based on the papers: M Baradaran, P Exner, J. Math. Phys. 63 (2022), 083502; M Baradaran, P Exner, J Lipovský, J. Phys. A: Math. Theor. 55 (2022) 375203; M Baradaran, P Exner, J Lipovský, arXiv preprint arXiv:2302.04601 (accepted for publication in Ann. Phys. 2023).

Jonathan Ben-Artzi, Cardiff University

Convergence rates in dynamical systems lacking a spectral gap

Abstract. It is well-known that the existence of a spectral gap is often a necessary first step in seeking to understand the long-time behavior of dynamical systems. In this talk, I will present recent results that replace this requirement with merely requiring that the density of

the spectrum near zero is controlled. Consequently, we can study the long-time dynamics of equations of the forms $u_t = -iHu$ (conservative case) and $u_t = -Hu$ (dissipative case), where H is self-adjoint (and $H \geq 0$ in the dissipative case). Applications include a uniform ergodic theorem for averages of solutions of the Schrödinger equation and new functional inequalities leading to optimal decay rates for solutions of the fractional heat equation in $\mathbb{R}^n$.

Giuseppe Cardone, University of Naples "Federico II", Naples, Italy

Elliptic operators with Steklov condition perturbed by Dirichlet condition on a small part of boundary

Abstract. We consider a boundary value problem for a homogeneous elliptic equation with an inhomogeneous Steklov boundary condition. The problem involves a singular perturbation, which is the Dirichlet condition imposed on a small piece of the boundary. We rewrite such problem to a resolvent equation for a self-adjoint operator in a fractional Sobolev space on the boundary of the domain. We prove the norm convergence of this operator to a limiting one associated with an unperturbed problem involving no Dirichlet condition. We also establish an order sharp estimate for the convergence rate. The established convergence implies the convergence of the spectra and spectral projectors. In the second part of the work we study perturbed eigenvalues converging to limiting simple discrete ones. We construct two-terms asymptotic expansions for such eigenvalues and for the associated eigenfunctions.

This is a joint work with D.Borisov, G.A.Chechkin and Yu.O.Koroleva.

Roland Duduchava, The University of Georgia & A. Razmadze Mathematical Institute, Tbilisi, Georgia. Email: *RolDud@gmail.com*

Laplace-Beltrami equation on Lipschitz hypersurface in the generic Bessel potential spaces

Abstract. The purpose of the presentation is to expose a new approach to the investigation of boundary value problems (BVPs) for the Elliptic Partial Differential Equations on the example of Laplace-Beltrami equation on a hypersurface $\mathcal{S} \subset \mathbb{R}^3$ with the Lipschitz boundary $\Gamma = \partial\mathcal{S}$, containing a finite number of angular points (knots) c_j of magnitude α_j, $j = 1, 2, \dots, n$. The Dirichlet, Neumann and mixed type BVPs are considered in a non-classical setting, when solutions are sought in the generic Bessel potential spaces (GBPS) $\mathbb{GH}^s_p(\mathcal{S}, \rho)$, $s > 1/p$, $1 < p < \infty$ with weight $\rho(t) = \prod\limits_{j=1}^{n} |t - c_j|^{\gamma_j}$ which is defined on the semi-axes $\mathbb{R}^+ := (0, \infty)$ by using the Mellin transform instead of the Fourier transform:

$$\|\psi \,|\, \mathbb{GH}^s_p(\mathbb{R}^+, t^\gamma)\| := \|\mathcal{M}^{-1}_\beta \langle\cdot\rangle^s \mathcal{M}_\beta \psi \,|\, \mathbb{L}_p(\mathbb{R}^+, t^\gamma)\| := \left[\int_0^\infty |\mathcal{M}^{-1}_\beta \langle\cdot\rangle^s \mathcal{M}_\beta \psi(t)|^p t^\gamma dt\right]^{\frac{1}{p}},$$

where $\langle\xi\rangle = (1 + \xi^2)^{s/2}$, $\xi \in \mathbb{R}$ and

$$\mathcal{M}_\beta \psi(\xi) := \int\limits_0^\infty t^{\beta - i\xi} \psi(t) \, \frac{dt}{t}, \;\; \xi \in \mathbb{R}, \quad \mathcal{M}^{-1}_\beta \varphi(t) := \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} t^{i\xi - \beta} \varphi(\xi) \, d\xi, \;\; t \in \mathbb{R}^+.$$

For an integer $s = m = 1, 2, \dots$ the space $\mathbb{GH}^m_p(\mathbb{R}^+, t^\gamma)$ is isomorphic to the generic Sobolev space $\mathbb{GW}^m_p(\mathbb{R}^+, t^\gamma)$, where functions have the finite norm

$$\|\varphi \,|\, \mathbb{GW}^m_p(\mathbb{R}^+, t^\gamma)\| := \left[\sum_{k=0}^{m} \|\boldsymbol{D}^k \varphi \,|\, \mathbb{L}_p(\mathbb{R}^+, t^\gamma)\|^p\right]^{1/p}, \qquad \boldsymbol{D}\varphi(t) := t\frac{d\varphi(t)}{dt}.$$

By the localization the problem is reduced to the investigation of Model Dirichlet, Neumann and mixed BVPs for the Laplace equation in a planar angular domain $\Omega_{\alpha_j} \subset \mathbb{R}^2$ of magnitude α_j, $j = 1, 2 \dots, n$. Further the model problem in the GBPS with weight $\mathbb{GH}^s_p(\Omega_{\alpha_j}, t^{\gamma_j})$ is investigated by means of Mellin convolution operators on the semi-axes $\mathbb{R}^+ = (0, \infty$. Explicit

criteria for the Fredholm property and the unique solvability of the initial BVPs are obtained and singularities of solutions at knots to the mentioned BVPs are indicated. In contrast to the same BVPs in the classical Bessel potential spaces $\mathbb{H}^s_p(\mathcal{S})$, the Fredholm property in the GBPS $\mathbb{GH}^s_p(\mathcal{S},\rho)$ with weight is independent of the smoothness parameter s.

Pavel Exner, Doppler Institute, Czech Technical University, Prague

Geometrically Induced Discrete Spectrum of Soft Quantum Waveguides

Abstract. The topic of the talk are Schrödinger operators with an attractive potential in the form of a channel of a fixed profile built along a smooth curve Γ in $\mathbb{R}^2$, in particular, the ways in which the geometry of the curve and the profile influence their spectrum. We present sufficient conditions under which the discrete spectrum of such operators is nonempty and mention several other related results and problems.

Delfina Gómez, Universidad de Cantabria, Spain

Asymptotics for some spectral problems in thin domains

Abstract. We consider a spectral problem for the Laplacian defined in a planar T-like shaped structure. The thickness of each component depends on a small parameter. We assume homogeneous Dirichlet boundary condition on the ends of the branches and homogeneous Neumann boundary condition on the remaining part of the boundary. We study the asymptotic behavior of the eigenvalues and the corresponding eigenfunctions of such a problem as the thickness tends to zero. We address extensions to 3D domains. Talk based in common works with Antonio Gaudiello and Maria-Eugenia Pérez-Martínez.

Joshua Isralowitz, University at Albany, SUNY

Agmon Theory and decay estimates for the fundamental solution of an elliptic system with matrix potential

Abstract. In this talk, we state a matrix weighted Fefferman-Phong inequality and use this to develop the Agmon theory and prove exponential decay estimates for the fundamental solution of a uniformly elliptic system with a matrix reverse Hölder potential. This is join work with Blair Davey

Larysa Khilkova, Fraunhofer ITWM, Germany

Asymptotic behavior of stable and unstable structures made of thin beams

Abstract. The aim of our work is to study the asymptotic behavior of an ε-periodic 3D structure made of "thin" beams of circular cross-section of radius r when the periodicity parameter ε tends to 0. By "thin", we mean that the radius r of the beams is much smaller than the periodicity parameter ε and that we deal with the case where ε and r/ε simultaneously tend to 0. The analysis is performed within the frame of linear elasticity theory and it is based on the known decomposition of the beam displacements into a beam centerline displacement, a small rotation of the cross-sections and a warping (the deformation of the cross-sections). This decomposition allows to obtain Korn inequalities. To study the asymptotic behavior of periodic beam-structures and derive limit problem we use the periodic unfolding method. We introduce two unfolding operators, one for the homogenization of the set of beam centerlines and another for the dimension reduction of the beams. In our work the limit homogenized problems are obtained for different types of elastic periodic beam-structures.

Andrii Khrabustovskyi, University of Hradec Králové

Operator estimates for Neumann sieve problem

Abstract. Let $\Omega \subset \mathbb{R}^n$ be a domain, which is intersected by a hyperplane Γ. We make a lot of small holes $D_{k,\varepsilon}$, $k = 1, 2, 3\dots$ in $\Gamma \cap \Omega$, where $\varepsilon > 0$ is a small parameter; when $\varepsilon \to 0$, the number of holes tends to infinity, while their diameters tends to zero. Let $\mathcal{A}_\varepsilon$ be the Neumann Laplacian in the perforated domain $\Omega_\varepsilon = \Omega \setminus \Gamma_\varepsilon$, where $\Gamma_\varepsilon = \Gamma \setminus (\cup_k D_{k,\varepsilon})$ ("sieve"). It is well-known that under some critical scaling of the holes radii, the operator $\mathcal{A}_\varepsilon$ converges in the strong resolvent sense to the Laplacian on $\Omega \setminus \Gamma$ subject to the so-called δ'-conditions on $\Gamma \cap \Omega$. In this talk we discuss some resent improvements of this result obtained in [A.K., Ann. Mat. Pura Appl. (2023), DOI: 10.1007/s10231-023-01308-z], where under rather general assumptions on the shapes and locations of the holes we derived estimates on the rate of convergence in terms of $L^2 \to L^2$ and $L^2 \to H^1$ operator norms. Some other close (not yet published) results will be also discussed.

Rakesh Kumar, Indian Institute of Technology Jodhpur, India

Operator Analysis of non-self-adjoint bent waveguide problem

Abstract. In modern times, integrated photonics devices have gained importance due to their high-speed signal processing capability. Optical waveguides are one of the basic integrated photonic devices used in several applications, such as communication, medical devices, sensors, and more. The prominent waveguides are straight and bent. These waveguides are investigated experimentally, numerically, and semi-analytically [1, 2].

The analytic study of the straight waveguides showed that the operator corresponding to its eigenvalue problem is self-adjoint. It has real eigenvalues, and eigenfunctions corresponding to distinct eigenvalues are orthogonal [3].

In this work analytical study for the bent waveguide is presented [4]. It is found that the operator involved in the governing eigenvalue problem is defined on the infinite domain, and it is non-self-adjoint [4]. There are not many general predictions about the properties of non-self-adjoint operators, such as the nature of eigenvalues, eigenfunctions, etc [5]. Further analysis of the bent waveguide problem showed that it has complex eigenvalues. In addition, eigenfunctions corresponding to distinct eigenvalues are orthogonal. This eigenvalue problem involves a bent radius as a parameter. When the parameter is large, this non-self-adjoint problem transforms into the self-adjoint problem, and complex eigenvalues change into real eigenvalues, i.e., the eigenvalue problem corresponds to bent waveguides transforming into straight waveguides. Moreover, the finiteness of the discrete spectrum is also studied based on the compactness of the operator involved in the bent waveguide problem.

Joint work with Prof. Kirankumar R. Hiremath (Supervisor).

1. M. Heiblum, and J. Harris, Analysis of Curved Optical Waveguides by Conformal Transformation, Journal of Quantum Electronics, 11, (2):75–83, IEEE, 1975.

2. K. R. Hiremath, M. Hammer, R. Stoffer, and L. Prkna, and J.Čtyrokỳ, Analytic Approach to Dielectric Optical Bent Slab Waveguides, Optical and Quantum Electronics, 37, (1-3), 37–61, Springer, 2005.

3. P. Joly and C. Poirier, Mathematical Analysis of Electromagnetic Open Waveguides, ESAIM: Mathematical Modelling and Numerical Analysis, 29, (5), 505–575, EDP Sciences, 1995.

4 R. Kumar, and K. R. Hiremath, Non-self-adjointness of bent optical waveguide eigenvalue problem, Journal of Mathematical Analysis and Applications, 12 (1), 126024, Elsevier, 2022.

5 E.B. Davies, Non-self-adjoint differential operators, Bulletin of the London Mathematical Society, 34 (5), 513–532, Cambridge University Press, 2002.

Pier Domenico Lamberti, Dipartimento di Tecnica e Gestione dei Sistemi Industriali (DTG), University of Padova

Spectral analysis of Steklov problems on varying domains

Abstract. Let $\Omega \subset \mathbb{R}^N$, $N \geq 2$, be a bounded domain with Lipschitz boundary. We consider the classical Steklov problem

$$\begin{cases} \Delta u = 0, & \text{in } \Omega\,, \\ u_\nu = \lambda u, & \text{on } \partial\Omega\,, \end{cases}$$

in the unknowns u (the Steklov eigenfunction) and $\lambda \in \mathbb{R}$ (the Steklov eigenvalue), where u_ν denotes the normal derivative on the boundary, and we study its variation upon perturbation of Ω. We identify a sharp condition for the convergence of the domains which allows to prove stability and instability results for eigenvalues and eigenfunctions. Our analysis includes the study of a boundary homogenization problem as a special case. Time permitting, we shall discuss analogous results for fourth order Steklov problems.

Based on a joint work with Alberto Ferrero.

Massimo Lanza de Cristoforis, Padova

A singularly perturbed problem for the Helmholtz equation. A functional analytic approach

Abstract. We consider a transmission problem for the Helmholtz equation in a domain with a small inclusion of size $\epsilon > 0$ and we analyze the behavior of the solutions as $\epsilon > 0$ tends to zero by an approach that is alternative to that of asymptotic expansions.

Joint work with Tuğba Akyel, Maltepe University, Istanbul.

Maria-Eugenia Pérez-Martínez, Universidad de Cantabria, Spain

Homogenization for alternating nonlinear Robin-Winkler boundary conditions

Abstract. We address a homogenization problem for the Laplace operator posed in a bounded domain of the upper half-space, a part of its boundary being in contact with the plane. On this part, the boundary conditions alternate from Neumann to nonlinear-Robin, being of Dirichlet type outside. The nonlinear-Robin boundary conditions are imposed on small regions periodically placed along the plane and contain a Robin parameter that can be very large. Different averaged boundary conditions are obtained depending on the different relations between parameters (period, size of the regions and Robin parameter). Extensions to strainer Winkler foundations are considered, focusing mainly on the "averaged Robin-Winkler" boundary condition and other "extreme cases".

Some references:

- D. Gómez, S.A. Nazarov and M.-E. Pérez-Martínez. Asymptotics for spectral problems with rapidly alternating boundary conditions on a strainer Winkler foundation. Journal of Elasticity. 142:89-120, 2020.

- D. Gómez, S.A. Nazarov and M.-E. Pérez-Martínez. Boundary homogenization with large reaction terms on a strainer-type wall. Z. Angew. Math. Phys. 73, 234, 2022.
https://doi.org/10.1007/s00033-022-01869-8

- D. Gómez, S.A. Nazarov and M.-E. Pérez-Martínez. In Proc. of The 16th International Conference on Integral Methods in Science and Engineering. Spectral homogenization problems in linear elasticity: the averaged Robin reaction matrix. Springer, 12 pp., to appear, 2023

Carmen Perugia, Department of Science and Technology, University of Sannio

The unfolding operator for the homogenization of a non-Newtonian fluid in a thin domain with oscillating boundary

Abstract. We consider an incompressible Bingham flow in a thin domain with rough boundary, under the action of given external forces and with no-slip boundary condition on the whole boundary of the domain. In mathematical terms, this problem is described by non linear variational inequalities over domains where a small parameter ϵ denotes the thickness of the domain and the roughness periodicity of the boundary. By using an adapted linear unfolding operator we perform a detailed analysis of the asymptotic behavior of the Bingham flow when ϵ tends to zero. We obtain the homogenized limit problem for the velocity and the pressure, which preserves the nonlinear character of the flow, and study the effects of the microstructure in the corresponding effective equations. There are several papers studying the asymptotic behavior of fluids in thin domains with rough boundary in the case of Newtonian fluids. However, for the non-Newtonian fluids, the situation is completely different. The main reason is that the viscosity is a nonlinear function of the symmetrized gradient of the velocity. We refer the reader to the very recent paper [2] and the references therein for the application of our study to problems issued from the real life applications. Indeed, predicting lava flow pathways is important for understanding effusive eruptions and for volcanic hazard assessment. One particular challenge is understanding the interplay between flow pathways and substrate topography that is often rough on a variety of scales.

[1] G. Cardone, C. Perugia, M. Villanueva-Pesqueira, Asymptotic behaviour of a Bingham flow in thin domains with rough boundary, Integral Equations Operator Theory, 93(3), (2021), pp.26.

[2] P. Richardson, L. Karlstrom, The multi-scale influence of topography on lava flow morphology. Bull. Volcanol. 81, 21 (2019).

Christine Pfeuffer, Martin-Luther University of Halle-Wittenberg

Invariance of the Fredholm Index of Non-Smooth Pseudodifferential Operators

Abstract. As nearly invertible operators Fredholm operators play an important role in the field of partial differential equations in order to obtain existence and uniqueness results. Hence great effort already was spent to get some conditions for the Fredholmness of pseudodifferential operators. However, there are very few results for the invariance of the Fredholm index of such operators.

In the smooth case Schrohe was able to show under certain conditions, that the Fredholm index of smooth pseudodifferential operators is invariant considered as a map between certain weighted Bessel potential spaces with symbols in the Hörmander-class $S^m_{1,0}(\mathbb{R}^n \times \mathbb{R}^n)$.

In applications also non-smooth pseudodifferential operators occur. The goal of this talk is to show the invariance of the Fredholm index for non-smooth pseudodifferential operators with symbols in the class $C^{\tilde{m},s}S^m_{1,0}(\mathbb{R}^n \times \mathbb{R}^n)$. To reach this aim we use the main idea of the result from Rabier about the Fredholm index for non-smooth differential operators. The main difficulty is to prove a regularity result for non-smooth pseudodifferential operators needed in the proof.

The talk is based on a joint work with H. Abels.

Roman Šimon Hilscher, Masaryk University

Singular Sturmian comparison theorems

Abstract. This work was obtained jointly with Peter Šepitka (Masaryk University, Brno). In 2010, Aharonov and Elias proved a singular comparison theorem for two second order differential equations satisfying the Sturmian majorant condition. In this talk we present how this result can be generalized to two linear Hamiltonian systems. At the same time we do not impose any controllability condition. The results are phrased in terms of the comparative index and the numbers of proper focal points of the (minimal) principal solutions of these systems at both endpoints of the considered interval. The main idea is based on an application of new transformation theorems for principal and antiprincipal solutions at infinity and on new limit properties of the comparative index involving these solutions. The results are new even for completely controllable linear Hamiltonian systems, notably also for even order Sturm–Liouville differential equations. In this way we also obtain an extension of the previous result of Aharonov and Elias. We also present some recent open problems related to this research.

Jari Taskinen, University of Helsinki

Spectral Laplace problem and heat equation in periodic domains

Abstract. We treat the classical linear heat equation in an unbounded, periodic waveguide $\Pi \subset \mathbb{R}^d$ by using Floquet transform methods. As well known, the behavior of the solution of the heat equation is determined by the underlying spectral Laplace problem, hence, we are led to analyse the properties of the spectrum which are relevant to the heat equation. The Floquet transform F turns the elliptic problem on the unbounded domain Π into a corresponding elliptic model problem on the (bounded) periodic cell ϖ of Π. We observe that estimates of the eigenvalues of the model problem yield sufficient estimates for the spectral bands so that the Floquet transform can also be directly applied to the heat equation. This leads to a heat equation with mixed boundary conditions on the periodic cell ϖ. We analyse the connection between the solutions of the two problems.

Particular attention is paid to the case of a disjoint component of the essential spectrum, where we describe the spectral projection onto the subspace $\mathcal{H}_S \subset L^2(\Pi)$ corresponding to the spectral component and also show that the translated Wannier functions form an orthonormal basis in $\mathcal{H}_S$. Then, applications to the heat equation follow immediately.

The talk is based on the manuscript

Marcus Rosenberg and Jari Taskinen, *Some aspects of the Floquet theory for the heat equation in a periodic domain,* submitted.

George Tephnadze, The University of Georgia, School of Science and Technology, Tbilisi, Georgia

Bi-Laplace-Beltrami equation on hypersurfaces and Γ*-convergence*

Abstract. This talk is devoted to investigate a mixed boundary value problem for the bi-Laplacian equation in a thin layer around a surface C with the boundary. We trace what happens in Γ-limit when the thickness of the layer converges to zero. It is shown how the mixed type boundary value problem (BVP) for the bi-Laplace equation in the initial thin layer transform in the Γ-limit into an explicit Dirichlet BVP for the bi-Laplace-Beltrami equation on the surface. For this we apply the variational formulation and the calculus of Günter's tangential differential operators on a hypersurface and layers. This approach allow global representation of basic differential operators and of corresponding BVPs in terms of the standard cartesian coordinates of the ambient Euclidean space $\mathbb{R}^n$.

Medea Tsaava, The University of Georgia

Mixed boundary value problems for the Helmholtz equation in a model 2D double angular domain

Abstract. The purpose of the present research is to investigate model mixed boundary value problems for the Helmholtz equation in a model 2D double angular domains $\Omega_{\alpha,\beta} := \Omega_\alpha \cup \Omega_{-\beta} \subset \mathbb{R}^2$, where Ω_α has magnitude $\alpha > 0$ and $\Omega_{-\beta} \subset \mathbb{R}^2$ has magnitude $-\beta > 0$. Angular domains Ω_α and $\Omega_{-\beta}$ have common boundary along the positive semi axes $\mathbb{R}^+ := (0, \infty)$. The BVP is considered in a non-classical setting, when solutions are sought in the Bessel potential spaces

$\mathbb{H}_p^s(\Omega_{\alpha,\beta})$, $s > 1/p$, $1 < p < \infty$. The problems are investigated using the potential method by reducing them to an equivalent boundary integral equation (BIE) in the Sobolev-Slobodečkii space on a semi-infinite axes $\mathbb{W}_p^{s-1/p}(\mathbb{R}^+)$, which is of Mellin convolution type. By applying the recent results on Mellin convolution equations in Bessel potential spaces obtained by V. Didenko & R. Duduchava in [1], explicit conditions of the unique solvability of this BIE in the Sobolev-Slobodečkii $\mathbb{W}_p^r(\mathbb{R}^+)$ and Bessel potential $\mathbb{H}_p^r(\mathbb{R}^+)$ spaces for arbitrary r are found and used to write explicit conditions for the Fredhoilm property and unique solvability of the initial model BVPs for the Helmholtz equation in the above mentioned non-classical setting.

This is joint work with Prof. R. Duduchava (The University of Georgia) and M. Tutberidze (The University of Georgia).

[1] V. Didenko, R. Duduchava, Mellin convolution operators in the Bessel potential spaces, *Journal of Analysis and Applications* (accepted) http://arxiv.org/pdf/1502.02756.pdf

Margarita Tutberidze, The University of Georgia, Tbilisi, Georgia. Email: *margarita.tutberidze@ug.edu.ge*

Dirichlet and Neumann boundary value problems for the Helmholtz equation in a double angle

Abstract. We investigate Dirichlet and Neumann boundary value problems for the anisotropic Helmholtz equation in a double angle of magnitude $\alpha > 0$ and $-\beta < 0$, having in common the positive semi axes
$mathbbR^+$. On the outer boundary is prescribed a Dirichlet or Neumann condition, while along the common boundary of angles (interface) $\mathbb{R}^+$ is prescribed the continuity)the transmission) conditions. We consider the non-classical $\mathbb{L}_p$-based Bessel potential space setting of the problem. for $1 < p < \infty$ We apply the potential method and reduce the boundary value problem to the system of boundary pseudodifferential equation, which is further reduced to an equivalent system of 6×6 Mellin-type convolution equations on $\mathbb{R}^+$ the Bessel potential space $\mathbb{H}_p^s(\mathbb{R}^+)$. By using the results obtained earlier by V. Didenko and R. Duduchava for such equations, we write symbol of the equation and derive the criteria for solvability (Fredholmness) of such systems of equations in the Bessel potential spaces $\mathbb{H}_p^s(\mathbb{R}^+)$. Moreover, we indicate the range of space parameters (s, p) for which the original Dirichlet and Neumann boundary value problems have unique solutions in the non-classical space settings.

The described results are obtained in collaboration with R. Duduchava and M. Tsaava.

Zurab Vashakidze, The University of Georgia, Tbilisi, Georgia

Wave Propagation in a Triangular Lattice with Discrete Sources Placed on Line Segments

Abstract. We investigate the propagation of time-harmonic waves through a triangular lattice with sources located on line segments. Specifically, the study focuses on the discrete Helmholtz equation, where the wave number k lies within the range of $(0, 2\sqrt{2})$, and the input data is prescribed on finite rows and columns of lattice sites without resorting to complex wave numbers. Similar to the continuum theory, the notion of a radiating solution is introduced, establishing a unique solvability result and Green's representation formula employing difference potentials. Furthermore, we apply a numerical computation method that demonstrates efficiency in solving problems related to the propagation of left-handed 2D inductor-capacitor metamaterials.

This talk relies on results attained through collaborative research with Prof. Dr. David Kapanadze from the Andrea Razmadze Mathematical Institute of Ivane Javakhishvili Tbilisi State University.

The work was supported by the Shota Rustaveli National Science Foundation of Georgia [grant number: FR-21-301, project title: "Metamaterials with Cracks and Wave Diffraction Problems"].

5.13. Operator Theory on Analytic Function Spaces.

Monday 2:30-4:30 in Language Center, Room 115 chair: Antti Perälä

2:30-2:55	Jari Taskinen *Bergman-Toeplitz operators on periodic planar domains*	
3:00-3:25	Armando Sánchez-Nungaray *Toeplitz operators with symmetric separately radial symbols on the unit ball*	
3:30-3:55	Hicham Arroussi *Toeplitz operators between large Fock spaces*	
4:00-4:25	Raul Quiroga-Barranco *Toeplitz operators and Polish groups*	

Tuesday 2:30-4:00 in Language Center, Room 115 chair: Antti Perälä

2:30-2:55	Santeri Miihkinen *On the exponential integrability of conjugate functions*	
3:00-3:25	Clifford Gilmore *Growth of Distributionally Irregular Functions*	
3:30-3:55	Yuri Karlovich *A C^*-algebra of convolution type operators with piecewise quasicontinuous data*	

Tuesday 4:30-6:00 in Language Center, Room 115 chair: Kehe Zhu

4:30-4:55	Boo Rim Choe *Linearly Connected Composition Operators on the Hardy Space*	
5:00-5:25	Inyoung Park *Reproducing kernel thesis for the weighted composition operators over the polydisk*	
5:30-5:55	Oleksiy Karlovych *The Coburn lemma and the Hartman-Wintner-Simonenko theorem for Toeplitz operators on abstract Hardy spaces*	

Thursday 2:30-4:00 in Language Center, Room 115 chair: Kehe Zhu

2:30-2:55	Haakan Hedenmalm *Hyperbolic Fourier series and the Klein-Gordon equation*	
3:00-3:25	Jouni Rättyä *Small Hankel operators on Bergman spaces induced by doubling weights*	
3:30-3:55	Pan Ma *Multiplication operators on invariant subspaces of Bergman space*	UPDATE

Thursday 4:30-6:00 in Language Center, Room 115 chair: Santeri Miihkinen

4:30-4:55	Adrián Llinares *Contractive inequalities between Dirichlet and Hardy spaces*	
5:00-5:25	David Norrbo *Compactness and related properties for weighted composition operators on BMOA*	
5:30-5:55	Viktor Didenko *Invertibility of Toeplitz plus Hankel operators on l^p-spaces*	

Friday 11:30-1:00 in Language Center, Room 115 chair: Santeri Miihkinen

11:30-11:55	Željko Čučković *On spectrum of Hankel operators on the polydisk*	
12:00-12:25	Trieu Le *On range of Berezin transform in several variables and applications*	
12:30-12:55	Juha-Matti Huusko *Linear differential equations with slowly growing solutions*	UPDATE

Friday 2:30-4:00 in Language Center, Room 115 chair: Santeri Miihkinen

2:30-2:55	Nina Zorboska *Toeplitz operators on weighted Hardy spaces*
3:00-3:25	Raffael Hagger *Toeplitz Operators on Polyanalytic Fock Spaces vol. 2*
3:30-3:55	Antti Perälä *Essential Positivity*

5.13.1. *Abstracts.*

Boo Rim Choe, Korea University

Linearly Connected Composition Operators on the Hardy Space

Abstract. We consider the space of all composition operators, acting on the Hardy space over the unit disk, in the uniform operator topology. We obtain a characterization for linear connection between composition operators. As one of applications, we see that the set of all compact composition operators is a polygonally connected component, in sharp contrast to the known fact that this set is properly contained in a path connected component. We also notice some results in conjunction with the Bergman space case. Several questions motivated by our results are included.

This presentation is based on a joint work with Koeun Choi, Hyungwoon Koo and Inyoung Park.

Željko Čučković, University of Toledo, USA

On spectrum of Hankel operators on the polydisk

Abstract. We give sufficient conditions for the spectrum of the Hermitian square of a class of Hankel operators on the Bergman space of the polydisk to contain intervals. (Joint work with Zhenghui Huo and Sonmez Sahutoglu)

Viktor Didenko, Southern University of Science and Technology, Shenzhen, China

Invertibility of Toeplitz plus Hankel Operators on l^p-Spaces

Abstract. Wiener-Hopf factorization has been used in [1,2,3] in order to study the invertibility of Toeplitz plus Hankel operators $T(a) + H(b)$ and Wiener-Hopf plus Hankel operators $W(a) + H(b)$ on classical Hardy spaces. However, this approach does not always work even for Toeplitz operators acting on spaces of sequences. The only exceptions are the operators with generating functions a and b from multiplier algebras such that the factorization factors belong to the same algebra. Nevertheless, the invertibility of the operators $T(a) + H(b)$ can be studied in a more general situation if certain auxiliary Toeplitz operators satisfy additional conditions.

The talk is based on joint work with Bernd Silbermann.

[1] V.D. Didenko and B. Silbermann, *The invertibility of Toeplitz plus Hankel operators*, J. Operator Theory **78**, 293–307 (2017).

[2] V.D. Didenko and B. Silbermann, *The invertibility of Toeplitz plus Hankel operators with subordinated operators of even index*, Linear Algebra Appl. **578**, 425–445 (2019).

[3] V.D. Didenko and B. Silbermann, *Invertibility issues for a class of Wiener-Hopf plus Hankel operators*, J. Spectr. Theory **11**, 847–872 (2021).

Clifford Gilmore, Université Clermont Auvergne

Growth of Distributionally Irregular Functions

Abstract. Distributional irregularity has been actively investigated in the past decade and many natural continuous linear operators turn out to be distributionally irregular. For instance the differentiation operator acting on the space of entire functions and the partial differentiation operators acting on the space of harmonic functions on $\mathbb{R}^N$, where $N \geq 2$.

The order of growth of distributionally irregular entire functions was first considered by Bernal-González and Bonilla [1]. In this talk we examine some recent results on the permissible growth rates of entire functions and harmonic functions that are distributionally irregular with respect to differentiation and partial differentiation operators.

This is joint work with Félix Martínez-Giménez and Alfred Peris (Universitat Politècnica de València).

[1] L. Bernal-González and A. Bonilla (2016) Order of growth of distributionally irregular entire functions for the differentiation operator, *Complex Var. Elliptic Equ.* **61(8)**, 1176–1186.

[2] C. Gilmore, F. Martínez-Giménez and A. Peris (2022) Rate of growth of distributionally chaotic functions, *Math. Inequal. Appl.* **25(1)**, 145–167.

Raffael Hagger, Kiel University

Toeplitz Operators on Polyanalytic Fock Spaces vol. 2

Abstract. Let μ denote the standard Gaussian measure on $\mathbb{C}$ defined by $\mathrm{d}\mu(z) = \frac{1}{\pi}e^{-|z|^2}\,\mathrm{d}z$ and $n \in \mathbb{N}$. The polyanalytic Fock space $\mathcal{F}_n^2$ consists of smooth functions in $L^2(\mathbb{C},\mu)$ with the property that $\frac{\partial^n f}{(\partial \overline{z})^n} = 0$. Clearly, $\mathcal{F}_1^2$ is just the usual Fock (or Segal–Bargmann) space. The space $\mathcal{F}_n^2$ can be decomposed into an orthogonal sum of so-called true polyanalytic Fock spaces $\mathcal{F}_{(k)}^2$, $k = 1, \ldots, n$. Each of these $\mathcal{F}_{(k)}^2$-spaces is essentially a copy of the analytic Fock space $\mathcal{F}_1^2 = \mathcal{F}_{(1)}^2$ in the sense that $\frac{1}{\sqrt{(k-1)!}}\left(-\frac{\partial}{\partial z} + \overline{z}\right)^{k-1}$ is an isometric isomorphism between $\mathcal{F}_{(1)}^2$ and $\mathcal{F}_{(k)}^2$. Nevertheless, there is some interesting operator theory to be discovered on these spaces. In particular, Toeplitz operators may behave much differently than expected. For a symbol $f \in L^\infty(\mathbb{C})$ they are defined as usual:

$$T_{f,(k)} : \mathcal{F}_{(k)}^2 \to \mathcal{F}_{(k)}^2, \quad T_{f,(k)}g := P_{(k)}(fg),$$

where $P_{(k)} : L^2(\mathbb{C},\mu) \to \mathcal{F}_{(k)}^2$ is the orthogonal projection. In my talk at IWOTA last year I showed that if $T_{f,(1)}$ is compact, then $T_{f,(k)}$ is necessarily compact as well, and then the obvious question came up whether the converse is true as well. It turns out that the converse fails in the most spectacular way possible. In fact, for each $k \geq 2$ there are bounded symbols f such that $T_{f,(1)}$ is unitary, but $T_{f,(k)}$ vanishes. In this talk I will explain this phenomenon and answer further open questions using some newly developed techniques from quantum harmonic analysis.

Based on joint work with Robert Fulsche.

Haakan Hedenmalm, KTH Royal Institute of Technology

Hyperbolic Fourier series and the Klein-Gordon equation

Abstract. This reports on joint work with M. Viazovska et al.

We discuss a discretized Goursat problem for the Klein-Gordon equation. This then connects with hyperbolic Fourier series, a way to represent functions or distributions on the line in terms a series based on complex exponentials $\exp(i\pi nt)$ and $\exp(-i\pi m/t)$ for integers m, n. This representation is unique in a wide range of ultradistributions dual to Gevrey class. The biorthogonal system solves the discretized Goursat problem. Some properties of the biorthogonal system are outlined, and a QUE-type conjecture is proposed.

Arroussi Hicham, University of Helsinki (FI) and Reading (UK)

Title of Talk: Toeplitz operators between large Fock spaces

Abstract. In this paper, for a certain class of weights $\mathcal{W}$ of exponential weights, we give a complete characterization of the bounded and compact Toeplitz operators between diferent large Fock spaces F^p_ω and F^q_ω, with $0 < p, q \leq \infty$ and $\omega \in \mathcal{W}$, in terms of average functions, certain generalized Berezin transforms and Carleson measures. We characterize the essential norms of Toeplitz operators from F^p_ω into F^q_ω for $0 < p, q \leq \infty$, as well.

Juha-Matti Huusko, University of Eastern Finland

Linear differential equations with slowly growing solutions

Abstract. In 1982, Pommerenke obtained a sharp sufficient condition for the analytic coefficient $A \in \mathcal{H}(\mathbb{D})$ which places all solutions f of

$$f'' + Af = 0 \tag{6}$$

to the classical Hardy space H^2. Pommerenke's idea was to use Green's formula twice to write the H^2-norm of f in terms of f'', employ the differential equation (6), and then apply Carleson's theorem for the Hardy spaces. Consequently, the coefficient condition was given in terms of Carleson measures. The leading idea of this (operator theoretic) approach has been extended to study, for example, solutions in the Hardy and Bergman spaces, Dirichlet type spaces and growth spaces, to name a few instances.

Our intention is to establish sufficient conditions for the coefficient of (6) which place all solutions to H^∞, $BMOA$ or to the Bloch space. In principle, Pommerenke's original idea could be modified to cover these cases, but in practice, this approach falls short since either it is difficult to find a useful expression for the norm in terms of the second derivative (in the case of H^∞) or the characterization of Carleson measures is not known (in the cases of BMOA and Bloch). Our approach takes advantage of the reproducing formulae, and is different to ones in the literature.

The talk is based on the paper J. Gröhn, J.-M. Huusko and J. Rättyä, *Linear differential equations with slowly growing solutions*, Trans. Amer. Math. Soc. 370 (2018), 7201-7227. https://arxiv.org/pdf/1609.01852.pdf

Yuri Karlovich, Universidad Autónoma del Estado de Morelos, México

A C^*-algebra of Convolution Type Operators with Piecewise Quasicontinuous Data

Abstract. Let $PQC_{\mathbb{R}}$ be the C^*-algebra of all piecewise quasicontinuous functions on the real line $\mathbb{R}$ that are equivalent to piecewise slowly oscillating functions at infinity. Let $\mathcal{B}$ stand for the C^*-algebra of all bounded linear operators acting on the Lebesgue space $L^2(\mathbb{R})$. The C^*-algebra $\mathfrak{A} \subset \mathcal{B}$ generated by all multiplication operators aI and by all convolution operators $W^0(b)$ with data functions $a, b \in PQC_{\mathbb{R}}$ is studied. First a Fredholm symbol calculus is constructed for the C^*-algebra $\mathcal{Z} \subset \mathfrak{A}$ generated by the operators $aW^0(b)$ with quasicontinuous data functions $a, b \in QC_{\mathbb{R}}$ that are equivalent to slowly oscillating functions at infinity. Then a Fredholm symbol calculus for the C^*-algebra $\mathfrak{A}$ is constructed and a Fredholm criterion for the operators $A \in \mathfrak{A}$ in terms of their Fredholm symbols is established. The talk is partially based on a joint work with C.A. Fernandes and A.Yu. Karlovich.

Oleksiy Karlovych, NOVA University Lisbon

The Coburn lemma and the Hartman-Wintner-Simonenko theorem for Toeplitz operators on abstract Hardy spaces

Abstract. Let X be a Banach function space on the unit circle $\mathbb{T}$, let X' be its associate space, and let $H[X]$ and $H[X']$ be the abstract Hardy spaces built upon X and X', respectively. Suppose that the Riesz projection P is bounded on X and $a \in L^\infty \setminus \{0\}$. We show that P is

bounded on X'. So, we can consider the Toeplitz operators $T(a)f = P(af)$ and $T(\overline{a})g = P(\overline{a}g)$ on $H[X]$ and $H[X']$, respectively. We show that if X is not separable, then one cannot rephrase Coburn's lemma as in the case of classical Hardy spaces H^p, $1 < p < \infty$, and guarantee that $T(a)$ has a trivial kernel or a dense range on $H[X]$. On the other hand, the following version of Coburn's lemma is true: the kernel of $T(a)$ on $H[X]$ or the kernel of $T(\overline{a})$ on $H[X']$ is trivial. The second main result is a generalisation of the Hartman-Wintner-Simonenko theorem saying that if $T(a)$ is normally solvable on the space $H[X]$, then $1/a \in L^\infty$. This is a joint work with Eugene Shargorodsky (King's College London, UK).

Trieu Le, University of Toledo

On range of Berezin transform in several variables and applications

Abstract. For an integrable function u on the unit ball, let $B(u)$ denote its Berezin transform. We investigate holomorphic functions $F_1, \ldots, F_m$ and $G_1, \ldots, G_m$ such that

$$B(u) = F_1\overline{G}_1 + \cdots + F_m\overline{G}_m.$$

Our results have direct applications to the study of algebraic properties of Toeplitz operators with pluriharmonic symbols on the Bergman space. In particular, we show that if ϕ, ψ are bounded pluriharmonic functions on the ball such that $T_\phi T_\psi$ has finite rank, then one of ϕ, ψ must be zero. Joint work with Akaki Tikaradze.

Adrián Llinares, Umeå University

Contractive inequalities between Dirichlet and Hardy spaces

Abstract. We say that the inclusion between two spaces of analytic functions is contractive if the norm of the corresponding inclusion operator is equal to 1. Although these inclusions are interesting in themselves, they have also attracted the attention of the experts because of their multiple applications. In this talk, we will show some of these contractive inequalities and discuss their most immediate consequences.

Pan Ma, Central South University

Multiplication operators on invariant subspaces of Bergman space

Abstract. In this talk, we will review some basic results of invariant subspaces of Bergman space and present some progress on multiplication operators on invariant subspaces of Bergman space.

Santeri Miihkinen, University of Reading

On the exponential integrability of conjugate functions

Abstract. The conjugate function $\tilde{f}$ (the periodic Hilbert transform) of an integrable function $f\colon \mathbb{T} \to \mathbb{R}$ on the unit circle $\mathbb{T}$ can be defined as the principal value integral

$$\tilde{f}(\theta) = \lim_{\epsilon \to 0} \frac{1}{2\pi} \int_{|\theta-\varphi|>\epsilon} \cot\left(\frac{\theta - \varphi}{2}\right) f(\varphi)\, d\varphi$$

for almost every θ.

Comparison of sizes of f and $\tilde{f}$ in different function spaces is a topic of significant interest in the literature. Although $f \in L^\infty$ does not imply that $\tilde{f} \in L^\infty$, the conjugation operator $f \mapsto \tilde{f}$ still has very strong boundedness properties. Namely, a classical theorem of Zygmund asserts that if $\|f\|_{L^\infty(\mathbb{T})} \leq \frac{\pi}{2}$ and $\lambda < 1$, then $\exp(\lambda\tilde{f})$ is integrable.

We investigate exponential non-integrability of conjugate functions, i.e., the reverse direction to Zygmund's result. The talk is based on a joint work with Hussain Gissy and Jani Virtanen (University of Reading).

David Norrbo, Postdoctorate

Compactness and related properties for weighted composition operators on BMOA

Abstract. A previously known function-theoretic characterisation of compactness for a weighted composition operator on BMOA is improved. Moreover, the same function-theoretic condition also characterises weak compactness and complete continuity. In order to close the circle of implications, the operator-theoretic property of fixing a copy of c_0 comes in useful. As a consequence, the weighted composition operator is compact if and only if it maps weakly unconditionally Cauchy sequences on unconditional convergent sequences.

Inyoung Park, Ewha Women's University

Reproducing kernel thesis for the weighted composition operators over the polydisk

Abstract. In this talk, we present the boundedness and compactness for the difference of weighted composition operators by examining only the behavior of the normalized reproducing kernels in the weighted Bergman spaces over the polydisk. Additionally, we obtain a function-theoritic characterization when the weight functions are in H^∞.

Antti Perälä, Umeå University

Essential Positivity

Abstract. We define essentially positive operators on Hilbert space as a class of self-adjoint operators whose essential spectra is contained in the nonnegative real numbers and describe their basic properties. Using Toeplitz operators and the Berezin transform, we further illustrate the notion of essential positivity in the Hardy space and the Bergman space. This is a joint work with Jani Virtanen.

Raul Quiroga-Barranco, Cimat, Mexico

Toeplitz operators and Polish groups

Abstract. Let $D \subset \mathbb{C}^n$ be a domain and μ a measure on D obtained by weighting the Lebesgue measure. If μ is suitably chosen, then the subspace $\mathcal{H}^2(D)$ of $L^2(D,\mu)$ that consists of holomorphic functions is closed and a reproducing kernel Hilbert space. This yields the so-called Toeplitz operators T_a obtained by compressing the multiplier operator M_a from $L^2(D,\mu)$ to $\mathcal{H}^2(D)$, where the symbol a belongs to $L^\infty(D,\mu)$. It has been found interesting to study C^*-algebras generated by Toeplitz operators with symbols from some given subspace $\mathcal{S} \subset L^\infty(D,\mu)$: let us denote such C^*-algebra by $\mathcal{T}(D,\mu,\mathcal{S})$. In particular, for a domain D with a large Lie group G of biholomorphisms, it has been observed that for suitable subgroups $H \subset G$ the space of H-invariant symbols (denoted by $L^\infty(D,\mu)^H$) yields corresponding C^*-algebras $\mathcal{T}(D,\mu,L^\infty(D,\mu)^H)$ that have interesting features and are accessible to their study. This uses the fact that the H-action on D induces a corresponding unitary representation on $\mathcal{H}^2(D)$.

We take this one step further and consider closed subgroups H of $\mathrm{U}(\mathcal{H}^2(D))$, the group of unitary operators acting on $\mathcal{H}^2(D)$. The group $\mathrm{U}(\mathcal{H}^2(D))$ is endowed with the weak-operator topology for which it is a Polish group as long as $\mathcal{H}^2(D)$ is separable, which is the case in all relevant examples. The groups involved are no longer Lie nor even locally compact in general. Furthermore, the corresponding unitary representations do not necessarily come from biholomorphism actions on D. However, by allowing to work with such ampler family of groups and representations we can obtain general results that extend the known ones as well as explaining some phenomena that has been observed.

Let $\mathcal{T} \subset \mathcal{T}(D,\mu,L^\infty(D,\mu))$ be an arbitrary C^*-subalgebra. Among the results that we obtain through our approach we have the following. There exists a closed subgroup $H \subset \mathrm{U}(\mathcal{H}^2(D))$ such that $\mathcal{T}$ is strong-operator dense in $\mathrm{End}_H(\mathcal{H}^2(D))$, where the latter denotes the von Neumann algebra of intertwining operators for H. In other words, every C^*-algebra generated by

some family of Toeplitz operators can be described using a unitary representation of some Polish group H. Furthermore, H can be chosen so that $\mathcal{H}^2(D)$ can be decomposed as a direct integral that decomposes both the actions of $\mathcal{T}$ and H in such a way that they mutually generate their commutants fiberwise. On the other hand, $\mathcal{T}$ is Abelian if and only if the unitary representation of H is multiplicity-free. Also, the weak-operator closure of $\mathcal{T}$ is maximal Abelian if and only if the group H can be chosen so that its weak-operator closure is maximal Abelian (as a group). In particular, this provides a new insight into the study of C^*-algebras generated by Toeplitz operators acting on the unit ball $\mathbb{B}^n$ obtained from maximal Abelian subgroups of biholomorphisms.

Jouni Rättyä, University of Eastern Finland, Finland

Small Hankel operators on Bergman spaces induced by doubling weights

Abstract. The boundedness of the small Hankel operator $h_f^\nu(g) = P_\nu(f\overline{g})$, induced by an analytic symbol f and the Bergman projection P_ν associated to ν, acting from the weighted Bergman space A^p_ω to A^q_ν is characterized on the full range $0 < p, q < \infty$ of parameters when ω and ν belong to the class $\mathcal{D}$ of radial weights admitting certain two-sided doubling conditions. Moreover, an asymptotic formula for the operator norm of h_f^ν is established in terms of a suitable norm of $f^{(n)}$ depending upon the inducing weights and parameters. Certain results obtained are equivalent to the boundedness of bilinear Hankel forms, which are in turn used to establish the weak factorization $A^q_\eta = A^{p_1}_\omega \odot A^{p_2}_\nu$, where $1 < q, p_1, p_2 < \infty$ such that $q^{-1} = p_1^{-1} + p_2^{-1}$ and $\widetilde{\eta}^{\frac{1}{q}} \asymp \widetilde{\omega}^{\frac{1}{p_1}} \widetilde{\nu}^{\frac{1}{p_2}}$. Here $\widetilde{\tau}(r) = \int_r^1 \tau(t)\,dt/(1-r)$ for all $0 \le r < 1$.

Armando Sánchez-Nungaray, Universidad Veracruzana

Toeplitz operators with symmetric separately radial symbols on the unit ball

Abstract. The principal goal is this work is to explore the relationship between the representation theory, as it is stablished by Raúl Quiroga, and the conmutativity of the C^*-algebra generated for the Toeplitz operators with symbols invariant under the groups given by semidirect producto of $\mathbb{T}^n$ with S_n and A_n, where $\mathbb{T}^n$ stand for the n-dimensional torus, S_n stand for for the symmetric group of permutations of the set $\{1, 2, \dots, n\}$, and A_n is the alternating group of S_n. It is important to point out that the types of symbols obtained by introducing these new subgroups of $U(n)$, the unitary group, generate new commutative algebras which have not been analyzed in current works on Toeplitz operators.

For the case of the symmetric separately radial symbols, this analysis correspond to the action semidirect product of $\mathbb{T}^n$ with S_n on the unit ball $\mathbb{B}^n$ on. We define an unitary operator R that permit to exhibit the simultaneous diagonalization of the Toeplitz operators with symmetric separately radial symbols into multiplication operators on $l^2(\mathbb{Z}^n_+)$.The representation theoretic approach rely on the Shur's lemma and it allows us to prove that the Toeplitz operators with symmetric separately radial symbols satisfies orthogonality relations. Also, the functions for the multiplication operators unitarily equivalent to Toeplitz operators with symmetric separately radial symbols are proved to be constant on the multi-indices that belong at the same $\mathcal{I}_\iota$.

A corresponding study is performed for alternating separately radial symbols, for this we considerer the action of semidirect product of $\mathbb{T}^n$ and A_n on $\mathcal{B}^n$.

Jari Taskinen, University of Helsinki

Bergman-Toeplitz operators on periodic planar domains

Abstract. We study spectra of Toeplitz operators T_a with periodic symbols in Bergman spaces $A^2(\Pi)$ on unbounded periodic planar domains Π, which are defined as the union of infinitely many copies of the translated, bounded periodic cell ϖ. We introduce Floquet-transform techniques and prove a version of the band-gap-spectrum formula, which is well-known in the framework of periodic elliptic spectral problems and which describes the essential spectrum of T_a in

terms of the spectra of a family of Toepliz-type operators $T_{a,\eta}$ in the cell ϖ, where η is the so-called Floquet variable.

As an application, we consider periodic domains Π_h containing thin geometric structures and show how to construct a Toeplitz operator $T_{\mathsf{a}} : A^2(\Pi_h) \to A^2(\Pi_h)$ such that the essential spectrum of T_{a} contains disjoint components which approximatively coincide with any given finite set of real numbers. Moreover, our method provides a systematic and illustrative way how to construct such examples by using Toeplitz operators on the unit disc $\mathbb{D}$ e.g. with radial symbols.

Using a Riemann mapping one can then find a Toeplitz operator $T_a : A^2(\mathbb{D}) \to A^2(\mathbb{D})$ with a bounded symbol and with the same spectral properties as T_{a}.

Nina Zorboska, University of Manitoba

Toeplitz operators on weighted Hardy spaces

Abstract. I will talk about the boundedness and compactness of Toeplitz and Toeplitz-type operators on a class of Hilbert spaces of analytic functions. The class includes the classical Bergman, Hardy and Dirichlet spaces. Since these spaces are also Reproducing Kernel Hilbert Spaces, it is natural to try to characterize the properties of the operators via the Berezin transform. As it turns out, for Toeplitz and Toeplitz-type operators, this approach works better for some spaces than for the others.

5.14. **Quantum Harmonic Analysis.**

Tuesday 2:30-4:00 in Language Center, Room 206 chair: Raffael Hagger

2:30-2:55	Franz Luef *τ-quantization and τ-Cohen classes of Feichtinger operators*
3:00-3:25	Michael Speckbacher *Eigenvalue estimates for Fourier concentration operators on two domains*
3:30-3:55	Irina Shafkulovska *Metaplectic action on modulation spaces*

Tuesday 4:30-6:00 in Language Center, Room 206 chair: Raffael Hagger

4:30-4:55	Robert Fulsche *Limit functions, limit operators and Wiener's Tauberian theorem in Quantum Harmonic Analysis*
5:00-5:25	Henry McNulty *A Short Time Fourier Transform and Coorbit Spaces of Operators*

Thursday 2:30-4:00 in Language Center, Room 206 chair: Robert Fulsche

3:00-3:25	Simon Halvdansson *Four ways to recover the symbol of a non-binary localization operator*
3:30-3:55	Helge J. Samuelsen *Fourier Restriction in Quantum Harmonic Analysis with Applications to Quantization*

Thursday 4:30-6:00 in Language Center, Room 206 chair: Robert Fulsche

4:30-4:55	Damian Kołaczek *Fractional order rescaled Wigner transformation*
5:00-5:25	Jukka Kiukas *Joint measurement of multiple non-commuting canonical observables in phase space*

5.14.1. *Abstracts.*

Robert Fulsche, Leibniz University Hannover

Limit functions, limit operators and Wiener's Tauberian theorem in Quantum Harmonic Analysis

Abstract. The results we are going to present in this talk are motivated by two seemingly unrelated questions, appearing naturally in quantum harmonic analysis: 1) When is the convolution of two functions, a function and an operator, or two operators, contained in a given closed, translation-invariant subspace of BUC, the bounded uniformly continuous functions, or a closed, translation-invariant subspace of $\mathcal{C}_1(\mathcal{H})$, the uniformly continuous operators? 2) What is an appropriate operator analogue of the class of slowly oscillating functions, appearing in Pitt's version of Wiener's Tauberian theorem?

We present a result, which simultaneously answers both questions and at the same time generalizes Wiener's classical Tauberian theorem, a version of Wiener's Tauberian theorem for operators that was recently described by F. Luef and E. Skrettingland, and R. F. Werner's correspondence theorem. In our results, we will make crucial use of the techniques of limit functions and limit operators.

The talk is based on joint work with F. Luef (NTNU Trondheim) and R. F. Werner (Leibniz University Hannover).

Simon Halvdansson, NTNU Norwegian University of Science and Technology

Four ways to recover the symbol of a non-binary localization operator

Abstract. In time-frequency analysis, localization operators restrict a signal to a certain subset of the time-frequency plane and are defined via a symbol which acts as a weighing factor. These operators can be realized as special cases of function-operator convolutions from quantum harmonic analysis with the symbol being the function and a rank-one operator. Throughout the last decade, the inverse problem of recovering the symbol from various measurements related to the operator has received attention. In the work discussed in this talk, we have shown four ways to perform this deconvolution. The talk will give a brief introduction to localization operators and show theorems as well as numerical examples where these deconvolutions are performed.

Jukka Kiukas, Aberystwyth University

Joint measurement of multiple non-commuting canonical observables in phase space

Abstract. Existence of observables which cannot be jointly measured is a fundamental feature of quantum theory. Observables represented by selfadjoint operators are jointly measurable exactly when they commute, and phase space quantum mechanics is naturally based on the non-commutativity of the canonical position-momentum pair. However, introducing noise to such observables by convolving them with probability measures leads to observables represented by general positive operator valued measures, in which case commutativity is no longer necessary for joint measurability, and the existence of joint observables is an interesting nontrivial problem.

In this talk I present a characterisation for joint measurability of multiple convolved canonical observables, each corresponding to a given direction in the phase space, focusing on the case where the number of observables exceeds the dimension of the phase space. The resulting joint observables can always be chosen suitably covariant with respect to the Weyl representation, which in turn forces constraints on the original observables. Interestingly, by applying a general framework developed recently by L. Dammeier and R.F. Werner, one can interpret this joint measurability structure in terms of canonical coordinate observables measured on a single state of an auxiliary quantum-classical hybrid system.

Damian Kołaczek, Department of Applied Mathematics, University of Agriculture in Krakow

Fractional order rescaled Wigner transformation

Abstract. Wigner-Weyl formalism has become an important tool in several areas of scientific research including quantum mechanics, harmonic analysis and signal processing. Over the years numerous generalizations of standard Wigner distribution function and associated Weyl pseudodifferential operators have appeared [1,2]. An interesting route was presented in [1], where authors developed fractional order generalization of ordinary Wigner transformation and Weyl pseudodifferential operators using metaplectic operators framework. We approach the problem of building fractional order generalization of this formalism in somewhat different manner. Our starting point is the Wigner transformation from [4], which is equivalent to the standard Wigner transformation up to rescaling of the variables. We built its fractional counterpart in analogous way as ordinary Fourier transform has been generalized into fractional Fourier transform and we study its basic properties in quantum mechanical context. In our approach we use Segal-Bargmann transform and the framework of metaplectic operators. We also consider fractional order generalization of Weyl pseudodifferential operators associated with our fractional transformation.

References:

[1] P. Boggiatto, G. de Donno and A. Oliaro, "Time-frequency representations of Wigner type and pseudo-differential operators", *Transactions of the American Mathematical Society* **362**, 4955 (2010), doi:10.1090/S0002-9947-10-05089-0

[2] E. Cordero, L. Rodino, "Wigner analysis of operators. Part I: Pseudodifferential operators and wave fronts", *Applied and Computational Harmonic Analysis* **58**, 85 (2022), doi:10.1016/j.acha.2022.01.003

[3] N. C. Dias, M. A. de Gosson and J. N Prata, "Metaplectic formulation of the Wigner transform and applications", *Reviews in Mathematical Physics* **25**, 1343010 (2013), doi:10.1142/S0129055X13430101

[4] J. C. Varilly and J. M. Gracia-Bondia, "The Wigner transformation is of finite order", *Journal of Mathematical Physics* **28**, 2390 (1987), doi:10.1063/1.527776

Franz Luef, Norwegian University of Science and Technology

τ-quantization and τ-Cohen classes of Feichtinger operators

Abstract. We investigate the τ-quantizations and Cohen's class distributions of a suitable class of trace-class operators, called Feichtinger's operators, and show that it is a convenient substitute for the class of Schwartz operators. Many well-known concepts and results for functions in time-frequency analysis have an operator-analog in our setting, e.g. that Cohen's classes are convolutions of Wigner functions with distributions or characterization of the class of Schwartz operators as an intersection of weighted variants of the class of Feichtinger operators.

Henry McNulty, NTNU

A Short Time Fourier Transform and Coorbit Spaces of Operators

Abstract. We introduce an operator valued Short-Time Fourier Transform for certain classes of operators with operator windows. This operator STFT acts in an analogous way to the Short-Time Fourier Transform for functions, in particular giving rise to a family of vector-valued reproducing kernel Banach spaces, the so called coorbit spaces, as spaces of operators. As a result of this structure the operators generating equivalent norms on the function modulation spaces are fully classified. These operator spaces enjoy the same atomic decomposition properties as the function spaces, and we use this to give a characterisation of the spaces using localisation operators.

Helge J. Samuelsen, NTNU

Fourier Restriction in Quantum Harmonic Analysis with Applications to Quantization

Abstract. A famous problem in Harmonic Analysis is the so-called Fourier restriction problem: Given a Borel measure μ on $\mathbb{R}^d$, Fourier restriction asks when restricting the Fourier transform to the support of μ defines a bounded operator from $L^p(\mathbb{R}^d)$ to $L^q(\mu)$. In this talk we will investigate Fourier restriction in the setting of Quantum Harmonic Analysis. By replacing $L^p(\mathbb{R}^d)$ by the Schatten class $\mathcal{S}^p$, we ask when restricting the Fourier-Wigner transform defines a bounded operator from $\mathcal{S}^p$ to $L^q(\mu)$. These restriction estimates gives rise to Schatten-class estimates for the Weyl quantization when considering the Fourier transform of measures as the Weyl symbols. This is based on joint work with Franz Luef.

Irina Shafkulovska, Univerisity of Vienna

Metaplectic action on modulation spaces

Abstract. In this talk we study the mapping properties of metaplectic operators $\widehat{S} \in \mathrm{Mp}(2d, \mathbb{R})$ on modulation spaces of the type $\mathrm{M}^{p,q}_m(\mathbb{R}^d)$. Our main result is a full characterization of the pairs of metaplectic operators and modulation spaces $(\widehat{S}, \mathrm{M}^{p,q}(\mathbb{R}^d))$ for which the operator $\widehat{S} : \mathrm{M}^{p,q}(\mathbb{R}^d) \to \mathrm{M}^{p,q}(\mathbb{R}^d)$ is *(i)* well-defined, *(ii)* bounded. It turns out that these two properties are equivalent, and they entail that $\widehat{S}$ is a Banach space automorphism.

Under mild conditions on the weight function, we provide a simple test to determine whether the well-definedness (boundedness) of the metaplectic operator $\widehat{S} : \mathrm{M}^{p,q}(\mathbb{R}^d) \to \mathrm{M}^{p,q}(\mathbb{R}^d)$ transfers to $\widehat{S} : \mathrm{M}^{p,q}_m(\mathbb{R}^d) \to \mathrm{M}^{p,q}_m(\mathbb{R}^d)$.

The talk is based on a joint work with Hartmut Führ (RWTH Aachen University).

Michael Speckbacher, University of Vienna

Eigenvalue estimates for Fourier concentration operators on two domains

Abstract. We derive eigenvalue estimates for concentration operators associated with the discrete Fourier transform and two concentration domains satisfying certain regularity conditions. These conditions are met, for example, when the discrete domain, contained in a lattice, is obtained by discretization of a suitably regular domain in the Euclidean space. As a limit, we obtain eigenvalue estimates for Fourier concentration operators associated with two suitably regular domains in the Euclidean space.

Our results cover for the first time non-convex and non-symmetric concentration models in the spatial and frequency domains, as demanded by numerous applications that exploit the expected approximate low dimensionality of the modeled phenomena. Our proofs build on Israel's work on one dimensional intervals and combines it with the use of redundant wave-packet expansions and a dyadic decomposition argument to obtain Schatten norm estimates for Hankel operators.

5.15. Random Matrix Theory and Mathematical Physics.

Monday 2:30-4:30 in Porthania, P674 chair: Christian Webb

2:30-3:25 Peter Miller
Extreme Superposition: Models for Large-Amplitude Rogue Waves

3:30-4:25 Shinji Koshida
From multiple SLE/GFF coupling to dynamical random matrices

Tuesday 2:30-4:00 in Porthania, P674 chair: Roozbeh Gharakhloo

2:30-3:25 Estelle Basor
Asymptotics of determinants of block Toeplitz matrices with singular symbols

3:30-3:55 Evangelos Nikitopoulos
Noncommutative Stochastic Calculus

Tuesday 4:30-8:00 in Porthania, P674

4:30-5:25 Tom Claeys
Solving an integrable PDE in terms of deformed sine kernel determinants

5:30-6:15 Giorgio Young
Ballistic transport for limit-periodic Schrödinger operators in one dimension

6:30-7:15 Scott Mason
Two-periodic weighted dominos and the sine-Gordon field at the free fermion point

7:15-8:00 Klara Courteaut
The planar Coulomb gas on a Jordan curve

Thursday 2:30-4:00 in Porthania, P674

2:30-3:25 Thomas Bothner
Bulk spacings in non-Hermitian matrix models

3:30-3:55 Baran Bayraktaroglu
Bosonization of the critical Ising model

Thursday 4:30-6:00 in Porthania, P674

4:30-5:25 Kevin Schnelli
Quantitative Tracy-Widom law for generalized Wigner random matrices

5:30-5:55 Jenia Rousseva
Reduction of Coherent States

Friday 11:30-1:00 in Porthania, P674

11:30-12:25 Christophe Charlier
Two new point processes on the unit circle

12:30-12:55 Nedialko Bradinoff
Benford's Law and the Circular β-Ensembles

Friday 2:30-4:00 in Porthania, P674

2:30-3:10 Ahmad Barhoumi UPDATE
Painlevé-III $D_6 \to D_8$ Confluence via Bäcklund Transformations.

3:15-3:55 Joona Oikarinen UPDATE
Gaussian multiplicative chaos and quantum field theories with exponential interactions

Friday 4:30-6:00 in Porthania, P674

4:30-5:25 Zhigang Bao
Phase Transition of Eigenvectors for Spiked Random Matrices

5:30-5:55 Jana Reker UPDATE
Functional Central Limit Theorems for Wigner Matrices

5.15.1. *Abstracts.*

Zhigang Bao, Hong Kong University of Science and Technology

Phase Transition of Eigenvectors for Spiked Random Matrices

Abstract. In this talk, we will first provide an overview of recent findings concerning eigenvectors of random matrices under fixed-rank deformation. We will then shift our focus towards analyzing the limit distribution of the leading eigenvectors of deformed models in the critical regime of the Baik-Ben Arous-Peche (BBP) phase transition. The distribution is determined by a determinantal point process with an extended Airy kernel. This result can be seen as an eigenvector counterpart to the BBP eigenvalue phase transition. The talk will be based on a joint work with Dong Wang.

Ahmad Barhoumi, University of Michigan, USA

Painlevé-III $D_6 \to D_8$ Confluence via Bäcklund Transformations.

Abstract. Painlevé equations are six nonlinear second order ODEs whose solutions are thought to be the special functions of the 21st century. They are characterized by the fact that their solutions have branch points which are independent of initial conditions (so-called non-movable singularities). Among the six equations, Painlevé III (P-III) is the simplest equation with two non-movable singularities. While its generic solutions are transcendental, it is known to possess families of special–function solutions: solutions written in terms of elementary and/or classical special functions. These are often generated by iterating Bäcklund transformations; transformations that map a solution of P-III to another solution of P-III with possibly modified parameters. This produces a family of solutions indexed by a natural number n, which counts the number of iterations. It is in this way that many families of solutions of interest (e.g. rational solutions and Bessel-function solutions) are constructed.

In this talk, I will describe an approach to studying the large-n behavior of solutions to P-III constructed in this way with a generic choice of seed function. One of the main findings is that the limiting function (under appropriate scaling) solves the "double-degenerate" version of P-III, known as P-III(D_8). This is joint work with Oleg Lisovyy, Peter Miller, and Andrei Prokhorov.

Estelle Basor, American Institute of Mathematics

Asymptotics of determinants of block Toeplitz matrices with singular symbols

Abstract. The asymptotics of determinants of finite Toeplitz matrices with scalar symbols (Fisher-Hartwig symbols) are now fairly well understood. This talk will describe some results for the case of block symbols with piecewise continuous symbols. The techniques are similar to the scalar case, but require some necessary modifications. This is joint work with Torsten Ehrhardt and Jani Virtanen.

Baran Bayraktaroglu, University of Helsinki (Finland)

Bosonization of the critical Ising model

Abstract. The Ising model is one of the simplest statistical models to define but also has a very rich structure. Previously, physicists predicted various key properties about the correlations of the critical Ising model in the scaling limit, and bosonization is one of those properties. We prove that the bosonization identity, i.e. the equality between a certain correlation involving the compactified Gaussian free field and the square of a correlation in the Ising model, is satisfied on planar domains. This talk is based on joint work with Konstantin Izyurov, Tuomas Virtanen, and Christian Webb.

Thomas Bothner, University of Bristol

Bulk spacings in non-Hermitian matrix models

Abstract. Random matrix eigenvalue spacings tend to show up in problems not directly related to random matrices: for instance, bumper to bumper distances of parked cars in a number of roads in central London are well represented by the so-called eigenvalue bulk spacing distribution of a suitable Hermitian matrix model. In this talk we will first survey several occurrences of these Hermitian spacing distributions and afterwards try to generalise them to non-Hermitian models. As it turns out, the theory of integrable systems, especially Painlevé special function theory, plays a crucial role in this field. Based on arXiv:2212.00525, joint work with Alex Little (Bristol).

Nedialko Bradinoff, KTH

Benford's Law and the Circular β-Ensembles

Abstract. Benford's law is a remarkable phenomenon that governs the digital expansion of both deterministic and random real quantities, coming from a broad variety of contexts, including terms of geometric progressions, stock prises, population numbers... Roughly, the leading digits of such quantities are likely to be heavily skewed towards smaller values. The focus of this talk is to describe a connection between Benford's law and another beautiful object, the Circular β-Ensemble. The CβE(N) is classical in random matrix theory and has been studied extensively to exhibit universal properties. A natural object that is studied in the context of the CβE(N) is its characteristic polynomial. In this talk I will describe how its absolute value obeys Benford's law in a strong sense as N tends to infinity. The talk is based on joint work with Maurice Duits.

Christophe Charlier, Lund University

Two new point processes on the unit circle

Abstract. In this talk I will present results on two new point processes on the unit circle. At first sight, these point processes look similar to the well-studied circular β ensemble, but in fact there are very different: instead of featuring the 2d Coulomb interactions $|e^{i\theta_j} - e^{i\theta_k}|^\beta$, they are characterized by the non-local interactions $|e^{i\theta_j} - e^{-i\theta_k}|^\beta$ and $|e^{i\theta_j} + e^{i\theta_k}|^\beta$, respectively. These point processes can be seen as β-ensembles of a new kind with a lot more randomness than in the classical β-ensembles. I will mostly be discussing the smooth linear statistics.

Tom Claeys, UCLouvain

Solving an integrable PDE in terms of deformed sine kernel determinants

Abstract. I will describe finite temperature deformations of the sine kernel Fredholm determinant, and I will explain how they allow to explicitly solve an integrable PDE in a remarkably simple way in terms of the Abel transform of the initial data. The talk will be based on work in progress with Sofia Tarricone.

Klara Courteaut, KTH

The planar Coulomb gas on a Jordan curve

Abstract. The eigenvalues of a Haar distributed unitary matrix (CUE) have the physical interpretation of a system of particles subject to a logarithmic pair interaction, restricted to lie on the unit circle, and at inverse temperature 2. In his thesis, Kurt Johansson generalized this model by replacing the unit circle with a sufficiently regular Jordan curve. He obtained the asymptotics of the Laplace transform of linear statistics. In this talk, I will present a recent paper with Johansson, in which we adapt the proof to any positive temperature and obtain the asymptotic partition function.

Shinji Koshida, Aalto University

From multiple SLE/GFF coupling to dynamical random matrices

Abstract. I will discuss coupling between two probabilistic models in two dimensions; multiple Schramm–Loewner evolution (SLE) and Gaussian free field (GFF). SLE is a random curve evolving in a two dimensional domain that is originally introduced for the purpose of describing a random interface in a critical lattice system. As its name suggests, multiple SLE is an extension of SLE that provides multiple random curves. GFF is another probabilistic model that gives one mathematical formulation of the free Bose field from quantum field theory. It has been known that there is a coupling between SLE and GFF. The focus of this talk is on extending it to the case of multiple SLE. Particularly, I take the version of multiple SLE that generates multiple curves simultaneously, driven by a many particle stochastic process. We will see that, under a certain setup, coupling happens if and only if the driving process is given by Dyson's Brownian motions, thereby connecting the three of multiple SLE, GFF, and dynamical random matrices. This talk is based on joint work with Makoto Katori (Chuo University).

Scott Mason, KTH

Two-periodic weighted dominos and the sine-Gordon field at the free fermion point

Abstract. The connection between the Gaussian free field and periodically weighted dimer models is well-known. Under a certain pullback, the Gaussian free field describes the fluctuations of the height function around their limit shape in the rough phase of the dimer model. We will present recent results on fluctuations of the height field of a dimer model at a rough-smooth transition. The continuum limit of the height field varies between the Gaussian free field and white noise. Renormalisation group heuristics suggest that the continuum field at this transition was a massive Gaussian free field, however this is not the case. The transition is described by a bosonized massive free fermion field. We show that the two-point correlations of the height field converge to those of the sine-Gordon field at the free fermion point.

Peter Miller, University of Michigan

Extreme Superposition: Models for Large-Amplitude Rogue Waves

Abstract. Rogue waves or freak waves are spatially-localized disturbances of a background field that are also temporally localized. In the setting of the focusing nonlinear Schrödinger equation, which is a universal model for the complex amplitude of a wave packet in a general one-dimensional weakly-nonlinear and strongly-dispersive setting that includes water waves and nonlinear optics as special cases, a special exact solution exhibiting rogue-wave character was found by D. H. Peregrine in 1983. Since then, with the help of complete integrability, Peregrine's solution has been generalized to a family of solutions of arbitrary "order" where more parameters appear in the solution as the order increases. These parameters can be adjusted to maximize the amplitude of the rogue wave for a given order. This talk will describe several recent results concerning such maximal-amplitude rogue wave solutions in the limit that the order increases without bound. For instance, it turns out that there is a limiting structure in a suitable near-field scaling of the peak of the rogue wave; this structure is a novel exact solution of the focusing nonlinear Schrödinger equation — the "rogue wave of infinite order" — that is also connected with the hierarchy of the third Painlevé equation. This is joint work with Deniz Bilman and Liming Ling.

Evangelos Nikitopoulos, University of California, San Diego

Noncommutative Stochastic Calculus

Abstract. Free probability – or more generally noncommutative probability – is a kind of mathematics that was invented to describe the large-N limits of $N \times N$ random matrix ensembles. When one considers an ensemble $X^{(N)} = \big(X_t^{(N)}\big)_{t\geq 0}$ of $N \times N$ matrix stochastic processes, one needs a theory of "free (or noncommutative) stochastic analysis" to describe the large-N limit of $X^{(N)}$. We describe a flexible general theory of noncommutative stochastic calculus that is useful for describing the large-N limits of solutions to $N \times N$ matrix stochastic differential equations. Our theory generalizes the theories of Biane-Speicher for free Brownian motion (the large-N limit of $N \times N$ Hermitian Brownian motion) and Donati-Martin for q-Brownian motion. Moreover, it unifies these theories with some aspects of the classical theory of stochastic calculus. This is joint work with D. Jekel and T. Kemp.

Joona Oikarinen, Universität Wien, Austria

Gaussian multiplicative chaos and quantum field theories with exponential interactions

Abstract. I will explain what is Gaussian multiplicative chaos (GMC) and how it appears in the path integral formulation of of two-dimensional Euclidean quantum field theories (EQFT) with exponential interactions. We will focus on the Liouville EQFT and its perturbations, which include the Sinh–Gordon model. These perturbations break the scaling symmetry of the Liouville model and lead to interesting behavior in the infinite volume limit. I will show how the asymptotic behavior of the partition function in the infinite volume limit ties into some open questions in the theory of GMC. Ongoing work with Nikolay Barashkov (University of Helsinki).

Jana Reker, Institute of Science and Technology Austria

Functional Central Limit Theorems for Wigner Matrices

Abstract. Consider the random variable $X := \mathrm{Tr}(f_1(W)A_1 \dots f_k(W)A_k)$ where W is a Hermitian Wigner matrix, $k \in \mathbb{N}$, and we choose regular functions $f_1, \dots, f_k$ as well as bounded deterministic matrices $A_1, \dots, A_k$. In this talk, we study the fluctuations around $\mathbb{E}X$ and give a functional central limit theorem on macroscopic and mesoscopic scales. Analyzing the underlying combinatorics further leads to explicit formulas for the variance of X as well as the covariance of X and $Y := \mathrm{Tr}(f_{k+1}(W)A_{k+1} \dots f_{k+\ell}(W)A_{k+\ell})$ of similar build. The results obtained match the structure of formulas in second-order free probability which were previously only available for f_j being polynomials.

Jenia Rousseva, University of Michigan

Reduction of Coherent States

Abstract. We demonstrate how to apply a quantum version of dimensional reduction to Gaussian coherent states in Bargmann space to obtain squeezed states on complex projective spaces. We also discuss semiclassical norm estimates and the Schrödinger dynamics of the reduced states.

Kevin Schnelli, KTH

Quantitative Tracy-Widom law for generalized Wigner random matrices

Abstract. We will discuss a quantitative Tracy-Widom law for the largest eigenvalue of generalized Wigner random matrices. More precisely, we will prove that the fluctuations of the largest eigenvalue of a generalized Wigner matrix of size N converge to its Tracy-Widom limit at a rate nearly $N^{-1/3}$, as N tends to infinity. Our result follows from a quantitative Green function comparison theorem, originally introduced by Erdős, Yau and Yin to prove the edge universality, on a finer spectral parameter scale with improved error estimates and without sec-

ond moment matching. The proof relies on the continuous Green function interpolation with the Gaussian invariant ensembles. Precise estimates on leading contributions from the second, third and fourth order moments of the matrix entries are obtained using iterative cumulant expansions along with uniform convergence estimates for correlation kernels of the Gaussian invariant ensembles. This is joint work with Yuanyuan Xu (IST Austria).

Giorgio Young, University of Michigan

Ballistic Transport for Limit-periodic Schrödinger Operators in One Dimension

Abstract. In this talk, I will discuss work on the transport properties of the class of limit-periodic continuum Schrödinger operators whose potentials are approximated exponentially quickly by a sequence of periodic functions. For such an operator H, and $X_H(t)$ the Heisenberg evolution of the position operator, we show the limit of $\frac{1}{t}X_H(t)\psi$ as $t \to \infty$ exists and is nonzero for $\psi \neq 0$ belonging to a dense subspace of initial states which are sufficiently regular and of suitably rapid decay. This is viewed as a particularly strong form of ballistic transport, and this is the first time it has been proven in a continuum almost periodic non-periodic setting. In particular, this statement implies that for the initial states considered, the second moment grows quadratically in time.

5.16. Spectral Inequalities and Null-Controllability.

Tuesday 2:30-4:00 in Metsätalo, Hall 6 chair: Michela Egidi

2:30-2:55	Jérôme Le Rousseau *Spectral inequality for the Laplace and bi-Laplace operator under general boundary conditions*
3:00-3:25	Philippe Jaming *Ingham and McGeehe-Pigno-Smith-Nazarov's inequalities in control theory*
3:30-3:55	Armand Koenig *Geometric control conditions for the fractional heat equation*

Tuesday 4:30-6:00 in Metsätalo, Hall 6 chair: Matthias Täufer

4:30-4:55	Christian Seifert *α-controllability, weak observability estimates, and exponential stabilizability for systems in Banach spaces*
5:00-5:25	Karsten Kruse *Final state observability estimates and cost-uniform approximate null-controllability for bi-continuous semigroups*
5:30-5:55	Ritika Singhal *Paley inequality and $L^p - L^q$ multipliers for the Weyl transform*

Thursday 2:30-4:00 in Metsätalo, Hall 6 chair: Jérôme Le Rousseau

2:30-2:55	Jone Apraiz *Controllability of parabolic equations on networks*
3:00-3:25	Jonathan Rohleder *Quantum trees which maximize higher eigenvalues are unbalanced*
3:30-3:55	André Laestadius *The unique-continuation property in density-functional theory*

Thursday 4:30-6:00 in Metsätalo, Hall 6 chair: Philippe Jaming

4:30-4:55	Mishko Mitkovski *Uncertainty, stabilization, and control*
5:00-5:25	Arick Shao *Control of parabolic equations with inverse square infinite potential wells*
5:30-5:55	Ivan Moyano *Spectral inequalities for Laplace-Beltrami and Schrödinger operators*

5.16.1. *Abstracts.*

Jone Apraiz, University of the Basque Country

Controllability of parabolic equations on networks

Abstract. During the last decades, the use of networks has been very helpful and effective in the study of pipes, neural systems, the flow of traffic on roads, the global economy or the human circulatory system. With this talk I would like to show you a contribution to this area from the fields of control theory and inverse problems.

In this talk we will consider the propagation of diffusion on a network with loops. Our objective is to control these networks by acting on the system that models the process of the heat diffusion in them, extending in this way the results of [2] and [3] to networks with loops.

The observability of the entire network will be achieved under certain hypotheses about the position of the observation domain. This will be done using a Carleman inequality. Then, we will use that observability to prove the null controllability of the network and to obtain the Lipschitz stability for an inverse problem consisting of retrieving a stationary potential in the heat equation from measurements on the observation domain.

If there is time for more, we will see a brief introduction about how the study of the spectrum of parabolic operators can give us more information about the controllability.

This work has been done in collaboration with Jon Asier Bárcena-Petisco, from the University of the Basque Country. The talk is based on the work [1].

[1] J. Apraiz and J. A. Bárcena-Petisco, *Observability and control of parabolic equations in networks with loops*, J. Evol. Equ. **23**, 37 (2023).

[2] J. A. Bárcena-Petisco, M. Cavalcante, G. M. Coclite, N. Nitti and E. Zuazua, *Control of hyperbolic and parabolic equations on networks and singular limits*, hal-03233211 (2021).

[3] L. Ignat, A. F. Pazoto and L. Rosier, *Inverse problem for the heat equation and the Schrödinger equation on a tree*, Inverse Prob. **28**, 1, (2011), 015011.

Philippe Jaming, University of Bordeaux

Ingham and McGeehe-Pigno-Smith-Nazarov's inequalities in control theory

Abstract. Ingham's Inequality states that, if $(\lambda_k)_{k\geq 1}$ is an increasing sequence of real numbers with $|\lambda_k - \lambda_\ell| \geq S$ for $k \neq \ell \geq k_0$ and $ST > 1$ and $(a_k)_k \in \ell^2$

$$\frac{1}{T}\int_{-T/2}^{T/2}\left|\sum_{k=1}^{\infty} a_k e^{2i\pi\lambda_k t}\right|^2 \mathrm{d}t \simeq \sum_{k=1}^{\infty}|a_k|^2.$$

Using this inequality, with V. Komornik, we showed that the solutions the periodic Schrödinger equation on $\mathbb{R}$ can be controlled from one moving poving point $u(t, x_0 + at)$, in arbitrary small time, provided the speed a is not an integer.

The main question we address here is the L^1-counterpart of these results. The L^1 analogue of Ingham's inequality is more complicated and reads

$$\sum_{k=1}^{\infty}\frac{|a_k|}{k} \lesssim \frac{1}{T}\int_{-T/2}^{T/2}\left|\sum_{k=1}^{\infty} a_k e^{2i\pi\lambda_k t}\right| \mathrm{d}t \lesssim \sum_{k=1}^{\infty}|a_k|.$$

and further requires that $|\lambda_k - \lambda_\ell| \geq S$ for $k \neq \ell \geq 1$.

This is the celebrated proof of the Littlewood conjecture by McGeehe-Pigno-Smith when the frequencies λ_k are integers and by Nazarov in the general case. We will present an improvement of the proof recently obtained with S. Chaba and K. Kellay as well as some open questions.

Armand Koenig, University of Toulouse III - Paul Sabatier

Geometric control conditions for the fractional heat equation

Abstract. We know that the null-controllability of the heat equation on a subdomain ω is equivalent to the thickness of ω (a geometrical notion aksing that ω is, in some sense, spread out in the whole space). In fact, if ω is thick, the fractional heat $(\partial_t + (-\Delta)^s)f = \mathbf{1}_\omega u$ is null-controllable as long as $s > 1/2$.

What about the case $s \leq 1/2$? The good geometrical notion on ω to study the null-controllability is stronger than the thickness, and we will discuss some sufficient condition that ensures the null-controllability, as well as some necessary condition.

Karsten Kruse, Technical University Hamburg

Final state observability estimates and cost-uniform approximate null-controllability for bi-continuous semigroups

Abstract. In this talk we consider final state observability estimates for bi-continuous semigroups on Banach spaces, i.e. for every initial value, estimating the state at a final time $T > 0$ by taking into account the orbit of the initial value under the semigroup for $t \in [0, T]$, measured in a suitable norm. We state a sufficient criterion based on an uncertainty relation and a dissipation estimate and provide two examples of bi-continuous semigroups which share a final state observability estimate, namely the Gauß–Weierstraß semigroup and the Ornstein–Uhlenbeck semigroup on the space of bounded continuous functions on Rd. Moreover, if time permits, we generalise the duality between cost- uniform approximate null-controllability and final state observability estimates to the setting of locally convex spaces for the case of bounded continuous control functions, which seem to be new even for the Banach spaces case. This contribution is co-authored by Christian Seifert and based on [1].

[1] Kruse, K. and Seifert, C., Final state observability estimates and cost-uniform approximate null-controllability for bi-continuous semigroups, *Semigroup Forum*, **106**(2): 421–443 (2023).

André Laestadius, Oslo Metropolitan University

The unique-continuation property in density-functional theory

Abstract. The unique-continuation property from sets of positive measure for the many-body magnetic Schrödinger equation holds significance in density-functional theory for magnetic fields. Density-functional theory uses the one-body particle density to describe correlated electrons and is one of the most commonly used computational methods in the field of quantum chemistry. In this talk, this property is discussed, particularly in connection with the Hohenberg-Kohn theorem, as well as challenges within this theoretical framework.

Jérôme Le Rousseau, Sorbonne Paris Nord University

Spectral inequality for the Laplace and bi-Laplace operator under general boundary conditions

Abstract. The spectral inequality of G. Lebeau and co-authors, D. Jerison, L. Robbiano, and E. Zuazua, were derived for second-order elliptic operators along with homogeneous Dirichlet boundary conditions. Neumann boundary conditions were also considered. We present such estimations under very general boundary conditions yielding similar applications for null-controllability. We also present extensions and open questions for the bi-Laplace operator. This is based on joint works with G. Lebeau, L. Robbiano, and E. Zongo.

Mishko Mitkovski, Clemson University

Uncertainty, stabilization, and control

Abstract. I will present several new manifestations of the harmonic analysis uncertainty principle and the corresponding consequences for control and stabilization problems for some classes of PDE's.

Ivan Moyano, University of Nice

Spectral inequalities for Laplace-Beltrami and Schrödinger operators

Abstract. In this talk we review some classical and recent results relating the uncertainty principles for the Laplacian with the controllability and stabilisation of some linear PDEs. The uncertainty principles for the Fourier transforms state that a square integrable function cannot be both localised in frequency and space without being zero, and this can be further quantified resulting in unique continuation inequalities in the phase spaces. Applying these ideas to the

spectrum of the Laplacian on a compact Riemannian manifold, Lebeau and Robbiano obtained their celebrated result on the exact controllability of the heat equation in arbitrarily small time. The relevant quantitative uncertainty principles known as spectral inequalities in the literature can be adapted to a number of different operators, including the Laplace-Beltami operator associated to C^1-metrics or some Schödinger operators with long-range potentials, as we have shown in recent results in collaboration with Gilles Lebeau (Nice) and Nicolas Burq (Orsay), with a significant relaxation on the localisation in space. As a consequence, we obtain a number of corollaries on the decay rate of damped waves with rough dampings, the simultaneous controllability of heat equations with different boundary conditions and the controllability of the heat equation with rough controls.

Jonathan Rohleder, Stockholm University

Quantum trees which maximize higher eigenvalues are unbalanced

Abstract. The isoperimetric problem of maximizing all eigenvalues of the Laplacian on a metric tree graph within the class of trees of a given average edge length is studied. While it has been known from earlier work that the lowest positive eigenvalue is maximized by equilateral star graphs, we show that the trees maximizing the higher eigenvalues are less balanced in their shape—an observation which is also known from numerical results on the optimization of higher eigenvalues of Laplacians on Euclidean domains.

Christian Seifert, Technical University Hamburg

α-controllability, weak observability estimates, and exponential stabilizability for systems in Banach spaces

Abstract. For given Banach spaces X, U we consider abstract control systems of the form $\dot{x}(t) = -Ax(t) + Bu(t)$ for $t \in (0, T]$ with $x(0) = x_0 \in X$, where $-A$ is the generator of a strongly continuous semigroup on X, $B\colon U \to X$ is a bounded linear operator and $T > 0$. For such systems and $\alpha \geq 0$, the question of α-controllability arises, i.e. whether for all initial conditions $x_0 \in X$ there exists a control function $u\colon [0, T] \to U$ such that the norm of $x(T)$ is at most α times the norm of x_0. I turns out that a version of α-controllability is equivalent to a weak obervability estimate for the associated dual system. We will explain the corresponding results and how they lead to stabilization of the system. Moreover, we will provide some examples.

The talk is based on joint work together with Michela Egidi, Dennis Gallaun, Peter Stollmann and Martin Tautenhahn.

Arick Shao, Queen Mary University of London

Control of parabolic equations with inverse square infinite potential wells

Abstract. We consider heat operators on a bounded convex domain, with a critically singular potential diverging as the inverse square of the distance to the boundary of the domain. We establish a general boundary controllability result for such operators in all spatial dimensions, in particular providing the first such result in more than one spatial dimension. The key step in the proof is a novel global Carleman estimate that captures both the relevant boundary asymptotics and the appropriate energy for this problem. The estimate is derived by combining two intermediate Carleman inequalities with distinct and carefully constructed weights involving non-smooth powers of the boundary distance. This is joint work with Alberto Enciso (ICMAT) and Bruno Vergara (Brown).

Ritika Singhal, Indian Institute of Technology Delhi, New Delhi, India

Paley inequality and $L^p - L^q$ multipliers for the Weyl transform

Abstract. The notion of Weyl multipliers was introduced by G. Mauceri. Under some regularity conditions, he found sufficient conditions for an operator $M \in \mathcal{B}(L^2(\mathbb{R}^n))$ to induce a bounded map C_M on $L^p(\mathbb{R}^n)$ defined for $f \in L^p(\mathbb{R}^n)$ as $W(C_M f) = MW(f)$ where $W(f)$ denotes the Weyl transform of f. We have studied the L^p-L^q boundedness of the Weyl multipliers for an abelian group G in terms of the decay property of the spectral information associated with the operator which does not require any regularity conditions. We aim at proving an analogue of Hörmander's multiplier theorem for any abelian group G. Also, we establish the Paley inequality, which will be used to prove the multiplier theorem and extends the continuity of the Weyl transform to Lorentz spaces. As an application to the Paley inequality, we also prove the Hardy-Littlewood theorem.

5.17. **Spectral Theory and Partial Differential Operators.**

Monday 2:30-4:30 in Language Center, Room 205 chair: Oleksandr Sakhnovich

2:30-2:55	Seppo Hassi *Boundary pairs for closed forms and generalized boundary triplets for nonnegative operators*
3:00-3:25	Selim Sukhtaiev *Resolvent expansions for self-adjoint operators via boundary triplets*
3:30-3:55	Yuri Latushkin *Families of boundary triplets, Hadamard formulas and Maslov index*
4:00-4:25	Bernhelm Booß-Bavnbek *Boundary Reduction - UCP - Calderón Projections - Spectral Estimates - Symplectic Geometry*

Tuesday 2:30-4:00 in Language Center, Room 205 chair: Ian Wood

2:30-2:55	Konstantin Pankrashkin *Unusual essential spectrum for Dirac operators with shell interactions*
3:00-3:25	Vladimir Lotoreichik *Spectral asymptotics of the Dirac operator on a thin shell*
3:30-3:55	Alexander Sakhnovich *Dirac systems with locally square-integrable potentials: direct and inverse problems for spectral functions*

Tuesday 4:30-6:00 in Language Center, Room 205 chair: Konstantin Pankrashkin

4:30-4:55	Aleksey Kostenko *A Glazman-Povzner-Wienholtz Theorem on graphs*
5:00-5:25	Ian Wood *Spectrum of the Maxwell Equations for a Flat Interface between Homogeneous Dispersive Media*
5:30-5:55	Marco Marletta *Essential spectra and spectral pollution for inhomogeneous, anisotropic dissipative Maxwell and Drude-Lorentz systems*

Thursday 2:30-4:00 in Language Center, Room 205 chair: Benjamin Eichinger

2:30-2:55	Chaofeng Zhu *Global Mountain Pass Points and Applications to Minimal Period Problems in Hamiltonian Systems*
3:00-3:25	Alim Sukhtayev *Renormalized oscillation theory for Hamiltonian pencils*
3:30-3:55	Jakob Reiffenstein *Order and density of eigenvalues of a canonical system*

Thursday 4:30-6:00 in Language Center, Room 205 chair: Alim Sukhtayev

4:30-4:55	Benjamin Eichinger *Universality limits with regularly varying scaling*
5:00-5:25	Rainer Hempel *On Schrödinger Operators in the Plane with Magnetic Fields of the Form* $\sin(r)$
5:30-5:55	Jonathan Rohleder *Laplacian eigenvalues and eigenfunctions: a non-standard variational principle*

Friday 11:30-1:00 in Language Center, Room 205 chair: Yuri Latushkin

11:30-11:55	Graham Cox
	Geometry and topology of spectral minimal partitions
12:00-12:25	Illia Karabash
	Impedance boundary conditions for wave equations and m-boundary tuples
12:30-12:55	Henk De Snoo
	Semibounded forms, representing maps and semibounded selfadjoint extensions

5.17.1. *Abstracts.*

Bernhelm Booß-Bavnbek, Roskilde University, Denmark

Boundary Reduction - UCP - Calderón Projections - Spectral Estimates - Symplectic Geometry

Abstract. A key result in spectral geometry is the calculation of the spectral flow of a curve of elliptic boundary value problems over a smooth compact manifold with boundary (belonging to the realm of functional analysis) by the Maslov index of related curves of Lagrangian subspaces over the boundary (belonging to the realm of symplectic geometry). I shall explain obstructions of a general validity of this Spectral Flow Theorem and how to overcome these obstructions by natural assumptions regarding the inner weak unique continuation property (UCP) that imply the continuity of families of Calderón projections. I shall give details of a simple proof of that crucial continuity where the widely spread ingredients listed in the title of my talk are interwoven. This is joint work with Chaofeng Zhu.

Graham Cox, Memorial University

Geometry and topology of spectral minimal partitions

Abstract. A minimal partition is a decomposition of a manifold into disjoint sets that minimizes a certain energy functional. In the bipartite case minimal partitions are closely related to eigenfunctions of the Laplacian, but in the non-bipartite case they are difficult to classify, even for simple domains like the square or the circle.

I will present new results that say a partition that minimizes energy locally is in fact a global minimum (in the bipartite case) and a minimum within a certain topological class of partitions in the non-bipartite case. I will also explain how to construct energy-decreasing deformations of a non-minimal partition, giving insight into the geometric structure of the true minimum. This is joint work with Gregory Berkolaiko, Yaiza Canzani, Peter Kuchment and Jeremy Marzuola.

Henk De Snoo, University of Groningen

Semibounded forms, representing maps and semibounded selfadjoint extensions

Abstract. By means of the notion of a representing map the representation theorems for semibounded sesquilinear forms in a Hilbert space can be formulated even when they are not closable. If the form is generated by a symmetric operator or relation the representing map of the form offers a way to factor the Friedrichs extension and the corresponding Krein (type) extensions. This is joint work with Seppo Hassi (Vaasa)

Benjamin Eichinger, TU Wien

Universality limits with regularly varying scaling

Abstract. In this talk I present an equivalence statement for universality limits for Christoffel-Darboux kernels, $K_n(z,w)$, with regularly varying scaling which is formulated purely in terms of the spectral measure μ. For $L > 0$, let $K_L(z,w)$ be obtained from $K_n(z,w)$ by linear interpolation. We show that existence of the limit

$$\lim_{L\to\infty} \frac{K_L\left(\xi_0 + \frac{z}{g(K_L(\xi_0,\xi_0))}, \xi_0 + \frac{w}{g(K_L(\xi_0,\xi_0))}\right)}{K_L(\xi_0,\xi_0)}$$

for some regularly varying function g is equivalent to existence of a regularly varying function h such that the following limits for the one-sided distribution functions exist

$$\lim_{\epsilon\to 0}\frac{\mu((\xi_0-\epsilon,\xi_0))}{h(\epsilon)},\quad \lim_{\epsilon\to 0}\frac{\mu([\xi_0,\xi_0+\epsilon))}{h(\epsilon)}$$

and are not both zero. This theorem naturally generalizes all previous results on bulk universality and hard edge universality limits with fixed measure. In particular, bulk universality and hard edge universality correspond to completely symmetric, respectively totally asymmetric behavior of the one-sided distribution functions. The limit kernels are presented in terms of confluent hypergeometric functions.

This talk is based on a joint work in progress with Milivoje Lukić and Harald Woracek.

Seppo Hassi, University of Vaasa

Boundary pairs for closed forms and generalized boundary triplets for nonnegative operators

Abstract. Let $\mathfrak{h}$ be a closed nonnegative form with dense domain in the Hilbert space $\mathfrak{H}$. Then $\operatorname{dom}\mathfrak{h}$ equipped with the inner product $(f,g)_1=\mathfrak{h}[f,g]+(f,g)$, $f,g\in\operatorname{dom}\mathfrak{h}$, is a Hilbert space denoted by $\mathfrak{H}_1=(\operatorname{dom}\mathfrak{h},\|\cdot\|_1)$. Let $\mathcal{H}$ be another Hilbert space and let $\widetilde{\Gamma}_0$ be a bounded (or closed) linear map from $\mathfrak{H}_1$ to $\mathcal{H}$ with dense range. Then $\ker\widetilde{\Gamma}_0$ is a closed subspace of $\mathfrak{H}_1$ and hence the form $\mathfrak{h}$ restricted to $\ker\widetilde{\Gamma}_0$ defines a closed form in $\mathfrak{H}$, which is denoted by $\mathfrak{h}_0$. By the first representation theorem for closed forms there are unique nonnegative selfadjoint relations H_0 and H_1 in the Hilbert space $\mathfrak{H}$ associated with the forms $\mathfrak{h}_0$ and $\mathfrak{h}$, respectively. Clearly, $H_1\leq H_0$ and by the second representation theorem $\operatorname{dom}H_1^{1/2}=\operatorname{dom}\mathfrak{h}$ and $\operatorname{dom}H_0^{1/2}=\operatorname{dom}\mathfrak{h}_0$. Moreover, H_1 is a densely defined operator, while H_0 is densely defined if and only if $\operatorname{dom}\mathfrak{h}_0=\ker\widetilde{\Gamma}_0$ is dense in $\mathfrak{H}$; this is assumed here for simplicity in which case H_0 and H_1 both are selfadjoint operators.

The pair $(\mathfrak{h},\widetilde{\Gamma}_0)$ is called a boundary pair associated with the form $\mathfrak{h}$. The notion of a boundary pair can be seen to emerge from the works of Kreĭn, Birman, and Višik. Boundary pairs for forms with some additional conditions have been introduced and studied e.g. by G. Grubb, V. E. Lyantse and O. G. Storozh, Yu. M. Arlinskii, A. Posilicano; the present general version appears in O. Post (2016).

In this talk, it is shown that by introducing the symmetric restriction $A=H_0\cap H_1$ the notion of boundary pairs for forms can be connected to the concept of unitary boundary triplets $(\mathcal{H},\Gamma_0,\Gamma_1)$ for the adjoint (maximal operator) A^*, where the combined mapping (Γ_0,Γ_1) is densely defined on (the graph of) A^* and unitary in the Kreĭn space sense. Different special types of boundary pairs can then be characterized with different classes of unitary boundary triplets introduced in our joint papers with Derkach, Malamud and de Snoo. For unitary boundary triplets we have proved a general version of Kreĭn-type resolvent formula, and this allows to carry out some spectral analysis for the underlying operators. All these notions appear naturally in the study of boundary value problems for ODEs and PDEs.

The talk is based on joint work with V. A. Derkach and M. M. Malamud.

Rainer Hempel, TU Braunschweig, Germany

On Schrödinger Operators in the Plane with Magnetic Fields of the Form $\sin(r)$

Abstract. We determine the essential spectrum of magnetic Schrödinger operators $H(\vec{A})=(-\mathrm{i}\nabla-\vec{A})^2$ in $L^2(\mathbf{R}^2)$ where $\vec{A}=\vec{A}(x,y)$ is a vector potential of class C^1 and the field $\mathcal{B}=\nabla\times\vec{A}$ is radially periodic, $\mathcal{B}(x,y)=\beta(r)$ for some periodic function β on the real line with zero mean. We use mild generalizations of calculations of Miller and Simon [MS], Hempel, Hinz and Kalf [HHK], Iwatsuka [I], Leinfelder [L], Erdös [E], and Barseghyan and Truc [BT]. We find that the essential spectrum of $H(\vec{A})$ has no gaps and coincides with the essential spectrum associated

with the field $\beta(x)$. Furthermore, if a certain auxilliary periodic operator on the real line has a spectral gap (a, b), then the pure point spectrum of $H(\vec{A})$ is dense in (a, b).

This is joint work with Marko Stautz, TU Braunschweig.

Illia Karabash, the University of Bonn

Impedance boundary conditions for wave equations and m-boundary tuples

Abstract. We characterize m-dissipativity of impedance-type boundary conditions with the use of a new type of a boundary tuple. We introduce m-boundary tuple as a generalization of a boundary triple in such a way that it fits boundary value spaces of the Maxwell system. However it is convenient also for other types of wave equations. Applications to several specific types of generalized impedance boundary conditions will be demonstrated. The talk is based on joint research with Matthias Eller.

Aleksey Kostenko, University of Ljubljana & University of Vienna

A Glazman-Povzner-Wienholtz Theorem on graphs

Abstract. The Glazman–Povzner–Wienholtz theorem states that the completeness of a manifold, when combined with the semiboundedness of the Schrödinger operator $-\Delta + q$ and suitable local regularity assumptions on q, guarantees its essential self-adjointness. Our aim is to extend this result to Schrödinger operators on graphs. We first obtain the corresponding theorem for Schrödinger operators on metric graphs, allowing in particular distributional potentials $q \in H^{-1}_{\mathrm{loc}}$. Moreover, we exploit recently discovered connections between Schrödinger operators on metric graphs and weighted graphs [1] in order to prove a discrete version of the Glazman-Povzner-Wienholtz theorem. A particular emphasis will be on the role of intrinsic metrics on weighted graphs. Based on joint work with M.Malamud and N.Nicolussi.

1. A. Kostenko and N. Nicolussi, *Laplacians on infinite graphs*, Memoirs Eur. Math. Soc. **3** (2023).

Yuri Latushkin, University of Missouri

Families of boundary triplets, Hadamard formulas and Maslov index

Abstract. In this work joint with Selim Sukhtaiev, we discuss asymptotic perturbation theory for varying self-adjoint extensions of symmetric operators obtained by using families of boundary traces, families of Lagrangian subspaces in boundary spaces, and families of bounded perturbations. Specifically, we derive a Riccati-type differential equation and the first and second order asymptotic expansions for resolvents of self-adjoint extensions determined by smooth one-parameter families of traces and Lagrangian planes. This asymptotic perturbation theory yields a symplectic version of the abstract Kato selection theorem and Hadamard-Rellich-type variational formula for slopes and second derivatives of multiple eigenvalue curves bifurcating from an eigenvalue of the unperturbed operator. The latter, in turn, gives a general infinitesimal version of the celebrated formula equating the spectral flow of a path of self-adjoint extensions and the Maslov index of the corresponding path of Lagrangian planes. The results are illustrated in the context of quantum graphs, periodic Kronig-Penney model, elliptic second order partial differential operators with Robin boundary conditions, etc.

Vladimir Lotoreichik, Czech Academy of Sciences

Spectral asymptotics of the Dirac operator on a thin shell

Abstract. In this talk, we will consider the Dirac operator with infinite mass boundary conditions on a tubular neighbourhood of a smooth compact hypersurface in $\mathbb{R}^n$ without boundary. We will discuss the asymptotic behaviour of the eigenvalues of this Dirac operator when the tubular neighbourhood shrinks to the hypersurface. It turns out that this asymptotic behaviour

is driven by a Schrödinger operator on the hypersurface involving electric and Yang-Mills potentials of geometric nature. The eigenvalues of the effective Schrödinger operator appear in the third term of the asymptotic expansion with respect to the thickness of the tubular neighbourhood. These results are obtained in collaboration with Thomas Ourmières-Bonafos.

Marco Marletta, Cardiff University

Essential spectra and spectral pollution for inhomogeneous, anisotropic dissipative Maxwell and Drude-Lorentz systems

Abstract. The methods required to study spectral pollution for dissipative Maxwell (and related) systems turn out to be closely connected to the methods required to study the essential spectrum for such systems. These are very different from the methods used for Schrödinger equations: in particular, the presence of dissipation - e.g. in the form of conductivity - means that the essential spectrum can be changed by changing the coefficients in the system on any arbitrarily small, non-empty open set. Our methods use instead a reduction of the system to a triangular block operator matrix, together with the concept of limiting essential spectrum developed by Sabine Bögli in 2015. We are able to show that any spectral pollution is confined either to the real axis or to a segment of the imaginary axis: any computed eigenvalue whose real and imaginary parts are both non-zero is genuine.

This talk describes joint work with various co-authors including Giovanni Alberti, Sabine Bögli, Francesco Ferraresso, Christiane Tretter, Ian Wood and the late Malcolm Brown.

Konstantin Pankrashkin, Carl von Ossietzky University of Oldenburg

Unusual essential spectrum for Dirac operators with shell interactions

Abstract. We consider three-dimensional Dirac operators with so-called critical shell interactions (δ-potentials) supported by compact smooth surfaces. It is known from many previous works that the operator shows a loss of regularity in its domain, but the influence of this effect on the spectral properties was not completely clear. We partially close this gap by computing the essential spectrum. It turns out that critical shell interactions give rise to a new interval in the essential spectrum, while the position and the length of the interval are explicitly controlled by the coupling constants and the geometric properties of the shell. This effect is completely new compared to lower dimensional critical situations (in which only a single new point in the essential spectrum was found). Based on joint work with Badreddine Benhellal (Oldenburg).

Jakob Reiffenstein, University of Vienna

Order and density of eigenvalues of a canonical system

Abstract. We investigate two-dimensional canonical systems $y'(t) = zJH(t)y(t)$ on $[0, L]$, i.e., with two limit circle endpoints. The order and density of the eigenvalue sequence of a boundary value problem associated to H correspond to the order and growth of the fundamental solution.

We present a new formula for the order. For large classes of Hamiltonians, we show how to evaluate the formula and even determine the growth of the fundamental solution up to multiplicative constants. This includes Hamburger Hamiltonians, which are in one-to-one correspondence to indeterminate Hamburger moment problems and Jacobi matrices (that are in limit circle case at infinity).

Jonathan Rohleder, Stockholm University

Laplacian eigenvalues and eigenfunctions: a non-standard variational principle

Abstract. In this talk we discuss a non-standard variational principle for the eigenvalues of the Neumann and Dirichlet Laplacians on bounded planar domains. The novelty is that the minimizers are gradients of eigenfunctions instead of the eigenfunctions themselves. We present applications to the hot spots conjecture and to eigenvalue inequalities.

Alexander Sakhnovich, University of Vienna

Dirac systems with locally square-integrable potentials: direct and inverse problems for spectral functions

Abstract. This work is an important development of our earlier works [2,3] (see also [1,4]). The case of locally bounded potentials was studied in [2] and the case of contractive Titchmarsh-Weyl matrix functions was treated in [3]. Here, we return to Titchmarsh-Weyl matrix functions belonging to Herglotz class, and in this way deal with direct and inverse problems for spectral matrix functions in the case of locally square-integrable potentials. We characterize the class of the spectral matrix functions as well. The structured operator which appears in the work is of convolution type and interesting connections of our results with Fourier frames theory are discussed.

1. F. Gesztesy and A.L. Sakhnovich, The inverse approach to Dirac-type systems based on the A-function concept, J. Funct. Anal. 279 (2020), Paper No. 108609.

2. A.L. Sakhnovich, Dirac type and canonical systems: spectral and Weyl-Titchmarsh functions, direct and inverse problems, Inverse Problems 18 (2002), 331–348.

3. A.L. Sakhnovich, Inverse problem for Dirac systems with locally square-summable potentials and rectangular Weyl functions, J. Spectr. Theory 5 (2015), 547–569.

4. A.L. Sakhnovich, L.A. Sakhnovich, and I.Ya. Roitberg, Inverse Problems and Nonlinear Evolution Equations. Solutions, Darboux Matrices and Weyl–Titchmarsh Functions, De Gruyter, Berlin, 2013.

Selim Sukhtaiev, Auburn University

Resolvent expansions for self-adjoint operators via boundary triplets

Abstract. In this talk we will discuss asymptotic perturbation theory for varying self-adjoint extensions of symmetric operators. Specifically, we derive a Riccati-type differential equation and second order asymptotic expansion for resolvents of self-adjoint extensions determined by smooth one-parameter families of Lagrangian planes. This asymptotic perturbation theory yields a symplectic version of the abstract Kato selection theorem and Hadamard-Rellich-type variational formula for slopes of multiple eigenvalue curves bifurcating from an eigenvalue of the unperturbed operator. Applications are given to quantum graphs, periodic Kronig-Penney model, elliptic second order partial differential operators with Robin boundary conditions, and physically relevant heat equations with thermal conductivity.

Alim Sukhtayev, Miami University

Renormalized oscillation theory for Hamiltonian pencils

Abstract. Working with a general class of linear Hamiltonian systems with nonlinear dependence on the spectral parameter, we show that renormalized oscillation results can be obtained in a natural way through consideration of the Maslov index associated with appropriately chosen paths of Lagrangian subspaces. By reduction to a generalized nonlinear eigenvalue problem, we apply our results to a class of models such as magneto-hydrodynamics systems and the Saint-Venant equations.

Ian Wood, University of Kent

Spectrum of the Maxwell Equations for a Flat Interface between Homogeneous Dispersive Media

Abstract. We determine and classify the spectrum of a non-selfadjoint operator pencil generated by the time-harmonic Maxwell problem with a nonlinear dependence on the frequency. More specifically, we consider one- and two-dimensional reductions for the case of two homogeneous materials joined at a planar interface. The dependence on the spectral parameter, i.e. the frequency, is in the dielectric function and we make no assumptions on its form. In order to allow also for non-conservative media, the dielectric function is allowed to be complex, yielding a non-selfadjoint problem. This is joint work with Malcolm Brown (Cardiff), Tomas Dohnal (Halle) and Michael Plum (Karlsruhe).

Chaofeng Zhu, Chern Institute of Mathematics, China

Global Mountain Pass Points and Applications to Minimal Period Problems in Hamiltonian Systems

Abstract. In this talk, we introduce the notion of the global mountain pass points. Then we show that under certain conditions, there exists either a non-trivial minimal point or a global mountain pass point. As an application, we show that for each $\tau > 0$, a strictly convex superlinear autonomous Hamiltonian system with brake symmetry has a periodic orbit with minimal period τ.

5.18. **Symmetries, Positivity and Representations.**

Tuesday 2:30-3:30 in Language Center, Room 406 chair: Jurij Volčič

2:30-2:55	Christian Ikenmeyer *All Kronecker coefficients are reduced Kronecker coefficients*	UPDATE
3:00-3:25	Klemen Šivic *Spaces of matrices with bounded number of eigenvalues*	UPDATE

Tuesday 4:30-6:00 in Language Center, Room 406 chair: Jurij Volčič

4:30-4:55	Saugata Basu *Homology of symmetric semi-algebraic sets*
5:00-5:25	Mario Kummer *Quadratic determinantal representations*
5:30-5:55	Gregor Podlogar *Real noncommutative rational invariants and multiplicative automorphisms of the free-skew fields*

Thursday 2:30-4:00 in Language Center, Room 406 chair: Visu Makam

2:30-2:55	Thu Hien Nguyen *Some simple conditions for entire functions to have only real zeros*
3:00-3:25	Bhumi Amin *Characterization of continuous linear maps on topological vector spaces*
3:30-3:55	James E. Pascoe *The inverse problem for kernel means*

Thursday 4:30-6:00 in Language Center, Room 406 chair: Visu Makam

4:30-4:55	Cordian Riener *The Wonderful Geometry of the Vandermonde map*
5:00-5:25	Greg Blekherman *Traces of Matrix Powers*
5:30-5:55	Jaka Cimprič *From semiprime ideals to semiprime submodules: a one-sided Nullstellensatz for matrix and free polynomials*

Friday 11:30-1:00 in Language Center, Room 406 chair: Visu Makam

11:30-11:55	Benjamin Lovitz *Computing linear sections of varieties: quantum entanglement, tensor decompositions and beyond*
12:00-12:25	Bill Helton *XOR Games; quantum vs commuting strategies*
12:30-12:55	Nujood Alshehri *Rational inner functions from $\mathbb{D}$ to the pentablock*

5.18.1. *Abstracts.*

Nujood Alshehri, Newcastle University

Rational inner functions from $\mathbb{D}$ to the pentablock

Abstract. The *pentablock* is the set in $\mathbb{C}^3$

$$\mathcal{P} = \{(a_{21}, \mathrm{tr}A, \det A) : A = [a_{ij}]_{i,j=1}^2 \in \mathbb{B}^{2\times 2}\},$$

where $\mathbb{B}^{2\times 2}$ denotes the open unit ball in the space of 2×2 complex matrices. The closure of $\mathcal{P}$ is denoted by $\overline{\mathcal{P}}$. The sets $\mathcal{P}$ and $\overline{\mathcal{P}}$ are polynomially convex and starlike about (0,0,0), but not convex. In this thalk we provide a description of rational maps from the unit disc $\mathbb{D}$ to $\overline{\mathcal{P}}$ that map the unit circle $\mathbb{T}$ to the distinguished boundary $b\overline{\mathcal{P}}$ of $\overline{\mathcal{P}}$. These functions are called *rational*

$\overline{\mathcal{P}}$*-inner functions*. We establish relations between $\overline{\mathcal{P}}$–inner functions and Γ-inner functions from $\mathbb{D}$ to the symmetrized bidisc Γ. We give a method of constructing rational $\overline{\mathcal{P}}$-inner functions starting from a rational Γ-inner function. We describe the construction of rational $\overline{\mathcal{P}}$-inner functions $x = (a, s, p) : \mathbb{D} \to \overline{\mathcal{P}}$ of prescribed degree from the zeros of a, s and $s^2 - 4p$ subject to the computation of Fejér-Riesz factorizations of certain non-negative trigonometric functions on the circle. We prove a Schwarz lemma for the pentablock, see [1].

The talk is based on joint work with my supervisor Dr Zinaida Lykova.

[1] N. M. Alshehri and Z. A. Lykova, A Schwarz lemma for the pentablock, *J. Geom. Anal.* **33** (2023) number 65.

Bhumi Amin, IIT Hyderabad

Characterization of continuous linear maps on topological vector spaces

Abstract. We extend the generalization of the Gleason-Kahane-Żelazko theorem for topological vector spaces to the case when the codomain of the map is a commutative Banach algebra. With the help of the characterization of continuous multiplicative linear maps on $C_b(X, \mathcal{B})$, where X is locally compact and $\mathcal{B}$ is a commutative Banach algebra, and using the theory of Bochner integral; we have given a characterization of continuous linear maps from the Bochner space $L^p(X, \mathcal{B})$ to the commutative Banach algebra $\mathcal{B}$. Later, we generalize these results and give representations for maps on an arbitrary topological vector space of algebra valued functions.

Saugata Basu, Purdue University

Homology of symmetric semi-algebraic sets

Abstract. Studying the homology groups of semi-algebraic subsets of $\mathbb{R}^n$ and obtaining upper bounds on the Betti numbers has been a classical topic in real algebraic geometry beginning with the work of Petrovskii and Oleinik, Thom, and Milnor. In this talk I will consider semi-algebraic subsets of $\mathbb{R}^n$ which are defined by symmetric polynomials and are thus stable under the standard action of the symmetric group $\mathfrak{S}_n$ on $\mathbb{R}^n$. The homology groups (with rational coefficients) of such sets thus acquire extra structure as $\mathfrak{S}_n$-modules leading to possible refinements on the classical bounds. I will also mention some connections with a homological stability conjecture.

Joint work (separately) with Daniel Perrucci and Cordian Riener.

Greg Blekherman, Georgia Tech

Traces of Matrix Powers

Abstract. I will consider the problem of characterizing inequalities satisfied by traces of powers of a real symmetric matrix A. I will focus on inequalities that are satisfied by symmetric matrices of all sizes. While we can describe inequalities satisfied by all matrices, the case of 0/1 matrices seems to be much more complicated, and is of interest in graph theory. Joint work with Jose Acevedo, Daniel Brosch, Sebastian Debus, Annie Raymond and Cordian Riener.

Jaka Cimprič, University of Ljubljana, Slovenia

From semiprime ideals to semiprime submodules: a one-sided Nullstellensatz for matrix and free polynomials

Abstract. Let R be a commutative ring with 1 and n a natural number. We say that a submodule N of R^n is *semiprime* if for every $f = (f_1, \dots, f_n) \in R^n$ such that $f_i f \in N$ for $i = 1, \dots, n$ we have $f \in N$. Our main result is that every semiprime submodule of R^n is equal to the intersection of all prime submodules containing it. It follows that every semiprime left ideal of $M_n(R)$ is equal to the intersection of all prime left ideals that contain it. For $R = k[x_1, \dots, x_d]$

where k is an algebraically closed field we can rephrase this result as a Nullstellensatz for $M_n(R)$: For every $G_1, \ldots, G_m, F \in M_n(R)$, F belongs to the smallest semiprime left ideal of $M_n(R)$ that contains $G_1, \ldots, G_m$ iff for every $a \in k^d$ and $v \in k^n$ such that $G_1(a)v = \ldots = G_m(a)v = 0$ we have $F(a)v = 0$. We also have a related Nullstellensatz for free algebras.

Bill Helton, UC San Diego

XOR Games; quantum vs commuting strategies

Abstract. The talk concerns a rigid class of equations which make sense both for matrix unknowns or for binary unknowns. The ultimate issue is to compare the satisfiability (solvability) of these equations. The problem comes from quantum games and amounts to understanding the advantage of allowing a 'perfect' quantum strategy over restricting players to using a classical strategy.

Satisfiability (SAT) problems are heavily studied in computer science for $m \times n$- systems of linear equations which one must solve mod 2. They generate a class of such systems randomly and find that asymptotically there is a critical threshold c, meaning that with high probability

$$m/n < c \text{ implies there is a solution;}$$

$$m/n > c \text{ implies there is no solution.}$$

For example, for 3XOR SAT c exists and is about 0.92; Dubois-Mandler 2003.

There is a k player XOR game and quantum game analog of this and historically these have been an influential guide to quantum vs classical behavior. That quantum entanglement exists was shown by experiments on a particular 2XOR game. 2XOR games were fully understood by Tsirelson in the 1980s, who showed they reduce to a SDP. About 15 years ago it was shown that quantum strategies could have unbounded advantage over classical strategies in the context of 3XOR games. Open remained: given a 3XOR game determine if it does or does not have a 'perfect' quantum strategy. Is this problem decidable?

Work with collaborators settles this by reducing quantum strategy production to new class of linear SAT like problems. The reduction starts at a high level with a noncommutative directional real nullstellensatz applied to a special class of toric ideals. Next comes a tricky calculation.

Having an algorithim allows us to run many experiments which confront us with questions about the critical threshold. So now we are developing theory for that. Our speculation is that

$$cSAT = cGAME = cQUANTUMGAME$$

holds for the thresholds. The talk will describe justification and current progress on this.

The work is joint with Adam Bene Watts , Jared Hughes, Daniel Kane, Igor Klep, Zehong Zhang.

Christian Ikenmeyer, University of Warwick

All Kronecker coefficients are reduced Kronecker coefficients

Abstract. Kronecker coefficients are the representation theoretic multiplicities of functions on spaces of tensors, and they describe the invariant space dimension of triple products of Specht modules. They have applications in the representation theory of the symmetric and general linear group, in quantum information theory, and in geometric complexity theory. They are a generalization of the so-called reduced Kronecker coefficients, which in turn are a generalization of the well-known Littlewood-Richardson coefficients.

We settle the question of where exactly do the reduced Kronecker coefficients lie on the spectrum between the Littlewood-Richardson and Kronecker coefficients by showing that every Kronecker coefficient is equal to a reduced Kronecker coefficient by an explicit construction. This implies

the equivalence of a question by Stanley from 2000 and a question by Kirillov from 2004 about positive combinatorial interpretations of these two families of coefficients.

This is joint work with Greta Panova, arXiv:2305.03003.

Igor Klep, University of Ljubljana

Ranks of linear pencils separate similarity orbits of matrix tuples

Abstract. Two matrices A, B are called similar if there is an invertible matrix P satisfying $AP = PB$. As is well known, complex matrices are up to similarity uniquely determined by their Jordan canonical form. This talk will discuss possible extensions to (joint) similarity of tuples of matrices. Tuples $(A_1, \ldots, A_n)$ and $(B_1, \ldots, B_n)$ are called similar if there is an invertible matrix P such that $A_j P = PB_j$ for all j. The classification of matrix tuples up to similarity has been deemed a "hopeless problem", but is widely studied due to its importance in multiple areas of mathematics, ranging from operator theory, invariant and representation theory and algebraic geometry to algebraic statistics and computational complexity. In this talk we shall present a new natural collection of separating invariants for matrix tuples, along the way solving a 2003 conjecture of Hadwin and Larson, which itself was an adaptation of a 1985 conjecture of Curto and Herrero.

This is based on joint work with Harm Derksen, Visu Makam and Jurij Volčič.

Mario Kummer, TU Dresden

Quadratic determinantal representations

Abstract. To a linear map $\varphi\colon \mathrm{Sym}_d(\mathbb{R}) \to \mathrm{Sym}_e(\mathbb{R})$ can be naturally associated a symmetric matrix $A(\varphi)$ of size $e \times e$ whose entries are quadratic forms in d variables. It was observed by Choi that the map φ is positive in the sense of operator theory if and only if $A(\varphi)$ is positive semidefinite at every point in $\mathbb{R}^d$, and it is completely positive if and only if $A(\varphi)$ is a matrix sum of squares. In particular, if φ is (completely) positive, then the form $\det(A(\varphi))$ of degree $2e$ in d variables is nonnegative (a sum of squares). In this talk we study the converse problem. For instance, there has been recent interest in whether every nonnegative ternary sextic arises in this way, i.e., can be written as $\det(A(\varphi))$ for some positive $\varphi\colon \mathrm{Sym}_3(\mathbb{R}) \to \mathrm{Sym}_3(\mathbb{R})$. It was conjectured by Buckley and Šivic that this is not the case for the Robinson polynomial. In a joint work with Clemens Brüser we prove, among others, this conjecture.

Benjamin Lovitz, Northeastern University

Computing linear sections of varieties: quantum entanglement, tensor decompositions and beyond

Abstract. We study the problem of finding elements in the intersection of an arbitrary conic variety in $\mathbb{F}^n$ with a given linear subspace (where $\mathbb{F}$ can be the real or complex field). This problem captures a rich family of algorithmic problems under different choices of the variety. The special case of the variety consisting of rank-1 matrices already has strong connections to central problems in different areas like quantum information theory and tensor decompositions. A robust variant of this problem asks *how far* the subspace is from the conic variety in Hausdorff distance. These problems are known to be NP-hard in the worst case, even for the variety of rank-1 matrices.

In this work, we propose and analyze algorithmic hierarchies for solving this problem. Surprisingly, despite the above hardness results we show that our algorithm solves the non-robust variant of this problem efficiently for "typical" subspaces. Here, the subspace $\mathcal{U} \subseteq \mathbb{F}^n$ is chosen *generically* of a certain dimension, potentially with some generic elements of the variety contained in it. We obtain similar results in the robust variant for certain physically-motivated cases of the variety $\mathcal{X}$. As corollaries, we obtain the following new results:

- Polynomial time algorithms for several *entangled subspaces* problems in quantum entanglement. While these problems are NP-hard in the worst case, our algorithm solves them in polynomial time for generic subspaces of dimension up to a constant multiple of the maximum possible.
- Uniqueness results and polynomial time algorithmic guarantees for generic instances of a broad class of low-rank decomposition problems that go beyond tensor decompositions. Here, we recover a decomposition of the form $\sum_{i=1}^{R} v_i \otimes w_i$, where the v_i are elements of the given variety $\mathcal{X}$. This implies new uniqueness results and genericity guarantees even in the special case of tensor decompositions.
- Generalizations of the Doherty-Parrilo-Spedalieri separability testing hierarchy to determining whether a quantum state is a mixture of pure states of limited entanglement specified by the variety $\mathcal{X}$.
- An alternative to the sum-of-squares (SOS) hierarchy for optimizing a real polynomial over the unit sphere, which has the practical advantage of only requiring a maximum eigenvalue computation at each level (as opposed to a full semidefinite program).

Thu Hien Nguyen, Julius-Maximilians-Universität Würzburg, V.N. Karazin Kharkiv National University

Some simple conditions for entire functions to have only real zeros

Abstract. The Laguerre–Pólya class is a special class of entire functions which appears to be the analytic closure of sets of univariate hyperbolic polynomials. We present some simple necessary and sufficient conditions for entire functions to belong to the Laguerre–Pólya class, or to have only real zeros, in terms of their Taylor coefficients. For an entire function $f(z) = \sum_{k=0}^{\infty} a_k z^k$, we define the second quotients of Taylor coefficients as $q_n(f) := \frac{a_{n-1}^2}{a_{n-2}a_n}$, $n \geq 2$ and find conditions on $q_n(f)$ for f to belong to the Laguerre–Pólya class. We also discuss the operators that preserve the Laguerre–Pólya class and its relation to the generating functions of totally positive sequences.

This is joint work with Anna Vishnyakova.

James E. Pascoe, Drexel University

The inverse problem for kernel means

Abstract. We discuss the inverse problem for the kernel embedding of measures. We identify which elements of a reproducing kernel Hilbert space which are in the cone generated by some set of kernel functions as polar dual of the Herglotz-type functions, the functions with positive imaginary part. Over certain spaces, such as Sobelev spaces, the duality to Herglotz functions reduces to a classical multivariate moment problem, and, over analytic spaces, we see more complex analytic type conditions. We give conditions for when Herglotz functions have representations in terms of kernel functions in terms of reflexive reproducing kernel Hilbert spaces. We identify the orbits of a dynamical system in terms of the Koopmanism philosophy: we give a way to decide when there is an orbit contained in some compact subset of the domain.

Gregor Podlogar, Faculty of Mathematics and Physics

Real noncommutative rational invariants and multiplicative automorphisms of the free-skew fields

Abstract. Free skew-field is the universal skew-field of fractions of the free algebra. Given an action of an abelian group on the free skew-field with complex coefficients, we can describe the skew-subfield of invariants. To get results for real coefficients we use some auxiliary action of the Galois group (cyclic of degree 2). This motivates us to study actions of cyclic groups, such that the (free) group for multiplication generated by the variables is closed under the action. Such

actions are defined by a periodic automorphisms of a free group. We show that for any such automorphism there exists a free generating set of the free skew-field, such that each generator are mapped in to scalar multiple of itself, i.a., such automorphisms are diagonalisable.

Cordian Riener, UiT The Arctic University of Norway, Tromsø

The Wonderful Geometry of the Vandermonde map

Abstract. The Vandermonde map is the polynomial map given by the power-sum polynomials. We study the geometry of the image of the nonnegative orthant under under this map and focus on the limit as the number of variables approaches infinity. We will show, the geometry of this limit is the key to new undecidability results in nonnegativity of symmetric polynomials and deciding validity of trace inequalities in linear algebra.

Joint work with Jose Acevedo (Georgia Institute of Technology, USA), Grigoriy Blekherman (Georgia Institute of Technology, USA) and Sebastian Debus (Otto-von-Guericke-University Magdeburg, Germany).

Klemen Šivic, University of Ljubljana

Spaces of matrices with bounded number of eigenvalues

Abstract. A seminal result of Gerstenhaber gives the maximal dimension of a vector space of nilpotent matrices. It also exhibits the structure of such a space when the maximal dimension is attained. Extensions of this result in the direction of vector spaces of matrices with a bounded number of eigenvalues have been studied. In the talk we consider the most general case. For any positive integers n and $k < n$ we give the maximal dimension of a vector space of $n \times n$ matrices with no more than k eigenvalues, which solves the conjecture proposed by Loewy and Radwan. We also exhibit the structure of the spaces for which the maximal dimension is attained.
The talk is based on the following preprint: M. Omladič, K. Šivic, *The solution of the Loewy-Radwan conjecture*, arxiv: 2209.09416.

5.19. **Operator Theory and Applications.**

Monday 2:30-4:30 in Porthania, P617 chair: Yong Zhang

2:30-2:55	Badriya AL-Azri *A Note on Marcinkiewicz Integral Operators on Product Domains*	
3:00-3:25	Nikolaos Alexandrakis *Classical and asymptotic results on zero-modes of the Weyl - Dirac operator: from simple cases to generalizations*	
3:30-3:55	Ahmad Al-Salman *Singular Integrals Along Surfaces*	
4:00-4:25	Stephen Sontz *Co-Toeplitz Operators defined as a categorical dual to Toeplitz operators*	

Tuesday 2:30-4:00 in Porthania, P617 chair: Giorgi Tutberidze

2:30-2:55	Anna Muranova *Discrete Laplacian over a non-Archimedean ordered field*	
3:00-3:25	Venku Naidu Dogga *Vertical operators on Bergman space over UHP*	
3:30-3:55	Miron Bekker *An operator approach to extremal problems on the Hardy and Bergman spaces*	

Tuesday 4:30-8:00 in Porthania, P617 chair: Petr Zemanek

4:30-4:55	Janko Bračič *Local commutants and ultrainvariant subspaces*	
5:00-5:25	Dimitri Bytchenkoff *Outer kernel theorem for bounded linear operators on co-orbit spaces associated with self-localised frames*	
5:30-5:55	Chung-Chuan Chen *Disjoint transitivity for translation operators on weighted Orlicz spaces*	
6:00-6:25	Jacek Chmieliński *On a property of approximate smoothness*	
6:30-6:55	Sukumar Daniel *Exponential spectrum in Banach space operators*	
7:00-7:25	Stephen Deterding *Boundary smoothness conditions for functions in $R^p(X)$*	
7:30-7:55	Lijia Ding *Bergman-type operators on bounded symmetric domains*	

Thursday 2:30-4:00 in Porthania, P617 chair: Miron Bekker

2:30-2:55	Atsuhide Ishida *Mourre inequality for Non-local Schödinger Operators*	
3:00-3:25	Krishna Kumar G. *Pseudospectral Mapping Theorem for Quoternionic Operators*	
3:30-3:55	Mitra Shamsabad *Fusion Frame Trees*	

Thursday 4:30-6:00 in Porthania, P617 chair: Stephen Deterding

4:30-4:55	Thankarajan Prasad *Some classes of operators related to subnormal operators*	
5:00-5:25	Yong Zhang *Amenability properties of weighted group algebras*	UPDATE
5:30-5:55	Sinem Yelda Sönmez *Weighted Bergman Spaces on Finitely Connected Domains*	

Friday 11:30-1:00 in Porthania, P617 chair: Anna Muranova

11:30-11:55	Mansi Anil Suryawanshi *Characterizations of Orthogonal Decompositions and Its Applications to Certain Classes of Isometries*
12:00-12:25	Giorgi Tutberidze *Some properties of the sequence of linear functionals on the space bounded variation*
12:30-12:55	Ferenc Weisz *Maximal operators on variable Hardy spaces and some applications in Fourier analysis*

Friday 2:30-4:00 in Porthania, P617 chair: Jacek Chmieliński

2:30-2:55	Yukihide Tadano *Continuum limit of discrete Schrödinger operators on lattices*
3:00-3:25	Petr Zemánek *Discrete symplectic systems and eigenfuctions expansion*
3:30-3:55	Satyabrata Majee *Wold-type decomposition for $\mathcal{U}_n$-twisted contractions*

5.19.1. *Abstracts.*

Badriya AL-Azri, Sultan Qaboos University

A Note on Marcinkiewicz Integral Operators on Product Domains

Abstract. Integral operators with homogeneous kernels are important part of harmonic analysis. There applications spread over many branches of mathematics. The aim of this work is to discuss the L^p estimates of a class of Marcinkiewicz integral operators on product domains along surfaces more general than polynomials and convex functions provided that Ω satisfies the condition

$$\sup_{(\xi',\eta')\in(\mathbb{S}^{n-1}\times\mathbb{S}^{m-1})} \int_{\mathbb{S}^{n-1}\times\mathbb{S}^{m-1}} |\Omega(u',v')| \left\{G(\xi',\eta')\right\}^{1+\epsilon} d\sigma(u')\, d\sigma(v') < \infty, \tag{7}$$

for some $\epsilon > 0$, where

$$G(\xi',\eta') = \log^+(|\xi'\cdot u'|^{-1}) + \log^+(|\eta'\cdot v'|^{-1}) + \log^+(|\xi'\cdot u'|^{-1})\, \log^+(|\eta'\cdot v'|^{-1}).$$

Nikolaos Alexandrakis, Lancaster University

Classical and asymptotic results on zero-modes of the Weyl - Dirac operator: from simple cases to generalizations

Abstract. In recent years, there has been an increasing interest in understanding the kernel of Weyl - Dirac operators. Physicists and engineers are interested in potential applications such as the development of electric devices based on graphene. Mathematicians, however, are interested in developing new ideas to solve similar problems and generalizing established results. Early landmark contributions include the related work by Erdős and Solovej in 2001, as well as the results of Balinsky, Evans and Elton in the nineties and early 2000s. These results largely rely on using a suitable Riemannian submersion to translate the original Weyl - Dirac operator (which is a 3-dimensional operator) into a 2-dimensional one. In this talk, we'll briefly present this technique, introduce a generalized version of it, and mention potential applications in the asymptotic regime.

Ahmad Al-Salman, Sultan Qaboos University

Singular Integrals Along Surfaces

Abstract. Singular integral operators are central part in harmonic analysis. Since the publication of the fundamental papers of Calderón and Zygmund ([4], [5]), the theory of singular integral operators has received a considerable amount of attention. For background information and related results, we advice readers to consult [1]-[6] and references there, among others. In this talk, we are interested in investigating the L^p boundedness of a class of singular integral operators in $\mathbb{R}^n$ with kernels supported in the product domain $\mathbb{R}^n \times \mathbb{R}^n$ which arises naturally when considering composition of singular integral operators in $\mathbb{R}^n$. Let $\mathbb{R}^n$, $n \geq 2$ be the n-dimensional Euclidean space and $\mathbb{S}^{n-1}$ be the unit sphere in $\mathbb{R}^n$ equipped with the induced Lebesgue measure $d\sigma$. For a suitable mapping $\gamma : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$, we let $T_{\gamma,\Omega}$ be the singular integral along γ which is defined by

$$T_{\gamma,\Omega} f(x) = \text{p.v.} \int_{\mathbb{R}^n \times \mathbb{R}^n} f(x - \gamma(u,v)) \frac{\Omega(u', v')}{|u|^n |v|^n} dudv, \tag{8}$$

where $\Omega \in L^1(\mathbb{S}^{n-1} \times \mathbb{S}^{n-1}), \Omega(tx, sy) = \Omega(x, y)$ for any $t, s > 0$, and

$$\int_{\mathbb{S}^{n-1}} \Omega(u', \cdot)\, d\sigma_n(u') = \int_{\mathbb{S}^{m-1}} \Omega(\cdot, v')\, d\sigma_n(v') = 0. \tag{9}$$

When $\gamma(u,v) = \mathcal{P}(u,v)$ is a polynomial mapping, the corresponding operator was considered in [1]. It is shown in [1] that if $\Omega \in L(\log L)^2(\mathbb{S}^{n-1} \times \mathbb{S}^{n-1})$, then the operator $T_{\mathcal{P},\Omega}$ is bounded on $L^p(\mathbb{R}^n)$ for $p \in (1, \infty)$ Moreover, it is pointed out in [1] that the condition $\Omega \in L(\log L)^2(\mathbb{S}^{n-1} \times \mathbb{S}^{n-1})$ is nearly optimal. When $\gamma(u,v) = \mathcal{P}(u,v) = u + v$, the corresponding operator (denoted by T_Ω) will be referred to by the classical operator. In this talk, we are interested in surfaces where $\frac{\partial^\alpha}{\partial u^\alpha}\gamma(u,v)\mid_{u=0} = 0$ or $\frac{\partial^\alpha}{\partial v^\alpha}\gamma(u,v)\mid_{v=0} = 0$ for all $\alpha, \beta \in \mathbb{N}^n$. More precisely, we shall consider the following general problem:

Problem . *For suitable functions* $\varphi_1, \varphi_2 : (0, \infty) \to \mathbb{R}$, *let* $\gamma_{\varphi_1,\varphi_2}$ be the surface given by $\gamma_{\varphi_1,\varphi_2}(u,v) = \varphi_1(|u|)u' + \varphi_2(|v|)v'$. U*nder what conditions on the functions* φ_1 *and* φ_2, *the corresponding operator* $T_{\gamma_{\varphi_1,\varphi_2},\Omega}$ *satisfies* $\left\| T_{\gamma_{\varphi_1,\varphi_2},\Omega}(f) \right\|_{L^p} \leq A_p \|f\|_{L^p}$ *for some* $1 < p < \infty$ *provided that* $\Omega \in L(\log L)^2(\mathbb{S}^{n-1} \times \mathbb{S}^{n-1})$.

References

[1] Al-Salman, A., Singular Integral Operators With Kernels Supported In Higher Dimensional Subvarieties, Banach J. Math. Anal. 16 (2022), no. 3, Paper No. 48, 19 pp.

[2] Al-Salman, A, Al-Qassem, H., Pan, Y., Singular integrals on product domains, Indiana Univ. Math. J. Vol 55, No. 1(2006), 369-387.

[3] Al-Salman, A., Pan, Y., Singular integrals with rough kernels in $Llog^+L(\mathbf{S}^{n-1})$, J. London Math. Soc. (2) 66 (2002) 153-174.

[4] Calderón, A. P. and Zygmund, A., On the existence of certain singular integrals Acta Math. 88 (1952), 85-139.

[5] Calderón, A. P., Zygmund, A., On singular integrals, Amer. J. Math. 78 (1956), 289-309.

[6] Duoandikoetxea, J., Multiple singular integrals and maximal functions along hypersurfaces, Ann. Ins. Fourier (Grenoble) 36 (1986) 185-206.

Atsuhide Ishida, Tokyo University of Science

Mourre inequality for Non-local Schödinger Operators

Abstract. We consider the Mourre inequality for the following self-adjoint operator $H = \Psi(-\Delta/2) + V$ acting on $L^2(\mathbb{R}^d)$, where $\Psi : [0,\infty) \to \mathbb{R}$ is an increasing function, Δ is Laplacian and $V : \mathbb{R}^d \to \mathbb{R}$ is an interaction potential. Mourre inequality immediately yields the discreteness and finite multiplicity of the eigenvalues. Moreover, Mourre inequality has the application to the absence of the singular continuous spectrum by combining the limiting absorption principle and, in addition, Mourre inequality is also used for proof of the minimal velocity estimate that plays an important role in the scattering theory. In this talk, we report that Mourre inequality holds under the general Ψ and V by choosing the conjugate operator $A = (p\cdot x + x\cdot p)/2$ with $p = -\sqrt{-1}\nabla$, and that the discreteness and finite multiplicity of the eigenvalues hold. This talk is a joint work with J. Lőrinczi (Hungarian Academy of Sciences) and I. Sasaki (Shinshu University).

[1] A. Ishida, J. Lőrinczi, I. Sasaki, Absence of embedded eigenvalues for non-Local Schrödinger operators. *J. Evol. Equ.* **22** (2022), no. 4, Parper No. 82, 30 pp.

Miron Bekker, Department of Mathematics, the University of Pittsburgh at Johnstown

An Operator Approach to Extremal Problems on the Hardy and Bergman Spaces

Abstract. S. Abbott and S. Abbott and B. Hanson developed an operator-theoretic approach to solve some extremal problems. We give a different proof of a theorem of S. Abbott and B. Hanson in the case when the corresponding operator is unbounded. We apply our theorem to the classical Kolmogorov and Szegö infimum problems. We also consider Kolmogorov and Szegö type infima, when integration over the unit circle is replaced by integration over the unit disk.

Janko Bračič, University of Ljubljana

Local commutants and ultrainvariant subspaces

Abstract. For an operator A on a complex Banach space X and a closed subspace $M \subseteq X$, the local commutant of A at M is the set $C(A;M)$ of all bounded operators T on X such that $TAx = ATx$, for all vectors $x \in M$. It is clear that $C(A;M)$ is a closed space of operators, however, it is not an algebra, in general. We show that $C(A;M)$ is an algebra if and only if the largest subspace $M_A \subseteq X$ such that $C(A;M) = C(A;M_A)$ is invariant for every operator in $C(A;M)$.

Reference: J. Bračič, *Local commutants and ultrainvariant subspaces*, J. Math. Anal. Appl. 506 (2022), no. 2, 19 pp.

Dimitri Bytchenkoff, Acoustics Research Institute of the Austrian Academy of Sciences and Faculty of Mathematics of the University of Vienna

Outer kernel theorem for bounded linear operators on co-orbit spaces associated with self-localised frames

Abstract. In this speech I shall report on the so-called outer kernel theorem for bounded linear operators mapping the co-orbit space of test functions induced by a localised frame for a Hilbert space on the co-orbit space of distributions induced by a localised frame for another Hilbert space. The proof of the theorem relies on using Galerkin's method to transform any given operator into a matrix, before synthesising the kernel of the operator from the matrix. I also show how the theorem, together with Schur's test, can be used for characterising the bounded linear operators.

Chung-Chuan Chen, National Taichung University of Education, Taiwan

Disjoint transitivity for translation operators on weighted Orlicz spaces

Abstract. We give some sufficient and necessary conditions for translation operators on the weighted Orlicz spaces of locally compact groups to be disjoint topologically transitive and disjoint topologically mixing. In particular, we show that in certain cases, operators are disjoint topologically transitive if, and only if, their direct sum is topologically transitive. This is a joint work with Prof. S. Oztop and Prof. S. M. Tabatabaie.

Jacek Chmieliński, Pedagogical University of Krakow

On a property of approximate smoothness

Abstract. The talk is partially based on a recent paper [1], in which the notion of approximate smoothness of a normed linear space was introduced. Some ap- plications of this property for general normed spaces and, in particular, for operator spaces will be given.

[1] J. Chmieliński, D. Khurana and D. Sain, *Approximate smoothness in normed linear spaces*, Banach J. Math. Anal. (2023), 17:41.

Sukumar Daniel, IIT Hyderabad

Exponential spectrum in Banach space operators

Abstract. In a Banach algebra $\mathcal{A}$, for a pair of elements the spectrum always commutes:

$$\sigma(ab) \setminus \{0\} = \sigma(ba) \setminus \{0\},$$

whereas the exponential spectrum does not in general, as shown by Klaja and Ransford in [1]. Towards, answering a question posed in the same article, we investigate the commutativity of exponential spectrum, mainly in $\mathcal{B}(\mathcal{X})$, where $\mathcal{X}$ is an infinite dimensional complex Banach space.

[1] Hubert Klaja and Thomas Ransford, *Non-commutativity of the exponential spectrum*, J. Funct. Anal. **272** (2017), no. 10, 4158–4164. MR 3626037

Sayan Das, University of California, Riverside

Noncommutative Poisson Boundaries

Abstract. In 1980's Alain Connes proposed that in order to study rigidity aspects of von Neumann algebras it is imperative to develop a theory of noncommutative Poisson boundaries. In this talk I will describe the construction of noncommutative Poisson boundaries due to Prof. Jesse Peterson and myself and present a double ergodicity theorem. I will also show how the double ergodicity theorem can be used to prove that every II_1 factor satisfies Popa's Mean-Value property, thereby answering a question he posed in 2019. I will also relate our construction to Izumi's notion of Poisson boundary of a unital, normal, completely positive map, and provide a solution to a recent question of Bhat, Talwar and Kar on unimodular eigenvalues of certain Markov Operators.

Stephen Deterding, Marshall University

Boundary smoothness conditions for functions in $R^p(X)$

Abstract. Let X be a compact subset of the complex plane and let $R^p(X)$, $2 < p < \infty$, denote the closure of the rational functions with poles off X in the L^p norm. We consider three conditions which provide different ways of showing how the functions in $R^p(X)$ can have a greater degree of smoothness at a boundary point $x \in X$ than might otherwise be expected: (A) The existence of an approximate Taylor's theorem at x, (B) The existence of an L^q function g that represents x such that $g/(z-x)$ is also an L^q function, and (C) The existence of a bounded point derivation at x. We will show that (B) and (C) are equivalent and imply (A) but (A) does not imply (B) or (C).

Lijia Ding, Zhengzhou University

Bergman-type operators on bounded symmetric domains

Abstract. Bergman-type operators are integral operators induced by modified Bergman kernels, which are classical operators in complex analysis and operator theory. In this talk, I shall discuss the related results of Bergman-type operators on irreducible bounded symmetric domains. In the first half of the talk, I shall give the characterization of L^p-L^q boundedness, L^p-L^q compactness, and Schatten class membership of Bergman-type operators on the usual unit ball. In the second half of the talk, I shall give the characterization of Schatten class Bergman-type operators on general irreducible bounded symmetric domains; as its application, I shall give a new integral estimate related to the Forelli-Rudin estimate.

Venku Naidu Dogga, Indian Institute of Technology Hyderabad, India.

Vertical operators on Bergman space over UHP

Abstract. Let Π be the upper half-plane. In this talk, we prove that every vertical operator on the Bergman space $A^2(\Pi)$ over the upper half-plane can be uniquely represented as an integral operator of very special form.

The talk is based on joint work with Co Author's: Shubham R Bais, Mohan P.

Connor Evans, Newcastle University

A model formula for bounded holomorphic functions on the symmetrized rectangle

Abstract. The well-known realization formula for a general element φ of the Schur class of the open unit disc $\mathbb{D}$ expresses φ by the formula

$$\varphi(z) = A + BX(1 - DZ)^{-1}C \text{ for all } z \in \mathbb{D},$$

where $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is a contraction on $\mathbb{C} \oplus \mathcal{H}$ for some Hilbert space $\mathcal{H}$. The realization formula is a powerful tool for both the function theory of the Schur class and for numerical computation. Accordingly, it is valuable to obtain realization formulae for other classes of functions. One class for which a realization formula is known is the Schur class of the symmetrized bidisc $\mathbb{G}$, that is, the class of holomorphic functions φ on $\mathbb{G}$ bounded by 1 in modulus, where the symmetrized bidisc $\mathbb{G}$ is the set

$$\mathbb{G} = \Big\{(z_1 + z_2, z_1 z_2) : |z_1| < 1, |z_2| < 1\Big\},$$

(see [1]).

In this talk, I describe a realization formula (and its associated model formula) for the Schur class of a variant $\mathbb{G}_r$ of $\mathbb{G}$, where $0 < r < 1$, and

$$\mathbb{G}_r = \Big\{(z_1 + rz_2, rz_1 z_2) : |z_1| < 1, |z_2| < 1\Big\},$$

which is called the "symmetrized rectangle." The formula is obtained by a symmetrization argument, starting from the known model formula for the Schur class of the bidisc.

This talk is based on joint work with Dr. Zinaida Lykova and Prof. Nicholas Young.

[1] J. Agler and N.J. Young. Realization of functions on the symmetrized bidisc. *J. Math. Anal. Appl.*, 453(9):227-240, 2017.

Krishna Kumar G., University of Kerala, India.

Pseudospectral Mapping Theorem for Quoternionic Operators

Abstract. G. Birkhoff and J. von Neumann stated that quantum mechanics could be formulated over the real, the complex, and the quaternionic numbers. The S-spectrum discovered by F. Colombo and I. Sabadini replaces the classical spectrum in the quaternionic spectral theory. This paper presents the analogue of the spectral mapping theorem for the pseudo S-spectrum of a quaternionic right linear operator. The usual quaternionic spectral mapping theorem is shown as a particular case of the result. The pseudo S-spectrum of certain classes of quaternionic right linear operators is characterized. A weak version of the pseudo S-spectral mapping theorem is also given. Finally, we offer examples to illustrate the findings and provide numerical experiments.

Research partially supported by SERB No. MTR/2021/000028, Department of Higher Education, Government of Kerala, India through PLEASE scheme and KSCSTE, Kerala, India No KSCSTE/1000/2021-FSHP-MS. The talk is based on joint work with Akhitha M S, Department of Mathematics, University of Kerala.

Satyabrata Majee, Indian Institute of Technology Roorkee

Wold-type decomposition for $\mathcal{U}_n$-twisted contractions

Abstract. Motivated by the definition of $\mathcal{U}_n$-twisted isometries, we introduce the notion of $\mathcal{U}_n$-twisted contractions. Let $n > 1$, and $\{U_{ij}\}$ for $1 \leq i < j \leq n$ be $\binom{n}{2}$ commuting unitaries on a Hilbert space $\mathcal{H}$ such that $U_{ji} := U_{ij}^*$. An n-tuple of contractions $(T_1, \ldots, T_n)$ on $\mathcal{H}$ is called $\mathcal{U}_n$-twisted contraction with respect to a twist $\{U_{ij}\}_{i<j}$ if $T_1, \ldots, T_n$ satisfy

$$T_iT_j = U_{ij}T_jT_i; \qquad T_i^*T_j = U_{ij}^*T_jT_i^* \quad \text{and} \quad T_kU_{ij} = U_{ij}T_k$$

for all $i, j, k = 1, \ldots, n$ and $i \neq j$.

In this talk, we present a recipe to calculate the orthogonal spaces of the Wold-type decomposition for $\mathcal{U}_n$-twisted contractions on Hilbert spaces. As a by-product, a new proof as well as complete structure for $\mathcal{U}_2$-twisted (or pair of doubly twisted) and $\mathcal{U}_n$-twisted isometries have been established. This is a joint work with Amit Maji.

Anna Muranova, University of Warmia and Mazury in Olsztyn

Discrete Laplacian over a non-Archimedean ordered field

Abstract. We investigate properties of a discrete Laplacian on graphs, whose edge-weights belong to a non-Archimedean (real-closed) ordered field, e.g. we show that the eigenvalues of Laplacian on finite graphs always belong to the same field and estimate the spectrum in terms of isoperimetric constants. Further, we introduce an analogue of probability operator and an analogue of capacitance together with the Dirichlet problem, and discuss their particular properties, arising from the existence of infnitesimals in a non-Archimedean field $\mathbb{K}$, i.e. $\varepsilon \in \mathbb{K}^+$ such that

$$\varepsilon \prec \frac{1}{n} = \frac{1}{\underbrace{1 + \cdots + 1}_{n \text{ times}}}$$

for any $n \in \mathbb{N} \subset \mathbb{K}$.

Thankarajan Prasad, Department of Mathematics, University of Calicut, Kerala, India

Some classes of operators related to subnormal operators

Abstract. In this talk, we discuss some natural extensions of well studied subnormal operators, namely n-subnormal operators and sub-n-normal operators. We discuss some inclusion relations among the above mentioned classes and some classes of operators related with these operators, namely n-quasinormal and quasi-n-normal and also some of its properties. Pietrzycki and Stochel(J. Funct Anal , 2021) proved if T is subnormal and also T^n is quasinormal, then T is quasinormal and it answers a question recently raised by Curto, Lee and Yoon(J. Funct Anal , 2020) Here we discuss an analog of the result by Pietrzycki and Stochel for the case of operators in the above-mentioned classes.

This talk is based on a joint work with Curto (in preparation) which focus to develop a theory for operators that are nth-roots of subnormal operators.

Mitra Shamsabad, Acoustics Research Institute, Austrian Academy of Sciences

Fusion Frame Trees

Abstract. A fusion frame is a sequence of subspaces, fulfilling a frame-like inequality. Those subspaces can themselves be again separated, and so the subspaces of a fusion frames can be equipped with fusion frames. This can be continued. We call this structure a fusion frame tree where the leaves of this fusion frame tree form a fusion frame for the branches. In this paper, we give relations between global fusion frame related operators and fusion frame ones based on local ones (leaves). Also, we will show which properties can be inherited between local and global fusion frames. Moreover, by combining duals of local fusion frames, we will get some duals for the global fusion frame.

Joint work with Peter Balazs (Acoustics Research Institute, Austrian Academy of Sciences, Austria).

Stephen Sontz, Centro De Investigacion En Matematicas (cimat), Mexico

Co-Toeplitz Operators defined as a categorical dual to Toeplitz operators

Abstract. Using the elements of a not necessarily co-commutative co-algebra, we define and study *co-Toeplitz operators*, which are categorical duals to Toeplitz operators as defined earlier by the author using symbols from a not necessarily commutative algebra. Examples are given.

Sinem Yelda Sönmez, Sabancı University

Weighted Bergman Spaces on Finitely Connected Domains

Abstract. In this talk, the weighted Bergman spaces on finitely connected planar domains are investigated. They are isomorphic to the product of weighted Bergman spaces on the unit disk. Using this argument, norm and pointwise estimates are proved for the reproducing kernel function of the weighted Bergman spaces. A characterization of the membership in the Schatten class of Toeplitz operators acting on a weighted Bergman space is provided.

Mansi Anil Suryawanshi, Indian Statistical Institute (ISI), Bangalore Center, India

Characterizations of Orthogonal Decompositions and Its Applications to Certain Classes of Isometries

Abstract. We classify tuples of (not necessarily commuting) isometries that admit von Neumann-Wold decomposition. As a corollary, we prove that each doubly-twisted isometry admits a von Neumann-Wold type orthogonal decomposition. Also we generalize Popovici's orthogonal decompositions for pairs of commuting isometries to general tuples of twisted isometries which also includes the case of tuples of commuting isometries. The former classification is partially inspired by a result that was observed more than three decades ago by Gaspar and Suciu. And the latter result generalizes Popovici's orthogonal decompositions for pairs of commuting isometries to general tuples of twisted isometries which also includes the case of tuples of commuting isometries. We will study C^*-algebraic properties and will exhibit concrete analytic models of of these families. Our motivation of twisted isometries stems from the classical rotation C^*-algebras and Heisenberg group C^*-algebras.

Giorgi Tutberidze, The University of Georgia, Tbilisi, Georgia. Email: *g.tutberidze@ug.edu.ge*

Some properties of the sequence of linear functionals on the space bounded variation

Abstract. The talk is devoted to investigating the sequence of linear functions in the space of finite variation functions. We prove that under certain conditions, this sequence is bounded. We also show that these results are sharp. In particular, the obtained results can be used to study the issues of convergence in the general Fourier series. Moreover, the obtained conditions are effective for bounded orthonormal systems.

Ferenc Weisz, Department of Numerical Analysis, Eötvös L. University, H-1117 Budapest, Pázmány P. sétány 1/C., Hungary, e-mail: weisz@inf.elte.hu

Maximal operators on variable Hardy spaces and some applications in Fourier analysis

Abstract. Let $p(\cdot): \mathbb{R} \to (0, \infty)$ be a variable exponent function satisfying the globally log-Hölder condition. We introduce the variable Hardy spaces $H_{p(\cdot)}(\mathbb{T})$ and $H_{p(\cdot)}[0, 1)$ and give their atomic decompositions. It is proved that the maximal operator of the Fejér means of the Fourier series and Walsh-Fourier series is bounded on these spaces. This implies some norm and almost everywhere convergence results for the Fejér-means, amongst others the generalization of the well known Lebesgue's theorem.

Yukihide Tadano, Tokyo University of Science

Continuum limit of discrete Schrödinger operators on lattices

Abstract. We consider continuum limit problems of discrete Schrödinger operators defined on lattices $\Gamma = L\mathbb{Z}^d$, where L is a non-singular matrix. We show that, once the set of edges and the weight function on edges are fixed so that they are Γ-periodic, the corresponding Schrödinger operators on the Euclidean space are obtained as the continuum limits of the above discrete Schrödinger operators in the generalized norm resolvent sense. We also discuss its application to estimation of the Hausdorff distance between the spectra of discretized and continuum operators from above. We note that, although the above result is not applicable to the hexagonal lattice, we can obtain the similar continuum limit even in the hexagonal lattice case by additional discussions.

This talk is based on joint work with Shu Nakamura (Gakushuin University).

Petr Zemánek, Department of Mathematics and Statistics, Faculty of Science, Masaryk University

Discrete symplectic systems and eigenfuctions expansion

Abstract. Eigenfunctions expansion is a very classical topic in the spectral theory of linear operators on a Hilbert space, and its origin can be traced back at least to the works of Euler, d'Alembert, D. Bernoulli and, especially, Fourier. Subsequently, in the works of Sturm and Liouville, the theory of the eigenfunctions expansion was built in a more general fashion of regular boundary value problems associated with the second-order linear differential equations. Its generalization to the case of a system of first-order differential equations was initiated by Hurwitz and all of these results can be unified by using linear Hamiltonian differential systems.

Although difference equations play also an important role in mathematical physics or continuum mechanics, the literature covering discrete counterparts of expansion theorems seems to be, surprisingly, quite humbler. In this talk, we present results of our current research concerning discrete symplectic systems, which include any even-order Sturm–Liouville difference equation and represent a natural discrete counterpart of linear Hamiltonian differential systems. In particular, we focus on an eigenfunctions expansion for a class of boundary value problems determined by the regular discrete symplectic system depending linearly on the spectral parameter $\lambda \in \mathbb{C}$, i.e.,

$$z_k(\lambda) = (\mathcal{S}_k + \lambda\, \mathcal{V}_k)\, z_{k+1}(\lambda), \quad k \in \mathcal{I}_{\mathbb{Z}}, \tag{S$_\lambda$}$$

where $\mathcal{I}_{\mathbb{Z}} = [0, N]_{\mathbb{Z}} := [0, N] \cap \mathbb{Z}$ is a finite discrete interval and, for every $k \in \mathcal{I}_{\mathbb{Z}}$, the coefficients $\mathcal{S}_k, \mathcal{V}_k$ are $2n \times 2n$ matrices such that

$$\mathcal{S}_k^* \mathcal{J} \mathcal{S}_k = \mathcal{J}, \quad \mathcal{V}_k^* \mathcal{J} \mathcal{S}_k \ \text{ is Hermitian,} \quad \text{and} \quad \mathcal{V}_k^* \mathcal{J}\, \mathcal{V}_k = 0 \tag{10}$$

for $\mathcal{J}$ being the $2n \times 2n$ orthogonal and skew-symmetric matrix $\mathcal{J} := \left(\begin{smallmatrix} 0 & -I \\ I & 0 \end{smallmatrix}\right)$ with $n \times n$ blocks of zero and identity matrices. The first condition in (10) means that $\mathcal{S}_k$ is a symplectic matrix, and all conditions in (10) can be simultaneously written by using the matrix $\mathbb{S}_k(\lambda) := \mathcal{S}_k + \lambda\, \mathcal{V}_k$ as the symplectic-type identity

$$\mathbb{S}_k^*(\bar{\lambda})\, \mathcal{J}\, \mathbb{S}_k(\lambda) = \mathcal{J},$$

which is valid for all $k \in \mathcal{I}_{\mathbb{Z}}$ and $\lambda \in \mathbb{C}$. The main result represents a significant generalization of the Expansion theorem given by Bohner, Došlý, and Kratz in [1] for the case of system (S$_\lambda$) with a very special linear dependence on λ.

[1] M. Bohner, O. Došlý, and W. Kratz, *Sturmian and spectral theory for discrete symplectic systems*, Trans. Amer. Math. Soc. **361** (2009), no. 6, 3109–3123.

Yong Zhang, University of Manitoba

Amenability properties of weighted group algebras

Abstract. Let G be a locally compact group and ω a weight function on G. We study the amenability, weak amenability and approximate/pseudo amenability of the weighted group algebra $L^1(G, \omega)$, discussing the know results and open questions in the area.

6. List of Participants

1. Konrad Aguilar, Pomona College, United States
2. Meiram Akhymbek, Institute of Mathematics and Mathematical Modeling, Kazakhstan
3. Badriya Al-azri, Sultan Qaboos University, Oman
4. Fatma Al-Musalhi, Sultan Qaboos University, Oman
5. Alexandru Aleman, Lund University, Sweden
6. Nikolaos Alexandrakis, Lancaster University, United Kingdom
7. Ahmad AlSalman, Sultan Qaboos University, Oman
8. Nujood Alshehri, Newcastle University, United Kingdom
9. Bhumi Amin, IIT Hyderabad, India
10. Jone Apraiz, University of the Basque Country, Spain
11. Joakim Arnlind, Linköping University, Sweden
12. Hicham Arroussi chakhade, University of Helsinki, Finland
13. Ghazaleh Asghari, University of Reading, United Kingdom
14. Zeinab Ashtab, Cinvestav-IPN, Mexico
15. Kari Astala, University of Helsinki, Finland
16. Are Austad, University of Southern Denmark, Denmark
17. Victor Bailey, Georgia Institute of Technology, United States
18. Joseph Ball, Virginia Tech, United States
19. Oscar Bandtlow, Queen Mary University of London, United Kingdom
20. Zhigang Bao, Hong Kong University of Science and Technology, Hong Kong
21. Marzieh Baradaran, University of Hradec Kralove, Czech Republic
22. Ahmad Barhoumi, KTH, Sweden
23. Julio A. Barrera-Reyes, CIMAT, Mexico
24. Estelle Basor, American Institute of Mathematics, United States
25. Saugata Basu, Purdue Unversity, United States
26. Charles Batty, University of Oxford, United Kingdom
27. Baran Bayraktaroglu, University of Helsinki, Finland
28. Miron Bekker, The University of Pittsburgh at Johnstown, United States
29. Carlo Bellavita, Aristotle University of Thessaloniki, Italy
30. Rajae Ben Taher, University Moulay Isamil-meknes- Morocco, Morocco
31. Jonathan Ben-Artzi, Cardiff University, United Kingdom
32. Catherine Beneteau, University of South Florida, United States
33. Chafiq Benhida, Université de Lille, France
34. Santu Bera, Indian Institute of Technology Kanpur, India
35. Tirthankar Bhattacharyya, Indian Institute of Science, India
36. Mainak Bhowmik, Indian Institute of Science, Bengaluru, India
37. Petr Blaschke, Silesian University In Opava, Czech Republic
38. Greg Blekherman, Georgia Tech, United States
39. Bernhelm Booss-Bavnbek, University Roskilde, Denmark
40. Babhrubahan Bose, Indian Institute of Science, Bengaluru, India
41. Thomas Bothner, University of Bristol, United Kingdom
42. Janko Bračič, University of Ljubljana, Slovenia
43. Nedialko Bradinoff, KTH Royal Institute of Technology, Sweden
44. Michał Buchała, Jagiellonian University, Poland
45. Andreas Buchinger, TU Bergakademie Freiberg, Germany
46. Hüseyin Budak, Duzce University, Turkey
47. Christian Budde, University of the Free State, South Africa
48. Dimitri Bytchenkoff, Austrian Academy of Sciences and University of Vienna, Austria
49. Giuseppe Cardone, Università di Napoli, Italy
50. Catalin Ion Carstea, National Yang Ming Chiao Tung University, Taiwan

51. Valentina Casarino, Università degli Studi di Padova, Italy
52. Nikolaos Chalmoukis, University of Milano - Bicocca, Italy
53. Christophe Charlier, Lund University, Sweden
54. Chung-Chuan Chen, National Taichung University of Education, Taiwan
55. Jacek Chmieliński, Pedagogical University of Krakow, Poland
56. Boo Rim CHOE, Korea University, South Korea
57. Paolo Ciatti, University of Padova, Italy
58. Jakob Cimpric, University of Ljubljana, Slovenia
59. Tom Claeys, Uclouvain, Belgium
60. Klara Courteaut, KTH Royal Institute of Technology, Sweden
61. Giovanni Covi, University of Bonn, Germany
62. Graham Cox, Memorial University, Canada
63. Joakim Cronvall, Lund University, Sweden
64. Zeljko Cuckovic, University of Toledo, Ohio, United States
65. Raul Curto, The University of Iowa, United States
66. Frej Dahlin, Lund University, Sweden
67. Hui Dan, Sichuan University, China
68. Sukumar Daniel, IIT Hyderabad, India
69. Sayan Das, University of California, Riverside, United States
70. Alberto Dayan, Saarland University, Germany
71. Hendrik de Snoo, University of Groningen, Netherlands
72. Jens de Vries, University of Twente, Netherlands
73. Anatole Dedecker, Université Paris Saclay and Aalto University, France and Finland
74. Chenxi Deng, Delft University of Technology, China
75. Harm Derksen, Northeastern University, United States
76. Stephen Deterding, Marshall University, United States
77. Philipp di Dio, Universität Konstanz, Germany
78. Viktor Didenko, Southern University of Science and Technology, Shenzhen, China
79. Kai Diethelm, Technical University of Applied Sciences Würzburg-Schweinfurt, Germany
80. Natalia Dilna, Institute of Mathematics Slovak Academy of Sciences, Slovakia
81. Lijia Ding, Zhengzhou University, China
82. Slavisa Djordjevic, Benemerita Universidad Autinoma De Puebla, Mexico
83. Alexander Dobrick, CAU Kiel, Germany
84. Venku Naidu Dogga, Iit Hyderabad, India
85. Pavel Dubovski, Stevens Institute of Technology, United States
86. Rolandi Duduchava, University of Georgia & A. Razmadze Mathematical Institute, Georgia
87. Jennifer Duncan, ICMAT, Spain
88. Michela Egidi, University of Rostock, Germany
89. Benjamin Eichinger, TU Wien, Austria
90. Christian Emmel, Stockholm University, Sweden
91. Per Enflo, Kent State University, United States
92. Sirkka-Liisa Eriksson, University of Helsinki, Finland
93. Connor Evans, Newcastle University, United Kingdom
94. George Exner, Bucknell University, United States
95. Pavel Exner, Czech Academy of Sciences, Czech Republic
96. Luisa Faella, University of Cassino and Southern Lazio, Italy
97. Junmei Fan, Peking University, China
98. Carla Farsi, University of Colorado Boulder, United States
99. Christopher Felder, Indiana University, United States
100. Arran Fernandez, Eastern Mediterranean University, Turkey

101. Francesco Ferraresso, University of Sassari, Italy
102. Milton Ferreira, Polytechnic of Leiria, Portugal
103. Lawrence Fialkow, SUNY New Paltz, United States
104. Robert Fulsche, Leibniz University Hannover, Germany
105. Krishna Kumar G., University of Kerala, India
106. Eva Gallardo Gutierrez, Complutense University of Madrid, Spain
107. Priyanga Ganesan, University of California San Diego, United States
108. Stephan Ramon Garcia, Pomona College, United States
109. Yufan Ge, Leiden University, Netherlands
110. Fereidoun Ghahramani, University of Manitoba, Canada
111. Soumitra Ghara, Indian Institute of Technology Kanpur, India
112. Roozbeh Gharakhloo, University of California, Santa Cruz, United States
113. Clifford Gilmore, Université Clermont Auvergne, France
114. Ramesh Golla, Indian Institute of Technology Hyderabad, India
115. Delfina Gómez, Universidad de Cantabria, Spain
116. Debashish Goswami, Indian Statistical Institute Kolkata, India
117. Raffael Hagger, Christian-Albrechts-Universität zu Kiel, Germany
118. Simon Halvdansson, NTNU Norwegian University of Science and Technology, Norway
119. Andreas Hartmann, Université De Bordeaux, France
120. Michael Hartz, Saarland University, Germany
121. Seppo Hassi, University of Vaasa, Finland
122. Haakan Hedenmalm, KTH Royal Institute of Technology, Sweden
123. Bill Helton, UC San Diego, United States
124. Rainer Hempel, TU Braunschweig, Germany
125. Sarah Hess, University of Konstanz, Germany
126. Dorothea Hinsen, TU Berlin, Germany
127. Jan Holland, Birkhäuser, Germany
128. Julian Hölz, University of Wuppertal, Germany
129. Jason Howell, State University of New York at Albany, United States
130. Kate Howell, State University of New York at Albany, United States
131. Juha-Matti Huusko, University of Eastern Finland, Finland
132. Tuomas Hytönen, University of Helsinki, Finland
133. Timo Hänninen, University of Helsinki, Finland
134. Christian Ikenmeyer, University of Warwick, United Kingdom
135. Atsuhide Ishida, Tokyo University of Science, Japan
136. Joshua Isralowitz, University At Albany, Suny, United States
137. Alexander Its, IUPUI, Indianapolis, United States
138. Elizabeth Its, IUPUI, Indianapolis, United States
139. Shubham Jain, Indian Institute of Technology, Kanpur, Uttar-Pradesh, India
140. Philippe Jaming, Université De Bordeaux, France
141. Aref Jeribi, University of Sfax, Tunisia, Tunisia
142. Kui Ji, Hebei Normal University, China
143. Muyan Jiang, UC Berkeley, United States
144. Abhay Jindal, Indian Institute of Science, Bengaluru, India
145. Michael Jury, University of Florida, United States
146. Jens Kaad, University of Southern Denmark, Denmark
147. H. Turgay Kaptanoglu, Bilkent University, Turkey
148. Hasan Kara, Duzce University, Turkey
149. Illia Karabash, University of Bonn, Germany
150. Oleksiy Karlovych, NOVA University Lisbon, Portugal
151. Yuriy Karlovych, Universidad Autónoma del Estado de Morelos, Mexico
152. Dmitry Khavinson, University of South Florida, United States

153. Alexander Kheifets, University of Massachusetts Lowell, United States
154. Larysa Khilkova, Fraunhofer Itwm, Germany
155. Andrii Khrabustovskyi, University of Hradec Králové, Czech Republic
156. Joonhyung Kim, Chungnam National University, South Korea
157. Sumin Kim, Sungkyunkwan University, South Korea
158. David Kimsey, Newcastle University, United Kingdom
159. Ran Kiri, Technion, Israel
160. Jukka Kiukas, Aberystwyth University, United Kingdom
161. Tillmann Kleiner, University of Stuttgart, Germany
162. Perry Kleinhenz, Michigan State University, United States
163. Igor Klep, University of Ljubljana, Slovenia
164. Katharina Klioba, Hamburg University of Technology, Germany
165. Armand Koenig, Université Toulouse Iii - Paul Sabatier, France
166. Damian Kołaczek, University of Agriculture in Kraków, Poland
167. Shinji Koshida, Aalto University, Finland
168. Aleksey Kostenko, University of Ljubljana and University of Vienna, Austria
169. Elena Koucherik, University of Missouri, United States
170. Athanasios Kouroupis, NTNU, Norway
171. Sebastian Król, Adam Mickiewicz University, Poznan, Poland
172. Karsten Kruse, Hamburg University of Technology, Germany
173. Poornendu Kumar, IISc Bangalore, India
174. Rakesh Kumar, Indian Institute of Technology Jodhpur, India, India
175. Mario Kummer, TU Dresden, Germany
176. Antti Kupiainen, University of Helsinki, Finland
177. Emma-Karoliina Kurki, Aalto University, Finland
178. Mikael Kurula, Åbo Akademi, Finland
179. Bojan Kuzma, University of Primorska, Slovenia
180. Hyun-Kyoung Kwon, University at Albany, SUNY, United States
181. Kalle Kytölä, Aalto University, Finland
182. Andre Laestadius, Oslo Metropolitan University, Norway
183. Giuseppe Lamberti, IMB, France
184. Pier Domenico Lamberti, University of Padova, Italy
185. Giovanni Landi, Trieste University, Italy
186. Massimo Lanza de Cristoforis, Universita' degli Studi di Padova, Italy
187. Yuri Latushkin, University of Missouri, United States
188. Aapo Laukkarinen, University of Helsinki, Finland
189. Matti Lassas, University of Helsinki, Finland
190. Trieu Le, The University of Toledo, United States
191. Jérôme Le Rousseau, Université Sorbonne Paris Nord, France
192. Mee-Jung Lee, Kookmin University, South Korea
193. Malte Leimbach, Radboud Universiteit Nijmegen, Netherlands
194. Andrei Lerner, Bar-ilan University, Israel
195. Ji Li, Macquarie University, Australia
196. Adem Limani, Universitat Autònoma De Barcelona, Spain
197. Yi-hsuan Lin, National Yang Ming Chiao Tung University, Taiwan
198. Mikael Lindström, Åbo Akademi University, Finland
199. Adrián Llinares, Umeå University, Sweden
200. Emiel Lorist, TU Delft, Netherlands
201. Vladimir Lotoreichik, Czech Academy of Sciences, Czech Republic
202. Benjamin Lovitz, Northeastern University, United States
203. Franz Luef, Norwegian University of Science and Technology, Norway
204. Annemarie Luger, Stockholm University, Sweden

205. Zinaida Lykova, Newcastle University, United Kingdom
206. Pan Ma, Central South University, China
207. Subhankar Mahapatra, Indian Institute of Technology Ropar, India
208. Alejandro Mahillo Cazorla, Universidad De Zaragoza, Spain
209. Satyabrata Majee, Indian Institute of Technology Roorkee, India
210. Rahul Majethia, Shiv Nadar Institution Of Eminence, Delhi-NCR, India
211. Amit Maji, Indian Institute of Technology Roorkee, India
212. Visu Makam, Radix Trading Europe B. V., Netherlands
213. Bartosz Malman, Mälardalen University, Sweden
214. Myrto Manolaki, University College Dublin, Ireland
215. Laurent Marcoux, University of Waterloo, Canada
216. Marco Marletta, Cardiff University, United Kingdom
217. Konstantinos Maronikolakis, University College Dublin, Ireland
218. Henri Martikainen, Washington University in St. Louis, United States
219. Robert Martin, University of Manitoba, Canada
220. Javad Mashreghi, Laval University, Canada
221. Scott Mason, University of Cambridge, Australia
222. Dorothy Mazlum, Birkhäuser, Germany
223. John McCarthy, Washington University, United States
224. Ken McLaughlin, Tulane University, United States
225. Henry Mcnulty, NTNU, Norway
226. Jonadab Nkemdilim Mekwunye, Benevolent Technical Senior Secondary School, Gambia
227. Bram Mesland, Leiden University, Netherlands
228. Santeri Miihkinen, University of Reading, United Kingdom
229. Peter Miller, University of Michigan, United States
230. Mishko Mitkovski, Clemson University, United States
231. Nedra Moalla, University of Sfax, Tunisia, Tunisia
232. Hakim Monaim, Moulay Ismail University, Morocco
233. Alessandro Monguzzi, University of Bergamo, Italy
234. Alfonso Montes-Rodríguez, University of Seville, Spain
235. Riccardo Morandin, TU Berlin, Germany
236. Kent Morrison, American Institute of Mathematics, United States
237. Ivan Moyano, Université Côte D'azur, France
238. Anna Muranova, University of Warmia and Mazury in Olsztyn, Poland
239. Lady Estefania Murcia Lozano, Cinvestav, Mexico
240. Kim Myyryläinen, Aalto University, Finland
241. Chi-Keung Ng, Nankai University, China
242. Thu Hien Nguyen, Julius-Maximilians-Universität Würzburg, Ukraine
243. Lars Niedorf, Kiel University, Germany
244. Zoe Nieraeth, BCAM, Spain
245. Evangelos Nikitopoulos, University of California San Diego, United States
246. David Norrbo, Åbo Akademi University, Finland
247. Medet Nursultanov, University of Helsinki, Finland
248. Ryan O'Loughlin, University of Leeds, United Kingdom
249. Tuomas Oikari, University of Jyväskylä, Finland
250. Joona Oikarinen, Universität Wien, Austria
251. Lauri Oksanen, University of Helsinki, Finland
252. Marcu-Antone Orsoni, University of Toronto, Canada
253. Tuomas Orponen, University of Jyväskylä, Finland
254. Zhidong Pan, Saginaw Valley State University, United States
255. Konstantin Pankrashkin, University of Oldenburg, Germany
256. Inyoung Park, Ewha Women's University, South Korea

257. Soohyun Park, Pusan National University, South Korea
258. James Pascoe, Drexel University, United States
259. Lassi Paunonen, Tampere University, Finland
260. Giulio Pecorella, University of Modena and Reggio Emilia, Italy
261. Marco Peloso, Università Degli Studi Di Milano, Italy
262. Hemant Pendharkar, University of South Florda, United States
263. Antti Perälä, Umeå University, Sweden
264. Carlos Perez, Univeristy of the Basque Country and BCAM, Spain
265. Maria-Eugenia Pérez-Martínez, Universidad De Cantabria, Spain
266. Carmen Perugia, Department of Science and Technology, University Of Sannio, Italy
267. Stefanie Petermichl, University of Würzburg, Germany
268. Christine Pfeuffer, Martin-Luther-Universität Halle-Wittenberg, Germany
269. Paolo Piazza, Sapienza Universita' Di Roma, Italy
270. Pawel Pietrzycki, Jagiellonian University, Poland
271. Piotr Pikul, Jagiellonian University In Kraków, Poland
272. Gregor Podlogar, University of Ljubljana, Slovenia
273. Gabriel Prajitura, Suny Brockport, United States
274. Paramita Pramanick, Indian Institute of Technology Kanpur, India
275. Andrew Pritchard, Newcastle University, United Kingdom
276. Marek Ptak, University of Agriculture in Krakow, Poland
277. Lassi Päivärinta, Tallinn University of Technology, Estonia
278. Raul Quiroga-Barranco, Cimat, Mexico
279. Andre Ran, Vrije Universiteit Amsterdam, Netherlands
280. Shubham Rastogi, Indian Institute of Science, Bangalore, India
281. Jouni Rättyä, Itä-Suomen yliopisto, Finland
282. Maria Carmen Reguera, University of Malaga, Spain
283. Jakob Reiffenstein, University of Vienna, Austria
284. Jana Reker, IST Austria, Austria
285. Guillermo Rey, Uam, Spain
286. Cordian Riener, UiT - The Arctic University of Norway, Norway
287. Maria Manuela Rodrigues, University of Aveiro, Portugal
288. Jonathan Rohleder, Stockholm University, Sweden
289. Floris Roodenburg, TU Delft, Netherlands
290. Marcus Rosenberg, University of Helsinki, Finland
291. William Ross, University of Richmond, United States
292. Jenia Rousseva, University of Michigan, United States
293. Olli Saari, Universitat Politècnica de Catalunya, Spain
294. Alexander Sakhnovich, University of Vienna, Austria
295. Eero Saksman, University of Helsinki, Finland
296. Mikko Salo, University of Jyväskylä, Finland
297. Jeet Sampat, Technion - Israel Institute of Technology, Israel
298. Helge Jørgen Samuelsen, Norwegian University of Science and Technology, Norway
299. Armando Sánchez-Nungaray, Universidad Veracruzana, Mexico
300. Jarosław Sarnowski, Nicolaus Copernicus University In Toruń, Poland
301. Haripada Sau, Indian Institute of Science Education and Research Pune, India
302. Gideon Schechtman, Weizmann Institute, Israel
303. Marcel Scherer, University of Saarland, Germany
304. Roland Schnaubelt, Karlsruhe Institute of Technology, Germany
305. Kevin Schnelli, KTH Royal Institute of Technology, Sweden
306. Christian Seifert, Hamburg University of Technology, Germany
307. David Seifert, Newcastle University, United Kingdom
308. Michio Seto, National Defense Academy, Japan

309. Irina Shafkulovska, University of Vienna, Austria
310. Serikbol Shaimardan, Ghent University, Belgium
311. Eli Shamovich, Ben-Gurion University of the Negev, Israel
312. Mitra Shamsabadi, Acoustics Research Institute, Austrian Academy of Sciences, Austria
313. Arick Shao, Queen Mary University of London, United Kingdom
314. Eugene Shargorodsky, King's College London, United Kingdom
315. Samuli Siltanen, University of Helsinki, Finland
316. Barry Simon, Caltech, United States
317. Roman Simon Hilscher, Masaryk University, Czech Republic
318. Apoorva Singh, Shiv Nadar Institution of Eminence, India
319. Ritika Singhal, Indian Institute of Technology Delhi, New Delhi, India
320. Sushil Singla, University of Primorska, Slovenia, Slovenia
321. Jaakko Sinko, University of Helsinki, Finland
322. Klemen Sivic, University of Ljubljana, Slovenia
323. Nathanael Skrepek, TU Freiberg, Germany
324. Lenka Slavíková, Charles University, Czech Republic
325. Baruch Solel, Technion, Israel
326. Sinem Yelda Sönmez, Sabancı University, Turkey
327. Stephen Sontz, Centro De Investigacion En Matematicas (cimat), Mexico
328. Hanna Britt Soots, University of Tartu, Estonia
329. Michael Speckbacher, University of Vienna, Austria
330. Olof Staffans, Åbo Akademi University, Finland
331. Michael Stessin, SUNY At Albany, United States
332. Cody Stockdale, Clemson University, United States
333. Selim Sukhtaiev, Auburn University, United States
334. Alim Sukhtayev, Miami University, United States
335. Mansi Suryawanshi, Indian Statistical Institute, Bangalore, India
336. Franciszek Hugon Szafraniec, Jagiellonian University, Poland
337. Yukihide Tadano, Tokyo University of Science, Japan
338. Jari Taskinen, University of Helsinki, Finland
339. Matthias Täufer, Fernuniversität In Hagen, Germany
340. Adi Tcaciuc, MacEwan University, Canada
341. George Tephnadze, University of Georgia, Georgia
342. Sanne ter Horst, North West University, South Africa
343. Prasad Thankarajan, University of Calicut, Kerala-673635, India, India
344. Hannah Thornton, CA, United States
345. Yuriy Tomilov, IM PAN, Warsaw, Poland
346. Sebastian Toth, Saarland University, Germany
347. Luis Manuel Tovar, Instituto Politécnico Nacional, Mexico
348. Camillo Trapani, Università Di Palermo, Italy
349. Christiane Tretter, University of Bern, Switzerland
350. Sascha Trostorff, CAU Kiel, Germany
351. Medea Tsaava, The University of Georgia, Georgia
352. Georgios Tsikalas, Washington University in St. Louis, United States
353. Kanat Tulenov, Ghent University, Kazakhstan
354. Giorgi Tutberidze, The University of Georgia, Georgia
355. Margarita Tutberidze, The University of Georgia, Georgia
356. Hans-Olav Tylli, University of Helsinki, Finland
357. Teemu Tyni, University of Toronto, Canada
358. Leo Tzou, University of Amsterdam, Netherlands
359. Gunther Uhlmann, University of Washington, Seattle, United States
360. Antti Vähäkangas, University of Jyväskylä, Finland

361. Koen Van Den Dungen, University of Bonn, Germany
362. Jan van Neerven, Delft University of Technology, Netherlands
363. Walter Van Suijlekom, Radboud University, Netherlands
364. Nicolas Vanspranghe, Tampere University, Finland
365. Zurab Vashakidze, University of Georgia, Georgia
366. Igor Verbitsky, University of Missouri, United States
367. Nelson Vieira, CIDMA - University of Aveiro, Portugal
368. Jani Virtanen, University of Reading and University of Helsinki, England and Finland
369. A. Virtanen, Imperial College London, United Kingdom
370. Alexander Volberg, Michigan State University, United States
371. Jurij Volcic, Drexel University, United States
372. Dragan Vukotic, Universidad Autonoma De Madrid, Spain
373. Emil Vuorinen, University of Helsinki, Finland
374. Nathan Wagner, Brown University, United States
375. Kai Wang, School of Mathematical Sciences, Fudan University, China
376. Yi Wang, Chongqing University, China
377. Gerald Wanjala, Sultan Qaboos University, Oman
378. Christian Webb, University of Helsinki, Finland
379. Lutz Weis, KIT- University of Karlsruhe, Germany
380. Ferenc Weisz, Eötvös Loránd University, Hungary
381. Brett D. Wick, Washington University in St. Louis, United States
382. Alexander Wierzba, University of Twente, Netherlands
383. Henrik Wirzenius, University of Helsinki, Finland
384. Hugo Woerdeman, Drexel University, United States
385. Michał Wojtylak, Jagiellonian University, Poland
386. Ian Wood, University of Kent, United Kingdom
387. Błażej Wróbel, Institute of Mathematics of the Polish Academy of Sciences, Poland
388. Fanglei Wu, University of Eastern Finland, Finland
389. Seonguk Yoo, Gyeongsang National University, South Korea
390. Giorgio Young, University of Michigan, United States
391. Nicholas Young, Newcastle University, U.K., United Kingdom
392. Jiayang Yu, School of Mathematics, Sichuan University, China
393. Aljaž Zalar, University of Ljubljana, Slovenia
394. Jacopo Zanchettin, Sissa, Italy
395. Sophie Emma Zegers, Charles University, Prague, Czech Republic
396. Petr Zemanek, Masaryk University, Czech Republic
397. Elhassan Zerouali, Mohammed V University In Rabat, Morocco
398. Yong Zhang, University of Manitoba, Canada
399. Zhan Zhang, University of Helsinki, Finland
400. Chaofeng Zhu, Chern Institute of Mathematics, Nankai University, China
401. Kehe Zhu, State University of New York at Albany, United States
402. Xiao Zhong, University of Helsinki, Finland
403. Bertin Zinsou, University of the Witwatersrand, South Africa
404. Nina Zorboska, University of Manitoba, Canada

7. Glossary and Short Descriptions of University Buildings

Conference Buildings

P	Porthania at Yliopistonkatu 3
L	Language Center at Fabianinkatu 26
M	Metsätalo at Unioninkatu 40

Social Program Locations

Sipuli	Restaurant at Kanavaranta 7
Market	Ferries to Suomenlinna at Market Square
City Hall	Reception at Pohjoisesplanadi 11-13

Days

M	Monday
Tu	Tuesday
W	Wednesday
Th	Thursday
F	Friday

Below are the descriptions of the three conference buildings and also of the Exactum building that houses the Mathematics Department in the Kumpula Campus.

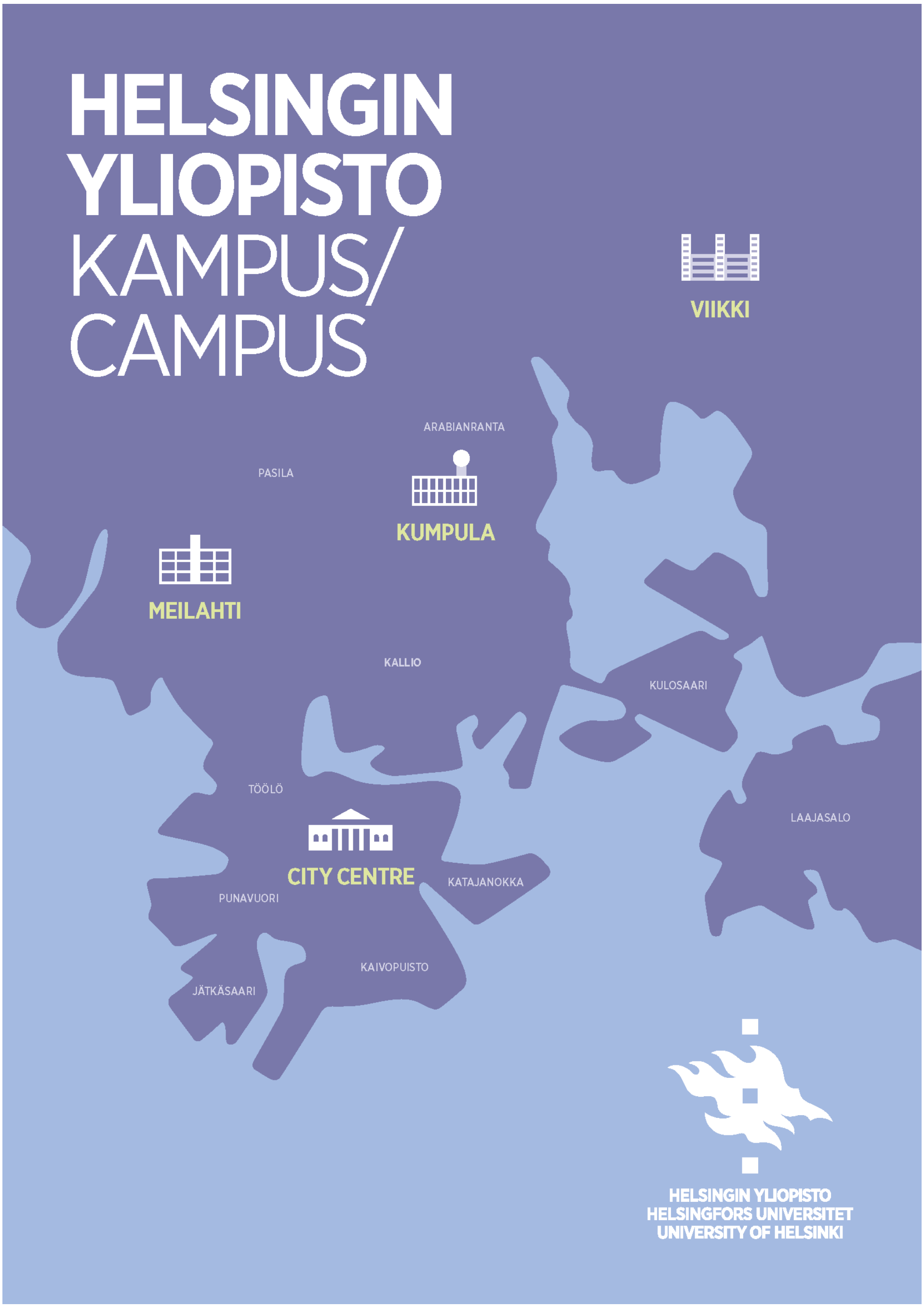
Completed in 1957, the Porthania building was named after Henrik Gabriel Porthan, a professor at the Royal Academy of Turku. The building is one of the major works of architect Aarne Ervi. Many of the technical solutions were brand-new: for example, concrete elements had never before been as extensively used in Finland. The preservation of the original architecture was the goal of a renovation completed in 2006.

Metsätalo was completed in 1939 for the University of Helsinki's forestry departments and the Finnish Forest Research Institute (Metla). Designed by architect Jussi Paatela, the building merges classicism and functionalism. The decor features different types of wood, including unusual tree species. The University's forestry departments relocated to Viikki in the late 1990s and early 2000s, and Metla vacated its wing in 2008. The facilities are now used by the language disciplines of the Faculty of Arts.

Designed by Kauno S. Kallio, the building at Fabianinkatu 26 was completed in 1907 with typical national romantic details. The facilities were used by the Suomen Liikemiesten Kauppaopisto business college until the late 1970s when the University took them over. The history of the building is still visible in its main entrance, which is decorated by the staff of Mercury, the symbol of the business college. The building was renovated in 2009 and is now used by the University's Language Centre.

Exactum, designed by Rainer Malkamäki and located on Kumpula Hill, was completed in the mid-2000s. It accommodates students, teachers and researchers of computer science, geosciences, geography, mathematics and statistics. Prior to moving to the Kumpula Campus, the Mathematics Department was situated next to the Porthania Building.

8. Maps of the City Center Campus



KESKUSTAKAMPUS
CENTRUMCAMPUS
CITY CENTRE CAMPUS

HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

KESKUSTAKAMPUS
CENTRUMCAMPUS
CITY CENTRE CAMPUS

1 Päärakennus / Huvudbyggnaden / Main Building
Fabianinkatu 30

2 Tiedekulma / Tankehörnan / Think Corner
Yliopistonkatu 4

IWOTA 3 Porthania
Yliopistonkatu 3

4 Kansalliskirjasto / Nationalbiblioteket / The National Library of Finland
Unioninkatu 36

5 Fabianinkatu 24

IWOTA 6 Kielikeskus / Språkcentrum / Language Centre
Fabianinkatu 26

7 Aleksandria
Fabianinkatu 28

8 Pääkirjasto / Huvudbiblioteket / Main library
Fabianinkatu 30

9 Topelia
Unioninkatu 38

IWOTA 10 Metsätalo / Forsthuset
Unioninkatu 40

11 Unioninkadun juhlahuoneistot
Unionsgatans festlokaler
Unioninkatu banqueting rooms
Unioninkatu 33

12 Snellmaninkatu 10

13 Unioninkatu 35

14 Svenska social- och kommunal högskolan / Swedish School for Social Science /
Snellmaninkatu 12

15 Snellmania /
Unioninkatu 37

16 Snellmaninkatu 14

17 Aurora /
Siltavuorenpenger 10

18 Minerva (uusi rakennus / ny byggnad / new building)
Siltavuorenpenger 5 A

19 Minerva (vanha rakennus / gammal byggnad / old building)
Siltavuorenpenger 5 A

20 Athena /
Siltavuorenpenger 3

21 Psychologicum
Siltavuorenpenger 1

VIERAILUKOHTEET
PLATSER ATT BESÖKA
ATTRACTIONS

Kaisaniemen kasvitieteellinen puutarha
Kajsaniemi botaniska trädgård
Kaisaniemi Botanic Garden
Fabianinkatu 30

Luonnontieteellinen museo (LUOMUS)
Naturhistoriska museet
Natural History Museum
Pohjoinen Rautatiekatu 13

Yliopistomuseo
Universitetsmuseum
University museum
Fabianinkatu 33

Observatorio
Observatory
Observatorium
Kopernikuksentie 1

KADUT
GATORNA
STREETS

Pitkäsilta / Långa bron
Unioninkatu / Unionsgatan
Vilhonkatu / Vilhelmsgatan
Kaisaniemenkatu / Kajsaniemigatan
Vuorikatu / Berggatan
Fabianinkatu / Fabiansgatan
Snellmaninkatu / Snellmansgatan
Siltavuorenpenger / Brobergsterrassen
Liisankatu / Elisabetsgatan
Yrjö Koskisen katu / Yrjö Koskinens gata
Rauhankatu / Fredsgatan
Kirkkokatu / Kyrkogatan
Yliopistonkatu / Universitetsgatan
Aleksanterinkatu / Alexandersgatan

Esteetön sisäänkäynti / Trapplösa ingången / Accessible entrance