ABSTRACTS

Lipschitz stable determination of polyhedral conductivity inclusions from boundary measurements

Elena Beretta

I will consider the problem of determining a polyhedral conductivity inclusion embedded in a homogeneous isotropic medium from boundary measurements and prove global Lipschitz stability for the polyhedral in- clusion from the local Dirichlet-to-Neumann map extending the results obtained previously in the two-dimensional case to the three-dimensional setting. This is a joint work with Andrea Aspri, Elisa Francini and Sergio Vessella.

Scattering from corners and other singularities

Emilia Blåsten

My previous work showed that in potential scattering, corners produce patterns in the far-field which cannot be cancelled by any other structure nearby or far away. This led to interesting finds such as unique shape determination of polyhedral or pixelated scattering potentials by the far-field made by any single incident wave. It also led to the study of how geometry of the domain affects the distribution of energy of the transmission eigenfunctions. Complete understanding is still away, and different geometrical configurations are being studied. In this talk I present shortly past results and also newer results related to general conical singularities and flat scattering screens.

The transport Oka-Grauert principle for simple surfaces

Jan Bohr

In inverse problems, a typical question asks to characterise the range of the underlying 'forward map'. I will address this question for a class of nonlinear inverse problems on simple Riemannian surfaces and describe a novel range characterisation that is reminiscent of the Ward correspondence for anti-self-dual Yang-Mills fields, but without solitonic degrees of freedom. The range characterisation turns out to be equivalent, via a novel twistor correspondence, to a non-existence theorem for holomorphic vector bundles on certain complex surfaces, resembling the classical Oka-Grauert theorem. This is joint work with Gabriel Paternain, arXiv:2108.05125.

On non-scattering wave numbers

Fioralba Cakoni

We examine necessary conditions for an inhomogeneous medium to permit non-scattering wave numbers, that is wave numbers for which there exist incident waves that render this medium invisible. Non-scattering wave numbers are subset (possibly empty) of the transmission eigenvalues. We show that the existence of non-scattering wave numbers (unlike the existence of transmission eigenvalues) generically imply a certain regularity of the inhomogeneity. We investigate this question for a large class of inhomogeneities. This talk is based on joint works with Michael Vogelius and Jingni Xiao.

An inverse boundary value problem for isotropic nonautonomous heat flows

Ali Feizmohammadi

We study an inverse boundary value problem on the determination of principal order coefficients in isotropic nonautonomous heat flows stated as follows; given a medium, and in the absence of heat sources and sinks, can the time-dependent thermal conductivity and volumetric heat capacity of the medium be uniquely determined from the Cauchy data of temperature and heat flux measurements on its boundary? We prove uniqueness in all dimensions under an assumption on the thermal diffusivity of the medium, which is defined as the ratio of the thermal conductivity and volumetric heat capacity.

Directional antilocality and Calderón inverse problems

María Ángeles García-Ferrero

The Calderón problem for the fractional Schrödinger equation, introduced by T. Ghosh, M. Salo and G. Uhlmann, satisfies global uniqueness with only one single measurement. This result exploits the antilocality property of the fractional Laplacian, that is, if a function and its fractional Laplacian vanish in a subset, then the function is zero everywhere.

Nonlocal operators which only depend on the function in some directions and not on the whole space cannot satisfy an analogous antilocality property. In theses cases, only directional antilocality conditions may be expected.

In this talk, we will consider antilocality in cones, introduced by Y. Ishikawa in the 80s, and its possible implications in the corresponding Calderón problem. Namely, we will study direct and inverse problems for stable, elliptic, nonlocal operators whose kernels are supported on cones and which satisfy a directional antilocality condition. In particular, we will focus on uniqueness results for the associated Calderón problem from both infinitely many and single measurements and on the new geometric restrictions which arise because of the antilocality only in cones.

This is a joint work with G. Covi and A. Rüland.

Scattering and lens rigidity in negative curvature

Colin Guillarmou

I will discuss recent results on scattering and lens rigidity for the geodesic flow of negatively curved metrics on compact manifolds with boundary. The talk is based on joint works with Bonthonneau, Jezequel and with Cekic, Lefeuvre.

Inverse problems with neutrinos

Joonas Ilmavirta

Neutrinos are very weakly interacting particles that have unique properties that allow indirect measurements that cannot be realized with any other particle or field. I will give an introduction to neutrino physics and describe two inverse problems with neutrinos. This is based on joint work with Gunther Uhlmann.

The Calderón problem for quasilinear conductivities

Yavar Kian

In this talk, we will consider the Calderón inverse problem of determining uniquely a quasilinear isotropic conductivity appearing in an elliptic equation on a bounded open set. More precisely, we consider the problem of determining a general quasi-linear conductivity depending simultaneously on the space variable, the solutions and the gradient of the solutions of an elliptic non-linear equation. We give a positive answer to this problem for some general class of conductivities subjected to some analytic dependency with respect to the gradient of the solutions of the equation. This talk is based on a joint work with Catalin Carstea, Ali Feizmohammadi, Katya Krupchyk and Gunther Uhlmann.

Stability estimates for an inverse problem for a hyperbolic PDE with space- and time-dependent coefficients

Venkateswaran Krishnan

We derive Lipschitz stability estimates for a formally determined inverse problem for a hyperbolic PDE with first and zeroth order coefficients dependent on space and time variables. The estimates are derived based on a modification of the Bukhgeim-Klibanov method which relies on Carleman estimates.

On isospectral connections

Thibault Lefeuvre

Kac's celebrated inverse spectral question ("Can one hear the shape of a drum?") on a closed Riemannian manifold consists in recovering a metric from the knowledge of the spectrum of its Laplacian. I will discuss a very similar question on negatively-curved manifolds, where the word "metric" is now replaced by "connection" on a vector bundle. In the low-rank range, I will explain a positive answer to Kac's inverse spectral problem. The results are based on a new method combining three distinct fields: algebraic topology (and number theory) through the notion of polynomial structures over spheres, partially hyperbolic dynamical systems, and microlocal analysis.

Inverse problems for nonlinear models

Tony Liimatainen

We discuss recent results regarding inverse problems for nonlinear models. The discussion will be kept relatively simple and focus on explaining how the nonlinearity can be used as a beneficial tool. Especially we will discuss an inverse problem for Riemannian minimal surfaces and inverse source problems. The talk is based on joint works with Catalin Carstea, Matti Lassas, Yi-Hsuan Lin and Lauri Oksanen.

Mapping properties of X-ray transforms near convex boundaries

Francois Monard

On a Riemannian manifold with boundary, the X-ray transform integrates a function or a tensor field along all geodesics through the manifold. The reconstruction of the integrand of interest from its X-ray transform is the basis of important inverse problems with applications to seismology and medical imaging.

The inversion of the X-ray transform is often done by inverting the normal operator (composition of the X-ray transform and its adjoint, the "backprojection" operator). The inversion problem includes the design of appropriate function spaces where to formulate forward and backward mapping properties of the X-ray transform, the backprojection operator, and their composites. Such spaces need to incorporate boundary behavior, and include Frechet spaces of 'polyhomogeneous conormal' type, or non-standard Sobolev scales (e.g., transmission spaces a la Hormander, or modeled after degenerate elliptic operator of Kimura type). In the design of such spaces, a landmark result by Pestov and Uhlmann, paving the way toward their 2005 result on the boundary rigidity of simple surfaces, was the design of a good Frechet setting where the backprojection operator is surjective.

In this talk, I will review recent results attempting to shed additional light on the (forward and backward) mapping properties of the X-ray transform and its normal operator(s) on convex, non-trapping manifolds. I will discuss recent joint works with Gabriel Paternain and Richard Nickl; Rafe Mazzeo; Rohit Mishra and Joey Zou.

Remarks on the Calderón problem

Lauri Oksanen

We discuss recent results on the Calderón problem on a Riemannian manifold, its Lorentzian version, and its analogue for non-linear elliptic equations. The talk is based on joint works with Spyros Alexakis, Cătălin Cârstea and Ali Feizmohammadi.

Gunther's work on geometric inverse problems in 2D

Gabriel Paternain

I will discuss some of the highlights of Gunther's work on geometric inverse problems in 2D with an eye towards recent developments.

The art of inverse problems

Samuli Siltanen

Mathematical inverse problems produce beautiful and curious images. But why? The starting point of an inverse problem is the desire to see something hidden. And not just that: we can only probe the hidden thing indirectly, gathering bits and pieces of information about it, and then hope to solve a mystery based on insufficient data and mathematical models of any extra information we might have about our mystery object. Three examples: (i) a doctor wants to see a slice image of a patient's cross section, but only has available a set of fan-beam X-ray data of her along multiple directions. (ii) A photojournalist took the best shot ever, but the camera was misfocused and it was quite dark. How to sharpen the noisy picture after the fact? (iii) Ancient navigators of the Marshall Islands in the Pacific Ocean learned to find their way by just feeling the waves. This skill, deeply founded in the mathematics of scattering, produced also a wonderful form of information graphics, or rather sculpture: stick diagrams of wave motion. This talk presents images coming from various inverse problems and discusses them both scientifically and aesthetically.

Nonlinear geometric optics and inverse problems for nonlinear wave equations

Plamen Stefanov

We study several wave type of nonlinear equations in the weakly nonlinear regime. Sending a single phase high frequency wave into the medium, and observing the transmitted one, we want to understand the structure of the transmitted one, its dependence on the nonlinearity, and whether we can recover the nonlinearity from such observations. We study semilinear wave equations and the quasilinear Westervelt equation modeling nonlinear acoustics. This talk is based on joint works with Antonio Sa Barreto and Nikolas Eptaminitakis.

A geometric user interface for analytic WF calculus for FIO

Leo Tzou

We find a geometric description for the propagation of regularity for a special class of elliptic FIOs in the analytic category. This can be seen as the initial step towards a calculus for analytic FIOs in the double fibration setting as in the smooth case. We describe various transforms in integral geometry that can fit into this setting.

Mathematics of magic angles

Maciej Zworski

Magic angles are a hot topic in condensed matter physics: when two sheets of graphene are twisted by those angles the resulting material is superconducting. I will present a very simple operator whose spectral properties are thought to determine which angles are magical. It comes from a 2019 PR Letter by Tarnopolsky–Kruchkov–Vishwan ath. The mathematics behind this is a blend of representation theory (of the Heisenberg group in characteristic three), Jacobi theta functions and microlocal analysis (Hörmander's bracket condition and analytic hypoellipticity). The results will be illustrated by colourful numerics which suggest many open problems (joint work with S Becker, M Embree, J Wittsten, 2020, S Becker, T Humbert 2022 and M Hitrik 2022).

Contributed talks

Monday 1.8. at 15.30–16.00

Divyansh Agrawal	Unique continuation results for Ray and Momentum Ray transforms on symmetric tensor fields
Duc-Lam Duong	Inverse problems for hyperbolic conservation laws - a Bayesian approach
Jaakko Kultima	Inverse scattering for quasi-linear biharmonic operator in 2D
Ping Liu	Mathematical foundation of sparsity-based multi-illumination super-resolution
Stephen McDowall	Luminescent solar concentrators
Oliver Petersen	Wave equations in subextremal Kerr-de Sitter spacetimes

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Tuesday 2.8. at 15.30–16.05

Pu-Zhao Kow	Non-scattering phenomena and quadrature domains
Antti Kykkänen	Pestov identities and X-ray tomography on non-smooth Riemannian manifolds
Shiqi Ma	An anisotropic inverse problem on certain Riemannian manifolds
Janne Nurminen	An inverse problem for the minimal surface equation
Suman Kumar Sahoo	The linearized Calderón problem for polyharmonic operators
Philipp Zimmermann	Inverse fractional conductivity problem
Joey Zou	The C^∞-isomorphism property for a class of singularly-weighted X-ray transforms

ABSTRACTS - contributed talks and posters

Unique continuation results for Ray and Momentum Ray transforms on symmetric tensor fields

Divyansh Agrawal (TIFR Centre for Applicable Mathematics)

Let $\mathcal{I}^k$ denote the first $(k+1)$-moments of a symmetric m-tensor field. It is known that $\mathcal{I}^k$ is injective over $(k+1)$-solenoidal fields, i.e., $\mathcal{I}^k f = 0$ iff $f = d^{k+1}v$ for some $(m-k-1)$-tensor field v. We focus on the partial data aspect of this problem: What if we only have information about $\mathcal{I}^k$ on a 'small' set? We prove that such information is also enough. Such a result was first proved for Ray Transform by Ilmavirta and Monkkonen for functions and vector fields ($m = 0$ and 1 respectively). This is based on a joint work with Prof. Venkateswaran P. Krishnan and Dr. Suman Kumar Sahoo.

Hölder Stability of the derivatives of the optical properties of an anisotropic medium at the boundary (Poster only)

Jason Curran (University of Limerick)

Diffuse Optical Tomography (DOT) is a promising medical imaging technique that shows potential to be a cheap, versatile alternative to other forms of medical imaging techniques or a good partner to already established medical imaging modalities [Hoshi et al - DOI: 10.1117/1.JBO.21.9.091312]. In the time harmonic case, DOT works by delivering near infrared light, at a fixed frequency $\omega = \frac{k}{c}$ a optical fibres, to an object's surface, where k is the wave number and c is the speed of light. Detectors on the body's surface then measure the propagation of the light through the object and using imaging reconstruction techniques, one can recover the optical properties (absorption coefficient - μ_a and scattering coefficient - μ_s) of the interior of the object.

Here we address the inverse problem in DOT of stably recovering the optical properties of an anisotropic medium $\Omega \subset \mathbb{R}^n$, for $n \geq 3$ under the so called diffusion approximation. Assuming that the scattering coefficient is known, we determine the absorption coefficient. Our argument is based on the construction of singular solutions of arbitrarily high order to the elliptic system governing the light propagation. We show Hölder stability for any derivative of the absorption coefficient at the boundary with respect to the Dirichlet-to-Neumann map.

Inverse problems for hyperbolic conservation laws - a Bayesian approach

Duc-Lam Duong (LUT University)

Many conservation laws in certain branches of physics (such as wave propagation, gas dynamics or traffic flow) are typically expressed by hyperbolic partial differential equations. With the presence of shockwaves, one needs special techniques to deal with these equations. We study the well-posedness and approximation of some inverse problems for hyperbolic conservation laws. More precisely, given observations of the entropy solution, we look at the problem of identifying the initial field (with known flux function) and the problem of recovering the flux function (with known initial field). Due to shockwaves, direct observations of the entropy solution are not "regulated" enough to fit in the Bayesian framework set out in Stuart (2010). To get round this, we consider the so-called trajectories for hyperbolic conservation laws and derive their existence and uniqueness using Filippov's theory. We prove that these trajectories are compatible with the front tracking approximation in the sense that the approximate trajectory given by this method converges uniformly. Moreover, for certain flux functions, illustrated by traffic flow, we obtain the convergence rate for the approximate trajectories with respect to changes in the initial field (or flux function). This rate is then translatable to the approximation of the posterior in the corresponding Bayesian inverse problems.

Non-scattering phenomena and quadrature domains

Pu-Zhao Kow (University of Jyväskylä)

We illuminate a penetrable obstacle by Herglotz wave (i.e. superposition of plane waves). If the obstacle has corner, then it must produce a unique non-trivial scattered wave, in other words "corners always scatter". In this talk, we focus on non-scattering domains. Such domains possibly have singularities on boundary, e.g., inward cusps and double points. It is notable that quadrature domains (for Helmholtz equation) are non-scattering. We also give an equivalent characterization of the Pompeiu problem (https://www.scilag.net/problem/G-180522.1). This talk is related to my recent work (https://arxiv.org/abs/2204.13934) with Simon Larson, Mikko Salo and Henrik Shahgholian.

Inverse scattering for quasi-linear biharmonic operator in 2D

Jaakko Kultima (University of Oulu)

We consider direct and inverse scattering problems for biharmonic operator in 2D. The bi-Laplacian (Δ^2) is perturbed by zero- and first-order quasi-linear perturbations. We discuss the solvability of the direct scattering problem and formulate the inverse problem of recovering certain combination of the perturbations from the knowledge of the scattering amplitude as scattering data. Saito's formula provides the uniqueness for this inverse problem, and in the case of full scattering data it can be directly inverted to obtain some numerical reconstructions. Finally, we will discuss back-scattering problem and present some reconstructions by using the method of Born approximation.

Pestov identities and X-ray tomography on non-smooth Riemannian manifolds

Antti Kykkänen (University of Jyväskylä)

How regular does a Riemannian metric have to be for the geodesic X-ray transform to be injective? A standard method for proving injectivity results for the geodesic X-ray transform is known as the Pestov identity method. We show that the same identity holds true on manifolds equipped with $C^{1,1}$ regular Riemannian metrics and we prove injectivity results on Riemannian manifolds that we call simple $C^{1,1}$ manifolds. When the Riemannian metric is C^{∞}-smooth the definition of a simple $C^{1,1}$ manifold is equivalent to the traditional definition of a simple manifold.

Mathematical foundation of sparsity-based multi-illumination super-resolution

Ping Liu (ETH Zurich)

It is well-known that the resolution of traditional optical imaging system is limited by the so-called Rayleigh resolution or diffraction limit, which is of several hundreds of nanometers. By employing fluorescence techniques, modern microscopic methods can resolve point scatterers separated by a distance much lower than the Rayleigh resolution limit. Localization-based fluorescence subwavelength imaging techniques such as PALM and STORM can achieve spatial resolution of several tens of nanometers. However, these techniques have limited temporal resolution as they require tens of thousands of exposures. Employing sparsity-based models and recovery algorithms is a natural way to reduce the number of exposures, and hence obtain high temporal resolution. Nevertheless, to date fluorescence techniques suffer from the trade-off between spatial and temporal resolutions.

Recently, a new multi-illumination imaging technique called Brownian Excitation Amplitude Modulation microscopy (BEAM) is introduced. BEAM achieves a threefold resolution improvement by applying a compressive sensing recovery algorithm over only few frames. Motivated by BEAM, our aim in this paper is to pioneer the mathematical foundation for sparsity-based multi-illumination super-resolution. More precisely, we consider several diffraction-limited images from sample exposed to different illumination patterns and recover the source by considering the sparsest solution. We estimate the minimum separation distance between point scatterers so that they could be stably recovered. By this estimation of the resolution of the sparsity recovery, we reveal the dependence of the resolution on the cut-off frequency of the imaging system, the signal-to-noise ratio, the sparsity of point scatterers, and the incoherence of illumination patterns. Our theory particularly highlights the importance of the high incoherence of illumination patterns in enhancing the resolution. It also demonstrates that super-resolution can be achieved using sparsity-based multi-illumination imaging with very few frames, whereby the spatio-temporal super-resolution becomes possible. BEAM can be viewed as the experimental realization of our theory, which is demonstrated to hold in both the one- and two-dimensional cases.

An anisotropic inverse problem on certain Riemannian manifolds

Shiqi Ma (University of Jyväskylä)

We consider an inverse problem of recovering a potential from the Dirichlet to Neumann map on certain general Riemannian manifolds. We extend the earlier results known for simple or conformally transversely anisotropic (CTA) manifolds.

Luminescent solar concentrators

Stephen McDowall (Western Washington University)

A luminescent solar concentrator (LSC) is a sheet of glass or plastic into which is embedded nanometer luminophores which absorb light over a range of wavelengths and re-emit light at red-shifted wavelengths. Most re-emitted light is waveguided by total internal reflection to the edges of the sheet where there are photovoltaic cells to convert captured light to electrical energy. While there are many challenges to making LSC's efficient enough to be useful, scattering is a major problem for achieving efficiency in large panels. When prototype LSC's are made there is uncertainty about what exactly has been fabricated. By making practically achievable measurements we seek to characterize the devices as accurately as possible and to determine the scattering and attenuation parameters of the device. We model photon transport within an LSC using a wavelength-dependent transport equation and solve the inverse problem of determining the material parameters from specific measurements. Analytically, the inverse problem yields a simple solution, however the types of measurements that can readily be made, and the potentially high level of noise in the measurements, make the problem more interesting.

An inverse problem for the minimal surface equation

Janne Nurminen (University of Jyväskylä)

In this work we study a novel inverse problem for the minimal surface equation, which is a quasilinear elliptic PDE. Consider a Riemannian manifold (M, g) where $M = \mathbb{R}^n$ and the metric is conformally Euclidean i.e. $g_{ij}(x) = c(x)\delta_{ij}$. Let $u : \Omega \subset \mathbb{R}^{n-1} \to \mathbb{R}$ be a smooth function that satisfies the minimal surface equation. Assume that we can make boundary measurements on the graph of u, that is we know the Dirichlet-to-Neumann (DN) map which maps the boundary value $u|_{\partial\Omega} = f$ to the normal derivative $\partial_\nu u|_{\partial\Omega}$. The Dirichlet data f is the height of minimal surface on the boundary. The normal derivative $\partial_\nu u|_{\partial\Omega}$ can be thought of as tension on the boundary caused by the minimal surface. In this talk we show that from the knowledge of the DN map we can determine information about the conformal factor. In particular, we can determine $\partial^k_{x_n} c(x', 0)$ for $k = 0, 1, \cdots,\ x' \in \mathbb{R}^{n-1}$. We use the method of higher order linearization that has received increasing attention lately.

Wave equations in subextremal Kerr-de Sitter spacetimes (Talk only)

Oliver Petersen (Uppsala University)

In 2013, Vasy proved that solutions to linear wave equations in Kerr-de Sitter spacetimes have asymptotic expansions in quasinormal modes up to an exponentially decaying term, assuming the angular momentum of the black hole satisfies certain bounds. This was the first step towards the proof of non-linear stability for slowly rotating Kerr-de Sitter black holes by Hintz and Vasy in 2018. In this talk, we briefly explain new ideas extending Vasy's result to the full subextremal range of Kerr-de Sitter spacetimes, by removing the extra conditions on the angular momentum of the black hole. This is joint work with Andras Vasy.

The linearized Calderón problem for polyharmonic operators

Suman Kumar Sahoo (University of Jyväskylä)

In this talk, we discuss a linearized Calderón problem for polyharmonic operators of order $2m(m \geq 2)$ in the spirit of Calderón's original work. We give a uniqueness result for determining coefficients of order $\leq 2m-1$ up to gauge, based on inverting momentum ray transforms. Momentum ray transforms are certain weighted ray transform which integrals symmetric tensor fields with a power t^k. This transforms were first introduced by Sharafutdinov. This is a joint work with Mikko Salo.

Inverse fractional conductivity problem

Philipp Zimmermann (ETH Zurich)

In this talk we survey recent results on the inverse fractional conductivity problem with partial data. The differences and similarities to the classical Calderón problem will be emphasised. We discuss the importance of the nonlocal structure underlying the problem. If time permits some possible future developments and open questions will be discussed.

The C^∞-isomorphism property for a class of singularly-weighted X-ray transforms

Joey Zou (University of California, Santa Cruz)

We consider the sharp mapping properties, particularly of behavior up to the boundary, of singularly-weighted normal operators associated to X-ray transforms on manifolds with boundary. We show that a particular family of weights on the Euclidean disk and on simple disks of constant curvature give rise to weighted normal operators which are isomorphisms on C^∞ (up to the boundary). The proof involves deriving the Singular Value Decomposition of a weighted X-ray transform and studying certain function spaces based on the singular vectors of the X-ray transform, which coincides with the eigenfunctions of a particular degenerately elliptic Kimura-type differential operator. Joint work with Rohit Kumar Mishra and Francois Monard.
