Recall

Conservation laws:

	strong	em	weak except	K^0, B^0, D^0
Isospin	+	no	no	no
P	+	+	no	no
C	+	+	no	no
T	+	+	+	no
CP	+	+	+	no
CPT	+	+	+	+
S	+	+	no	no
c				
⋮				

Quark model for hadrons

SU(2) isospin: symmetry concerning u, d

SU(3) flavour: u, d, s form the triplet representation

$\Rightarrow$ form hadrons $3 \times \bar{3}$ or $3 \times 3 \times 3$

two diagonal generators:
isospin, hypercharge

Need of new quantum number, colour:

Δ^{++}: $|uuu\rangle$ baryon $\Rightarrow$ as fermion must have totally antisymmetric wave function

$J = \frac{3}{2}$, $L = 0$ $\Rightarrow$ spins $|\uparrow\uparrow\uparrow\rangle$

$\therefore$ $\left.\begin{array}{l}\text{spin w.f. symmetric} \\ \text{space w.f. } \quad " \end{array}\right\} \Rightarrow$ colour w.f. antisymmetric

Quarks and gluons carry colour

$\Rightarrow$ <u>confined to colourless combinations in hadrons</u>

Magnetic moments of baryons

- measured for $\frac{1}{2}^+$ octet (except Σ^0), which have $L=0$
- quark magnetic moment ($\vec{B} \parallel \hat{z}$, $E = g\mu_q \vec{S}\cdot\vec{B} \overset{g=2}{=} \mu_q 2S_z B$)

$$\mu_q = e_q \frac{e}{2m_q} \equiv e_q \frac{M_p}{m_q} \mu_N \; ; \quad \mu_N = \frac{e}{2M_p}$$

= nuclear magneton

For $\Lambda = uds$, $S_{ud} = 0$ (x, next page)

$$\Rightarrow \mu_\Lambda = \mu_s = -\frac{1}{3} \frac{M_p}{m_s} \mu_N$$

For quark configurations aab, $S_{aa} = 1$, symmetric

For the proton

$$\mu_p = \sum_{i=1}^{3} \langle p\uparrow | \mu_i | p\uparrow \rangle$$

$$= \frac{1}{18}\Big[(\mu_u - \mu_u + \mu_d) + (-\mu_u + \mu_u + \mu_d) + 4(2\mu_u - \mu_d) \Big] \times 3$$

from permutations

$$= \frac{1}{3}(4\mu_u - \mu_d)$$

For the neutron $u \leftrightarrow d$

$$\mu_n = \frac{1}{3}(4\mu_d - \mu_u)$$

When $m_u = m_d \Rightarrow \mu_u = -2\mu_d \Rightarrow \frac{\mu_n}{\mu_p} = -\frac{2}{3}$

experimentally $\frac{\mu_n}{\mu_p} \simeq -0.685$

(*) Note: Total w.f. antisymmetric; colour w.f antisymmetric => rest = symmetric:

space × spin × flavour

↑

for light baryons $\ell = 0$ => symmetric

Two particles with quark content uds: Λ^0, Σ^0

Σ^0 belongs to isospin triplet with

$\underbrace{uu}s$

flavour w.f. symmetric => spin w.f. symmetric

=> $S = 1$

For the isospin multiplet only difference is $u \leftrightarrow d$

=> $S_{ud \text{ in } \Sigma^0} = 1$

Thus the remaining uds combination has $S_{ud \text{ in } \Lambda^0} = 0$

μ_Λ, μ_p agree with measurements for quark masses $m_s \simeq 510$ MeV, $m_{u,d} \simeq 336$ MeV

From these one finds predictions for the other magnetic moments of $\frac{1}{2}^+$ baryon octet [MS]:

	Prediction	experiment $[\mu_N]$
p	2.79	2.793
n	-1.86	-1.913
Λ	-0.61	-0.613 ± 0.004
Σ^+	2.69	2.42 ± 0.05
Σ^-	-1.04	-1.157 ± 0.025
Ξ^0	-1.44	-1.250 ± 0.014
Ξ^-	-0.51	-0.679 ± 0.031

Sources of differences: the states may slightly mix with $L>0$ states; they could also slightly mix with more complicated states, like $qqqq\bar{q}$ (pentaquark)

Heavy quarks

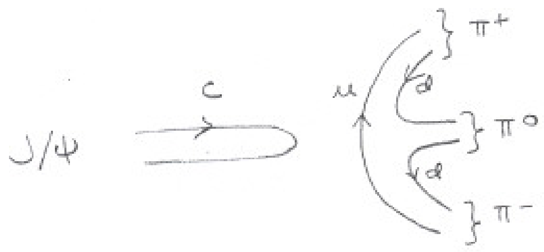
"The November revolution": J/ψ ($c\bar{c}$) was found at Brookhaven (Ting) and at SLAC (Richter) in 1974.
c-quark was known to be needed to solve some technical problems in weak interactions. J/ψ is a very narrow resonance ($m_{J/\psi} = 3.1$ GeV, $\Gamma_{J/\psi} = 69$ keV)

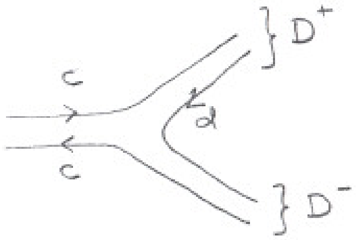
For heavier $c\bar{c}$-states than J/ψ the decay

$$\to D(\bar{q}c) + \bar{D}(q\bar{c}) \quad , \quad q = u, d$$

is possible, but for J/ψ this is inhibited by kinematics.

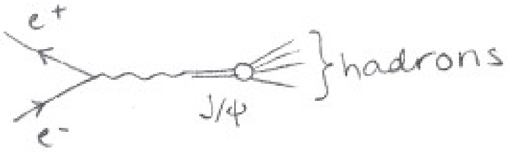
Instead, the hadronic decay $J/\psi \to \pi^+\pi^-\pi^0$ is possible:





<u>OZI-rule</u>: disconnected quark-line diagrams are highly suppressed when compared the connected ones.
(Okubo, Zweig, Iizuka)
$\Rightarrow$ long lifetime or narrow resonance

For J/ψ $J^{PC} = 1^{--}$, just like for photon. Main production mechanism in e^+e^- is:



Peak in the cross section, when $E_{e^+e^-} \simeq m_{J/\psi}$

<u>Bottomium</u> was found in 1977 (Lederman).
$\Upsilon(b\bar{b})$ ($m_\Upsilon \simeq 9460$ MeV, $\Gamma_\Upsilon \simeq 53$ keV)

<u>Top</u> 1995 at Tevatron, Fermilab.

<u>Spectroscopy</u> : next page

Table 14.3: $q\overline{q}$ quark-model assignments for the observed heavy mesons. Mesons in bold face are included in the Meson Summary Table.

$n\,^{2s+1}\ell_J$	J^{PC}	I = 0 $c\overline{c}$	I = 0 $b\overline{b}$	I = $\frac{1}{2}$ $c\overline{u}$, $c\overline{d}$; $\overline{c}u$, $\overline{c}d$	I = 0 $c\overline{s}$; $\overline{c}s$	I = $\frac{1}{2}$ $b\overline{u}$, $b\overline{d}$; $\overline{b}u$, $\overline{b}d$	I = 0 $b\overline{s}$; $\overline{b}s$	I = 0 $b\overline{c}$; $\overline{b}c$
$1\,^1S_0$	0^{-+}	$\boldsymbol{\eta_c(1S)}$	$\eta_b(1S)$	$\boldsymbol{D}$	$\boldsymbol{D_s^\pm}$	$\boldsymbol{B}$	$\boldsymbol{B_s^0}$	$\boldsymbol{B_c^\pm}$
$1\,^3S_1$	1^{--}	$\boldsymbol{J/\psi(1S)}$	$\boldsymbol{\Upsilon(1S)}$	$\boldsymbol{D^*}$	$\boldsymbol{D_s^{*\pm}}$	$\boldsymbol{B^*}$	B_s^*	
$1\,^1P_1$	1^{+-}	$h_c(1P)$		$\boldsymbol{D_1(2420)}$	$\boldsymbol{D_{s1}(2536)^\pm}$	$\boldsymbol{B_1(5721)}$	$\boldsymbol{B_{s1}(5830)^0}$	
$1\,^3P_0$	0^{++}	$\boldsymbol{\chi_{c0}(1P)}$	$\boldsymbol{\chi_{b0}(1P)}$	$\boldsymbol{D_0^*(2400)}$	$\boldsymbol{D_{s0}^*(2317)^{\pm\dagger}}$			
$1\,^3P_1$	1^{++}	$\boldsymbol{\chi_{c1}(1P)}$	$\boldsymbol{\chi_{b1}(1P)}$	$D_1(2430)$	$\boldsymbol{D_{s1}(2460)^{\pm\dagger}}$			
$1\,^3P_2$	2^{++}	$\boldsymbol{\chi_{c2}(1P)}$	$\boldsymbol{\chi_{b2}(1P)}$	$\boldsymbol{D_2^*(2460)}$	$\boldsymbol{D_{s2}^*(2573)^\pm}$	$\boldsymbol{B_2^*(5747)}$	$\boldsymbol{B_{s2}^*(5840)^0}$	
$1\,^3D_1$	1^{--}	$\boldsymbol{\psi(3770)}$			$D_{s1}^*(2700)^\pm$			
$2\,^1S_0$	0^{-+}	$\boldsymbol{\eta_c(2S)}$						
$2\,^3S_1$	1^{--}	$\boldsymbol{\psi(2S)}$	$\boldsymbol{\Upsilon(2S)}$					
$2\,^3P_{0,1,2}$	$0^{++}, 1^{++}, 2^{++}$		$\boldsymbol{\chi_{b0,1,2}(2P)}$					

† The masses of these states are considerably smaller than most theoretical predictions. They have also been considered as four-quark states (See the "Note on Non-$q\overline{q}$ Mesons" at the end of the Meson Listings). The open flavor states in the 1^{+-} and 1^{++} rows are mixtures of the $1^{+\pm}$ states.

The potential between c and $\bar{c}$ or b and $\bar{b}$ can be understood reasonably well in terms of non-relativistic treatment. (However, relativistic effects, like spin-dependent interactions, are not negligible.)

In the cm-frame of $q\bar{q}$ pair, the Schrödinger eq.

$$-\frac{1}{2\mu}\nabla^2\psi(\bar{x}) + V(r)\psi(\bar{x}) = E\psi(\bar{x}),$$

where $r = |\bar{x}|$ = distance between quarks, $\mu = \frac{1}{2}m_q$.

The particle masses $M(q\bar{q}) = 2m_q + E$ depend on principle quantum number n and angular momentum L.

If potential is Coulomb like: $V(r) \propto r^{-1}$ then only dependence on n

Harmonic oscillator potential would be $V(r) \propto r^2$

Heavy quarkonia spectra can be fitted with

$$V(r) = -\frac{a}{r} + br$$

$$a = 0.48, \quad b = 0.18\ \text{GeV}^2, \quad 0.2\ \text{fm} \leq r \leq 0.8\ \text{fm}$$

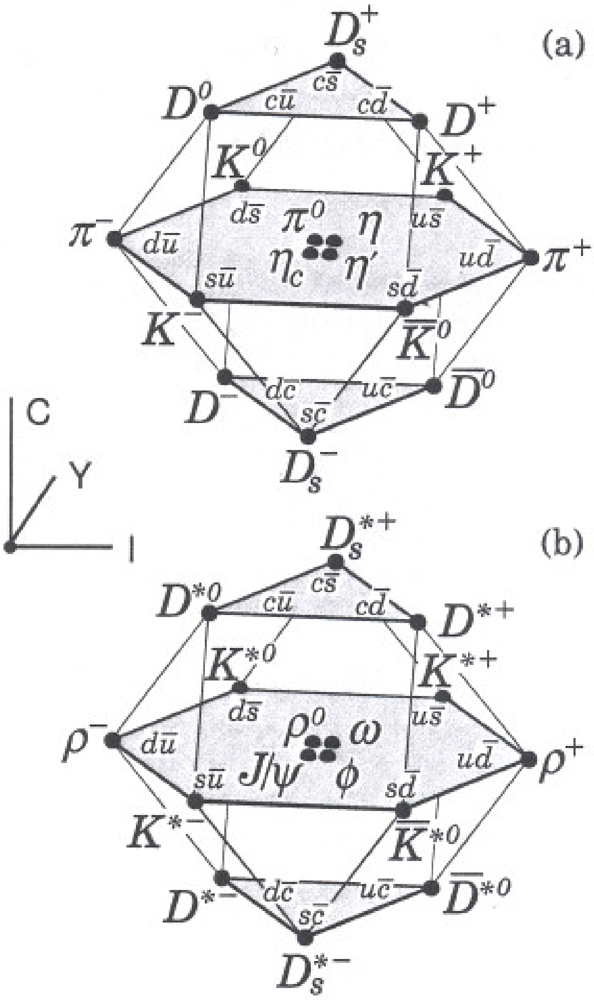
Also $V(r) = a \ln br$, $a = 0.7$ GeV, $b = 0.5$ GeV gives satisfactory results.

Potential increases for increasing r, but is very small for small r => <u>asymptotic freedom</u>

When c-quark was invented, it was natural to fit the hadrons in SU(4) flavour.

c-quark is so heavy, $m_c \sim 1.5$ GeV, that SU(4) is not very useful.

$\boxed{\text{PDG}}$



$4 \times \bar{4}$

$= 1 + \frac{4\cdot5\cdot3\cdot2}{4\cdot2}$

$= 1 + 15$

Figure 14.1: SU(4) weight diagram showing the 16-plets for the pseudoscalar (a) and vector mesons (b) made of the u, d, s and c quarks as a function of isospin I, charm C and hypercharge $\mathsf{Y} = \mathsf{S}+\mathcal{B} - \frac{\mathsf{C}}{3}$. The nonets of light mesons occupy the central planes to which the $c\bar{c}$ states have been added.

which are satisfied for the vector mesons. However, for the pseudoscalar (and scalar mesons) Eq. (14.11) is satisfied only approximately. Then Eq. (14.9) and Eq. (14.10) lead to somewhat different values for the mixing angle. Identifying the η with the f' one gets

$$\eta = \psi_8 \cos\theta_P - \psi_1 \sin\theta_P \;, \tag{14.15}$$

$$\eta' = \psi_8 \sin\theta_P + \psi_1 \cos\theta_P \;. \tag{14.16}$$

Following chiral perturbation theory the meson masses in the mass formulae (Eq. (14.9)) and (Eq. (14.10)) should be replaced by their squares. Table 14.2 lists the mixing angle θ_{lin} from Eq. (14.10) and the corresponding θ_{quad} obtained by replacing the meson masses by their squares throughout.

Colour factors

Each quark flavour comes in three colours: R, G, B

Colour is assumed to be exactly conserved, and the corresponding theory is called quantum chromodynamics, or QCD.

The mediators in QCD are 8 gluons:

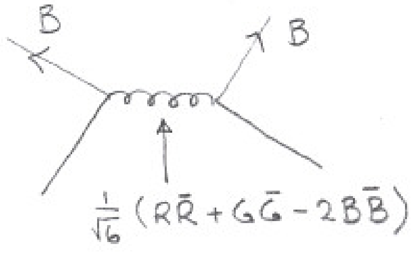
$$\boxed{R\bar{G},\ R\bar{B},\ G\bar{R},\ G\bar{B},\ B\bar{R},\ B\bar{G},\ \sqrt{\tfrac{1}{2}}\,(R\bar{R}-G\bar{G}),\ \sqrt{\tfrac{1}{6}}\,(R\bar{R}+G\bar{G}-2B\bar{B})}$$

corresponding to 8 generators of $SU(3)_{colour}$-group

Analogously to QED, where em coupling between two quarks is $e_{q_1} e_{q_2} \alpha$ ($e_{q_i} = \frac{2}{3}$ or $-\frac{1}{3}$), in QCD the coupling between two colour charges is $\frac{1}{2} c_1 c_2 \alpha_s$, where c_1 and c_2 are colour coefficients at the vertices and α_s is strong coupling constant.

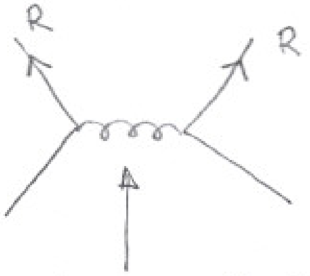
The colour factor is $C_F = \frac{1}{2} |c_1 c_2|$

Examples of colour factors:

1) 

$$c_1 c_2 = \frac{-2}{\sqrt{6}} \times \frac{-2}{\sqrt{6}} = \frac{2}{3}$$

2)



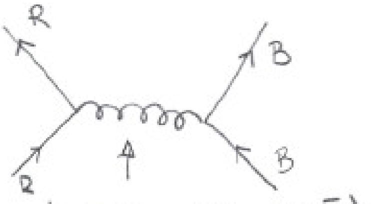
$\frac{1}{\sqrt{6}}(R\bar{R} + G\bar{G} - 2B\bar{B}) \rightarrow c_1 c_2 = \frac{1}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}} = \frac{1}{6}$

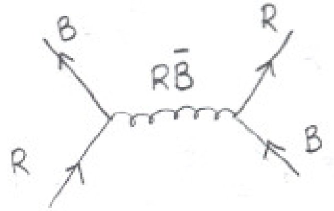
or

$\frac{1}{\sqrt{2}}(R\bar{R} - G\bar{G}) \rightarrow c_1 c_2 = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$

Total $= \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$, same as in 1), as must be due to colour symmetry

3)





$\frac{1}{\sqrt{6}}(R\bar{R} + G\bar{G} - 2B\bar{B})$

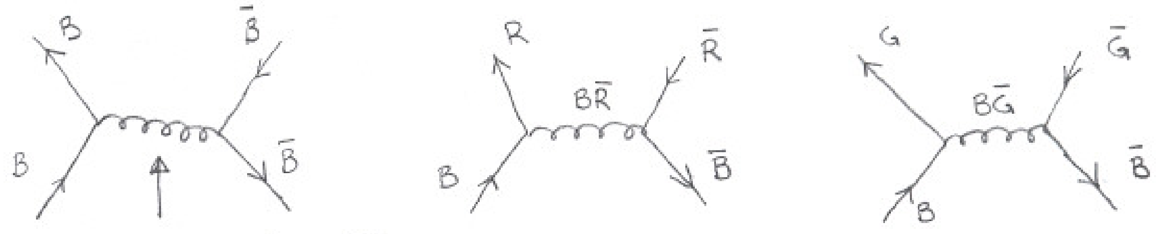
$\rightarrow c_1 c_2 = \frac{1}{\sqrt{6}} \cdot \frac{-2}{\sqrt{6}} = -\frac{1}{3}$

$c_1 c_2 = 1$

If the colour w.f. should be antisymmetric in the exchange of quarks, total $c_1 c_2 = -\frac{1}{3} - 1 = -\frac{4}{3}$, if symmetric, total $c_1 c_2 = -\frac{1}{3} + 1 = \frac{2}{3}$

4) What about gluon exchange between a quark-antiquark system, which is in colour singlet, $\sqrt{\frac{1}{3}}(R\bar{R} + G\bar{G} + B\bar{B})$ (i.e. a meson)?

All colours on equal footing, consider e.g. B-$\bar{B}$ interaction:



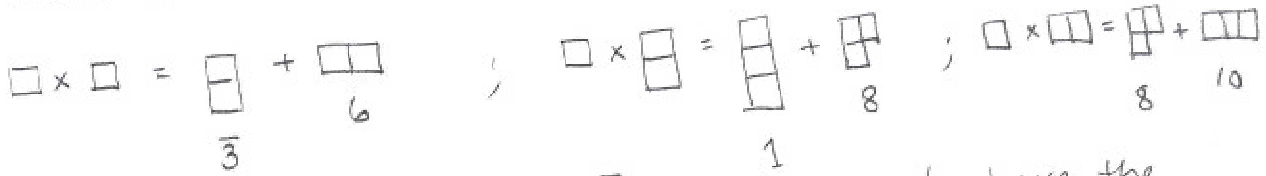
$\frac{1}{\sqrt{6}}(R\bar{R}+G\bar{G}-2B\bar{B})$ $\quad$ $c_1c_2 = (+1)(-1)$ $\quad$ $c_1c_2 = -1$

$\rightarrow c_1c_2 = \frac{-2}{\sqrt{6}} \cdot \frac{+2}{\sqrt{6}} = \frac{-2}{3}$ $\quad$ $= -1$

Total $c_1c_2 = \underset{\substack{\uparrow \\ \text{For } G\bar{G} \text{ or } R\bar{R} \\ \text{in initial state}}}{3} \times \underset{\substack{\uparrow\uparrow \\ |i\rangle \text{ and } |f\rangle \\ \text{have } \frac{1}{\sqrt{3}}}}{\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}}\left(-\frac{2}{3}-1-1\right) = -\frac{8}{3} \Rightarrow C_F = \frac{4}{3}$

5) Two quarks exchanging a gluon within a baryon.

In $SU(3)_{colour}$, the quarks are in $3 \otimes 3 \otimes 3$, which must contain the colour singlet:



Thus the first $3 \otimes 3$ is $\bar{3}$, in order to have the colour singlet baryon.

$\bar{3}$ is antisymmetric, thus $c_1c_2 = -\frac{4}{3} \Rightarrow C_F = \frac{2}{3}$