Introduction to Particle Physics I Spring 2012 Exercise 4

Due at 12.00 on Tuesday 21 February

1. The hadron $\Sigma_C^+(2455)$ is observed to decay by $\Sigma_C^+ \to \Lambda_C^+ + \pi^0$ with a rate typical of strong interactions, where the $\Lambda_C^+(2285) = udc$ is an $I = 0$ hadron. Deduce the values of the quantum numbers and hence the quark content of the Σ_C^+. Does it posses of any isospin partners, and if so, what is their quark content?

2. In the simple model of Σ baryon mass splittings (see the lectures), the electromagnetic interaction energy between two quarks a and b was assumed to be of order $\delta e_a e_b$ where e_a and e_b are the quark charges in units of e. Deduce the value of δ obtained from the observed Σ baryon masses in this approximation, and use it to make a rough estimate of the typical distance r between the quarks by assuming that the Coulomb interaction dominates.

3. Construct the quark model wavefunction $|n\uparrow\rangle$. The charge operator is defined as $Q = \sum_i Q_i$, where Q_i are the charges of the quarks in units of the proton charge e. The sum is over the constituent quarks of the hadron. Show that
$$\langle p\uparrow|Q|p\uparrow\rangle = 1, \quad \langle n\uparrow|Q|n\uparrow\rangle = 0.$$

4. Instead of having a totally symmetric spin-flavour wave function for proton, one could construct a totally antisymmetric proton wave function,
$$|p\uparrow\rangle = \sqrt{\frac{1}{2}}\left[p_A\chi(M_S) - p_S\chi(M_A)\right]$$
and forget about colour. Write this function in an explicit form. Obtain also $|n\uparrow\rangle$. Show that
$$\frac{\mu_n}{\mu_p} = -2.$$
This option is experimentally excluded. (Furthermore, here μ_p is negative, although it has been measured to be positive.)

5. Calculate β for electron and proton, if $E_{kin} = 100$ MeV. How much should the electron kinetic energy be, in order to have β of electron the same as β of proton with proton kinetic energy $E_{kin} = 100$ MeV? Do the same calculations for $E_{kin} = 100$ GeV. What can you deduce?