Natural units

length: 1 fermi = 10^{-15} m (~ radius of proton)

cross section: 1 barn = 10^{-28} m^2; 1 mb = 10^{-31} m^2, ...

$$\sigma_{tot}(pp) \simeq 40 \text{ mb} = 4 \cdot (10^{-15} m)^2 \simeq \pi r_p^2$$

energy: 1 electron volt (eV) = $1.6 \cdot 10^{-19}$ J (= $1.6 \cdot 10^{-12}$ erg)

time: 1 second | reaction time ~ $\frac{1 fm}{c} \sim \frac{10^{-15} m}{3 \cdot 10^8 m/s} \simeq 0.3 \cdot 10^{-23}$.

Basic constants (see Particle Data Group listings)

velocity of light $c = 2.99792458(12) \cdot 10^8$ m/s

Planck constant $\hbar = h/2\pi = 1.0545887(57) \cdot 10^{-34}$ Js

Boltzmann constant $k = 1 eV/11604.50(36)$ °K ($E = kT$)

Usually in particle physics $\boxed{\hbar = c = 1}$ and add suitable powers of $\hbar$, c when needed.

∴ Unit of energy, mass and momentum = GeV;
unit of length and time = GeV^{-1}

Ex. $m_e = 0.511$ MeV

$$= 0.511 \cdot 10^6 \cdot 1.6 \cdot 10^{-19} \frac{kg\, m^2}{s^2} \cdot \left(3.00 \cdot 10^8 \frac{m}{s}\right)^{-2}$$

$$= 0.911 \cdot 10^{-30} \text{ kg}$$

$$1 \text{ fm} = 1 \text{ fm}/c = \frac{10^{-15} m}{3 \cdot 10^8 m} s = 0.33 \cdot 10^{-23} s$$

$$= 1 \text{ fm} \frac{1}{\hbar c} = \frac{10^{-15} m \cdot 1.6 \cdot 10^{-19}}{1.05 \cdot 10^{-34} eV\, s \cdot 3 \cdot 10^8 m/s} = \frac{1}{197 \text{ MeV}}$$

$\Rightarrow$ $\boxed{\hbar c = 197 \text{ MeV} \cdot \text{fm}}$

Then from the Heisenberg uncertainty relation

$\Delta x \, \Delta p \gtrsim \hbar = 1$ $\Rightarrow$ localization of a particle within 1 fm means 197 MeV uncertainty in momentum.

Also $\boxed{E^2 = p^2 + m^2}$

Interactions and charges

Gravity

Potential energy $V = G_N \frac{m_p^2}{r}$ $\quad G_N = 0.67 \cdot 10^{-38} \text{ GeV}^{-2}$

$m_p \cong 1 \text{ GeV}, \quad r \sim 10^{-15} \text{ m} \sim 5 \frac{1}{\text{GeV}}$

$\Rightarrow V \sim 10^{-39} \text{ GeV}$

$$\sqrt{G_N} = \frac{1}{M_{Planck}} \sim 0.8 \cdot 10^{-19} \frac{1}{\text{GeV}} \sim 5.3 \cdot 10^{-44} \text{ s} \sim 1.6 \cdot 10^{-35} \text{ m}$$

$$\frac{1}{\sqrt{G_N}} = M_{Planck} \sim 10^{19} \text{ GeV}$$

Gravity becomes important at energies $\sim \mathcal{O}(M_{Planck})$ or at length scales $\sim \mathcal{O}(10^{-35} \text{ m})$.

$\therefore$ Not relevant in present particle physics experiments.

Weak interaction (mediated by $W^{\pm}, Z$)

Responsible for radioactivity:

$n \to p e^- \bar{\nu}_e$ (lifetime ~ 885.7 ± 0.8 s ~ 14.76 min

$\pi^+ \to \mu^+ \nu_\mu$ $\tau \sim 2.6 \cdot 10^{-8}$ s

$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$, $\tau \sim 2.2 \cdot 10^{-6}$ s

⋮

all the ν reactions:

$\nu_\mu p \to \mu^- p \pi^+$,

$\bar{\nu}_e e^- \to \bar{\nu}_e e^-$,

⋮

Cross section small:

$\sim 10^{-42}$ m^2

$= 10^{-8}$ μb $= 10^{-2}$ pb = 10f

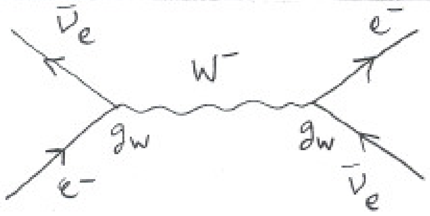
Strength is given by Fermi coupling constant

$$G_F = 1.16637 \cdot 10^{-5} \text{ GeV}^{-2}$$

$$\frac{1}{\sqrt{G_F}} \sim 300 \text{ GeV} \quad ; \quad \sqrt{G_F} = 0.7 \cdot 10^{-18} \text{ m}$$

Interaction strength is also described by

$\alpha_W = \frac{g_W^2}{4\pi}$, where g_W is weak interaction coupling constant.



$m_W \cong 80.425 \pm 0.038$ GeV

$m_Z \cong 91.1876 \pm 0.0021$ GeV

found in 1983 at CERN

From the uncertainty relation the range of the weak interaction

$$R_W \sim \frac{\hbar c}{m_W c^2} \sim \frac{197 \cdot 10^{-15} \text{ MeV m}}{80 \cdot 10^3 \text{ MeV}} \sim 2 \cdot 10^{-18} \text{ m}$$

Electromagnetic interaction (mediated by γ)

$\gamma p \to \pi^+ n$

$e^- p \to e^- p \pi^+ \pi^-$

$e^+ e^- \to e^+ e^-$

$e^+ e^- \to \mu^+ \mu^-, \tau^+ \tau^-$

⋮

cross section $1 - 100\ \mu b$

$\pi^0 \to \gamma\gamma$ decay, $\tau \sim 10^{-16} s$

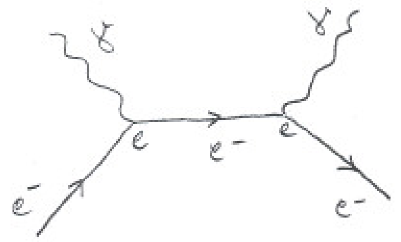
⋮

Binding energy of the electron in the hydrogen.

$$|E| = \alpha^2 \frac{m_r c^2}{2} \sim 13.605\ eV$$

$$\Rightarrow \alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

Thomson scattering:

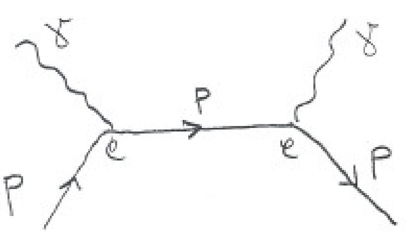


$$\sigma_{Th} \propto \alpha^2 (4\pi R_e^2) ;$$

$R_e = \frac{\hbar}{m_e c} = \frac{1}{m_e}$ = electron Compton wavelength

$\sigma_{Th} = 0.665$ barn

similarly



$$\propto \alpha^2 (4\pi R_p^2) ; \quad R_p = \frac{1}{m_p}$$

Strong interaction (mediated by gluons)

$\pi^+ = u\bar{d}$, $\pi^- = d\bar{u}$

$p = uud$, $n = udd$

$K^+ = u\bar{s}$

$\Lambda^0 = uds$

$\Delta^{++} = uuu$

E.g.

$\pi N \to \pi N$

($\pi = \pi^\pm, \pi^0$; $N = n, p$; charge is conserved)

$\pi^- p \to K^- p \Lambda^0$

$K^+ p \to K^+ p \pi^0$

$pp \to pp K^+ K^- \pi^0$

⋮

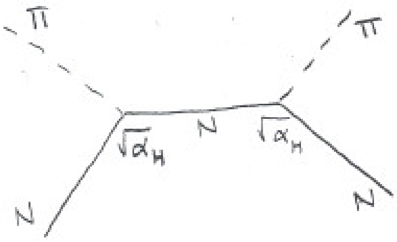
decays $\Delta^{++} \to p\pi^+$

⋮

cross section

$> 1 mb$

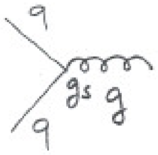
$\tau \lesssim 10^{-23} s$



$\sigma \sim \alpha_H (4\pi R_p^2)$

$\alpha_H \sim .15$: not a good expansion parameter in perturbation theory

Replace hadrons by quarks: $q\bar{q}$, qqq



$\alpha_s = \frac{g_s^2}{4\pi} = 0,1187$