Gravitational waves from first order phase transitions[1]

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What was known

- First-order phase transitions (say, at the electroweak scale in extended models) are a source of GWs.
- The power spectrum of gravitational waves produced *during* bubble collisions can be calculated by the 'envelope approximation'[2].

What this work adds

- Overlapping acoustic waves in the plasma of light particles are a stronger source than the collisions themselves.
- The scale and behaviour of this source is characterised by moments of the fluid power spectrum.

Next steps

- Gravitational waves from electroweak phase transitions are a strong candidate for detection by eLISA and other missions.
- Quantitative predictions for particular models are required, this will require new methods.

Basics: field+fluid system

- Stress energy tensor for field ϕ and fluid with 4-velocity U^μ

$$T^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left[\tfrac{1}{2}\partial_\alpha\phi\partial^\alpha\phi\right] + [\epsilon + p]\,U^\mu U^\nu + g^{\mu\nu}p.$$

- Effective potential with parameters γ, α, λ and T_0

$$V(\phi,T) = \frac{1}{2}\gamma(T^2 - T_0^2)\phi^2 - \frac{1}{3}\alpha T\phi^3 + \frac{1}{4}\lambda\phi^4.$$

- Rest-frame energy density ϵ and pressure p, with $a = (\pi^2/90)g$

$$\epsilon = 3aT^4 + V(\phi,T) - T\frac{\partial V}{\partial T}; \qquad p = aT^4 - V(\phi,T).$$

- Evolution equations are (W is the relativistic γ-factor)

$$-\ddot\phi + \nabla^2\phi - \frac{\partial V}{\partial\phi} = \eta W(\dot\phi + V^i\partial_i\phi)$$

$$\dot E + \partial_i(EV^i) + p[\dot W + \partial_i(WV^i)] - \frac{\partial V}{\partial\phi}W(\dot\phi + V^i\partial_i\phi) = \eta W^2(\dot\phi + V^i\partial_i\phi)^2$$

$$\dot Z_i + \partial_j(Z_iV^j) + \partial_i p + \frac{\partial V}{\partial\phi}\partial_i\phi = -\eta W(\dot\phi + V^j\partial_j\phi)\partial_i\phi$$

– the η parameter varies the friction.

- Evolve *unprojected* perturbations

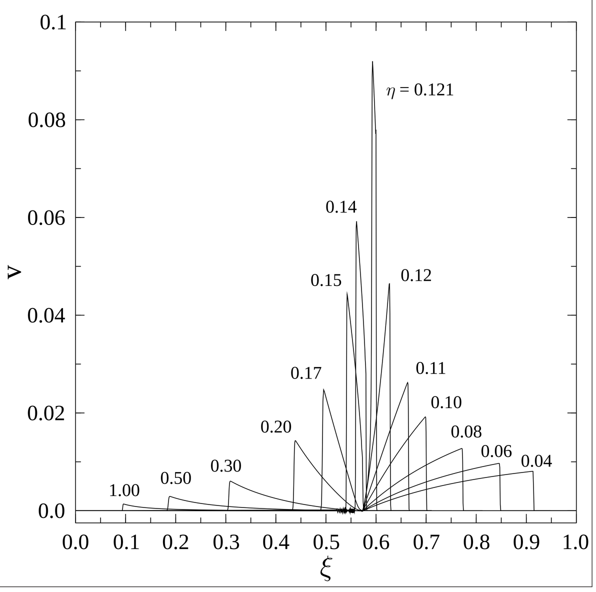
$$\ddot u_{ij} - \nabla^2 u_{ij} = 16\pi G(\tau^\phi_{ij} + \tau^{\rm f}_{ij}),$$

where $\tau^\phi_{ij} = \partial_i\phi\partial_j\phi$ and $\tau^{\rm f}_{ij} = W^2(\epsilon+p)V_iV_j$ are the field and fluid sources respectively. Use the projection technique of Ref. 3 to get the true metric perturbations.

Bubble nucleation and growth

- Simulated nucleation takes place by attempting to nucleate bubbles of scalar field with probability $P = P_0\exp(\beta(t - t_0))$ per unit volume and time.
- For different values of η the bubble wall moves at different velocities $v_{\rm w}$.
- Can form *detonations* ($v_{\rm w} > c_{\rm s}$) or *deflagrations* ($v_{\rm w} < c_{\rm s}$) depending on η.
- The potential parameters control the *strength* of the phase transition $\alpha_{T_{\rm N}}$; more latent heat $\to$ higher fluid velocities.
- We work with two choices of potential that we term *weak* ($\alpha_{T_{\rm N}} \approx 0.01$) and *intermediate* ($\alpha_{T_{\rm N}} \approx 0.1$).

- At right, fluid velocity profile as a function of scaled radius $\xi = r/t$ (taken from Ref. 4) for the *weak*-strength potential parameters
- The bubble wall velocity $v_{\rm w}$ increases as η is decreased; both detonations and deflagrations are possible.
- For our *intermediate* case, the fluid velocities v are about ten times larger, but the qualitative behaviour is same.



What is the appropriate length scale?

- Define dimensionless quantities $\overline{U}_\phi$ and $\overline{U}_f$ that show relative importance of the field and the fluid respectively to gravitational wave production.

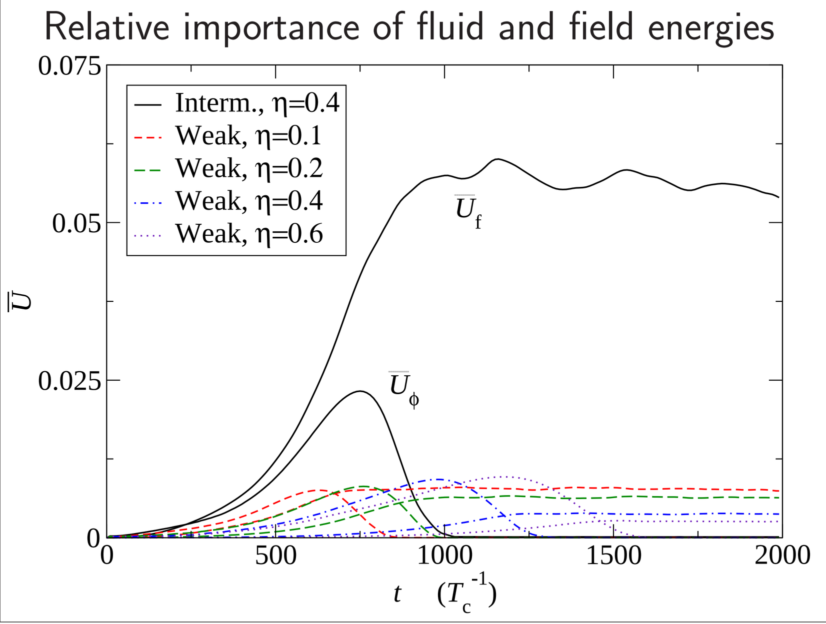
$$(\bar\epsilon + \bar p)\bar U_\phi^2 = \frac{1}{V}\int d^3x\,\tau^\phi_{ii} \quad\text{and}\quad (\bar\epsilon + \bar p)\bar U_{\rm f}^2 = \frac{1}{V}\int d^3x\,\tau^{\rm f}_{ii}$$

- Also, define the *integral scale*,

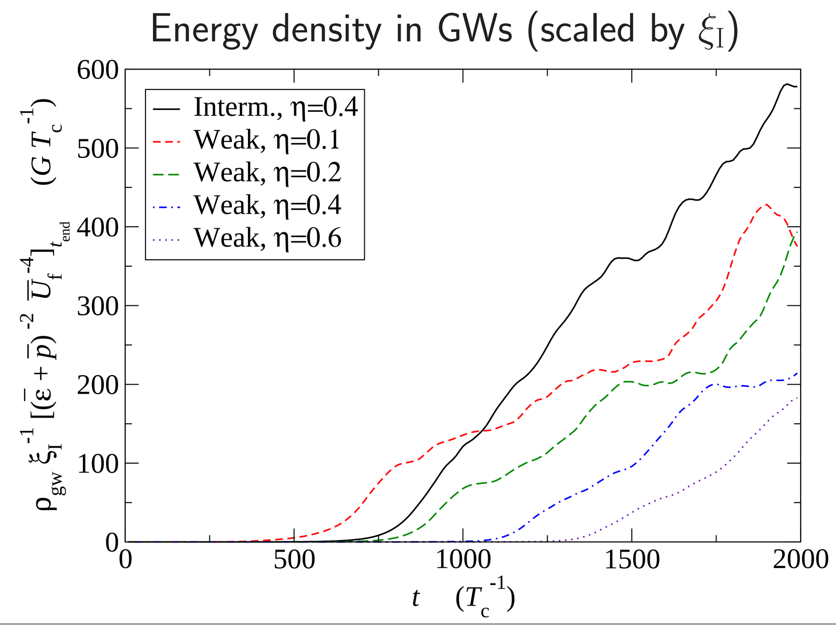
$$\xi_I = \frac{\int d^3k\,k^{-1}\,|v_{\mathbf k}|^2}{\int d^3k\,|v_{\mathbf k}|^2}$$

which gives dominant length scale of fluctuations in the fluid without reference to the fluid profile shape.

Relative importance of fluid and field energies

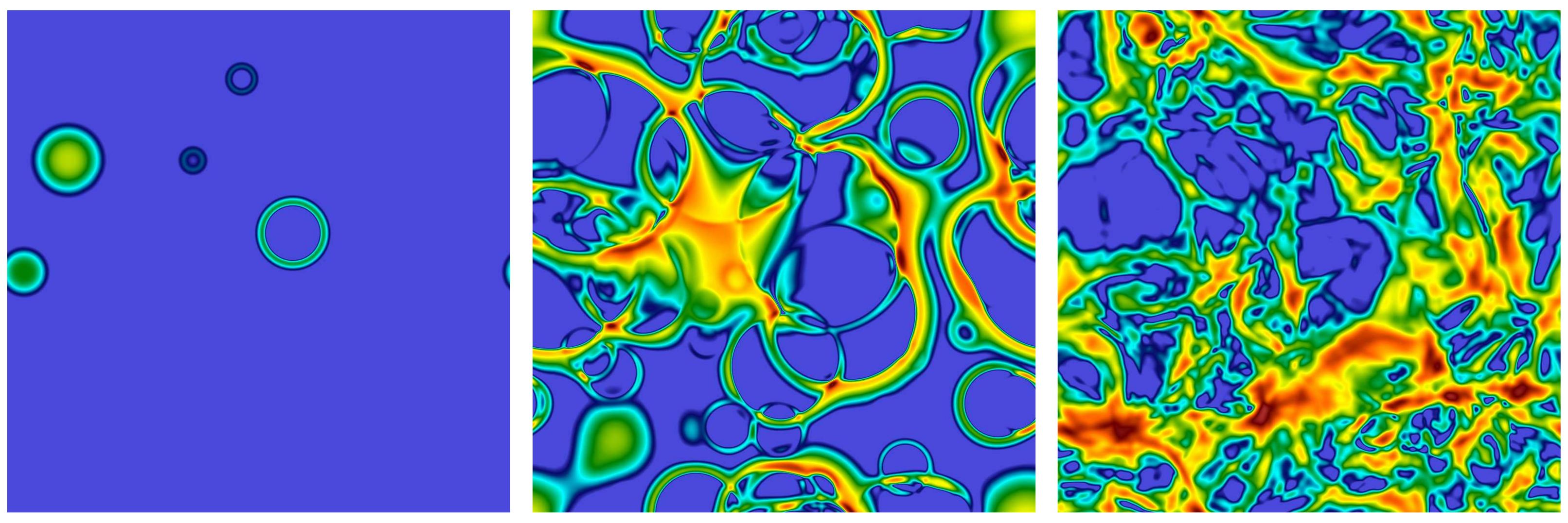


Energy density in GWs (scaled by $\xi_{\rm I}$)



Key result: gravitational wave energy grows linearly (due to fluid).

Results: fluid energy density



Slices of fluid energy density $E/T_{\rm c}^4$ for the $\eta = 0.4$ intermediate-strength deflagration at $t = 500\,T_{\rm c}^{-1}$, $t = 750\,T_{\rm c}^{-1}$ and $t = 1000\,T_{\rm c}^{-1}$ respectively. Simulation volume $1024^3\,T_{\rm c}^{-3}$.

Recent work

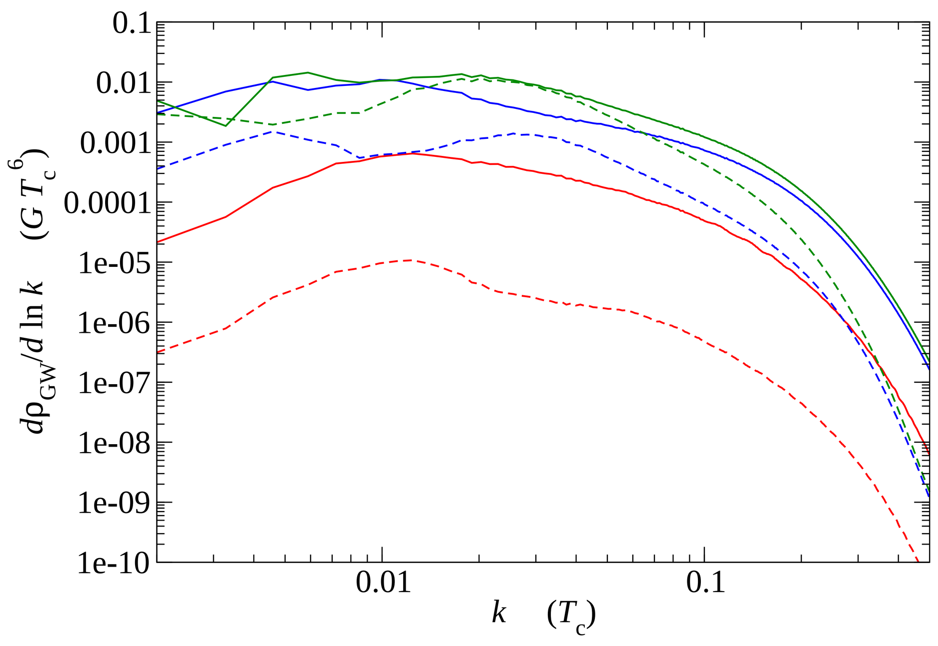
- Early work (see above, and Ref. 1) was in small simulation volumes
- Therefore our more recent simulations have shifted emphasis
  - Separate field and fluid power spectrum calculation into two separate problems
  - Concentrate on fluid behaviour
  - Larger simulation volumes
  - Nucleate bubbles simultaneously
- Aim is to see power laws and other scale-invariant behaviour by simulating very large volumes (up to $8400^3\,T_{\rm c}^{-3}$); comparison with results in Ref. 5.
- By rescaling our parameters, we can make the bubble spacing physical.
- Diagrammatic approach? See Ref. 6.

Recent results: gravitational wave power spectrum; power laws

- The power spectrum per logarithmic frequency interval is given by

$$\frac{d\rho_{\rm GW}(k)}{d\ln k} = \frac{1}{32\pi GL^3}\frac{k^3}{(2\pi)^3}\int d\Omega\,\left|\dot h_{lm}(t,\mathbf{k})\right|^2$$

GW power spectrum for $\eta = 0.2$ deflagration:
1000 bubbles nucleated simultaneously, simulation volume $4800^3\,T_{\rm c}^{-3}$.



Power spectra are plotted at times $t = 500\,T_{\rm c}^{-1}$ (red), $1000\,T_{\rm c}^{-1}$ (blue) and $1500\,T_{\rm c}^{-1}$ (green). Solid lines show the full gravitational wave power spectrum; dashed lines show the power sourced by the fluid only.

- Despite the size of our box and the number of bubbles, we have limited resolution in the IR, so no k^3 scaling is visible.
- The putative k^{-1} scaling is destroyed by the exponential decay at higher k.
- Turnover from power law to exponential occurs at inverse wall thickness.
- Scalar field dynamics *are* therefore properly captured by the envelope approximation[2].
- In future work we can concentrate on understanding the fluid source and avoid computing the field source.

Key references

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