

ON THE NECESSITY OF PROPERTY (α) FOR SOME VECTOR-VALUED MULTIPLIER THEOREMS

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ABSTRACT. The notion of R -bounded operator families has proven to be crucial to several aspects of vector-valued Harmonic Analysis and its applications to PDEs. Concrete and important R -bounded sets have been identified among operators acting on UMD Banach spaces, which in addition enjoy G. Pisier's property (α) . We prove the necessity of (α) for a number of conclusions of this kind, complementing analogous results of G. Lancien in the product-space theory of Fourier multipliers and H^∞ -calculus.

1. INTRODUCTION AND RESULTS

A Banach space X is called UMD if all martingale difference sequences in $L^p(\Omega, X)$, where $1 < p < \infty$ and Ω is any probability space, are unconditionally convergent; see [4]. It has been known since the 1980s that this probabilistic condition on X is both necessary and sufficient for the boundedness of several classical operators of Harmonic Analysis on spaces of X -valued functions [2, 3, 5, 10].

In the recent years, the notion of R -bounded operator families has proven to be crucial to several aspects of vector-valued Harmonic Analysis and its applications to PDEs [1, 6, 9, 12, 14, 21]. We recall the definition.

1.1. Definition. Let $(\varepsilon_j)_{j=1}^\infty$ be independent symmetric ± 1 -valued random variables on some probability space. We refer to them as *Rademacher variables*. The corresponding expectation operator is denoted by \mathbb{E} or \mathbb{E}_ε .

A family of operators $\mathcal{T} \subset \mathcal{L}(Y)$ is called *R -bounded* if there is a finite constant C such that for all $f_j \in Y$, $T_j \in \mathcal{T}$ and $N \in \mathbb{N}$ there holds

$$\mathbb{E} \left\| \sum_{k=1}^N \varepsilon_k T_k f_k \right\|_Y \leq C \mathbb{E} \left\| \sum_{k=1}^N \varepsilon_k f_k \right\|_Y.$$

The smallest C is denoted by $\mathcal{R}(\mathcal{T})$ and called the *R -bound* of \mathcal{T} .

Various concrete and important examples of such operator collections on $L^p(\mathbb{T}, X)$ have been identified in [9, 20]; however, these results assume another condition of the underlying Banach space X , in addition to UMD: G. Pisier's property (α) [17].

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1.2. Definition. Let $(\varepsilon_j)_{j=1}^\infty$ and $(\eta_k)_{k=1}^\infty$ be two independent sequences of Rademacher variables, with the related expectation operators \mathbb{E}_ε and \mathbb{E}_η .

We say that a Banach space X has *property* (α) if there is a finite constant C such that for all $x_{jk} \in X$, all $\alpha_{jk} \in \{+1, -1\}$ and all $N \in \mathbb{N}$, there holds:

$$\mathbb{E}_\varepsilon \mathbb{E}_\eta \left| \sum_{j,k=1}^N \alpha_{jk} \varepsilon_j \eta_k x_{jk} \right|_X \leq C \mathbb{E}_\varepsilon \mathbb{E}_\eta \left| \sum_{j,k=1}^N \varepsilon_j \eta_k x_{jk} \right|_X.$$

We denote the smallest admissible C above by $\alpha(X)$.

Our present contribution is to clarify the rôle of property (α) in the various R -boundedness results by completing the proof of the following characterization, several implications in which are already well-known, as we recall.

1.3. Theorem. *Let X be a Banach space, $1 < p < \infty$, and $\gamma > 0$. Then the following conditions are equivalent:*

- (U) X is a UMD space with property (α) .
- (M) Every uniform set of Marcinkiewicz multipliers is R -bounded on $L^p(\mathbb{T}, X)$.
- (L) Every uniform set of Littlewood–Paley multipliers is R -bounded on $L^p(\mathbb{T}, X)$.
- (F) The functional calculus of $-\Delta$ is R -bounded on $L^p(\mathbb{T}, X)$.
- (P) The imaginary powers $(-\Delta)^{is}$, $0 < s < \gamma$, are R -bounded on $L^p(\mathbb{T}, X)$.

We recall that *Fourier multipliers* on $L^p(\mathbb{T}, X)$ are operators

$$(1.4) \quad T_\lambda : f \mapsto \sum_{k \in \mathbb{Z}} \lambda(k) \hat{f}(k) e_k,$$

where $\lambda = (\lambda(k))_{k \in \mathbb{Z}}$ is scalar sequence, $\hat{f}(k) \in X$ is the k th Fourier coefficient of f , and $e_k(x) = e^{i2\pi k \cdot x}$. One may also consider *operator-valued Fourier multipliers*, which have the same formal expression, but it is allowed that $\lambda(k) \in \mathcal{L}(X)$. However, in this note we agree that a Fourier multiplier T_λ has a scalar-valued multiplying sequence λ unless the attribute “operator-valued” is used explicitly. Our emphasis is on the necessary conditions for multiplier theorems, and what is necessary for a special case is obviously necessary for a more general statement.

We call T_λ a *Marcinkiewicz multiplier* if for all $m \in \mathbb{Z}$

$$(1.5) \quad |\lambda(m)| + \sum_{j \in I_m} |\lambda(j+1) - \lambda(j)| \leq K, \quad \text{where} \\ I_0 := \{0\}, \quad I_m := \text{sgn}(m)[2^{|m|-1}, 2^{|m|}) \cap \mathbb{Z} \quad \forall m \in \mathbb{Z} \setminus \{0\}.$$

If moreover $\lambda(j)$ is a constant for $j \in I_m$, then T_λ is called a *Littlewood–Paley multiplier*. A collection of either kind of multipliers is called uniform if they satisfy (1.5) with the same K . Thus the implication $(M) \Rightarrow (L)$ is obvious.

There is a well-established theory of H^∞ -calculus of abstract Banach space operators A (cf. [11, 14]), but we here only recall the case of the Laplacian $A = -\Delta = -d^2/dx^2$. Since $-\Delta$ is formally the Fourier multiplier T_λ with $\lambda(j) = 4\pi^2 j^2$, it is reasonable to define $\phi(-\Delta)$ as T_λ with $\lambda(j) = \phi(4\pi^2 j^2)$. If $\phi \in H^\infty(S_\theta)$, the space of bounded holomorphic functions in the sector $S_\theta = \{z \in \mathbb{C} \setminus \{0\} : |\arg(z)| < \theta\}$ for some $\theta \in]0, \pi]$, it is easy to check that $\phi(-\Delta)$ is a Marcinkiewicz multiplier with $K = C(\theta) \|\phi\|_\infty$, and hence $(M) \Rightarrow (F)$.

There are some general relations of (R) -bounded H^∞ -calculus and (R) -bounded imaginary powers of an operator A in the abstract framework of *sectorial operators* [11, 14]. To describe these results, recall that A is said to have (R) -bounded

imaginary powers of type θ if $\{e^{-\theta|s|}A^{is} : s \in \mathbb{R}\}$ is (R) -bounded. Since

$$\phi_s(z) = e^{-\theta|s|}z^{is} = \exp(-\theta|s| - \arg(z)s + is \log|z|) \in H^\infty(S_\theta),$$

uniformly in $s \in \mathbb{R}$, it is clear that the (R) -bounded $H^\infty(S_\theta)$ -calculus implies (R) -bounded imaginary powers of type θ ; in particular, $(F) \Rightarrow (P)$. Conversely, if the underlying space has *finite cotype* (which e.g. every UMD space does), then R -bounded imaginary powers of type $\theta < \pi$ already imply bounded (intentionally without R) $H^\infty(S_\theta)$ -calculus for every $\vartheta \in]\theta, \pi]$, and this improves to R -bounded calculus if the space even enjoys property (α) ([13]; cf. [14]). But there is little use for this fact in the present context, since the whole point is proving, and not presupposing, (α) . It is also worth observing that, unlike in the mentioned abstract results, we do not require any condition (such as $\theta < \pi$) on the type of the R -bounded imaginary powers in the condition (P) above.

The implication $(U) \Rightarrow (M)$ was first discovered by A. Venni [20], and it has been elaborated in [9]. Of course, these results build on the earlier theorems for individual operators due to J. Bourgain [3]. That UMD is necessary for (L) (even with ordinary boundedness instead of R -boundedness) is also due to Bourgain [2], and its necessity for (P) (also without R) was proved by S. Guerre-Delabrière [10]. What remains for us, hence, is to demonstrate the necessity of (α) for both (L) and (P) . While its necessity for (F) already follows from this, we also prove it directly for the purposes of illustration.

Our hope is to give the reader, in addition to the precise theorem as stated, a general impression of the kind of vector-valued results for which the assumption of property (α) is unavoidable, and also to provide a small arsenal of tricks which one may try to apply in order to justify the intuition by proof in such cases which we have not treated in the present paper. Let us mention explicitly that Theorem 1.3 remains valid with \mathbb{T} replaced by \mathbb{R} ; for instance, one may apply the well-known transference techniques to pass from multiplier theorems on $L^p(\mathbb{R}, X)$ to their analogues on $L^p(\mathbb{T}, X)$. See [19], Theorem VII.3.8, for the case $X = \mathbb{C}$, which is easily generalized to the vector-valued setting by reworking the same proof with absolute values replaced by norms.

Besides the various R -boundedness results, property (α) also plays a (not completely unrelated) rôle in the *product-space* theory of Fourier multipliers [22] and in the *joint H^∞ -calculus* of several operators [12, 15]. A characterization of the condition (U) , analogous to Theorem 1.3, in terms of the product theory is already known; this circle of implications was closed by G. Lancien [16] who showed the necessity of (α) for a number of results of such type. The present paper complements his work.

2. PROOFS

Let X be a Banach space, and $1 < p < \infty$.

2.1. Proposition. *If (L) holds, then X has property (α) .*

Proof. We prove this claim by reducing it to Lancien's result about multipliers on $L^p(\mathbb{T}^2, X)$. Let $\lambda = (\lambda(j, k))_{j, k \in \mathbb{Z}}$ be an arbitrary two-dimensional Littlewood–Paley sequence, i.e., $\lambda \in \ell^\infty(\mathbb{Z}^2)$ and $\lambda(j, k)$ is a constant for $(j, k) \in I_m \times I_n$, $m, n \in \mathbb{Z}$.

We define the operator sequence $\Lambda := (\Lambda_j)_{j \in \mathbb{Z}} \in \mathcal{L}(L^p(\mathbb{T}, X))$, where $\Lambda_j = T_{\lambda(j, \cdot)}$ is the Fourier multiplier on $L^p(\mathbb{T}, X)$ related to the (scalar-valued) Littlewood–Paley sequence $(\lambda(j, k))_{k \in \mathbb{Z}}$. Thus $\mathcal{R}(\{\Lambda_j : j \in \mathbb{Z}\}) < \infty$ by assumption.

But now the operator-valued Littlewood–Paley theorem [21] tells us that $T_\Lambda F = \sum_{j \in \mathbb{Z}} \Lambda_j \hat{F}(j) e_j$ is a bounded Fourier multiplier on $L^p(\mathbb{T}, L^p(\mathbb{T}, X))$. (The one-dimensional operator-valued Littlewood–Paley theorem holds in all UMD spaces, and we already know from the assumed condition (L) that X , and then $L^p(\mathbb{T}, X)$, must have UMD.)

Upon identifying $L^p(\mathbb{T}, L^p(\mathbb{T}, X))$ with $L^p(\mathbb{T}^2, X)$, we find that the operator-multiplier T_Λ is identified with the two-dimensional scalar-multiplier

$$T_\lambda f(x, y) = \sum_{j, k \in \mathbb{Z}} \lambda(j, k) \hat{f}(j, k) e_j(x) e_k(y).$$

Hence we have just shown that an arbitrary two-dimensional Littlewood–Paley multiplier is bounded on $L^p(\mathbb{T}^2, X)$. As observed by Lancien [16], this implies that X has property (α) . For the reader's convenience, let us recall here the short argument which, at the bottom, is based on a result of Pisier [18]. By Kahane's inequality, one may replace the L^1 norm in the definition of property (α) by any L^p norm, $1 < p < \infty$, possibly changing the constant by a multiplicative factor. Then, by Pisier [18], one may further replace the Rademacher variables ε_j, η_k by a lacunary sequence of exponential $e_{2^j}(x), e_{2^k}(y)$ on \mathbb{T}^2 . But the uniform boundedness of the two-dimensional Littlewood–Paley multipliers, $\|T_\lambda f\|_{L^p(\mathbb{T}^2, X)} \leq C \|f\|_{L^p(\mathbb{T}^2, X)}$, reduces precisely to this reformulation of (α) when applied to lacunary functions $f(x, y) = \sum_{1 \leq j, k \leq N} x_{jk} e_{2^j}(x) e_{2^k}(y)$. \square

2.2. Proposition. *If (F) holds, then X has property (α) .*

Proof. Our proof is based on L. Carleson's interpolation theorem (see [8], Chapter VII), which was also used by Lancien [16] in his proof of an analogous statement in the product theory. Recall that $(z_j)_{j=0}^\infty \subset \Omega$ is called an *interpolating sequence* for the domain $\Omega \subset \mathbb{C}$ if every interpolation problem $\phi(z_j) = a_j$ with $(a_j)_{j=0}^\infty \in \ell^\infty$ has a solution $\phi \in H^\infty(\Omega)$. In this case there is a finite M , the *constant of interpolation*, such that corresponding to every $a = (a_j)_{j=0}^\infty$, there is at least one $\phi \in H^\infty(\Omega)$ with $\|\phi\|_\infty \leq M \|a\|_\infty$. A complete characterization of interpolating sequences in a half-plane is given in [8], Theorem VII.1.1; we only need the special case that $z_j = 2\pi \cdot 2^j$ is interpolating for $\Omega = S_{\pi/2}$. By the obvious conformal invariance of interpolating sequences, it follows that $\zeta_j = z_j^2 = 4\pi^2 \cdot 2^{2j}$ is interpolating for S_π . In particular, given $(\alpha_{jk})_{j,k=0}^\infty \in \{-1, +1\}^{\mathbb{N} \times \mathbb{N}}$, there are $\phi_j \in H^\infty(S_\pi)$ with $\phi_j(4\pi^2 \cdot 2^{2k}) = \alpha_{jk}$ and $\|\phi_j\|_\infty \leq M$.

Let $f_j := \sum_{k=1}^N x_{jk} e_{2^k}$, where $x_{jk} \in X$. Then

$$\phi_j(-\Delta) f_j = \sum_{k=1}^N \alpha_{jk} x_{jk} e_{2^k},$$

and the R -boundedness estimate of the $H^\infty(S_\pi)$ -calculus of $-\Delta$,

$$\mathbb{E}_\varepsilon \left\| \sum_{j=1}^N \varepsilon_j \phi_j(-\Delta) f_j \right\|_{L^p(\mathbb{T}, X)} \leq C M \mathbb{E}_\varepsilon \left\| \sum_{j=1}^N \varepsilon_j f_j \right\|_{L^p(\mathbb{T}, X)},$$

now reads

$$\mathbb{E}_\varepsilon \left\| \sum_{j,k=1}^N \varepsilon_j e_{2^k} \alpha_{jk} x_{jk} \right\|_{L^p(\mathbb{T}, X)} \leq CM \mathbb{E}_\varepsilon \left\| \sum_{j,k=1}^N \varepsilon_j e_{2^k} x_{jk} \right\|_{L^p(\mathbb{T}, X)}.$$

As in the previous proof, by Pisier [18], we may replace the lacunary exponentials e_{2^k} on \mathbb{T} in the previous inequality by the Rademacher variables η_k on some probability space, and by Kahane the L^p norm by an L^1 norm, at the cost of changing C to $c_p C$. Then we have reached precisely the defining inequality of property (α) . \square

2.3. Proposition. *If (P) holds for some $\gamma > 0$, then X has property (α) . More precisely, there is $C_p < \infty$ such that*

$$\alpha(X) \leq C_p \lim_{\gamma \downarrow 0} \mathcal{R}(\{(-\Delta)^{is} : 0 < s < \gamma\}),$$

where the R -bound is on $L^p(\mathbb{T}, X)$.

Proof. A new twist in the present proof is the fact that our assumption only involves a relatively restricted operator family. Above, we constructed appropriate (and somewhat artificial) operators, whose R -boundedness was just a reformulation of property (α) . Now we have to work with the given concrete operators, and we obtain property (α) as the limit of a sequence of inequalities that our assumptions give. The argument is inspired by S. Guerre-Delabrière's proof [10] that UMD is necessary for the boundedness (without $R!$) of the $(-\Delta)^{is}$.

We exploit the simple but useful idea (cf. [9]) of reformulating R -boundedness conditions in terms of uniform boundedness on a larger space $\text{Rad}(X)$, which is defined as the completion of the linear space of finite sums $\sum_{j=0}^N \varepsilon_j x_j$, $x_j \in X$, in the L^p norm. By Kahane's inequality, the choice of $p \in [1, \infty[$ here only affects $\text{Rad}(X)$ up to isomorphism. With this notation, the R -boundedness of $(-\Delta)^{is}$, $0 < s < \gamma$, on $L^p(\mathbb{T}, X)$ is equivalent to the uniform boundedness, for all sequences $(s_j)_{j=0}^\infty \in]0, \gamma[^\mathbb{N}$, of the operators

$$((-\Delta)^{is_j})_{j=0}^\infty : (f_j)_{j=0}^\infty \mapsto ((-\Delta)^{is_j} f_j)_{j=0}^\infty$$

on $L^p(\mathbb{T}, \text{Rad}(X))$. Observe that $T := ((-\Delta)^{is_j})_{j=0}^\infty$ is an operator-valued Fourier multiplier with the symbol

$$m(\xi) = (|2\pi\xi|^{i2s_j})_{j=0}^\infty \in \mathcal{L}(\text{Rad}(X)).$$

By a result of Ph. Clément and J. Prüss [7], or more precisely its periodic version in [1], the boundedness of T on $L^p(\mathbb{T}, \text{Rad}(X))$ implies the R -boundedness of $\{m(\xi) : \xi \in \mathbb{Z} \setminus \{0\}\}$ on $\text{Rad}(X)$, with R -bound $C_p \|T\|_{\mathcal{L}(L^p(\mathbb{T}, \text{Rad}(X)))} =: C$. Thus

$$\mathbb{E}_\varepsilon \mathbb{E}_\eta \left| \sum_{j,k=1}^N \varepsilon_j \eta_k e^{i2s_j \log(2\pi|\xi_k|)} x_{jk} \right|_X \leq C \mathbb{E}_\varepsilon \mathbb{E}_\eta \left| \sum_{j,k=1}^N \varepsilon_j \eta_k x_{jk} \right|_X$$

for all $N \in \mathbb{N}$, $x_{jk} \in X$, $\xi_k \in \mathbb{Z} \setminus \{0\}$, and $s_j \in]0, \gamma[$. Let us denote $t_k := 2 \log(2\pi |\xi_k|) = 2 \log(2\pi) + 2 \log_2(|\xi_k|) / \log_2(e)$. We make the following choice of s_j and t_k :

$$s_j := \pi s 2^{Mj}, \quad t_k := \frac{1}{s} \sum_{\ell=1}^N \delta_{\ell k} 2^{-\ell M} + 2 \log(2\pi), \quad s := \log_2(e) 2^{-NM - M - 10},$$

where $\delta_{\ell k} \in \{0, 1\}$ and $M \in \mathbb{N}$ is large enough so that $2^{-M} < \gamma$, and hence $s_j \in]0, \gamma[$. Note that then

$$\log_2(|\xi_k|) = \frac{\log_2(e)}{2} \frac{1}{s} \sum_{\ell=1}^N \delta_{\ell k} 2^{-\ell M} = \sum_{\ell=1}^N \delta_{\ell k} 2^{(N-\ell+1)M+9} \in \mathbb{N},$$

so that $|\xi_k| \in \mathbb{Z}_+$, as it should.

With the choice made, we have

$$\begin{aligned} s_j t_k &= \pi \sum_{1 \leq \ell < j} \delta_{\ell k} 2^{M(j-\ell)} + \pi \delta_{jk} + \pi \sum_{j < \ell \leq N} \delta_{\ell k} 2^{M(j-\ell)} + 2\pi \log(2\pi) s 2^{Mj} \\ e^{i s_j t_k} &= e^{i\pi \delta_{jk}} \exp \left(i\pi \sum_{j < \ell \leq N} \delta_{\ell k} 2^{M(j-\ell)} + i 2\pi \log_2(2\pi) 2^{(j-N-1)M-10} \right), \\ |e^{i s_j t_k} - e^{i\pi \delta_{jk}}| &\leq \pi \sum_{\ell > j} 2^{M(j-\ell)} \leq \frac{\pi}{2^M - 1}. \end{aligned}$$

Thus we find that

$$\begin{aligned} \mathbb{E}_\varepsilon \mathbb{E}_\eta \left| \sum_{j,k=1}^N \varepsilon_j \eta_k e^{i\pi \delta_{jk}} x_{jk} \right|_X &\leq \mathbb{E}_\varepsilon \mathbb{E}_\eta \left| \sum_{j,k=1}^N \varepsilon_j \eta_k e^{i s_j t_k} x_{jk} \right|_X + \mathbb{E}_\varepsilon \mathbb{E}_\eta \left| \sum_{j,k=1}^N \varepsilon_j \eta_k (e^{i\pi \delta_{jk}} - e^{i s_j t_k}) x_{jk} \right|_X \\ &\leq C \mathbb{E}_\varepsilon \mathbb{E}_\eta \left| \sum_{j,k=1}^N \varepsilon_j \eta_k x_{jk} \right|_X + \sum_{j,k=1}^N \frac{\pi}{2^M - 1} |x_{jk}|_X. \end{aligned}$$

The left-hand side and the first term on the right are independent of M , while the last term vanishes as $M \rightarrow \infty$. On the other hand,

$$\max s_j = s_N = \pi \log_2(e) 2^{-M-10} \rightarrow 0, \quad M \rightarrow \infty,$$

so that we only need the R -bounds of $(-\Delta)^{\mathbf{is}}$ for arbitrarily small positive values of s to make this estimate. Finally, when δ_{jk} are arbitrary numbers in $\{0, 1\}$, the exponentials $e^{i\pi \delta_{jk}}$ are arbitrary numbers in $\{+1, -1\}$. Thus we obtain the defining inequality of property (α) with the asserted estimate for the constant. \square

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