COSMOLOGY Homework 2 Fall 2018

Due on Saturday September 22 by 14.00.

1. **Practice with natural units.**

   (a) The Planck mass is defined as $M_{\rm Pl} \equiv \frac{1}{\sqrt{8\pi G_{\rm N}}}$, where $G_{\rm N}$ is Newton's gravitational constant. Give the Planck mass in units of kg, J, eV, K, $\rm m^{-1}$, and $\rm s^{-1}$.

   (b) The energy density of the cosmic microwave background is $\rho_\gamma = \frac{\pi^2}{15}T^4$ and its photon density is $n_\gamma = \frac{2}{\pi^2}\zeta(3)T^3$, where ζ is Riemann's zeta function ($\zeta(3) \approx 1.20$). What is the energy density in units of $\rm kg/m^3$ and the photon density in units of $\rm m^{-3}$, i) today, when $T = 2.725$ K, ii) when the temperature was $T = 1$ MeV? What was the average photon energy, and what was the wavelength and frequency of such an average photon?

   (c) Suppose the mass of an average galaxy is $m_G = 10^{11} m_\odot$ and the galaxy density in the universe is $n_G = 3 \times 10^{-3}\rm Mpc^{-3}$. What is the galactic contribution to the average mass density of the universe, in $\rm kg/m^3$?

   (d) The critical density of the universe is $\rho_c \equiv \frac{3}{8\pi G}H_0^2$, where H_0 is the Hubble constant; let us adopt the value 70 km/s/Mpc. What is the critical density in units of $\rm kg/m^3$ and in $\rm MeV^4$? What fraction of the critical density is contributed by the microwave background (today), by starlight (see problem 1.2), and by galaxies?

2. **Redshift in Newtonian cosmology.** Continuing from problem 1.3, consider the case of critical density ($K = 0$). Denote the distance of galaxy G from the origin by r_G.

   (a) Solve for $r_G(t)$.

   (b) An observer at the origin sees the light from the galaxies redshifted due to the Doppler effect. For electromagnetic radiation, when the source is moving away from the observer at speed v, we have $\lambda_{\rm obs}/\lambda_{\rm em} = \sqrt{\frac{1+v}{1-v}}$. Show that at short distances we obtain the Hubble law: $z = Hr_G$. What approximations do you have to make?

3. **Curved space.**

   (a) Consider the spatial part of the Robertson–Walker metric (at time t when $a(t) = a$) in the two cases $K > 0$ and $K < 0$. What is the volume of the spherical region whose distance from the origin is between s and $s + ds$? What is the deviation from the Euclidean ($K = 0$) result when $s = 0.1R_{\rm curv}$, $s = R_{\rm curv}$, $s = 3R_{\rm curv}$, and $s = 10R_{\rm curv}$? (Recall that the curvature radius is $R_{\rm curv} \equiv a/\sqrt{|K|}$.)

   (b) Show that the volume of a hypersphere with curvature radius $R_{\rm curv}$ is $2\pi^2 R_{\rm curv}^3$.