

$1 \text{ eV} = 11600 \text{ K} = 1.60 \times 10^{-19} \text{ J} = 5.07 \times 10^6 \text{ m}^{-1} = 1.52 \times 10^{15} \text{ s}^{-1} = 1.78 \times 10^{-36} \text{ kg}$   
 $c = 1 = 3.00 \times 10^8 \text{ m/s}$   
 $\hbar = 1 = 197 \text{ MeVfm}$   
 $1 \text{ pc} = 3.09 \times 10^{16} \text{ m}$   
 $1 \text{ a} = 3.156 \times 10^7 \text{ s}$   
 $h \equiv H_0/(100 \text{ km/s/Mpc})$   
 $(100 \text{ km/s/Mpc})^{-1} = 9.78 \times 10^9 \text{ a}$   
 $= 2998 \text{ Mpc} = 4.69 \times 10^{32} \text{ eV}^{-1}$   
 $T_0 = 2.725 \text{ K} = 2.348 \times 10^{-4} \text{ eV}$   
 $T_{\nu 0} = (4/11)^{1/3} T_0$   
 $z_{\text{dec}} = 1100$   
 $m_e = 0.511 \text{ MeV}$   
 $m_N = 938 \text{ MeV}$   
 $B = m_p + m_e - m_H = 13.6 \text{ eV}$

$\zeta(3) = 1.20206$   
 $G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}$   
 $M_{\text{Pl}} \equiv (8\pi G_N)^{-1/2} = 2.436 \times 10^{21} \text{ MeV}$   
 $g_n = g_p = g_e = 2, \quad g_H = 4$   
 $Q = m_n - m_p = 1.293 \text{ MeV}$   
 $g_*(T \ll m_e) = 3.363$   
 $g_{*S}(T \ll m_e) = 3.909$   
 $g_*(1 \text{ MeV}) = g_{*S}(1 \text{ MeV}) = 10.75$   
 $\tau_n = t_{1/2}/\ln 2 = 880 \text{ s}$   
 $n + \nu_e \leftrightarrow p + e^-$   
 $\mu_e \ll T \quad (\text{when } T > 30 \text{ keV})$

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$$

$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

$$n_i = \left\{ \begin{array}{l} 1 \\ 3/4 \end{array} \right\} \frac{\zeta(3)}{\pi^2} g_i T^3 \quad (T \gg m_i)$$

$$n_i = g_i \left( \frac{m_i T}{2\pi} \right)^{3/2} e^{-\frac{m_i - \mu_i}{T}} \quad (T \ll m_i)$$

$$3 \frac{\dot{a}^2}{a^2} + 3 \frac{K}{a^2} = 8\pi G_N \rho + \Lambda$$

$$3 \frac{\ddot{a}}{a} = -4\pi G_N (\rho + 3p) + \Lambda$$

$$\dot{\rho} + 3 \frac{\dot{a}}{a} (\rho + p) = 0$$

$$\rho = \frac{\pi^2}{30} g_* T^4$$

$$s = \frac{2\pi^2}{45} g_{*S} T^3$$

$$g_*^{1/2} t T^2 = 1.51 M_{\text{Pl}}$$

$$n - \bar{n} = \frac{g T^3}{6\pi^2} \left[ \pi^2 \frac{\mu}{T} + \left( \frac{\mu}{T} \right)^3 \right] \quad (T \gg m)$$

$$n - \bar{n} = 2g \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T} \sinh \frac{\mu}{T} \quad (T \ll m)$$

$$\Gamma = n \langle \sigma v \rangle$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \text{arsinh}(x)$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\varepsilon = \frac{M_{\text{Pl}}^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta = M_{\text{Pl}}^2 \frac{V''}{V}$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$$

$$\rho = \frac{1}{2} \dot{\varphi}^2 + V(\varphi), \quad p = \frac{1}{2} \dot{\varphi}^2 - V(\varphi)$$

$$H^2 = \frac{V(\varphi)}{3M_{\text{Pl}}^2}$$

$$3H\dot{\varphi} = -V'(\varphi)$$

$$\mathcal{R} = -H \frac{\delta\varphi}{\dot{\varphi}}$$

$$\mathcal{P}_g(k) \equiv \left( \frac{L}{2\pi} \right)^3 4\pi k^3 \langle |g_{\mathbf{k}}|^2 \rangle$$

$$\left( \frac{2\pi}{L} \right)^3 \sum_{\mathbf{k}} \rightarrow \int d^3k$$

$$\mathcal{P}_\varphi(k) = \left( \frac{H}{2\pi} \right)_{aH=k}^2$$

$$\mathcal{P}_{\mathcal{R}}(k) = \left( \frac{H}{\dot{\varphi}} \right)^2 \left( \frac{H}{2\pi} \right)_{aH=k}^2$$

$$n(k) - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}$$

$$\left( \frac{\delta T}{T} \right)_{\text{obs}} = \frac{1}{4} \delta\gamma - \mathbf{v} \cdot \hat{\mathbf{n}} + \Phi(t_{\text{dec}}, \mathbf{x}_{\text{ls}}) + 2 \int \dot{\Phi} dt$$

$$e^{i\mathbf{k} \cdot \mathbf{x}} = 4\pi \sum_{lm} i^l j_l(kx) Y_{lm}(\hat{\mathbf{x}}) Y_{lm}^*(\hat{\mathbf{k}})$$

$$\ddot{\Phi}_{\mathbf{k}} + H(4 + 3c_s^2) \dot{\Phi}_{\mathbf{k}} + c_s^2 \frac{k^2}{a^2} \Phi_{\mathbf{k}} + [2\dot{H} + (3 + 3c_s^2)H^2] \Phi_{\mathbf{k}} = 0$$

$$\delta_{\mathbf{k}} = -\frac{2}{3} \frac{k^2}{(aH)^2} \Phi_{\mathbf{k}} - 2 \frac{1}{H} \dot{\Phi}_{\mathbf{k}} - 2\Phi_{\mathbf{k}}$$

$$\Phi_{\mathbf{k}} = -\frac{3 + 3w}{5 + 3w} \mathcal{R}_{\mathbf{k}}, \quad (w \equiv p/\rho)$$

$$\int d\Omega Y_{lm}^*(\theta, \phi) Y_{l'm'}(\theta, \phi) = \delta_{ll'} \delta_{mm'}$$

$$\sum_m |Y_{lm}(\theta, \phi)|^2 = \frac{2l+1}{4\pi}$$

$$\int_0^\infty \frac{dz}{z} j_l(z)^2 = \frac{1}{2l(l+1)}$$

**The particles in the Standard Model and  $g_*(T)$**   
*Particle Data Group, 2018*

Quarks	$t$	$173.0 \pm 0.4$ GeV	$\bar{t}$	spin $\frac{1}{2}$ 3 colours	$g = 2 \cdot 2 \cdot 3 = 12$	
	$b$	$4.18 \pm 0.03$ GeV	$\bar{b}$			
	$c$	$1.275^{+0.025}_{-0.035}$ GeV	$\bar{c}$			
	$s$	$95^{+9}_{-3}$ MeV	$\bar{s}$			
	$d$	$4.7^{+0.5}_{-0.3}$ MeV	$\bar{d}$			
	$u$	$2.2^{+0.5}_{-0.4}$ MeV	$\bar{u}$			
						72
Gluons	8 massless bosons			spin 1	$g = 2$	16
Leptons	$\tau^-$	$1776.86 \pm 0.12$ MeV	$\tau^+$	spin $\frac{1}{2}$	$g = 2 \cdot 2 = 4$	
	$\mu^-$	105.658 MeV	$\mu^+$			
	$e^-$	510.999 keV	$e^+$			
						12
	$\nu_\tau$	$< 2$ eV	$\bar{\nu}_\tau$	spin $\frac{1}{2}$	$g = 2$	
	$\nu_\mu$	$< 2$ eV	$\bar{\nu}_\mu$			
	$\nu_e$	$< 2$ eV	$\bar{\nu}_e$			
						6
Electroweak gauge bosons	$W^\pm$	$80.379 \pm 0.012$ GeV		spin 1	$g = 3$	
	$Z^0$	$91.1876 \pm 0.0021$ GeV				
	$\gamma$	0 ( $< 1 \times 10^{-18}$ eV)				$g = 2$
						11
Higgs boson	$H^0$	$125.18 \pm 0.16$ GeV		spin 0	$g = 1$	1
						<hr/> $g_f = 72 + 12 + 6 = 90$ $g_b = 16 + 11 + 1 = 28$

**History of  $g_*(T)$**

$T \sim 200$ GeV	all present	106.75	
$T < 170$ GeV	top annihilation	96.25	
$T \sim 160$ GeV	EW crossover	(no effect)	
$T < 125$ GeV	$H^0$	95.25	
$T < 80$ GeV	$W^\pm, Z^0$	86.25	
$T < 4$ GeV	bottom	75.75	
$T < 1$ GeV	charm, $\tau^-$	61.75	
$T \sim 100$ MeV	QCD crossover	17.25	(u,d,g $\rightarrow$ $\pi^{\pm,0}$ , $37 \rightarrow 3$ )
$T < 100$ MeV	$\pi^\pm, \pi^0, \mu^-$	10.75	$e^\pm, \nu, \bar{\nu}, \gamma$ left
$T < 500$ keV	$e^-$	(7.25)	$2 + 5.25(4/11)^{4/3} = 3.36$