Higgs Physics Exercise 7 Autumn 2013

(To be returned by 12.00 on Friday 1.11.)

1. For massless scalar electrodynamics, the Lagrangian is given as (see Coleman and Weinberg, Phys.Rev. D7 (1973) 1888)

$$L = -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}(\partial_\mu \phi_1 - eA_\mu \phi_2)^2 + \frac{1}{2}(\partial_\mu \phi_2 + eA_\mu \phi_1)^2 - \frac{\lambda}{4!}(\phi_1^2 + \phi_2^2)^2.$$

   Find the 1-loop effective potential for this theory,

$$V = \frac{\lambda}{4!}\phi_c^4 + \left(\frac{5\lambda^2}{1152\pi^2} + \frac{3e^4}{64\pi^2}\right)\phi_c^4 \left(\ln\frac{\phi_c^2}{M^2} - \frac{25}{6}\right).$$

2. Find out the solutions of the renormalization group equations for λ' and the charge e' in the scalar electrodynamics,

$$\begin{aligned} e'^2 &= e^2/(1 - e^2 t/24\pi^2), \\ \lambda' &= \frac{1}{10}e'^2[\sqrt{719}\tan(\frac{1}{2}\sqrt{719}\ln(e'^2) + \theta) + 19], \end{aligned}$$

   where θ is the integration constant.
   (Hint. Look at the paper by Coleman and Weinberg, Phys. Rev. D 7 (1973) 1888.)