

Soft condensed matter physics, spring 2010

Exercise 3

The answers are to be returned no later than Tue 9.2.2010 at 16:00. The answers are discussed in the exercise session Thu 11.2.2010 at 16:00, room D115.

Brownian motion, random walks and diffusion

1. The Brownian motion of a small (micron-size) particle suspended in an isotropic solvent is one of the simplest examples of stochastic fractals. Due to the fluctuative movement of the solvent molecules and their collisions with the particle, the particle jumps in apparently random directions and each jump has some characteristic mean length a (Brownian motion is thus often called *random walk*).

a) Show that the mean squared end-to-end distance of unconstrained random walk is $\langle r^2 \rangle = Na^2$ where N is the number of steps. (2 pts.)

b) What is the fractal dimension of unconstrained random walk? Explain. (1 pt.)

2. The model of random walk can be used as a very idealized model of a linear polymer in good solvent. When the degree of polymerization is very high (N is very large), show that

$$\frac{\langle r^2 \rangle}{\langle R_g^2 \rangle} = 6,$$

where R_g is the radius of gyration of the polymer.

3. For 1D random walk, the exact probability function is a binomial distribution

$$W(m, N) = \left(\frac{1}{2}\right)^N \frac{N!}{(N/2 + m/2)!(N/2 - m/2)!},$$

where $W(m, N)$ is the probability that the particle starting from the origin will reach the point $x = ma$ after N jumps.

a) Show that for $m \ll N$, the probability function can be approximated as

$$W(m, N) = \left(\frac{2}{\pi N}\right)^{1/2} \exp\left(-\frac{m^2}{2N}\right).$$

Hint: $\ln(W)$, and Stirling's approximation to get rid of the factorials.

b) If N is large enough, the distribution can be considered as a function of a continuous variable $x = ma$ (e.g. $W(x, N)dx$). What is the variance σ^2 in this case? (1 pt.)

4. The Fick's second law of diffusion relates the rate of change of concentration C at a point to the spatial variation of the concentration at that point:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2},$$

where D is the diffusion coefficient (assumed constant in this case). Show that the following function is a solution to the diffusion equation:

$$C(x, t) = \frac{B}{\sqrt{Dt}} \exp\left(-\frac{x^2}{4Dt}\right), \quad (1)$$

where B is a constant independent of both x and t .

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5. a) If in a random walk the time between each step is τ and t is the total time, what is the diffusion coefficient of a random walk (compare problems 3b & 4)? (1 pt.)
- b) Show that for Eq. 1 in problem 4 the mean squared distance $\langle x^2 \rangle = 2Dt$. How does this relate to the random walk mean squared end-to-end distance? (2 pts.)

Recommended reading on the subjects at hand

Richard A.L. Jones, *Soft Condensed Matter*, chapters 4 & 5.

William D. Callister, Jr, *Material science and engineering*, chapter 5.

Michel Daune, *Molecular Biophysics*, chapter 7.