Existential Graphs:
What a diagrammatic logic of cognition might look like

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There exists here different logical structures from the ones we are ordinarily used to in logic and mathematics. Thus logic and mathematics in the central nervous systems, when viewed as languages, must structurally be essentially different from those languages to which our common experience refers.

(John von Neumann, *The Computer and the Brain*, 1958)

This paper examines the contemporary philosophical and cognitive relevance of Charles Peirce's diagrammatic logic of Existential Graphs (EGs), the 'moving pictures of thought'. The first part brings to the fore some hitherto unknown details about the reception of EGs in the early 1900s that took place amidst the emergence of modern conceptions of symbolic logic. In the second part, philosophical aspects of EGs and their contributions to contemporary logical theory are pointed out, including the relationship between iconic logic and images, the problem of the meaning of logical constants, the cognitive economy of iconic logic, the failure of the Frege–Russell thesis, and the failure of the Language of Thought hypothesis.

Key Words: Existential Graphs, Peirce, Diagrammatic Logic, History of Modern Symbolic Logic, Icons, Logic of Cognition.

1 Introduction

This paper examines the background and the philosophical and cognitive relevance of Charles Peirce's diagrammatic logic of Existential Graphs (EGs). In 1896–1911, he developed a number of such logics, and distinguished them as the alpha, beta, gamma and delta parts. The theories of EGs contain, among others, the diagrammatic counterparts to propositional logic, fragments of first-order logic with identity, modal and quantified multi-modal logics, higher-order logics, meta-assertions similar to Gödel numbering, and logics for non-declarative assertions.

Despite these advancements clearly ahead of their time, EGs have played an unusual and largely veiled role in the development of modern conceptions of symbolic logic. The first part of the paper brings to the fore what I take to be the representative aspects of that curious yet slighted history. A century ago, Peirce's proposed alternative met with both neglect and praise. Time was hardly ripe in the pre-computerized era, but even today, diagram logics have faced with ambivalent reception. Even the alternative and unconventional approaches to computability have been dominated by the tradition defined by symbolic outlook on logic.

Diagrammatic logics have not figured in the major historiographies that have travelled over the developments that resulted in the modern notions of symbolic logic. You will find nothing in *van Heijernoort (1967)* or in *Kneale & Kneale (1962)*. You will find only a fleeting mention on Peirce's EGs in *Hilpinen's (2009)* paper in *The Development of Modern Logic* (*Haaparanta 2009*), which in other respects is a notable update of Kneale & Kneale's book. *Hammer's (2002)* brief review of the basic systems of EGs in the *Handbook of Philosophical Logic* ends with a dissuading note, stating that "a diagrammatic logic is simply a logic whose target objects are diagrams rather than sentences. Other than this, diagrammatic logics and logics involving expressions of some language are not different in kind" (*Hammer 2002: 421*). The purpose of the present paper is to argue that these two approaches to logic are not only different in kind but also in the substance the amount of which is gradually beginning to come into sight.

In the second part, I will address the question of the philosophical and cognitive status of EGs—Peirce's 'moving pictures of thought'—as the 'logic of our cognition'. I will examine the background and the philosophical and cognitive relevance of the diagrammatic logic of EGs, including their iconicity and the role of images, the question of the meaning of logical constants and their cognitive economy, the failure of the Frege–Russell thesis, and the failure of the Language of Thought hypothesis. I will assume that the basic notations of EGs are known (see the bibliography of *Liu 2010*).

2 What the existential graphs were

2.1 Saying it in pictures

EGs were first of all, a new way of expressing logical notions, as well as the deductive component, in a graphical, diagrammatic, spatial, topological and iconic instead of the unilinear, conventional and symbolic manner. It parts company with the way natural language is written and spoken. However, Peirce did not come to contemplate diagrams merely to have an alternative notation by means of which to do logical modelling, linguistic analysis or deductive reasoning. He was simply not well versed to think in words, or to come to the grasp of the meaning of linguistic expressions, if one has to follow the symbolic and serial mode of expression. The next quotation is a famous acknowledgement of such incapacity:

> The words or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which seem to serve as elements in thought are certain signs and more or less clear images which can be "voluntarily" reproduced and combined. There is, of course, a certain con-

nection between those elements and relevant logical concepts. … The above mentioned elements are, in my case, of visual and some of muscular type. Conventional words or other signs have to be sought for laboriously only in a secondary stage, when the mentioned associative play is sufficiently established and can be reproduced at will. … The play with the mentioned elements is aimed to be analogous to certain logical conceptions one is searching for.

This was not from Peirce's, this was from Einstein's pen (*Hadamard 1949: 142-143*). However, Peirce expressed the very same sentiment as follows:

I do not think I ever *reflect* in words: I employ visual diagrams, firstly because this way of thinking is my natural language of self-communion, and secondly, because I am convinced that it is the best system for the purpose. (*MS 619: 8, 1909,* "Studies in Meaning")

Earlier, he had defined such schematisations to be diagrams that are certain iconic representations of facts and which may, but need not be, visual:

We form in the imagination some sort of diagrammatic, that is, iconic, representation of the facts, as skeletonized as possible. The impression of the present writer is that with ordinary persons this is always a visual image, or mixed visual and muscular; but this is an opinion not founded on any systematic examination. (*CP 2.778, 1901,* "Notes on Ampliative Reasoning")

Like Einstein, Peirce felt objectionable to labor so much over when twirling diagrammatic or image-like thought-signs into an alien format of symbols. The mind has a natural tendency to excommunicate them. The main purpose for employing visually representable graphs was for Peirce to be able to analyse what Einstein took to be "signs" and "more or less clear images" produced by the mind in thought in an analytic fashion even when symbolic signs may fail us.

Next, I will turn to some hitherto by and large unacknowledged historical matters concerning the reception of Peirce's implementation of his vision.

2.2 *The reception*

EGs had a notable yet largely unnoticed and irregular place in the history of the 20th century symbolic logic. The prospects were all good. In 1902, Peirce publishes the article "*Symbolic Logic* or *Algebra of Logic*", co-authored with Christine Ladd-Franklin, in the influential and widely referenced Baldwin's *Dictionary of Philosophy and Psychology* (*Peirce 1902: 640-651*, printed, with omissions, in *CP 4.372-4.393*). That article is in fact a comprehensive exposition of the propositional (alpha) and the first-order (beta) parts, including a complete proof system for the alpha graphs, and not only. All logical notions and conventions are given a careful philosophical justification. Peirce subsumes diagram logics under the wider notion of an "analytical system of logical symbols", the purpose of which is "simply and solely the investigation of the theory of logic, and not at all the construction of a calculus to aid the drawing of inferences" (*Peirce 1902: 645*). Since he recognises it "as a defect of a system intended for logical study that it has two ways of expressing the same fact" (*ibid.: 645*), diagrammatic notions are ideal in unifying what under the 'calculus' conception would necessitate using different logical constants for the expression of the same underlying fact. For example, the soon-to-be-emerging design of formalised logic by Bertrand Russell had the defect of ripping apart a certain underlying fact of the logical universe of discourse, turning it into the separate notations for existence (existential

quantification), predication (propositional function/predicate terms and bound variables) and identity (a special two-place relation). All of these are in EGs expressed by the same, iconic sign of the line of identity. The result in Peirce's estimation is "by far the best *general* system which has yet been devised" (*Peirce 1902: 649*) and "the only perfectly analytic method of logical representation known" (*MS 284, 1905,* "The Basis of Pragmaticism"). Rival notions for symbolic logic, which were soon to find their foundational value in being able to ape mathematical calculi, would according to Peirce have to be rated not "as much higher than puerile" (*MS 499, 1906,* "On the System of Existential Graphs Considered as an Instrument for the Investigation of Logic").[1]

In the *Monist*, a widely distributed American journal of philosophy ever since its launch in 1890,[2] Peirce publishes material on EGs (*Peirce 1906,* "Prolegomena to an Apology for Pragmaticism"; Peirce *1908*, "Some Amazing Mazes"). He did not include any systematic exposition of the theory in these contributions, as they were meant merely to exhibit the preferred framework for his intended proof of pragmaticism.[3] Some precursory diagrammatic systems had appeared in 1897 in an extensive treatise on algebraic and diagrammatic logic Peirce wished to supersede Ernst Schröder's and Alfred Bray Kempe's earlier attempts (*Peirce 1897*). According to Peirce, Schröder and Kempe had misplaced their orientation towards an all too formal treatment of logic, at the expense of semantic and pragmatic considerations such as how the "diagram is to be connected with nature" (*CP 3.423, 1892,* "The Critic of Arguments").

In manuscripts dating back to 1896 he announces having come to the grasp of definitive diagrammatic systems of logic and the theory of quantification which would considerably advance all the earlier approaches.[4] The detailed exposition of EGs nevertheless appeared only in unpublished draft form. Ten years later, Peirce shows considerable originality in recreating the interactive (game-theoretic) semantics for his graphical logic (*Peirce 1906*; see *Pietarinen 2006a, Ch.3*), which developed upon his earlier suggestions as to the meaning of quantifiers in the context of the logic of relatives. Francis C. Russell, Peirce's distinguished friend in Chicago, was in his own words "counselled to give them a lit-

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[1] Peirce's allusion is to Peano's pasigraphy: "Peano's system is no calculus; it is nothing but a pasigraphy; and while it is undoubtedly useful, if the user of it exercises a discrete freedom in introducing additional signs, few systems of any kind have been so wildly overrated, as I intend to show when the second volume of Russell and Whitehead's *Principles of Mathematics* appears" (*MS 499*). For Peirce logic is "not intended for a plaything"; it is neither a universal system of expression nor a calculus in its limited sense: "This system [of logical algebras and graphs] is not intended to serve as a universal language for mathematicians or other reasoners, like that of Peano. [And this] system is not intended as a calculus, or apparatus by which conclusions can be reached and problems solved with greater facility than by more familiar systems of expression" (*CP 4.424, c.1903*). To these two requisites Peirce adds that he has endeavoured to exclude any consideration of human psyche that could have been involved in those traits of thinking that led to the inventions of the signs employed in his systems of diagram logics.

[2] Though apparently not in Cambridge (see below)!

[3] The proof has been reconstrued in *Pietarinen & Snellman (2006)* and *Pietarinen (2010a)*.

[4] I have detailed in *Pietarinen (2006a)* how EGs dawned upon Peirce in view of his work on quantificational theory at Johns Hopkins University in collaboration with Oscar H. Mitchell and other students of his. It is noteworthy how in the 1897 *Monist* paper he comes to suggest 'Skolem normal forms' for first-order logic, which accentuates the importance of quantifies dependencies, that is, the functional dependence of Σs on Πs. He accomplishes the same feat in the beta part of EGs in terms of the nesting of identity lines.

tle attention", and he attempts to explain what he takes to be the "complex" systems of EGs to the lay reader of the *Monist* journal in a few pages (*Russell 1908: 410-415*). Russell's motivation was that an understanding of these graphs and an ability to think in terms of icons is essential in order for anyone to be able to follow Peirce's "perfectly flawless deduction that is so admirable that no words are available to characterize it in fit measure" (*Russell 1908: 414*). He referred to Peirce's complex system of "card tricks" and "cyclic arithmetic" in his "Some Amazing Mazes" published in two parts in the same issue. In that paper, Peirce applies new forms of higher-order graphical logic in a penetratingly analytic manner to iconically represent the main ideas of his long proof about ring-theoretic properties of systems of cards.

C. I. Lewis, who in his own work was following in Peirce's footpaths in several respects,[5] was among those well acquainted with these publications when writing his account of the history of symbolic logic (*Lewis 1918*; *Murphey 2005*; *Brady 2000*). Having studied Peirce's manuscripts at Harvard and having gotten acquainted with his 1903 writings on the gamma part that outlined some systems of modal logics, Lewis in his 1918 study of the developments on algebraic and symbolic notions of logic refers only to Peirce's 1897 paper among those concerning diagrammatic logic. Lewis's subsequent studies nowhere mention Peirce's work on diagrams despite their apparent authority, together with Hugh MacColl's modal-logical studies, on Lewis's work.[6]

In the Old Continent, prompted by Victoria Welby's correspondence with Peirce in 1903-11, C. K. Ogden, a student and a colleague of Russell and Wittgenstein who was working as Welby's secretary, begins studying the graphs in Cambridge in late 1910. In the correspondence with Welby he praises Peirce's discoveries:[7]

> I think Dr Peirce's letters wonderful, and shall do my best to understand his Existential graphs! I wish he would bring "Significs" into the title of his new work which I hope is nearing completion. (*Ogden to Welby, 12 January 1911*).

> I am much impressed by Dr Peirce and am proposing to try to master his general position. At present I want to get some idea of his Existential Graphs, for he seems to consider them of great importance for Significs, and I am inclined to pay some attention to his advice, after reading his "Classification of Signs", in the letter borrowed last term. It was for the Existential Graphs that I took the Monist for 1908 July (as I am ashamed to say that periodical is not to be seen anywhere in Cambridge!).[8] (*Ogden to Welby, 30 March 1911*).

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[5] Such as in accepting metalogical approach to ascertain pragmatic purposes of deductive systems, and in agreeing with the centrality of the "significance of concepts" rather than the "non-absolute character of truth" (*Lewis 1970: 12*).

[6] Moreover, *Lewis (1923)* studies systems and possible worlds defining them in terms of closely reminiscent of what was later to be known as maximally consistent or Hintikka sets. The semantic notion of possible worlds for modal logics germinated in Peirce's studies (*Pietarinen 2006b*).

[7] The Welby–Ogden Correspondence, The Welby Fonds, York University Archives, Toronto. All quotations from the correspondence refer to copies received from the York University Archives (YUA).

[8] The original handwritten letter has a continuation which is omitted from the original typescript in the YUA: "But I find I have made a mistake and that when he refers in the letter to "my Monist exposition of Existential Graphs" he means the October'1906 number [*Prolegomena to an Apology for Pragmaticism*] and not the other [*Some Amazing Mazes*]: which is

> I very much want to get at his logical theory of signs etc. It seems to me that if it were to be combined in a less abstruse form with your exhaustive material and viewpoint, the result should prove irresistible. (*Ogden to Welby, 29 April 1911*).

In the first, January 1911 quotation, Ogden refers to the contribution Peirce was solicited to write for a book in honor of Welby, *Essays on Significs*, to be edited by John W. Slaughter and George F. Stout (*Schmitz 1990*; *Pietarinen 2009*). The second quotation in fact gives away another interesting piece of information: the *Monist* journal in which Peirce's key papers were published, was apparently not available at the University of Cambridge. As to the third quotation, Ogden means not only Welby's ideas on significs but her marginal notes written on Peirce's 1906 Prolegomena paper which Ogden wanted to be clarified (*Ogden to Welby, 12 April 1911*). His interest in EGs intensifies over the spring, so much so that in 2 May 1911 Welby reports Peirce having found him "a disciple in Cambridge" (*SS: 138*).

> Those years, Peirce himself was no less confident on the prospects of his graphical innovation:

> [T]he system of Existential Graphs … is the simplest possible system that is capable of expressing with exactitude every possible assertion. […] [I]ts expressions are diagrams upon a surface, and indeed must be regarded as only a ~~picture~~ projection upon that surface of a sign extended in three dimensions. Three dimensions are necessary and sufficient for the expression of all assertions; so that, if man's reason was originally limited to the line of speech (which I do not affirm), it has now outgrown the limitation. (*MS 654: 5-6, 17-19 August 1910,* "1st Preface to Essays on Meaning")

A year later he remarks:

> At great pains, I learned to think in diagrams, which is a much superior method [to algebraic symbols]. I am convinced that there is a far better one, capable of wonders; but the great cost of the apparatus forbids my learning it. It consists in thinking in *stereoscopic moving pictures*. (*MS L 231, 22 June 1911, Peirce to Kehler*)

The reference to the stereoscopic moving pictures is noteworthy, since stereoscopic pictures were indeed the latest discoveries in cinematographic technologies in the beginning years of the 1900s (*Gosser 1977*; *Hayes 1989*). Peirce had always been acutely attracted to the development of new technologies. He even wrote a subentry "Stereoscopic diagrams" under "Diagrams" for *The Century Dictionary* (*CD II: 1589*). He found it natural to think that new technologies shape the ways human beings are capable of thinking, and that they provide useful analogies not to be missed in cognitive studies on the logic of thinking. I have suggested (*Pietarinen 2004a*) that the diagrammatic logic of stereoscopic moving pictures corresponds to the logic of two-dimensional moving pictures as IF ('independence

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only a continuation and had to be understood by itself. I hope you will allow me to borrow that number for a short time as well? I am quite ashamed to be so greedy—but I much want to master the difficulty once and for all, and the single number which I have is of little use!" Ogden soon gets hold of the Prolegomena issue as well and becomes "very busy with the Existential Graphs", being "just beginning to see their meaning", which he admits to be "rather a formidable undertaking" (*Ogden to Welby, 12 April 1911*).

friendly') logic corresponds to the one-dimensional notation of logic. Peirce's remark on stereoscopic diagrams in contemporary terms is an IF extension of diagram logics.[9]

2.3 Existential graphs and symbolic logic

Alas, Slaughter and Stout never published the *Essays on Significs*. Living in poverty and with severe health problems, Peirce never managed to complete his contribution to his own satisfaction. What Ogden predicted to be a highly original contribution did not become available. The two unpublished manuscripts entitled "Assurance through Reasoning" (*MSS 669; 670, 1910-11*) are what we have from Peirce's archives to that effect.[10] They might have been the intended drafts for the *Essays* or else provided the necessary logical preliminaries for the larger topic of "Logical Critics" and "Significs" he was planning to complete in book form (*Pietarinen 2009*).[11] Be that as it may, these papers were buried under the piles and boxes of his unpublished manuscripts at Harvard for almost a century. Yet they are among the best introductions Peirce ever wrote on EGs. These two papers discuss the modal gamma part up to quantified multi-modal systems. He may have thought the quantified modal logics to be the new system of diagrammatic logics "capable of wonders", and these papers were at Lewis's disposal during his formative years.

The setbacks accumulated. Ogden and Richard's own book project, *The Meaning of Meaning*, was over ten years overdue. When finally published in 1923, it was supplemented with some fragmentary excerpts of the Welby–Peirce Correspondence taken from Ogden's set of copies of the correspondence he received from Welby. Included is Peirce's wish for Welby to "study my Existential Graphs," which "quite wonderfully open up the true nature and method of logical analysis; – that is to say, of definition."[12] What follows is a buffet of snippets on Peirce's theory of signs, derived mostly from Ogden's handwritten copies of Peirce's 12 October 1904 and 14 March 1909 letters to Welby (*SS: 32* and *108-130*) and from his copy of the "Prolegomena" paper.[13] However, Ogden did not include any exposition of Peirce's logic, which he could have easily done given the material in his possession. No hint at how to use graphs to logically analyse natural language assertions was provided. No trace of Ogden's interest in Peirce's graphs that sparked off in late 1910 through his exchange with Welby remains in the book. True, *The Meaning of Meaning* was to some extent written and prepared for publication already in 1910. But Ogden's interest in logic was apparently short lived all the same, lasting all but one semester. The published version of the book failed to catch the attention it was predicted to have when it was conceived for the first time. Had the book appeared on time, during the era preceding Peirce's and

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[9] More precisely, take sheets of assertions as layers in a 3D space and predicates as cylinders connecting those layers. An equivalent method is to take the lines of identities to have direction: for any two non-connected lines, the flow of semantic information can be both the standard outside-in (Peirce's "endoporeutic") as well as the converse, inside-out direction.

[10] Transcriptions of *MS 669* and *MS 670* are available at www.helsinki.fi/~pietarin/courses/.

[11] In 17 September 1911 Ogden writes to Welby to have seen "no signs of Dr Peirce's Book or of the volume Dr Slaughter is editing: or even of Dr Schiller's Logic." Welby, who died in March 1912, is silent on these matters in their subsequent exchange.

[12] Quoted in *Ogden & Richards (1923: 282)*.

[13] The abundance of typographical errors in Ogden's quotations testifies this.

Welby's deaths, and in a different format and with a proper commentary on all the relevant material Ogden had at his disposal, it should have made a more positive and lasting contribution.

By the mid-1920s, the focus had already turned to other matters. Fueled by the reception of Wittgenstein's *Tractatus (1921)*, Bertrand Russell soon launched a new campaign in logic, promoting a reorientation towards uninterpreted formal languages of logic.[14] Gradually, it turned its back on the algebraic tradition, the birthplace of diagrammatic logic, in its redefinition of what symbolic logic should amount to. Peirce's 1902 entry on symbolic logic in Baldwin's *Dictionary* conjoined it with diagrammatic logic and thus with algebraic thinking. A singular reason for Russell's sea change was Peirce's flippant October 1903 book notice in the *Nation* on the *Principles of Mathematics*, which F. C. S. Schiller reported had driven Russell "hugely annoyed" at once (*Schiller to Welby, 26 November 1903*; *Pietarinen 2009*).

Peirce, Schiller, Christine Ladd-Franklin, Philip Jourdain and Norbert Wiener have all gone on record for actively protesting Russell's remaking of the symbolic as formal logic. Peirce's former student, Ladd-Franklin, pressed the skirmish with Russell for many years after Peirce's death. In the 1917 *Proceedings of the American Philosophical Association* she wrote Russell to be an "unstable guide" for a philosophical mathematician and logician; that his *Principia* contains many "infelicities" and "fallacies"; that its symbolism is "inept"; and that it is not a new work but rather a "Volume II" of the *Principles of Mathematics* (*Ladd-Franklin 1918: 177*)—as Peirce indeed had earlier predicted in his 1903 book notice.[15]

However, most who knew something about EGs and their philosophical and semiotic underpinnings were either soon gone or else lost interest. Russell eased up too, but it was too late: the ground was already set for another campaign. Though *Reichenbach (1947)* is one of the rare instances from that second era that remarks on Peirce's logic and his EGs positively, Quine and his allies produced very unsympathetic reviews of Peirce's *Collected Papers*. They are really fun to quote:

> [Peirce] recognizes the superiority of the ordinary one-dimensional symbolism both in clarity and in facility of manipulation. (*Quine 1935*)

> Peirce would regard "for all *x*" as predicating, of an indefinite *x*, namely whatever *x* the listener might propose, which is less clear and less expedient … [The] fundamental presuppositions of [Peirce's] logic are Faith, Hope, and Charity. (*Quine 1933*)

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[14] Let us add here Wittgenstein's own dismissive assessment of the worthiness of the entire project undertaken in *The Meaning of Meaning*.

[15] "Whoever wishes a convenient introduction to the remarkable researches into the logic of mathematics that have been made during the last sixty years, and that have thrown an entirely new light both upon mathematics and upon logic, will do well to take up this book. But he will not find it easy reading. Indeed, the matter of the second volume will probably consist, at least nine-tenths of it, of rows of symbols" (*Peirce 1903, p. 308*, *15 October* , Review of Russell's *Principles of Mathematics* and Welby's *What is Meaning*). See also my "Notable Women of Logic and Semiotics: Ladd-Franklin and Welby", forthcoming in *Semiotica*.

> [Peirce] loved too much the dialectical elaboration of ideas into strange bye-paths, and his pretence of scientific candor is a dubious counterpoise to the lack of significance consequent to his uncontrolled reasoning. (*Feuer 1936*)

The first remark is simply false, as the August 1910 quotation from Peirce brings out. The tables are turned on the second by Peirce's initiation of game-theoretic semantics (*Pietarinen 2003, 2006a*). Here is a telling snippet to that effect:

> "Any man will die," allows the interpreter, after collateral observation has disclosed what single universe is meant, to take any individual of that universe as the Object of the proposition, giving, in the above example, the equivalent "If you take any individual you please of the universe of existent things, and if that individual is a man, it will die". (*EP 2:408, 1907*)

And the third remark, by Lewis S. Feuer, then a 24-year-old post-doc from Harvard… Well, just study Peirce's logical systems and judge for yourself what they can do for contemporary science, or consult *Sowa (2006)*.

Given the authoritativeness of these men, however, symbolic logic took a different turn from the plans Peirce had set out in his 1902 dictionary article. That article emphasised the fruitfulness of studying logic in terms of icons, comparable to a semantic perspective, while the soon-to-be-prevailing Frege–Russell conception was calculated to begin the theory development with uninterpreted constants and rules of inference. Peirce surely recognized the interest in such purely formal rules as such. He termed them the "Code of Archegetic Rules" of transformation (*MS 478: 151*).[16] He delineated the "purely mathematical definition" of EGs "regardless of their interpretation" (*MS 508*), which he perceived to be useful in studying the proof-theoretic component of the general theories of the alpha, beta and gamma.[17] But such an uninterpreted language alone would not meet the ends and purposes of *theorematic* logical reasoning, reasoning with the discovery of new elements. Likewise, accepting uninterpreted non-logical vocabularies in logic would have compromised the entire project of conceiving "logic as the theory of semeiotic", without which scientific discovery would according to Peirce hardly be possible at all (*MS 336, c.1904*, "Logic viewed as Semeiotics"; cf. *MS 337*).

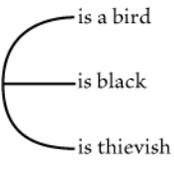## 3 What the existential graphs are

3.1 Rudiments

The syntax of EGs is simple: it consist of (1) a sheet of assertion (SA) representing the universe of discourse, (2) a juxtaposition of graphs upon the SA, and (3) a cut making an incision of a graph from

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[16] The system of archegetic transformation rules is sound, since "the rules are so constructed that the permissible transformations are all those, and all those only, by which it is logically impossible to pass from a true graph to a false one." This metalogical explanation itself "is no part of the rules, which simply permit, but do not say why" (*MS 478: 150*). The system of rules moreover is what we can express in terms of semantic completeness, since "none of its rules follows as a consequence from the rest, while all other permissibilities are consequences of its rules" (*MS 478: 151*).

[17] The gamma part here concerns the "potentials" and not the broken-cut modal logics. Potentials give rise to higher-order graphs in which quantificational lines refer not to individuals but to what in Peirce's terms are "strange kinds" of "proper names" that are "substantive possibilities" devoid of individualities (*MS 508*, "Syllabus B.6").

the SA. This gives rise to the alpha system, and the beta part adds (4) the line of identity (LI) and (5) spots. Peirce terms a branching line or the one that crosses cuts a ligature. Spots are simple qualitative regions of the SA. LIs span between areas of graphs (loose extremities at the interiors of cuts) and spots (line connected to the hooks at the peripheries of spots). With juxtaposition and cuts, disjunctive and conditional (the "scroll") information is easily captured. Negative and positive areas (areas enclosed within an odd or an even number of cuts) segregate universal and existential quantification. The gamma part adds a number of new kinds of signs, such as a broken cut for a modal operator or special spots and LIs for abstraction and higher-order reasoning.

For example, the beta graph represents the assertion "Some black bird is thievish",
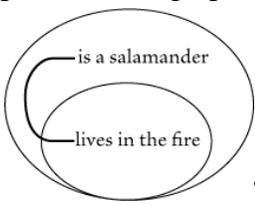
and the graph "Every salamander lives in fire". Here spots are marked by linguistic rhemas, such as "_is a salamander" that has one hook occupied by an LI. The graph
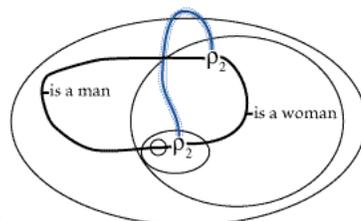

comes in handy in depicting the resumptive quantification in "no two persons love each other", while the graph in which the LI depicting the reflexive construction is cut
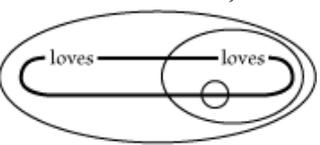
states that "nobody loves anybody who does not love somebody else" (CP 4.418-529).[18]

The rules of inference are likewise straightforward: they are surgical operations in terms of a complete set of illative transformations: (1) a double cut can be added and erased; (2) on a positive area any graph or a portion of a LI can be erased; (3) on a negative area any graph can be inserted or a broken LI mended; and (4) a copy of a graph can be iterated in the same area or any area in its nest and

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[18] Peirce makes in his writings on EGs a number of very modern observations, including those pertaining to generalised quantifiers. For example, the quantification in "there are at least as many women as men" is not first-order expressible, as it
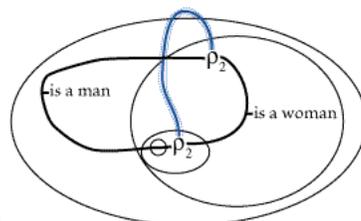
is represented in a gamma graph by a special enthickened LI identifying the two three-place directed spots (the "potentials"), (CP 4.470; see Pietarinen 2007).

conversely, any action resulting from iteration can be reversed, that is, copies of graphs can be deiterated.

3.2 Iconicity: the philosophical bedrock

Peirce's goal with his EGs was to develop a comprehensive iconic logic of cognition. To accomplish this, the workings of mind's information processes must be analysed in a rigorous and structure-preserving fashion even when symbolic expressions fall short of fulfilling that purpose. And they shall fall short, he avers, since "there are countless Objects of consciousness that words cannot express; such as the feelings a symphony inspires or that which is in the soul of a furiously angry man in the presence of his enemy" (*MS 499*).

Iconic logic requires the essential representational and inferential aspects of the processes of the mind to be articulated by diagrammatic signs. According to Peirce, logical diagrams indeed are such precise snapshots of thoughts the mind produces. On the contents of the mind diagrams give only "rough and generalized" pictures (*CP 4.582*), which nevertheless are logically as precise as any conceptual or abstract framework can possibly reveal. The reason is, Peirce explains, that diagrams are icons that reflect continuous connections between "rationally related objects" (*MS 293:11*). Our knowledge about rational connections comes not from experience or mathematical certainty, but from something "which anybody who reasons at all must have an inward acquaintance with" (*MS 293:11*; *Pietarinen 2005b*).

The role model is a practicing scientist, who takes the meaning of experiments to be in the results they produce:

> Consider what effects that *might conceivably* have practical bearings—especially in modifying habits or as implying capacities—you conceive the object of your conception to have. Then your (interpretational) conception of these effects is the whole (meaning of) your conception of the object. (*MS 324: 11-12, 1907*, Letter intended to *The Nation*)

With EGs, one is equipped to represent and investigate analytically "all that is in any way or in any sense present to the mind" (*CP 1.284*). Such graphs are vital in proving the principle of pragmaticism and Peirce appealed to them in his recurrent attempts to demonstrate that philosophy to be the true theory of meaning:

> The study of that system must reveal whatever common nature is necessarily shared by the significations of all thoughts. [EGs] furnish a test of the truth or falsity of Pragmaticism [by disclosing] what nature is truly common to all significations of concepts. (*MS 298, 1905*, "Phaneroscopy")

I will not go on to present the detailed reconstruction of the intended argument here, but it ties in with Peirce's interactive, game-theoretic conception of semantics and his notion of habits as stable, self-controlled tendencies. Since habits have a counterfactual nature, they agree with the contemporary notion of strategies (*Pietarinen 2010a*; *Pietarinen & Snellman 2006*):

> Now, the identity of a habit depends on how it might lead us to act, not merely under such circumstances as are likely to arise, but under such as might possibly occur, no matter how improbable they may be. (No

matter if contrary to all previous experience.) (*CP 5.400*, *1877*, last sentence in parentheses added in 1893, "How to Make Our Ideas Clear")

This is nearly conclusive evidence that habits play the same role as strategic plans of action do in contemporary game theory.

3.2 The logics of EGs

A modest body of literature has emerged on the main ideas of the alpha and beta systems.[19] Overall, EGs can be quite expressive and already Peirce took them to supersede propositional and first-order languages. His own classification was predominantly threefold, though later on he hinted at the fourth, delta part that would focus on modalities:

1. Alpha Graphs (propositional logic)
2. Beta Graphs (fragments of predicate logic with identity)
3. Gamma Graphs, including:
    (a) Modalities (logical, subjective, epistemic, etc.)
    (b) Higher-order reasoning ("Aristotle has all the virtues of a philosopher")
    (c) Meta-logical graphs (graphs of graphs, Gödel-type encoding of the syntax)
    (d) Non-declarative assertions (e.g., commands, interrogatives, emotions, interpretations of music).[20]

A couple of remarks not found in the earlier literature on EGs are in order. What the corresponding fragment of first-order logic is depends on the details of the beta language. For example, the usual presentations, including most of Peirce's own writings on the matter, assume all relation terms (graphically "the spots") to be symmetric. Peirce is nevertheless aware that one should add a special proviso to be able to speak about all relations, including asymmetric ones. The 1902 dictionary entry observes that, "in taking account of relations, it is necessary to distinguish between the different sides of the letters" (*Peirce 1902: 649*). When we do linguistic analysis, the lines connected to spots in beta graphs are normally to be observed not only from outside in ("endoporeutically") but also from left to right just as English speakers read natural language. By 1905 Peirce acknowledges that relations could be generalised as soon as we give "relative significations to spots", so that, "if a spot signifies an asymmetric relation it is necessary to distinguish connection with one part of it as meaning something different from connection with another side". He adds that "colors or other qualities of lines" could be recognised to build up "a corresponding variety of asymmetric relations" (*MS 284: 90*).

In alpha graphs, there is no need for the operation of commutation, because one does not recognise "any order of arrangement [of propositional terms] as significant" (*Peirce 1902: 645*). But the extended beta system has specific spots and lines denoting asymmetric relations. As the sheets of assertion upon which graphs are scribed will be oriented, the property that the graphs can be observed from any angle without losing their isotopy-equivalence is lost.

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[19] See e.g. *Pietarinen (2006a), Roberts (1973), Shin (2002)* and *Zeman (1964)*, as well as http://www.helsinki.fi/˜pietarin/courses/ for explanations of the basic concepts not covered here.

[20] *Pietarinen (2008)* suggests EGs for the representation of commands and imperatives.

Second, in beta there are no free variables. They could be introduced by fiat as specific selectives (*Pietarinen 2010e*), but more recommendable is to use an iconic method. I propose that free variables are dots attached to the hooks within the interiors of the spots. Unlike bound variables, free variables are not attached to the hooks at the peripheries of the spots outside the boundaries of spots, because an attachment to the hooks outside the boundary refers to predication, and free variables do not predicate anything. When they become bound, they can be extended from the hooks inside the spots to the corresponding hooks outside of the boundary.

Third, the beta EGs do not distinguish well between proper names and singular terms. Peirce treats both as predicate terms (spots) having specific qualities that delineate some regions of space in the sheet of assertion. This necessitates a complication in the transformation rules, as we do not want to infer from, say, "Barack Obama is a man" that "It is not the case that something is a man", in other words "Everything is not a man". We come across such illicit inferences if we were to substitute free ends of the identity line within a negative area for a proper name that attaches to a singular term, as we might then apply the standard erasure and deiteration rules to the line which at once permits the inference. A natural solution is to keep apart the notions of names ("selectives") and singular terms ("spots") and never substitute spots for names.

Within the gamma realm we encounter a number of issues which I will mostly forego here. Suffice it to mention that Peirce occasionally referred to the planned delta part, which one needs "to add in order to deal with modals" (*MS 500: 3, December 1911*, "A Diagrammatic Syntax"). What was the delta part predicted to be? He already had several modal logic systems in place, including quantification and multi-modal logics. But they were all introduced intermittently, and he was not able to expose their fundamental nature. He probably envisioned a unifying graphical account for all modality types, one that would encompass the tinctures, identity lines (quantification), and potentials, together with an interpretation that would agree with his scholastic realism—which in contemporary terms is for all practical and logical purposes a possible-worlds semantics (*Pietarinen 2006b*). And presumably it was that unificatory challenge which was to be relegated to the delta part. However, we need to remark that even if all the modal notions were to be removed from the gamma part it would still comprise a whole spectrum of logically diverse graphs, including higher-order logics, logic of collections, imperatives, erotetic logic, and even metaphors (*Pietarinen 2008, 2010e*).

4   And some graphs that have not yet come to pass

Moving on to the philosophical and cognitive significance of EGs, might we view them as an early attempt towards the logic of our cognitive processes? What could they communicate about the concepts of information and information processing? What do they say about the meaning of logical constants?

4.1 The "moving pictures of thought" and their cognitive relevance

EGs provide grounds for Peirce's announcement that they represent our moving pictures of thought:

1. The notion of information in EGs is in the iconic appearance of graphs. Are graphs thus an aid to drawing deductive inferences, over and above those accomplished by symbolic rules of inference? Peirce believed that reasoning is iconic and in making inferences we are experimenting with diagrammatic representations. But even the simplest deductive inference may involve creative consideration of where and what new individuals to add into the course of the proof, as for instance existential instantiation in countermodel constructions demonstrates. We should expect deductive inferences to be facilitated when presented as diagrammatic transformations. That this may indeed be the case is suggested by some optimisation problems in automated theorem-proving, where diagrammatic forms are beneficial though unlikely to solve the fundamental limits of mechanical traits of reasoning. According to Peirce, satisfactory deductive inference, let alone ampliative modes of reasoning, cannot at the end be accomplished by anything else than a "living intelligence" (*MS 499*).

2. Peirce sought for the simple, "indecomposable elements of thought" that could constitute the building blocks of the complexes of our cognitive systems (*MS 284: 43, 1905*, "The Basis of Pragmaticism"; *MS 325: 3, n.d.*, "Pragmatism Made Easy"). I argued in Pietarinen (2005a) that logically, indecomposable elements are the atomic graphs, or "spots" in Peirce's terminology. These spots are not diagrams but *images*, firstnesses of iconic signs that live on the *phaneron* (*Pietarinen 2010b*). Unlike the interpretation of diagrams, the interpretation of images is singular and physiognomic. Peirce emphasises that the result need not be a simple quality (*MS 280: 17, 1905*, "The Basis of Pragmaticism"). I take this to mean that indecomposable elements, represented by the spots, are the iconic counterparts to what the interpretation of non-logical constants of the logical alphabet is in the symbolic realm. Spots, the specific bounded regions of SAs, and having some specific qualities by which they are distinguished from the surrounding space, are just as iconic as logical constants, though not in terms of being involved with observations of diagrammatic structures but in terms of being involved with qualitative imagery. Their intended interpretations are made possible precisely by virtue of them being such images. These interpretations provide the boundary conditions within which the overall interactive semantics may then be built upon. The intended interpretations may change following the changes in spots defined in terms of spatial and metric and not only in terms of topological regions.

3. EGs imply the failure of the Frege–Russell ambiguity thesis. Mentioned in the beginning of the paper, the thesis states that the verb for *being* is multiply ambiguous: it has several uses such that the underlying logic reflects those multiple uses. However, in EGs the line of identity represents predication, identity, existence, and class-inclusion, all in one go. One logical sign captures all these varieties of *being*.[21] The unification and simplification of logical notation has not only the benefit of increased economy and efficiency of expressing diagrammatic assertions but is also a dramatic return to the age-old Aristotelian being qua being, namely being that is represented as lines qua lines and not through anything else. However, the fact that there are multiple yet logically equivalent readings of graphs does not imply ambiguity in such representations. Contra *Shin (2002)*, we need not puzzle over the "visually clear" and "intuitive" ways of "reading off" these graphs. What we think of as natural-language ambi-

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[21] We can add anaphora here, too: "A dean dances in the park. He sings" = "A dean dances in the park and *is* singing". Aside from predication, identity, and existence, the same notation also takes care of coreference.

guity does not carry over to the iconic realm of diagrammatic expressions. As diagrams enjoy cognitive economy not encountered in the symbolic systems of logical languages based on conventions, this is just to say that the correct way of understanding EGs is not through translations to sentences of propositional, first-order or modal logics.[22]

4. What, then, is the meaning of logical constants? Where does such a meaning come from? A new answer can be sought for in the diagrammatic iconicity of logical expressions: negation, conjunction, implication, and quantification are iconic signs, and hence capable of expressing their own meaning. Negation is an operation of incision of an area of a graph from the space of assertion in question followed by a reversal of that area; conjunction is juxtaposition of assertions in the space; given two nested cuts, implication is ability to continue a passage from the area of an outer cut area to the area of the inner cut; and quantification is a dot or a continuous line the extremities of which hit upon certain elements in the domain of discourse of the topological manifold of all potential assertions. Hence the meaning of logical constants does not follow from inference or transformation rules. A further argument is that it is impossible to diagrammatise the TONK connective by any transformation rules. TONK takes the introduction side from the disjunction rule and the elimination side from the conjunction rule and merges these into the one mock rule. However, there is no way of erasing a negated graph from a negative area which is not a result of any iteration.[23] As far as the meaning of logical constants is concerned, therefore, the iconicity of logical signs means attuning to the properties of the space within which EGs are scribed and hence is a feature that has to precede the deductive and transformative conception.

The fact that logical constants may be spatial and need not follow the linearity of time was much later reaffirmed: "We speak in real time, and real time progresses linearly. … But formal languages are not spoken (at least not easily). So there is no reason to be influenced by the linearity of time into being narrow-minded about formulas. And linearity is the ultimate in narrowness" (*Enderton 1970: 393*). Enderton refers here to partially ordered quantifiers, which as noted may well provide a symbolic counterpart to those "stereoscopic" graphs Peirce's alluded to in his June 1911 letter (*MS L 231*).

Peirce in fact maintains in the same letter that the rational parts exhibited in diagrammatic syntax "are really related to one another in terms of relations analogous to those of the assertions they represent." Hence, "in studying this syntax we may be assured that we are studying the real relation of the parts of the assertions and reasoning," which is not the case "with the syntax of speech" (*MS L 231: 10*). He notes the syntax of speech to be restrictively linear, much like two-dimensional algebra is in comparison with, say, the topological higher-dimensional algebras.

At present we have the 'heterogeneous' logics at our disposal (*Barwise & Etchemendy 1995*; *Shin 2004*). But they are not iconic in the full sense of the term. They combine diagrammatic with symbolic signs and replace some of the constituents, such as predicates, which in EGs are non-diagrammatic icons of images, with symbols. Conversely, symbolic logic is heterogeneous in the sense in which al-

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[22] *Sánches (1987)* studies natural logic, an attempt to take logic closer to the actual structures of natural language. In that regard the goal it is similar to Peirce's graphs, to take a step closer to the iconic structures of thought. I am indebted to Johan van Benthem for pointing out the relevance of Sánches's studies.

[23] At least not unless we scribe diagrams on higher than three-dimensional SAs.

gebraic, model-theoretic or inferential thinking all have to do with diagrammatic conceptualisations. Of course, EGs do not claim to be completely and purely iconic, either, but to strive to be "as iconic" representations of logical thought "as possible". Iconicity represents relations by "visible relations analogous" to the intended, actual relations in the model (*MS 492: 22*). Such profound iconicity is related to the idea of diagram construction and assertions as utterances employing "any method of graphic communication" (*MS 492: 24*). Peirce operationalises this communicative aspect of semantics by imagining a dialogue that takes place between the utterer and the interpreter, which are "intelligent agents" of our "make-believe" (*MS 3*; *MS 280*; *Pietarinen 2006a*).

5. It would nevertheless be a gross error to mistake the theories of EGs for the class of theories such as mental models (*Lakoff & Turner 1989*, *Johnson Laird 2002*), cognitive spaces (*Gärdenfors 2000*) or image schemas (*Hampe 2005*). These cognitive theories take spatial arrangements of conceptualisations to be the meanings of assertions. According to the iconicity of logical diagrams, however, the meaning is not in the representations, because they are signs, that is, representations that make themselves interpretable. In other words, to understand complex diagrammatic expressions requires a theory of semantics and pragmatics. To take meanings of complex assertions to be in representations would imply nominalism and internalism according to which meaning *is* conceptualization in schemas, image-like qualities, mental models or what have you. There is nothing distinguishing your images as being right or wrong about something, or as being true or false in a model. Instead, EGs are evidence for externalism in the strong pragmatist sense that takes meaning to be connected with extra-linguistic habits of actions. There is a world of difference between cognitive linguistics and semantic/pragmatic theories of meaning instigated by Peirce and developed further by Paul Grice (*Pietarinen 2004b*).

Consequently, the so-called 'Language of Thought' hypothesis is ready to be taken off the board. For a postulation of internal, symbolic language beneath the logical level is from the point of view of EGs implausible. A brain-wired internal code cannot determine which of the multiple readings of icons would be the intended ones. From the point of view of Peirce's theory of signs, it does not even make sense to pose the possibility of a symbolic level beneath an iconic one in the first place, because symbols involve indexicality and indexicality involves iconicity. That is to say that icons—images, diagrams and metaphors alike—are the closest we get at in terms of a logical representation of cognitive thought operations and processes of gaining new information. To claim otherwise is to subscribe to the separation of cognition and meaning, which admittedly has radicalised the 20th century thought (not only in the realm of symbolic logic in terms of its formal purification but also in such structuralist and formalist traditions in the studies of arts, culture and semiotics that aimed at building ideological barriers between language, thought and the world instead of uniting them).[24] To put logic and cognition together again is to forever close the door to those lost paths.

4.2 Logical matters

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[24] *Greimas and Courtés (1982)* should function as a warning sign. It is not an occupational hazard that their entry on "Semiotics" has no reference to Peirce, for example.

The issue with regard to recent logical developments concerns the relationship between non-classical and deviant logics as well as the possible extensions and variations of the standard systems of EGs. Here is a concise list of such issues:

1. Intuitionistic versions take the *cut*, which is the icon of negation, to be an *incision* and not a *reversal* (*Pietarinen 2006a: 169*). Thus a doubly cut proposition does not yield the proposition itself. *Zalamea (2008)* offers an alternative proposal as to how to get at an intuitionistic version of EGs by changing the iconic representation of the conditional.

2. New modal systems for the gamma part can be developed by systematic variation of transformation rules. Some of them were studied long ago in *Zeman (1964)* but never taken further. We need to place generic constraints on the transformation rules in order to generate different systems, and to study the relationship of such transformation rules to the accessibility relation in modal logic.

3. Peirce proposed representing higher-order notions, such as the relations of *anteriority* and *succession*, still are routinely regarded as Frege's sole discoveries, by a modification of gamma graphs with spots as "potentials" that use abstraction and lines of identities as "objective possibilities" (*MS 508*, "Syllabus B.6"). He also suggested some transformation rules for the ensuing higher-order graphs, and notes that they appear to result in incomplete systems of rules (*ibid.*)—we of course know that second-order logic is semantically incomplete. Since the semantics can be modified to weaker versions for semantically incomplete logics (*Krynicki & Mostowski 1995*), the search for useful proof systems for higher-order languages need not be a dead end, however.

4. We ought to inquire about impossibility proofs as well. Is there something not representable by icons but indispensable in symbolic communication? One candidate is fixed points such as in modal μ-calculus: it is not at all obvious what the essentially iconic core in recursion and fix-point operators is. Another issue hard to diagrammatise has to do with multiplicative connectives familiar from linear logics. On the other hand, it is worth keeping in mind that these are both paradigm examples of systems that are born when the formal and structural assails the semantic and pragmatic.

5   Conclusion

Existential Graphs have a good claim to be the logic of our cognition, along the lines of presenting the "moving picture of the action of the mind in thought" (*MS 298: 1, 1905*, "Phaneroscopy") and the "system for diagrammatizing intellectual cognition" (*MS 292: 41, 1906*, Draft of "Prolegomena"). To fully argue for these claims necessitates a combination of logical and cognitive issues, some of which have been raised here, including the role of icons and images in logical theories, the meaning of logical constants and their cognitive economy, the failure of the Frege–Russell thesis, and the failure of the Language of Thought hypothesis.

Combined with the curious history and the growing awareness of the impact of iconic and diagrammatic logics to real issues in science—including the contribution pragmatism has to mathematics (see *Pietarinen 2010c*)—it is now critical to return to the roots and resume the joint investigation of the philosophical, cognitive, and logical facets of systems that can demonstrate the 'logic in action'. And

to begin just where Peirce left off when he waved goodbye to attempts to envisage such action solely through our preconceived symbolic spectacles.

Acknowledgments

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