Chapter 3

PEIRCE'S GAME-THEORETIC IDEAS IN LOGIC

> THINKING ALWAYS PROCEEDS IN THE FORM OF A DIALOGUE, — a dialogue between different phases of the *ego*, — so that, being dialogical, it is essentially composed of signs, as its Matter, in the sense in which a game of chess has the chessmen for its matter. (MS 298: 6, 1905, *Phaneroscopy*).

1. Introduction

My title of this chapter is, and is intended to be, openly anachronistic in the sense that game theory as we know it today had, of course, not yet been developed during Peirce's lifetime. However, my purpose is to demonstrate that, on many occasions, Peirce did conceive his logical and semeiotic ideas in ways that allow faithful translation into game-theoretic terminology. One particularly illustrative example of this has been noted in earlier literature: For example, Brock (1980) and Hilpinen (1982, 2004) have shown that in Peirce's logical system — the system that through its later developments came to be known as first-order logic — existential and universal quantifiers, as indeed connectives and negation, are quite explicitly understood as integral parts of an interpretation game or a dialogue between two parties or "functionaries" (MS 500: 13), most often termed the Utterer and the Interpreter.

He alluded to countless terms to describe his dialogical approach to logic and cognition, including

the utterer – the interpreter
the proponent – the opponent
the defender – the attacker
the speaker – the hearer
the addressor – the addressee
the assertor – the critic
the graphist – the grapheus

the Artifex of Nature[1] – the Interpreter of Nature
the symboliser – the thinker
the scribe – the user
the affirmer – the denier
the ego – the non-ego
the quasi-utterer – the quasi-interpreter
the quasi-proponent – the quasi-opponent
the delineator – the interpreter
the concurrent – the antagonist
the compeller – the resister
the agent – the patient
the putter forth – the auditor
the writer – the reader
the Me – the Against-Me
the interlocutor – the receiver.

Also in use were the descriptions "an omniscient being desirous of supporting his assertion" versus "the most knowing antagonist of the assertion".[2] Both members of these pairs are the "repositories of thought" or "theatres of consciousness" (MS 318: 18). Adjectives prefixed to these subjects include "enlightened" (MS 539: 25, c.1903), "right", "rational" and "virtual".[3] The term "virtual" refers to intrapersonal conversation as opposed to person-to-person speaker-addressee relation, and is in this sense as real as the latter.[4]

The roles exemplified by these terms are essential for any genuine semiosis or sign interpretation to take place. One of their main purposes is to establish the meaning of logical expressions asserted in propositions. In many instances, we may go further and study the idea of dialogues or semeiotic interactions themselves from the viewpoint of the mathematical theory of games, which of course was not available in Peirce's time. These games have subsequently found wide use in formal and natural language semantics, computation, and in the philosophy of logic and language.

My aim in this chapter is to explore a couple of aspects concerning the relation between the side of Peirce's thinking from the perspective of game-theoretic conceptualisation, and his logical studies dating especially from the later period of his life. For one thing, such affinity is evident in his overall diagrammatic approach to logic and reasoning, and in his theory of existential graphs (EGs). Despite these findings, his logic does not share many of the modern features of the game-theoretic interpretation, such as winning strategies, crucial in connecting the truth or falsity of formulas to the processes involved in their interpretation. I will argue, however, that early anticipation of the game-theoretic notion of strategy is buried in his concept of a habit.

The modern era of game-theoretic semantics started with the contributions of Henkin (1961), Hintikka (1973a), and Scott (1993). The explicit connection between truth-values and the existence of winning strategies was noted in Hintikka (1973a). Some further philosophical and linguistic applications of game-theoretic semantics are investigated in Hintikka & Kulas (1983); Hintikka & Sandu (1991, 1997). The otherwise extensive and well-represented study of Houser et al. (1997) does not recognise this important dialogical or game-theoretic side of Peirce's logic.

That language and other symbolic systems can be compared with games, interactions or contests of different kinds is, of course, an ancient idea. The generalisation of the idea of the polar opposition of two into metaphysical portions is found in the *chôra* and *kosmos* of Plato's philosophy, in the feminine and the masculine, the distracted and the ordered, the changing and the permanent, the diachronic and the synchronic, and the inseparable and infolded parts of the fluctuating universe's primeval constituents. At times, one of the oppositions dominates, but soon the other will take over. Apart from the theologico-cosmological overtones, which will not be discussed here, the pattern has recurred in philosophical attempts to understand ideas ranging from the Newtonian concept of force to Friedrich Nietzsche's (1844–1650) will to power, and from Cartesian motion to all that rationalists would explain by action versus passion. It permeated Kant's philosophy and the Hegelian Ego, much of the latter only to be kidnapped and put into psychoanalytic services.

As a metaphor for argumentation, Aristotle's Topics and its later incarnations such as *ars obligatoria* are set up as dialogical duels. In the 20th century, dialogue logics resorted to strategisation in their goal of clearing up the concept of proofs. The idea retained its topicality in many of the theories of computation. Attempts have been made to recast the notion of computation itself in terms of interaction between the Computing System and its Environment (Nature). Such attempts might reflect the embodied notions of cognition, but may also be traced to Peirce's understanding of the mind as the "*sign-creatory in connection with a reaction machine*" (MS 318: 18, original emphasis).[5] I will provide a more extensive review of the range of fields in which the notion of games has found its intellectual home in exact and natural sciences in Chapter 10.

Reflecting some genuine plays and recreations such as chess, language has been the object of interest not only in Wittgenstein's philosophy of language, but also in the work of the semioticians of the early 20th century. Ferdinand de Saussure, a notable pioneer of structural linguistics, considered the game of chess to be the man-made counterpart of what language provides in a form that has emerged by natural processes. He was soon accompanied by Louis Hjelmslev, Roman Jakobson and many others in the semiological tradition. The closer comparison that de Saussure went on to make is essentially the presentation of a list of some analogies between language and chess, such as their dynamics,

conventionality of rules, and positionality (history-freeness). The difference he suggests is in the notion of deliberation: while in chess the player intends various moves, in language moves are spontaneous and fortuitous. If we interpret this as the difference between what is strategic and what is non-strategic, de Saussure ends up advocating a difference that runs the risk of erasing practically all he wants to be chess-like in language from it.

The main reason why de Saussure advocated synchronic linguistics was that he took games to be secondary to rules. The converse, according to which games precede the rule-governed system of constitutive rules, is what diachronic linguistics professes. All game-theoretic actions contribute to the meaning, including the history of how the positions are arrived at. I argue in Chapter 8 that this was Wittgenstein's mature position.

Many others then followed de Saussure and Wittgenstein, inspired by the potential of comparing language and thinking to the game of chess to a varying degree and with varying success. Peirce was ahead of the others, however, and he put the comparison into a profound logical perspective.

Language games are not only an illustration of Vaihingerial "as if" philosophy, or something that was termed by Richard Harré "the weakest of all forms of theory — the use of metaphors", namely the last resort of theoreticians of science engaged in the process of trying to select the "candidate for reality" from among the multiplicity of models (Harre, 1961, p. 26) (Harré was attacking the idea of mathematical and formal models as such candidates). They should also be put to the test by harnessing them to methods that pertain to the mathematical theory of games. This theory was largely developed by John von Neumann and his successors, but there were others, too (Chapter 7). It soon started to gain credence in neighbouring disciplines, most notably in economics.

Few have acknowledged Peirce's significant role in the development of economic theories (Hodgson, 1999; Wible, 1998, 2000). Peirce and James influenced Thorstein Veblen's (1857–1929) economic thinking. Veblen boasted economics to be post-Darwinian evolutionary science. The concepts of equilibria and utility were then used during the early phases of economic modelling, but were precisely formalised in game-theoretic terms much later, although this was at least faintly envisaged by some early economists of the 19th century. For instance, Peirce studied and appreciated Francis Ysidro Edgeworth's (1845–1926) Mathematical Psychics, an early work on econometrics and the application of mathematics and science to psychological phenomena, and a precursor of equilibria analysis of social phenomena, even game theory, published in 1881. The extent of this influence does not appear particularly broad, however, and Peirce was quick to dismiss Edgeworth's views on probability as quite untenable, albeit unorthodox and atypical.[6]

Despite this dismissal, the ideas of these two men were not altogether different. Edgeworth realised that if one abstracts away from institutionalised

economics, one may treat the remaining material (the core in gamespeak) as applicable to a much broader range of cases. Likewise, Peirce's notion of a habit (see below) was not evolving on individuals' actions, but rather starting out from the motivational and dispositional attitudes of those individuals, the "habit-taking" tendencies of all beings.[7]

As an independent prospect, games returned to the philosophical scene as a logical theory of the interpretation of formal languages, vindicating Peirce's vision in several respects. The rest of this chapter is devoted to the question of what this vision was.

2. The emergence of the notion of strategy

At a very early stage, Peirce appears to have advanced the connections between dialogical or game-like conceptualisations and logical notions. It is mostly his unpublished writings that contain explanations of the meaning of an algebraically notated quantifier as an interaction between the Utterer and the Interpreter. Many descriptions of this idea also surface in his published work (the special terminology is explained in later chapters):

> Begin by saying: "Take any things you please, namely," and name the letters representing bonds not encircled; then add, "Then suitably select objects, namely," and name the letters representing bonds each once encircled; then add, "Then take any things you please, namely," and name the letters representing bonds each twice encircled. Proceed in this way until all the letters representing bonds have been named, no letter being named until all those encircled fewer times have been named; and each hecceity [proper name, A.-V.P.] corresponding to a letter encircled odd times is to be suitably chosen according to the intent of the assertor of the medad proposition, while each hecceity corresponding to a bond encircled even times is to be taken as the interpreter or the opponent of the proposition pleases. (3.479, c.1896–07, *The Logic of Relatives*).

To the same effect, consider also: "In the sentence 'Every man dies,' 'Every man' implies that the interpreter is at liberty to pick out a man and consider the proposition as applying to him".[8]

It also turns out that Peirce envisioned an elementary game-theoretic interpretation of modal notions, as shown in the passage in which he considered the question of identities of individuals in terms of whether it is possible that similar principles to do with general, essential or determinate characters that distinguish actual objects from one another carry over to general characters that pertain to possible objects:

> How can a character be *general* which cannot possibly belong to more than one possible object? Let us devote a minute to the examination of the nature of the impossibility of two objects precisely alike in all respects being two, in the case of necessities, actualities, and possibilities. A necessary proposition is one which makes its predication of whatever case the interpreter of it may imagine, as contradistinguished from a universal proposition which allows the interpreter a choice only among existent cases. Two forms of proposition predicating the same determinations of any object chosen in either way are one and the same proposition. So two general terms applicable to whatever the interpreter may choose under the same limitations are the same. (NEM 2:516; cf. Hilpinen 1995).

An interesting open question here is whether Peirce thought of something like the accessibility relation familiar in modal logic as a prerequisite for conceiving

alternative states of affairs. This passage tentatively suggests that such relations may not be essential for possible scenarios that participants can "imagine". I will return to this question later on in this and the next two chapters.

The motivation for using the terminology of interactive game situations was, for Peirce as indeed for many of those who succeeded him, to spell out the meaning of quantified statements, rather than to explore in detail how games can actually be played. Such prioritising of the activities of contestants over actually playable situations may provide a clue as to why he did not come to use the term 'game' in relation to his logic. Only later has it become clear how Peirce's logic may be neatly pigeonholed.

Peirce did conduct a number of studies on other kinds of games, such as chess, Tit-Tat-Too, various card games, betting, games of chance (in relation to probability calculus), and many others.[9]

While he did not make any genuine use of the actual term 'game' in his logical investigations, he frequently admitted to having been deeply impressed by Friedrich C.S. Schiller's conception of play in Aesthetische Briefe (MS L 463: 25), a work in which Schiller discusses 'World-spirit's *Spiel-trieb*' (dubbed by Peirce "mere amusement" or "the play of musement", Ger. 'play-drive'). He even judged 'game' as one of those entries in the Century Dictionary that deserve a fairly extensive definition.[10]

Recreational plays and games are quite different, but they also have similarities. No one would deign to play a game with those who do not observe the rules, and are thus do not take it seriously, but alas, if they react to the outcomes, other participants are prone to accuse them of taking it too seriously. There is a trade-off between the amount of musement that can be garnered from a single game and the complexity of its constitutive and regulative rules.

As far as Peirce's use of, or intention to use, game-related concepts is concerned, the reason why he did not come to develop the connection between logic (semeiotics) and games (dialogues) further was the lack of one of the most important game-theoretic conceptualisations, namely the notion of strategy. Hintikka (1998, p. 515) has remarked that "the concept of habit was one of the notions [Peirce] used to serve some of the same purposes as the notion of strategy has been introduced by later thinkers to serve". Indeed, the theory of games deals entirely with games of strategy, and other recreational, non-strategic games are destined to fall outside.

However, there are reasons to believe that Peirce's game-theoretic characterisation of logical notions — which he devised not only for quantifiers and modalities, but also for logical connectives[11] and negation[12] — are far more advanced than has been acknowledged in the literature so far. He did appear to perceive, however remotely, the need for a concept of strategy, albeit in the disguise of his sweeping notion of a habit.

Some preliminary evidence for this suspicion is to be found in places in which Peirce discusses habits in relation to interpretation: "The interpreter will have formed the habit of acting in a given way whenever he may desire a given kind of result" (5.491). This statement is interesting, because here he addresses one participant of the game of language, the interpreter, and emphasises his or her decisions based on the concept of desire. In addition, he writes elsewhere: "A habit arises, when, having had the sensation of performing a certain act, $\mathfrak{m}$, on several occasions $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}$, we come to do it upon every occurrence of the general event, $\mathfrak{l}$, of which $\mathfrak{a}, \mathfrak{b}$ and $\mathfrak{c}$ are special cases".[13] One possible interpretation of this is to take the character of a strategy to be an abstraction of a regulative rule that looks away from any single position in which the player may be located in the relevant semantic game. A motivation is that the outcome of the game — perhaps in terms of payoffs given in the normal-form matrix or in the extensive-form terminal nodes (Chapter 7) — is typically assigned only to total strategies, not to any isolated and individual action or a set of actions, since more relevant than mere observable actions are how actions are constituted. In other words, game theory is concerned with seeking solution concepts that associate payoff profiles with outcomes so that some indication of the right or good courses of action may be perceived.

There is further evidence in Peirce's writings to support the view that habit contains, implicitly, aspects of strategic behaviour and action: "Action cannot be a logical interpretant, because it lacks generality . . . But how otherwise can a habit be described than by a description of the kind of action to which it gives rise, with the specification of the conditions and of the motive?" (5.491).[14] In the terminology of semantic games (Chapter 7), the motives Peirce refers to are the purposes of the two players, the verifier and the falsifier, the former aiming to verify a sentence or an expression and the latter aiming to falsify it.

Further, he writes: "It would be necessary, in order to define a man's habit, to describe how it would lead him to behave and upon what sort of occasion — albeit this statement would by no means imply that the habit *consists* in that action".[15] If we take it that a logical interpretant of a sign is its meaning, what Peirce is in effect saying is that no single action or sequence of actions, that is, no choice or sequence of choices as consecutive moves in a game, can spell out the meaning of the signs in question, because it does not put in the picture how one arrives at such choices. There has to be abstraction and generalisation. In order to do that one needs to effectively employ a strategy that leads to those actions. But strategic content is often difficult to identify, and one needs to content with analysing some extraneous output that becomes visible after the strategies have been applied.

For example, logical interpretants are formed by habits upon which an agent can exercise some real effect. A habit, then, organises interpretants into a general concept that only in certain cases gives rise to sentence meanings.

According to Peirce, this is not to say that there is no way we can know the inner structure of habit, however: "Even from the human mind we only collect external information about habit. Our knowledge of its inner nature must come to us from logic" (NEM 4:142).

Habits are the operational heart of pragmatism and the general rule of its logic. Relevant for Peirce here are "voluntary habits", which are subject to "some measure of self-control", under the necessary condition of "circumstances [having] a triadic influence [in] strengthening or weakening the disposition to do the like on a new occasion".[16] According to Peirce, this amounts to voluntary habit being "conscious habit". What is more, "Meaning of a general physical predicate consists in the conception of the habit of its subject that it implies. And such must be the meaning of a physical predicate".[17] Quite similar "conception of the habit" is involved in the game-theoretic process of interpreting sentences of language or logic.

Moreover, "the habits must be known by experience which however exhibits singulars only".[18] Likewise, strategies are exemplified by individual actions. Since "Our mind must generalize these [singulars]", and that generalisation is achieved by the a "triadic consciousness" or "purpose to act in certain ways (including motive) on certain conditions",[19] we may reinterpret the pragmatist rule of logic in the setting of purposeful, strategic action guided by the triadic relationship between modalities, actions and payoffs.[20]

Admittedly, many other things may be distilled from this all-purpose notion of a habit, but seeing it as a normative rule of action indeed comes close to what nowadays is meant by a strategy. As general rules of action, habits are non-deterministic functions from possible situations to actions. In the game-theoretic setting they acquire purpose specified in the payoff structure. To be noted well is that the relevance of the expectations concerning future courses of events to the constitution of meaning does not lead to semantic holism, since the players do not have perfect foresight and thus cannot compute the habitual relevance of all actions.

Further support for these conclusions comes from studies that address the history and evolution of the concept of habit, such as Camic (1986)'s sociological study. These studies have not quite noted the affinity between habits and strategic actions, but indirectly vindicate it by discussing the across-the-board explanatory character of habits that was in use in early studies of psychological and biological behaviour and adaptation, as well as in socio-economic considerations of Western social thinkers. What happened to that notion was that it was soon "intentionally expunged from the vocabulary of sociology as American sociologists attempted to establish the autonomy of their discipline by severing its ties with the field of psychology, where (esp. in connection with the growth of behaviourism) a restricted notion of habit had come into very widespread usage" (Camic, 1986, p. 1077). Indeed, the original meaning of 'behaviour'

came from the Latin *habere* ('to have'), but this was distorted in behaviourism to harmonise with action and doing, not with being in possession of some general capacity or respect.

What, then, are the measurable quanta that are won or lost in the plays that produce the actions Peirce describes in his passages just quoted? If we look at his notion of assertion, it is fascinating how close to the idea of strategic interaction it comes. Assertions bear a responsibility to its utterer: if they are not true, the utterer will be penalised. Similar normative and disciplinary aspects also apply to the receiving end of the communication, the interpreter.

In order to set these into a game-theoretic scheme, one simply needs to quantify the value of the strategy a player follows, taking the amount of punishment that the actions may bear into account in describing such payoffs. What Peirce endorses is the normativity of language, in other words taking language as a system of norms prescribing its correct use and semantic meaning relations. From the use and meaning normativity the normative component may then escalate to the prosodic, phonetic, morphological, grammatical and syntactic structures of language. The pronouncement of logic as a normative science is, of course, much better known and better documented in Peirce's writings than the normativity of language, but it is only natural that the latter is no different.

In addition to the quantificational part, there need to be rules that define the legal choices in the game, plus some generator that acts accordingly as principles of rationality. The most obvious generator for Peirce was the mind, which, as noted, he referred to as "sign-creatory", something resonant with machine-like systems that put forward and reproduce endless cycles of further signs. For him, rationality was a term perhaps best seen as a principle welded into the notion of a habit, in accordance with the principle of the *summum bonum*, the ultimate good that looms on the horizon. The regulative rules that define which moves are permissible in a game and which are not presents no great difficulty; if the context is natural language then it contains a variety of rules appropriate for the interpretation of natural-language expressions. Some such rules have been described in the literature on game-theoretic semantics (Hintikka & Kulas 1983; Pietarinen 2001b). As a secondary component, the set of defining rules also comprises principles that govern conversation and communication between different parties in typical situations of language use in general. If the context is logic, the rules will be those given by the semantic game for the logical language in question (Chapter 7). The sense in which evolutionary considerations enter the picture when the semiosis tends to infinity is covered in Chapter 11.

Furthermore, epistemic dimensions permeate Peirce's habit. He introduced them to facilitate its use as an organisational principle that could bring an organism's modes of response, or its interaction with an environment, into a unified and strategic rule of action. These organisms need not be conscious, though the control of habits is more or less voluntary. Such habitual responses are

becoming increasingly relevant in evolutionary approaches to language and in game-theoretic approaches to biology, where principles of reflection and rationality are being mitigated.

Such activities, and the frequent references Peirce makes to epistemic, temporal, dynamic and interactive notions, suggest that one ought to place aspects of a habit into a category that roughly corresponds to the contemporary paradigm of 'computation as interaction' (Chapter 8). Meaning of a program, sentence or logical formula may be viewed in computational theories endorsing this classification as a relation between a sign (program) and the interpreter (environment).

It would be productive to explore these connections further. For example, one could think of interactive computing (or game semantics in the so-called programme of 'geometry of interaction') as an epistemic game that computes strategy functions (habits), and of game-theoretic semantics as giving meaning to constituents as a system or environment. There is a ring of the pragmatic maxim here. Accordingly, I will examine some interrelations further in Chapter 8, bringing Wittgenstein into the picture. Further, Chapter 13 concerns other recently emerged forms of communication that can be positioned in the pragmatic tradition. However, I will not discuss Peirce's speculations about the possibility of logical reasoning machines, or whether it can be shown that such machines are impossible (which Peirce did not believe). What the contemporary street value of his suggestions about logical machines is has to be left for further deliberation.

Peirce's notion of a habit surfaces most frequently in his semeiotic and metaphysical work, not in his logical studies. It shows up in psychological (roughly, habit as a disposition), attitudinal (habit as a belief), and physical (habit as regularity in Nature) contexts. However, he did not connect this notion with his otherwise rather advanced game-theoretic outlook on logic in any unequivocal manner. The connection is bound to remain incomplete. For example, the relation between winning strategies and the truth of a proposition is absent. Such a connection is of paramount importance to the modern theory of semantic games (Chapter 7). Admittedly, some fragmentary indications exist: "The duality of the *ego* and *non-ego* is the chief constituent of the idea of the Truth" (MS 515: 24), but this is as far as the connection extends.

I think that part of the reason for this failure is Peirce's mixing of what were later on characterised as syntactic, semantic and pragmatic elements in logic. This cocktail is particularly evident in his groundbreaking theory of EGs that provided a remarkably advanced view of later modal-theoretic approaches to different logics, including first-order and modal logics. Peirce's logic has a clear tendency toward notions that, from the later perspective, are recognised as semantic, but the demarcation between syntax, semantics and pragmatics is not a marked one, and in all likelihoods was not even intended to be something definite.

Despite its rarity in logical contexts, a habit provoked interesting remarks in 4.572 [1905, *Prolegomena*]: "The logical relation of the Conclusion to the Premisses might be asserted; but that would not be an Argument, which is essentially intended to be understood as representing what it represents only in virtue of the logical habit which would bring any logical Interpreter to assent to it". If instead of reading "logical habit" in this passage, we were to read "strategy", we would gain a lucid explication of the meaning of signs (here: arguments) as interpreted by strategic actions undertaken by one of the participants engaged in the process of dialogically or game-theoretically depicted interpretation.

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According to Wittgenstein, we recall, a game, like language and a rule, is an institution (Wittgenstein, 1978, VI, 32). Peirce writes in similar tones. However, for both of them, the notion of a game may impose either too much or too little structure. As far as Peirce was concerned, the term 'game', if we are permitted to use it anachronistically in the sense of the theory of games, is too rigid in much the same way as the real line of numbers is too rigid (or too small in multitude) to uphold the true notion of continua, but too loose in the sense that it does not bring into the picture what the reality of expectations, the would-bes of future actions, might consist of and how they would affect one's decisions. For Wittgenstein, the whole affair of games runs the risk of arbitrariness, that is, they appear to employ arbitrary rules for any given particular situation. For instance, non-cooperative games would need to be repeated in order for one to even begin to gain glimpses of how individuals follow a rule. This latter drawback is much the same as expressed by Peirce: the epistemic considerations pertaining to future deliberations are still missing in the major classes of games.

Indeed, even after some seventy-five-odd years of the existence of game theory, very little is known about all that out-of-equilibria that dispense with hyper-rationality, perfect communication, information, memory, and competition, and that include dynamic or non-stationary payoffs, very large sets of players, syndicates, negotiation with incomplete information, cheating, lying, threat, sympathy and so on.

Even if such notions could one day be accommodated into game theory, with the obvious risk of rendering it technically abstruse and something the critics would no longer be well acquainted with, the theory is not likely to account for all kinds of language games, since there are always those that have no regard for many of these notions. Not all interactions involve judgements to be made about percepts, as many function by way of habitual and spontaneous responses. Some such activities, the critics may well maintain, just are not amenable to mathematical scrutiny.

I will not profess to any full response to such criticism, but some of the issues concerning Peirce's and Wittgenstein's views on language, logic and

games are exposed in Chapter 8. The idea of institutions, or societies of agents and inquirers, prevails in game-theoretic arguments concerning the evolution of semantics and pragmatics of language (Chapters 11 and 12) and the role and use of communities of agents in computation (Chapter 14).

It is worth noting that in the original account of Darwin's theory of evolution, habits might have been playing the role of adaptation. A passage from his autobiography is illustrious:

> I soon perceived that Selection was the key-stone of man's success in making useful races of animals and plants. But how selection could be applied to organisms living in a state of nature remained for some time a mystery to me. In October 1838 . . . I happened to read for my amusement 'Malthus on Population', and being well prepared to appreciate the struggle for existence which everywhere goes on from long-continued observation of *the habits of animals and plants*, it at once struck me that under these circumstances favourable variations would tend to be preserved and unfavourable ones to be destroyed. The result of this would be the formation of new species. (Quoted in Plotkin 1994, pp. 28–29, emphasis added).

Evolution that Peirce endorsed was nevertheless quite different and of a broader range and signifigance than it was under Darwin's conception.

3. The economics of research and evolutionary metaphysics

Even though Peirce moved away from any particular 'solution concepts' for a game of interpretation, his theory of ideal inquiry, which, seen from the institutional perspective, might suggest a candidate for such a solution. The scientific inquiry is performed by a community of agents that in the long run tends to agree on the final opinions concerning the truth of their scientific goals. Likewise, the institutional theory of economics may be viewed as a solution to a game of inquiry, a game in which one puts questions to Nature and holds it to be a relatively reliable source of information. Experimentation on diagrams, the moving pictures of thought as will be observed in Chapters 4 and 5, is one such form of communal activity, albeit that it may well take place within the single mind of one inquirer, consisting of a series of phases of dialogue and deliberation rather than any actual cooperation in societal settings.

Without such an institutionalised and communal approach to experimentation, the limiting goals of inquiry may not succeed. No individual is capable of approaching this limit to anything other than an extremely insignificant degree. In order to come to terms with this brute fact of life, Peirce frequently spoke about the economy of research. The intention was to outline a methodology that can be best upheld within the group of investigators seeking such tests that delineate good hypotheses among a class of possible ones. He introduced the notions of caution, breadth and incomplexity as the principles according to which inquiry is, economically speaking, best conducted.

The economics of inquiry was Peirce's brainchild. He perceived that science is of human making, not free from considerations of what self-interested and self-desired agents would do in social and economic contexts. He argued that

as any commodity in the market, knowledge and information could be priced, managed, marketed and sold: "Knowledge, even of a purely scientific kind, has a money value".[21] Similar views were propagated later by Freidrich Hayek, Ludwig von Mises, and other predominantly Viennese economists, soon to be joined by von Neumann and other ludents. Knowledge beseech a habit-change to take place in the *homo œconomicus honorabilis*, but it also marks a measurable process that we try to harness and turn into a useful good for commercialisation.

What Peirce suggested, and what Nicholas Rescher and others have advanced, is the cost-benefit analysis of what researchers do, what they should do, how would they develop the most reliable methods, and how they should communicate in the most efficient way, given the expenditure of finite amount of time effort, energy and money. A typical domain for the cost-benefit analysis is the minimisation of costs of testing hypotheses while maximising the information that comes out from the tests.[22] The general question that it addresses is that, given a set of prima facie plausible hypotheses produced in a given research project, given investible capita (either fiscal or cognitive), which of the hypotheses disqualify? The point is that this analysis is conducted with the benefit of all science, or the totality of the community of inquirers, in mind, not with reference to some specific group of individuals and their welfare. In this sense, it is the answer to the Gospel of Greed that depressed Peirce in societies in which progress no longer comes from individuals merging their individualities in sympathy with their neighbours.[23]

Peirce distinguished three components in his argument for research economy in relation to hypotheses. First, there is the quality of caution, according to which a hypothesis is broken up into its smallest logical components. For instance, big questions are better to be divided into series of small questions, and why-questions are better to be divided into series of yes-no questions.

According to the component of breadth, a hypothesis is evaluated by its applicability for the same underlying phenomena occurring in other related subjects across contexts and circumstances. Several explanations of the same phenomena should be evaluated according to their consequences that can meaningfully be sorted and classified.

Finally, the quality of incomplexity (absence of complexity, simplicity, artlessness) says, among other things, that because hardly any hypothesis is optimal, such not altogether successful and complete hypotheses ought at least to "give a good leave" (EP 2:110). That is to say, in case one wishes to refute a hypothesis, it could be viewed as setting an example of a good conduct to be followed, by attempting as large a 'break' as possible from it, for example, or by necessitating the opponent the 'use of sides', thus referring forward to new hypotheses. Underdetermination, being a result of accident rather than of design, is the property of billiard balls that Nature it likely to bring off.[24]

On the other side of the metaphysical fence there stands the evolutionary philosophy. Peirce's approach to metaphysics was in essence that the universe consists of continuous processes, not things, and that these processes are continuously connected. Thoughts and intellectual ideas alike are likewise instances of such synechist processes. The universe is governed by three interrelated principles: absolute chance, mechanical necessity, and their synergetic evolutionary love, agapism, of which chance and necessity are degenerate forms. Chance begets order, and order begets laws. Chance, the disorderly property of the chôra of the universe, cannot and ought not to be explained by any theories that we possess. But laws, including natural laws, are not immutable and eternal. They are evolutionary. They are habits of cosmos just as logical truth of a proposition is a habit of its assertor. Habits grow. Universes are progenies or experiments in which laws of the most fundamental sciences are contested, challenged and litigated. We happen to live in one of them. Others, where physical or biological constants may have had different values, had different regularities and irregularities, and were not so lucky and did not survive.

What follows is that no law of physics or invention of experimental science is needed to understand natural laws. In recent cosmological theories of quantum gravity, similar lines of thought are present as in Peirce's metaphysics. They allow room for physical laws that may undergo change, and that the universe is a network of relatives rather than things. They attempt to demonstrate that not by experiments, but by finding a consistent mathematical description in terms of some constructive logic such as topos theory. They allow for observer-related perspective to systems, like non-commutative geometry, which allows for uncertainties in geometric structures. These technical developments are unlikely to answer to the philosophical questions of what natural laws are, how they originated, and what their explanatory value is, but they are a "condiment to excite" the "own proper observation" prompted by these questions (1.241).

The prospects of agapistic evolutionary philosophy were not rosy. Like habits, it was soon superseded by utilitarianism and neoclassical economics promoting free market, bolstered — not entirely justifiably, because of the omnipresence of chance and trial-and-error — by the Darwinian theory of evolution. Why, then, did Peirce accept the economic approach to research in terms of the cost-benefit analysis, apparently so in some discrepancy with agapism?

The answer is that the project Peirce dubbed the economics of research actually contains more than just maximisation of expected utilities given an adequate span of time, energy, intellectual capital and labour. It has to do with the methodology of inquiry. It is thereby linked with methodeutic (speculative rhetoric, formal rhetoric) of normative logic and semeiotics (Chapter 1).

To begin with the methodology, the project states that only plausible hypotheses come to be considered by abduction, much in the same way as good chess players only care to consider a tiny fragment of possible scenarios. The

means of doing so have been implanted into humans during the millennia of their existence, and in this historical development their habits have been nurtured and methods arisen that give instructions of how to block the generation of any (that is, any conceivable but implausible) hypothesis. This historical part of economics is governed by chance (tychism), necessity and their synthesis, and is related to the habitual metaphysical evolution of natural laws.

The relation between habits and pragmatic maxim is that the habits, which may be viewed as propositional attitudes of beliefs of agents, refer to those circumstances upon which agents are prepared to act. There is thus an element of possibility to them. Just as possible scenarios or figments of worlds represent states of affairs for which we must be prepared, so are habits, grown and unravelled as we explore the contingencies, the hypothetical states of affairs that constitute the practical bearings or conceivable effects of the concept that we strive to understand and evaluate.

Alongside with the habits of research that mainly contribute to abduction, the economy of research recommends explicit strategies for conducting actual, institutionalised research. These include methods of taking the likelihood of plausible hypotheses into account in tasks related to induction, performing experiments on complex visual representations of scientific data, or in setting up costly machinery to test infinite divisibility of matter. Further, Peirce explains, "Among hypotheses choose one whose elements are well understood, so that unknown complications, and consequent expense of energy cannot arise. Prefer general hypotheses to special ones, provided the more general are so by being simpler; if they are so by being complex, it is necessary to consider the economics of testing them more particularly" (MS L 75). It is in these optimisation and decision-making tasks that the cost-benefit analysis is best applicable.

The bifurcation of economics into the evolutionary and utility-expenditure compartments implies that Peirce's project of the economy of research is not conflicting with his agapism. The *summum bonum* of open-systems inquiry is in most respects increased by heeding the principles of economy (Chapter 13).

Furthermore, according to him, methodeutic, alongside with critic, is cheap to employ, and by virtue of the principle of economics of research should hence be used throughout. As it studies the power of signs as they appeal to the mind, and as it tries to uncover how the interpretants are linked with symbols, indices and icons, this component of inquiry thus leads us straight to the heart of pragmatism. Methodeutic is also related to albeit more general than pragmatics, which has predominantly been a linguistic discipline. Hence pragmatics of language ought to be studied in connection with methodological questions (Chapter 12). Like methodeutic, its main purpose is to find out how the energies contained in assertions are transmitted into the minds of their interpreters, and what their consequences are. In this sense, speaking of the assertoric force of statements is apt, but its area of application broader than

what the received speech act theories suggest. For instance, questions should be studies in tandem with assertions, also having a degree of energy — potential if not kinetic — in their capacity of requests for information. This is the route that paves the way in which economic considerations are pertinent to logic and the methodeutic of interrogation in inquiry. After all, according to Peirce, "It is a question whether [economics] is not a branch of logic" (MS L 75).

4. Graphs, semeiotics and language

The theory of existential graphs To return to the notion of a habit in logical contexts, it is virtually non-existent in Peirce's *chef d'œuvre*, or in the main component of his logical analysis, his EGs (4.347–584, 1901–02; Roberts 1973a; Zeman 1964), despite the fact that EGs are not at all devoid of game-theoretic ideas. He remarks himself on the role of such activities in this context. For instance, in relation to his tinctured (multi-modal) EGs, game-theoretic concepts rear their heads in his interpretation of the whole theory as collaboration between two parties. These parties or functionaries are to be understood in a broader sense, not necessarily only as persons or animate players but also as mental attitudes or states of mind in a single individual, such as the 'Ego' and the 'Non-Ego' (MS 515: 22, 24).[25]

In fact in most contexts, Peirce laid out the meaning of EGs in terms of two such functionaries: the Graphist who scribes a graph instance on a sheet, and the Grapheus, the interpreter of the graph, who is allowed to manipulate the given graphs according to the rules of the system.[26] He calls the actual starting point of the investigation "the Phemic sheet" (the universe of discourse) designated by one of the participants. Other sheets are placed upon the Phemic sheet as the need arises. Negation, for example, is interpreted as a game-theoretic role switch: "Should the Graphist desire to negative a Graph, he must scribe it on the *verso*, and then, before delivery to the Interpreter, must make an incision" (4.556, 1905). He adds in a footnote: "I am tempted to say that it is the reversal alone that effects the denial" (these statements return with vengeance in Chapter 4). In fact, we can go as far as near assimilation of all the five conventions given for tinctured EGs in 4.552–563 [1905] with general definitions of a game and its intended structure. Furthermore, the four methods of interpretation, or permissions (4.565–569, 1905) are remarkably similar to the constitutive rules of such a game, that is, to the rules prescribing which moves are legitimate and which are not. Similar comments may be voiced on the conventions and permissions crafted for other systems of EGs besides the beta part.

These remarks should suffice to substantiate the finding that Peirce, to some extent at least, did conceive of modalities as related to game-theoretic activities, and that the game approach in general is useful in theorising about modalities. This comes close to the semantic evaluation of modal possible-worlds models as a piecemeal exploration of accessible and conceivable states. Even further, a

moment's reflection on Peirce's tinctured EGs gives us a decent idea about how they could be interpreted in terms of possible-worlds semantics: by varying the tinctures instead of the books of sheets we would get other types of alternatives accessible from the actual sheet, namely those of some other agents whose modalities or propositional attitudes also need to be represented. Taken in totality, then, Peirce's theory of EGs marked the beginning of the development of modern possible-worlds semantics, while at the same time incorporating several game-theoretically recognisable ideas into an overall logical and semeiotic system of diagrammatisation.

The gamma part of EGs (4.510–584; Chapter 4) played a key role in anticipating, and to some extent even in contributing to, the later development of the semantics of modal notions. However, the primary reason why gamma graphs did not actually succeed in forming a semantically articulated modal system was the lack of a clearly defined accessibility relation. Yet, some scattered remarks suggest that something like an accessibility relation between "states of information" (Peirce's term, 4.517, 1903) was what Peirce had in mind, for he occasionally referred to such relations as "selectives", drawn as lines between the states. Sometimes these relations were even crossed to distinguish a particular state.[27]

Furthermore, this idea bears a resemblance to indexical notions of time and location. Nowadays, such notions are customarily couched in two- or multidimensional modal semantics.

The use of selectives also serves to assign meanings to proper names, something that Peirce took to be produced by two or more instances of a name attached to the graphs. The selectives are meant to denote the identities of individuals, such as by their proper names. At times, Peirce expressed some doubts about the status of selectives in EGs, and thought of them as redundant given the machinery of subject names linked with rhemas. It has also been claimed that selectives function like bound variables in first-order logic. It would be more accurate to say that they represent instantiations or substitutions of the dots of the rhemas that play the role of bound variables.[28]

Although mainly developed in the beta part of EGs, identity in the gamma part as a continuous connection between spots is not far removed from the contemporary concept of identification in semantics for predicate modal logic (Chapter 4). For Peirce, the notion of identification meant that the interpreter has to meet with a proper name many times in several contexts, or else he fails to be fully acquainted with it. The first cycle of interpretation connected with a name starts with a selective (that is, the outermost occurrence of the name), which then has to be presented to the interpreter repeatedly, on different occasions and in different contexts (4.568, 1905). Note also that, "By a Proper Name, I mean a Sign whose Object a name of anything considered as a single thing; and this thing which the Proper Name denominates must have been one which

the Interpreter was already acquainted by direct or indirect experience".[29] "A single thing" does not need to be a denotation of a proper name as a single existent individual, but may also represent an idea or a character.

A logical approach to epistemic notions was not alien territory to Peirce, for he took it that a modal proposition is "about the universe of facts that one is in a state of information sufficient to know" (4.520, 1903). He also recognised that propositions have to be evaluated against *conceivable states of information*: "Suppose, however, we wish to assert that there is a conceivable state of information of which it would not be true that, in that state, the knower would not be in condition to know that [the graph] g is true" (4.520). He used a special cross mark sign in his gamma graphs to distinguish a particular state of information from the one to which it refers. The selectives to which these marks refer have, as he remarked, "the additional peculiarity of having a definite order of succession", and thus "are of great use in cleaning up the confused doctrine of *modal propositions* as well as the subject of logical breath and depth".[30]

These remarks may be deciphered by setting them in a modern context. What he was thinking about was logical depth in terms of modal depth as nested occurrences of modalities. Its counterpart in existential-graph semantics is the succession of the states of information by means of the special cross marking. The nesting of knowledge, or a version of the KK-thesis (that is, 'knowing entails knowing that one knows', an issue extensively discussed in the first semantic treatment of the logic of knowledge and belief in Hintikka (1962) and in the subsequent literature, and with its precursors in the later middle ages, see Knuuttila 1993), is traceable in the form of "peculiar and interesting little rules, owing to the fact that what one knows, one has the means of knowing that one knows" (4.521, 1903). However, Peirce refuted the straightforward rule that "whatever one knows, one knows that one knows, which is manifestly false" (4.521). It is immediately after these remarks — probably the only place in his writings — that he introduced the arrow-like notation signifying the fact that one state of information follows another.

In the light of these findings, it is justified to conclude that, in addition to tinctured EGs, the gamma part contains elements of possible-worlds semantics, although this did not come to have a distinctly defined accessibility relation in the sense in which it is used today (Zeman, 1986).

It should be noted, however, that Peirce referred to the projected delta part of his EGs a few times. It is not known what the delta part was to be about, but in all probability it was meant to deal with modal predicate logic in a more explicit manner than the gamma part, and to repair many of its shortcomings, while the gamma part, with which he struggled in his later years, revising it several times, was converging towards a system that could represent and reason about abstractions, collections and higher-order notions. There is textual evidence for this, for one of the few places in which he refers to the delta

part is in the context in which he thought this was what one still had to "add . . . in order to deal with modals" (MS 500: 3) No known document discloses what kind of system Peirce had in mind with his delta graphs. In places he mentions the possibility of extending the graphical system beyond assertions, lamenting that he came to confine his sheets to assertions, and suggesting that other sheets would do as well — including platforms for non-declarative moods such as interrogatives and imperatives, and even those concerning feelings that abound in the arts of music and painting (MS 500). These do not belong to the province of necessary reasoning, such as the anatomy of deductive reasoning in mathematical arguments, however, and for that reason were excluded from diagrammatic studies by Peirce as late as 1911.

The incompleteness of his theory of modalities and his failure to fully recognise the need for a special relation of accessibility between various states of information (bearing in mind that we do not know what was planned in the delta part of EGs) serve to further illustrate the fact that he did not draw together all the detached pieces and ideas he had developed — presumably largely independently of whether he realised some connection between his ideas or not, and presumably largely independently of whether he had developed such ideas in his semeiotic programme of signs or in his more calculi-oriented, although by no means exclusively calculi-like, diagrammatic logical corpus.

A fuller discussion of these points, tracing the exact timing of Peirce's remarks on his dissatisfaction with gamma graphs and on his new projected work on EGs in relation to his other plans and the technological and other innovations of that era, is found in Chapters 4–6.

Logical semeiotics in perspective It was not only the obvious reasons involving the disorderly state of Peirce's writings, his constant lack of time and misplaced orientation, together with the regrettable failure of his colleagues to urge him to put his vast observations into a more coherent presentation, that were behind his failure to collate his rich ideas. There were deeper reasons, illustrated perhaps by a few words about his general outlook on logic.

Logic, as he came to conceive of the notion, formed only part of a much larger project of understanding thought and reality, including things that can be established by means of rules of inference, and things that can be established by means of concepts that we nowadays recognise as semantic, but is not exhausted by either of these methods. This larger semeiotic project, which he attempted to bring to light in the incomplete *Logic viewed as Semeiotics* (1.286–287, c.1904), promised to reveal several logics and calculi that he (and to some extent his collaborators and contemporaries) had developed, many of which clearly involved a semantic, or more strikingly, a model-theoretic component.[31]

Nevertheless, what we usually recognise as semantic in logic may not have sufficed for Peirce when the key problem is not only one of understanding the

language–world or language–model relationships, but also of saying something meaningful about the model–world associations. As to the latter, the project that has been dubbed logical semantics, at least since Tarski made his contributions, would have had little to add to Peirce's general project in which the foundational questions address the role of human thought and action in mediating these links, in tandem with the pragmatic overtones that such roles will echo. In view of this, it is likely that Peirce would have remained singularly unaffected by the course research in logic took at the beginning of the twentieth century and later, rapidly becoming bogged down in a sanitised discipline in which pragmatic issues were isolated from the semantic domain, and in which there was a tendency to avoid any contamination that Peirce could have caused.

One distinguishing feature of the received game-theoretic interpretation of logic is that it evaluates formulas by starting with the outermost component and then proceeding from the outside in, ending when an atomic formula is reached (Hintikka, 1973a). Interestingly, we can trace this approach back to Peirce's treatment of EGs. He coined the method "endoporeutic" (*endon* 'within'; *poros* 'passage, pore', see Chapter 6). For example: "The rule of interpretation which necessarily follows from the diagrammatization is that the interpretation is 'endoporeutic' (or proceeds inwardly)" (MS 514: 16). This method was shown to be at work in Peirce's account of the evaluation of proper names, for instance. More precisely, the first occurrence of a proper name to the interpreter (the selective) has to be the outermost one, proceeding towards further, contextually-constrained occurrences and their interpretations. I have found very few passages in which Peirce elaborated this term in so many words, however. He even mentions the "Endoporeutic Principle" (MS 293: 53), but just a handful of scattered references to it exists elsewhere. Nevertheless, in many places in which the term is not explicitly mentioned, it is clearly being assumed as the principle behind the right direction of the flow of information in logical and linguistic interpretation processes. It is also a principle that would not endorse the so-called Frege principle of compositionality.[32]

There is thus room for conjecture. Had the endoporeutic method become more popular, we might have witnessed the game-theoretic development of logic in full, instead of the more prevalent Tarski semantics.[33] It is of some interest that it was only much later that the usefulness of game-theoretic methods was demonstrated in corners of logic in which the more prevalent methods failed. In retrospect, such developments have vindicated Peirce in that one of the most prominent methods in logical semantics in the early part of the last century only merits an isolated chapter in the study of logic in general, and a fortiori represents only a special case in Peirce's general semeiotic and endoporeutic programme of logic.

What kind of characteristics did Peirce assume, then, for the activities involved in uncovering the meaning of logical statements? There is not much

evidence to suggest that he took games to form any fixed system with a predetermined structure and rules of operation. Yet in his (tinctured) EGs, for example, there are definitions of the game structure and the rules that players must obey. In addition, the various drafts of MS 295: 55–59 show the evolution of the transformation rules for EGs into rules that have a game-like character. But where did he stand on other general characteristics of such games? Is it meaningful to ask whether he thought that they should be ones of perfect or imperfect information? Should they be competitive or non-competitive? What about cooperation versus non-cooperation?

Yes, these are meaningful questions. The question about players' information has, in fact, already been answered (Hilpinen, 1982). For in MS 9: 4, Peirce remarked, "Whichever of the two makes his choice of the object he is to choose, after the other has made his choice, is supposed to know what that choice was. This is an advantage to the defence or attack, as the case may be". Hilpinen comments that, in modern terminology, this means that such games are ones of perfect information, and so is the logic. This is consonant with the fact that Peirce took the law of excluded middle to hold in the non-vague part of logic: "A Proposition is either True or False".[34]

On the question of the competitiveness of the players, Peirce remarked: "The utterer is essentially a defender of his own proposition, and wishes to interpret it so that it will be defensible. The interpreter, not being so interested, is *relatively* in a hostile attitude, or looks for the interpretation least defensible" (MS 9: 3–4). Hilpinen (1982) notes that Peirce meant his system to be what we currently recognise as congenial to zero-sum or strictly competitive games, in which players have competing and conflicting aims that they try to achieve (Chapter 7). Indeed, Peirce typically does not assume cooperation between the Interpreter and the Utterer, and so this conclusion is justified. However, it needs to be added that, at times, he characterised the game between the Graphist and the Interpreter, as in his tinctured EGs, as "collaborative" (4.552), and thus no similar competitive setting was being assumed therein.

Natural language Peirce was using a great deal of natural-language examples to draw motivation for as well as to give content to his logical investigations. Yet, natural language is one of the least-analysed aspects of his semeiotics, although his work provides a rich source for uncovering the meaning of sentence in terms of confrontation of two contestants:

> Instead of the selection of the instance being left — as it is, when we say "any man is not good" — to the opponent of the proposition, when we say "some man is not good," this selection is transferred to the opponent's opponent, that is to the defender of the proposition. Repeat the some, and the selection goes to the opponent's opponent's opponent, that is, to the opponent again, and it becomes equivalent to *any*. (3.481).

Here, the role switch is applied in the interpretation process to the phrases *some* versus *any*. Peirce frequently considered these quantifiers, especially when

adducing examples of expressions that prompt the choice of an individual by one of the parties in the game.

> In my paper on the Classification of arguments, I endeavored to make out that *Some* could be so conceived that its iteration abolished it ('~~An~~ double particularization makes universal'). This I did, I believe, by conceiving ~~the select~~ *Some* to mean that a selection was to be thrown upon the interlocutor.
>
> All A is B. Take any A *you* please and it is B.
> Some A is B. Transfer to your interlocutor (me) the choice of an A and it is B.
> Some Some A is B. Retransfer to you the choice of A and it will be B.
>
> There are but few languages in which two negatives make an affirmative. If *not* means 'less than one' or 'fewer than one', fewer than fewer than one is simply fewer than one. The new signs I propose make *Some Some*, *All*. (MS L 237, 12 November 1900, *Letter to Christine Ladd-Franklin*).

The quantifier *any* was moreover held by him to be a universal quantifier just as *All*, although he recognised *Any*'s commonplace free-choice use.

There is a great deal of further evidence of the kinship between Peirce's logic and the more recent game-theoretic conceptualisations of natural language. According to Peirce, the sentence

> "Any man will die," allows the interpreter, after collateral observation has disclosed what single universe is meant, to take any individual of that universe as the Object of the proposition, giving, in the above example, the equivalent "If you take any individual you please of the universe of existent things, and if that individual is a man, it will die". (EP 2:408).

This is similar, both in spirit and in content, to the interpretation game-theoretic semantics assigns to sentences containing the universal *any* (Saarinen 1979):

> If the game has reached the sentence
>
> X – any Y who Z – W,
>
> then Nature may choose an individual and give it a proper name (if it did not have one already), say 'b'. The game is continued with respect to
>
> X – b – W, b is a(n) Y, and (if) b Z.

It is not only simple individual nominals, but also generalised quantifications that fall within the purview of game-theoretic interpretations. The prehistory of generalised quantifiers is indeed vital, yet still uncharted. As is well known, Frege considered quantifiers as variable-binding operators denoting second-order relations. What seems not to have been noted before is that the need to have generalised quantifiers in logic was already shown by Peirce more than a hundred years ago. For one thing, he refers on many occasions to "hemilogical quantifiers" in addition to universal and existential ones. They were taken to mean phrases such as *all but one*, *all but two* and so on. For example, the algebraic quantifiers Π', Π'' ... were taken to mean products of all individuals except one, except two, and so on (SIL: 203).[35] He even attempted to characterise sentences containing phrases such as "there are at least three things in the universe that are lovers of themselves" according to such hemilogical quantifiers (SIL: 203).

Further evidence of the importance he attached to the generalised notion of quantifiers is to be found in the following:

> Two varieties of [selective pronouns] are particularly important in logic, the *universal selectives*, . . . such as *any, every, all, no, none, whatever, whoever, everybody, anybody, nobody.* These mean that the hearer is at liberty to select any instance he likes within limits expressed or understood, and the assertion is intended to apply to that one. The other logically important variety consists of the *particular selectives*, . . . *some, something, somebody, a, a certain, some or other, a suitable, one.*
>
> Allied to the above pronouns are such expressions as *all but one, one or two, a few, nearly all, every other one,* etc. Along with pronouns are to be classed adverbs of place and time, etc.
>
> Not very unlike these are, *the first, the last, the seventh, two-thirds of, thousands of,* etc. (2.289, 1893, *Speculative Grammar: The Icon, Index, and Symbol*).

Peirce did not interpret these quantifiers in relational terms as ones that quantify sets and express relations that would hold between things and predicates. Rather, it is more likely that, had he continued his development of generalised quantifiers, we would have witnessed the development of game-theoretic semantics for them on a par with games for the usual existential and universal quantifiers during his lifetime.[36] The projected course of such developments can be gleaned from the following passage.

> A subject should be so described as to be neither Universal nor Particular; as in *exceptives (Summulae)* as "Every man but one is a sinner." The same may be said of all kinds of numerical propositions, as "Any insect has an even number of legs." But these may be regarded as Particular Collective Subjects. An example of a Universal Collective subject would be "Any two persons shut up together will quarrel." A collection is logically an individual. (2.324, c.1902–03, *Speculative Grammar: Propositions*).

Accordingly, Peirce came to stress the importance of having generalised quantifiers in logical approaches to language and meaning. In his terminology, the logic concerns particular collective subjects. While some issues related to his view on collections were discussed in Chapter 1, there is ample room for more research on the largely unexplored terrain of these directions into which, as I have argued, he continued to push the study of the logic of collective subjects.

5. Conclusions

If we take logic to constitute a major part of human decision making and cognitive ability, then the game-theoretic approach is utterly natural. In the nomenclature of one of the major parts of game theory, the theory of extensive forms of games (Chapter 7), a strategy has to specify an action for each history, even for those that are on the off-equilibrium paths and never lead to a win, and the strategy has to be specified for every possible choice in a game. Interestingly, this can be emphatically compared with similar sentiments in 5.400 [1893, *How to Make Our Ideas Clear*]: "Now, the identity of a habit depends on how it might lead us to act, not merely under such circumstances as are likely to arise, but under such as might possibly occur, no matter how improbable they may be".

That one needs to take the would-bes, the possible future courses of events, into account is preserved in extensions of the notion of equilibrium that applies to games in which there is imperfect information, such as sequential equilibrium. Moreover, one needs to act as rationally and as optimally in those as in histories

that lie on the equilibrium paths. Here, strategies codify interim beliefs plus an assessment of the beliefs concerning other players' types and past and future actions. What else is this extension, then, but a broadened conception of a strategy that just makes more use of habits than non-sequential strategies, and includes the aspects of habits that relate to beliefs, while resting peacefully within the purview of this broad notion?

One of the themes that permeate Peirce's philosophy is the notion of continuity, his synechism, both in the mathematical sense represented by a pseudo-continuum, and in the metaphysical sense, which he considers the real continuum. This suggests that continuity ought to be respected in any of the game-theoretic schemes in which Peirce's ideas may be reflected. One candidate here would be the class of differential games. These are games in which time is continuous rather than discrete, and decisions have to be made at each instant or point along its continuous measure.

Alternatively, a class of continuous games exists in use in which continuity relates to the idea that pure strategies form a continuum. Whether any of these would constitute a viable continuation of Peirce's outcroppings is yet to be seen, but these points should be investigated with an eye on Peirce's notion of the continuum, far exceeding that of the real line, that "pseudo-continuum" of multitude $\mathfrak{c}$ that did not suffice for him (and so one needs to bring infinitesimals into play, cf. NEM 2:169, Putnam 1995). What such 'non-standard' differential or continuous theories of games would amount to have been little investigated.[37]

The upshot of these observations is that there are many instances in which strategic plan of action and the habit have overlapping significations. Some repercussions of this convergence are imminent in current theories of computation, however blind these theories may be to the complex history and development of this intellectual idea, in which the connection between a thought — say a symbolic term — and its meaning is found habit-changes that such a thought provokes, mostly in observational outcomes of a computational process.

However, this is only one of the many points at which elements of the theory of strategic decisions and Peirce's thoughts on logic and semeiotics meet, and I hope to have uncovered a few more. Such a kinship remains mostly logical in nature, and does not show up particularly often in his economic studies, even though I have here indicated a few such links, and even though "It is a question whether [economics] is not a branch of logic". Soon after his death, the theory of games started to emerge in the works of Ernst Zermelo, Émile Borel, John von Neumann, and many others. The first explicit technical connection between games and logic was that between Skolem functions and winning strategies, discovered by Henkin (1961). I will return to these discoveries in Chapter 7.

Notes

1 Referring to his theory of existential graphs (see sect. 4 below and later chapters), Peirce wrote: "The Graphist is really Plastic Nature, or the Artifex of Nature; and the special permissions are the experiences given to the interpreter of Nature, to the man, to which he is at liberty to attend, or not to attend at all, or to attend and immediately put out of sight, as he will" (MS 280: 23). Peirce means hylozoism of Richard (Ralph) Cudworth (1617–1688) and the idea of an inferior soul or substance by which purposeful behaviour may be attributed to the activities of physical nature. Among the assorted pages of MS 280 we find that "the Graphist *must* be regarded as corresponding to the 'Plastic Nature' of Cudworth, or else to the Artifex of Nature" (30 a.p., emphasis added). Peirce had marked down the neo-Platonist philosophy of hylozoism slightly earlier, in August 1904, in a draft of The Monist article *What Pragmatism Is* (EP 2:331–345) entitled *Nichol's Cosmology and Pragmaticism*, within the context of a dialogue between the Questioner and the Pragmaticist: "*Questioner*: The narrowness of your view of reality only appears more and more strikingly as you go on. You ~~simply~~ are, as you ~~say~~ [yourself well phrased it], simply color-blind to ~~the being~~ [the idea of existence] in itself. *Pragmaticist*: Hylozoism, the doctrine that all matter feels, is an idle ~~theory~~ [and senseless apology for a theory] as long as there is not [a] way of bringing it to the test of experiment; but as soon as such a way shall be found it will ~~be~~ [become] a working hypothesis particularly well worth trying" (MS 329: 22, Copy C).

2 MS 430: 62, 1902, *Minute Logic. Chapter III. The Simplest Mathematics*.

3 MS 25, 1897, *Multitude and Number*.

4 "In all discourse, or reasoning, there are *virtually* two parties. Either there are actually two parties, as when one speaker addresses an audience of one or more persons; or else one person reasons out something with himself, and even then, the difference between his conceptions and opinions before and after a given operation of thought results in his influencing himself much as one person influences another; so that we may say that even in this case there are *virtually* two parties" (MS 25: 2).

5 This and several other related pages, which contain material not found elsewhere in other versions and drafts under the title of *Pragmatism*, were not printed in the transcription of MS 318 in EP 2: 398–433. The claim in the headnote, written by the editors of the Peirce Edition Project, that Peirce's proof of pragmatism is complete in this transcription is thus a little doubtful.

6 Edgeworth was among the key players in the neoclassicist 'utility' movement that grew out of the 'energetics' of physical sciences, popular since about the mid-1800s. Its idea was that energy and its fluctuations are central not only in mathematical and physical investigations, but that they also nurture the human soul, aiming at the maximisation of that nurture. This suggests some tempting analogies with the 'informatics' movement in our late-20th-century computational sciences.

7 6.262, 1891, *Man's Glassy Essence*.

8 5.542, c.1902, *Belief and Judgment*.

9 See, for instance, MSS 1135 (c.1903) and 1525–1537. They had little or no bearing on his logical and semeiotic studies, however.

10 See CD III:2447, CDS XI:0509. Apart from Whatley's *Logick*, Schiller's book was the first philosophical book Peirce is said to have read.

11 MS 290, 1905, *Issues of Pragmaticism*.

12 MS 1147: 3, subentry *Negation*, cf. DPP; 3.480–3.482, c.1896–97, *The Logic of Relatives*.

13 5.297, 1868, *Some Consequences of Four Incapacities*.

14 This statement has been interpreted by the Peirce Edition Project to express the conclusion of Peirce's proof of pragmatism.

15 2.665, 1910, *Critical Logic: The Doctrine of Chances*.

16 MS 318: 48, Note at Pragmatism; cf. EP 2:431.

17 MS 318: 44, Note to Pragmatism continued.

18 *ibid.*: 44.

19 *ibid.*: 44.

20 "No number of existential meanings can be adequate to the meaning of an intellectual concept, since the latter is general; and no collection of individuals, nowever multitudinous, can be adequate to a general" (MS 318: 20).

21 1.122, 1896, *The Scientific Attitude*.

22 See e.g. Rescher (1978a,b, 1989).

23 6.294, 1891, *Evolutionary Love*.

24 Similar qualities will be applied to the methodology of historical pragmatics in Chapter 12.

25 See also: "Though these two functionaries [the utterer and the interpreter] may live in one brain, they are nevertheless two" (MS 500: 13), and "Whenever a person thinks over any question in his own mind, he carries on a sort of conversation. His mind of one minute appeals to his mind of the next minute to agree with it and say whether so-and-so is not reasonable; and then the mind of the next minute ~~says~~ [may say] either 'Certainly by all means, and I wish all future minutes of my mind to take note that this is my decided opinion, after close examination' or else he may say 'Well, that seems so, at first glance, but I don't feel quite so sure of it as that mind of the last minute wished me to be' or he may even think 'Well, look you, my mind of the future, before whom my last minute mind and I, *this* minute's mind are arguing (for we both submit to you as knowing more than either of us do) — it appears to me that that last minute's mind was a goose and entirely failed to perceive the real state of the case', etc etc." (MS 514: 45–46, 1909, [*Fragments on Existential Graphs*]).

26 4.431, 1903; MS 293: 50, c.1906; MS 295: 41, 45, c.1906, Rejected pages for the Monist article of 1906; cf. Roberts 1973a, p. 92.

27 4.518, 1903, *The Gamma Part of Existential Graphs*; MS 467, 1903, *Lowell Lectures. Lecture IV*. Roberts (1973a, p. 86) and Zeman (1964, p. 253) were the first to note the use of a special sign Peirce had in connecting states of information with an arrow-like pointer (a special spot that has been abstracted) attached to them. Peirce's intended meaning was that one state of information follows another. As Zeman notes, one interpretation of this precedence notation is certainly as an accessibility relation. Peirce did not seem to have exploited this device further.

28 This terminology is explained in Chapter 4.

29 MS 612: 33–34, *Common Ground*, 12 November 1908.

30 5.518, 1903, *Consequences of Critical Common-Sensism*.

31 For example, the work of Peirce's gifted student Oscar Howard Mitchell was particularly notable in the development of the notion of the quantifier, and of the logic of multiple dimensions, see Mitchell (1982). His premature death at the age of 38, gone scarcely noticed, surely affected the pace in which the first-orderisation of elementary logic was arrived at by Hilbert and others.

32 The notion of compositionality indicates just one way in which language is built up and understood, not as a necessity but as a property that has become highly popular (Chapters 4, 6).

33 Following Peirce, we could dub Tarski semantics 'ectoporeutic'.

34 4.547, 1905, *Prolegomena*.

35 According to Peirce–Mitchell logic, the sign Σ, denoting the algebraic sum of relative products of terms, corresponds to the existential quantifier, and the sign Π, denoting the product of relative sums of terms, is the universal quantifier. One of the first instances of the name 'Quantifier' is in 3.396 [1885, *On the Algebra of Logic*].

36 Pietarinen (2001b) defines, among other things, some game rules for generalised quantifiers.

37 Aumann (1964) originated the study of infinitesimal notions of players.