THE GENESIS OF PEIRCE’S BETA PART OF EXISTENTIAL GRAPHS

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When it comes to the origins and the development of diagrammatic approach in logic and representation of natural language, one cannot make do without looking into Charles Peirce’s theory of Existential Graphs. That theory introduces a number of fundamental ideas and issues serving as a wake-up call to all contemporary semanticists, historians and philosophers of logic.

Our paper concerns issues to do with the emergence of some of the most fundamental ideas in Peirce’s diagrammatic logic, many of them still unpublished. How did he gravitate at the diagrammatic approach to logic? It is best to frame this question by asking more precisely: How did he arrive at the full-fledged diagrammatic quantification, the theory that corresponds – or has been claimed to correspond in the previous literature (but see Pietarinen 2005, 2011a for clarification) – to what we tend to call the first-order language and its logic? To answer this, one must of course look into the development of his main achievement, the 1897 Beta part of Existential Graphs. After all, he took his system to be, “though far from perfect”, “the most nearly perfect (for the purposes of the science of logic only), system of representation of logical relations which has hitherto been discovered” (MS S 283; Pietarinen 2011a). I here omit discussing the alpha and gamma parts, that is, the propositional and the modal and higher-order diagram logics, respectively, and focus on the first-order case and the diagrammatic notion of the quantifier as well as the question of how such an understanding of quantification was quite remarkable and ahead of its time in representing natural language assertions in the ways it was intended to do.

Historically, it looks as if there was a gradual development through several logicians in the late 19th century, including W.K. Clifford, J.J. Sylvester, Caley, Hamilton, John Venn and A.F. Kempe, which especially in the works of Clifford and Sylvester meant using chemical diagrams to represent algebraic invariants.¹ And it looks as if these developments were slated for the appearance of graphical logics very much like Existential Graphs. Had Peirce not managed to create it, then someone else would have. None of the elements in the works of these Peirce’s contemporaries suffices to explain the genesis of diagrammatic quantification in full, however. They lacked some crucial innovations that Peirce was able to come up with during the early years of the 1890s.²

¹ Sylvester publishes a groundbreaking article on this analogy in the first issue of the American Journal of Mathematics which he launched in 1878 and so came to be one of the founding fathers of graph theory.
² We mentioned Peirce’s account of the history of Euler graphs in MS 479: He accuses Hamilton for misrepresenting the origins to be in Weise’s Nucleus Logicae Weisiane of 1712, though Weise died in 1708 and the book is mostly written by J. C. Lange. The same accusation on unreliability of Hamilton is made by Venn almost verbatim in Symbolic Logic (1894, p. 477, 2nd ed., and p. 423, 1st ed. 1881, the latter reference according to Peirce), and Peirce aptly acknowledges Venn’s rectification. Peirce then notes how another Lange, Friedrich Albert Lange attributes the origins of what we tend to term Euler diagrams to Juan Vives and his tradition. According to Peirce, Euler should be acknowledged, however, as the first logician to develop the idea into the direction of a calculus, which subsequently was improved by Venn.
The steps that led to the creation of Beta Graphs were based on elements that at the end were, I argue, Peirce’s own discoveries. I also argue that his logic was in a certain specific sense related to the behavior of quantifiers that was ahead not only of his fellow logicians but that he also corrected a delimitative mistake in the idea of a quantifier that was to persist in the logical theory long after Peirce and even into our times.

The question “Why and how did the invention of EGs came about?” was recently addressed in Sun-Joo Shin’s SEP article (Shin 2010). According to her, EGs emerged out of the fact that Peirce wanted to come up with iconic forms of logic. This statement is not wrong, but it is unsatisfactory. True, his earlier work had taken the index as the defining sign of the nature of a quantifier, and since that idea met with problems and also since diagrammatic thinking and reasoning had always been there at the background of his earlier algebraic studies of logic, it surely is the case that a strong motivation for a greater role icons could play in one’s logical theory was called for. But the motivation itself does not account for the nature of insights imminent for comprehensive diagram logic. That diagrams are icons is a correct but insignificant observation. What does iconicity consist of logically speaking? To explain the emergence, it is not sufficient to go through the main ingredients of EGs, such as representing relation and predicate terms or lines of identities visually, and then to highlight the iconic nature of these components. At bottom, iconicity has relatively little to do with visual aspects of EGs. Properties of space in which logical graphs take place are to be experienced or imagined rather than seen. And imagination concerns real possibilities. (And real possibilities are opposite to what is fictional.) According to Peirce, the shortfall of language lies precisely in the fact that it gives rise to too pictorial forms of interpretation. It is merely an accidental fact that we happen to scribe projections of logical graphs in two-dimensional, visually observable form. There is much more to logic diagrams than just being counterparts to symbolic expressions that appeal to some specific perceptual modes of recognition. Pietarinen (2010), for instance, shows how there can be auditory versions of logical diagrams. In general, thus, the nature of diagrams is not to be understood by their algorithmic translation to some more familiar symbolic notation by “reading off” of what their visually appealing features may be.

How did it all happen, then? What is an icon when it comes to logic? How are logical diagrams to be understood? Before the appearance of Existential Graphs, Peirce had already forged first-order logic and its quantification theory, and he had readily noted several interesting issues about it. This was in 1885. For instance, interpreting quantification substitutionally is doomed to fail, because the universes of discourse may be “innumerable” (uncountable), and so first-order quantifiers cannot be understood through their substitution instances (see Pietarinen 2012 for the argument). Peirce would move from the algebraic approach to one that could employ the main ideas of graphs, while all along being clear that both approaches are really two sides of the same, diagrammatic way of thinking about logical notions, both having their advantages and

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Peirce then notes that the logic of relations answers why the diagram method works in the way it does and why it is so appropriate to the analysis of the meaning of natural-language assertions. This MS continues with a detailed study of Venn’s improvements (see also CP 4.353).

3 See MS 573: “Another fault of ordinary language as an instrument of reasoning is that it is more pictorial, than diagrammatic. It serves the purposes of poetry literature well, but not those of logic.”
disadvantages. By 1885, he had already insisted upon the fact that icons, together with indices and symbols, are indispensible in logical reasoning and representation. So it cannot be that the motivation for greater iconicity alone would serve to explain the peculiarities that were needed to have a proper quantification in the language of EGs.

During the year 1893 (MS 410, “Analysis of Propositions”, Ch.II from How to Reason/‘Grand Logic’, §74), which was written four years before his announcement of the discovery of both the Entitative and Existential Graphs, he analyzed hundreds of natural-language sentences hoping to get clearer of their logical constitution and meaning. These sentences are all interesting and challenging, many of them are quite complex, and some are even familiar from much later studies in natural-language semantics, such as the scope and anaphora issues introduced in sentences such as “Every woman loves some child of hers” and complex donkey sentences – the former here depicted by Peirce as a kind of ‘neural network’ or a structure of the model of the sentence rather than being any fully-fledged logical representation of what this sentence asserts:

His 1893 analysis of propositions proceeds along semantic and model-theoretic lines. He formulates the “states of things” that one needs to have in order to understand the nature of necessary reasoning. Moreover, conditional propositions are taken to be categorical: “If Enoch and Elijah died, the Bible errs”, is his famous example. And subjects can as well be replaced with “a pebble of a tree equally well”, if some “word-mongers” do not appreciate the talk on “states of things” as they would require another hypothetical proposition: “Every pebble co-existent with Enoch and Elijah as dying men is co-existent with an error in the Bible”. This indeed strikes as a very model-theoretic way of looking at logic. Thus, according to Peirce, with respect to the sentence “Every mother loves some child of hers”,

4 In MS 650 Peirce argues for the superiority of maps over oral descriptions: “Well, what I propose to put you into possession of is a way of making a diagram of any fact you please, and to this I shall add a way of writing a description of a fact somewhat resembling an algebraic expression. The diagrammatic method I call the method of Existential Graphs; the other I call my Universal Algebra of Logic.”
the diagram exhibits a state of things in which every mother loves one of her children. Being a mere icon, the diagram can do no more. But every person is connected with \( i \); and \( C \) and \( E \), the persons loved by their mothers, are connected with \( j \). Against the diagram is written \( \prod_i \Sigma_j \) and this asserts that whatever person be taken, persons related to the rest as \( C \) and \( E \) can be found. Even with this attachment the diagram does not very distinctly assert the proposition intended; but it does serve to show very clearly that inherence, which it is the peculiar function of categorical propositions to express, is nothing but a special variety of connection.

He adds in a footnote that “This diagram has been very obviously suggested by the ideas of Kempe’s Memoir on Mathematical Form”. (Cf. Roberts 1973, p. 24.) This semantic approach nevertheless misses, as Peirce himself notes, the exact form of the meaning of what the natural language sentence in question is taken to assert.

As to another example, he wanted to analyze the sentence, “Every house has a tenant”. Of this he remarks that the “order of the selection of things is material for quantified sentences”. This is a fundamental observation, and something that is completely lacking in Frege’s notion of the quantifier, for instance. By “order” he means that part of the meaning of quantifiers is in their interaction, namely their priority scope and functional dependence and not just how and where they bind variables by indices that refer to things in the universe of discourse. By “selection of things”, he refers to the semantic side of the systems, which is virtually equivalent to that of game-theoretic semantics (Hilpinen 1982; Pietarinen 2003, 2011b). That these ingredients are “material” Peirce obviously means the differences in the semantic value of the assertion in case either the selections or the order of selections were to change.

More interesting than the actual form and representation of the sentences is the role these factors play in overall logical theories. Crucially, thus, Peirce notes that there needs to be an interpretive rule making sure that, “whenever we have to enunciate a proposition about a number of subjects which are differently selected, to take pains so to express ourselves that there can be no mistake as to the order in which the different subjects are to be selected” (MS 410). What he discovers here (and he had largely accomplished this already by the 1885 theory of quantification) was that the decisive thrust of quantifiers is not that they are operators binding variables and ranging over things in the mutually agreed universes of discourse, but the idea of a dependent quantifier.

The crucial idea of a dependent quantifier shows a powerful grasp of what is at issue in the conceptual side of the situation. Without the kind of functional variability that it introduces much of the expressive power of first-order logic is lost. This fact Peirce surely recognized in the course of developing the algebraic logic. But how to bring it about in diagrammatic systems that he thought largely to surpass the algebraic way of looking at logical theories?

The first thing to note is that, by 1893 he would already have hit upon the idea of quantifier as the line (lines of identity) connecting different regions of the space (spots, rhemas).\(^5\) The space is normally taken to be two dimensional sheet of assertion in which graphs are built upon. By means of connecting lines, it was possible for Peirce to conceive the ideas of identity,

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\(^5\) The connecting lines are really crucial for being able to go beyond Peirce’s predecessors in diagrammatic logics, since “the system would be inferior to the (ill-named) Euler’s diagrams unless extended with lines of identities” (MS 430, 1902, p. 7).
quantification and predication using just one, single notation for all these three different uses of the copula. This fact stands in stark contrast to the Frege-Russell approach in which the different uses of copula were separated from one another by introducing different logical notations for them. But it is not obvious that the underlying logic for different uses of the verb for being would differ in the sense Frege and Russell proposed. Hintikka, among others, has been arguing that nobody between Aristotle and Russell really though so (Hintikka 2004, Analyses of Aristotle). In Peirce’s Beta part, one thus has a straightforward refutation of the Frege-Russell trichotomy. Peirce had noted in proposing a minimal notation and its cognitive economy that the logical system of graphs “does not involve any inessential elements in its representations” (MS 905). He would also not “care for copulas” (MS L 237, Letter to Ladd-Franklin, 17 November 1900), as logical terms are predicative and as according to him very few languages ordinarily use the verb for est or being. Which one of the several imports of the proposition – as when we state that A is B – is the import of the proposition is “the story of the gold and silver shield once again” (MS L 237, Letter to Ladd-Franklin, 1901).

Here are some representative examples of EGs, with their largely self-explanatory (iconic) notation:

By the early 1890s, graphical expression of relatives was well known by way of an analogy of such bonds to valential representations of chemical compounds. This connection largely derives from Peirce’s colleague J. J. Sylvester’s explorations in early graph theory. The connection has
been well noted and studied in the earlier literature and it does not concern us as such (see Houser et al. 1997) – though there is something to be said on it after a fashion.

What mainly concerns us is that there was one crucial idea that was missing from the proposed analogy. And it was equally missing in all the antedating works, too, including the works by Sylvester, Venn or Kempe. Namely, what was lacking was the idea of dependent lines, in other words the idea that would become the diagrammatic correlate to the quantifier that would conform to Peirce’s interpretive rule about the order of selections.

The new innovation that Peirce would come across by 1896 is the notion of a separation in the space/sheet of assertion. The closed Jordan lines of separation serve both as severing their interiors from the assertion space surrounding them (which is the iconic idea of negation) and providing parity (distinction between positive and negative spaces) to all things appearing in that space. These fine separational lines, or cuts, are not regarded as being “scribed” on the sheet of assertion in the sense in which graphs replicas, that is, atoms, combinations of atoms and identity lines are scribed, but are “a sort of photographic negative which is copied in the replica” (MS 5 26). Cuts, unlike graph replicas, are not seen or perceived and they are not subject to manipulation by the rules of transformation.

In comparing the earlier, 1893 diagram for the sentence “Every mother loves some children of hers”, which lacks any sign for negation, and the EG representation which employs parity considerations, we note that the latter is considerable simpler, it is more language-like, and much more prone to give rise to a logic close to natural language and cognition:

![Diagram]

There are in fact a number of fine distinctions in one’s iconic logic that the separation line, or the cut, brings about. Peirce analyses these distinctions in the 1902 manuscript 430 in five parts, three pertaining to the alpha and two to the beta extension. Here he calls the separation an oval and the distinctions are as follows (MS 430, p. 53-64):

Why have we made use of these ovals in our schematism? What do they effect? Careful attention will show that, even when there are no lines of identity, they fulfill three distinct offices, and that in introducing these lines we have imposed upon them two more. They fulfill all five with success. Only, in performing the last they slightly hamper that freedom of manipulation which mathematics requires; -- for, in the course of this chapter, the reader will perceive more and more clearly that all mathematical inquiry advances by means of experimenting upon schemata. The first office which the ovals fulfil is that of negation. [...] The second office of the ovals is that of associating the conjunctions of terms. [...] This is the office of parentheses in algebra. [...] The ovals are able to combine these offices because the last does not refer to single terms; so that we have only to use the ovals so as rightly to associate the elementary parts of the assertion we wish to express; and then, if any such parts have the wrong quality (which is the technical term for the distinction between affirmative and negative), it only needs to have an oval drawn around it so as to enclose nothing else. [...] The third office of the ovals is to distinguish the modes of
conjunction of the parts of propositions. [...] The circumstance which enables the ovals to add this office to the first two is that by means of negations all six modes of conjunction are expressible by means of any single one; and without using the negation of the whole are expressible by two, copulative conjunction, which is that of grandinat pluitque and disjunctive conjunction, which is that of grandinat aut pluit, both of which modes are associative. [...] The consequence is that it is only necessary (or even desirable) to distinguish which of two conjunctions is first used unless they are, the one copulative, the other disjunctive. Thus, every time an associative sign is wanted, the mode of conjunction changes from copulation to disjunction or the reverse. All that is needed, then, is to see that the outer mode of conjunction is right, and all the others follow a rule. If the outermost conjunction is not that one of the two which is expresses by simple juxtaposition (which is copulation in the schematic of existential graphs) we simply enclose the whole in an outer oval, and then correct the quality of each smallest part.

The fourth office of the ovals is to indicate the order of succession of the identifications. In order to understand what this means let us compare these two graphs [...] Examining the first we find a tail protruding from an oval. That means that something, X, exists which is not of the description found in the oval. What is found in the oval? Again a tail protruding from an oval, which were it unenclosed would signify that there is something, Y, of which what is within the inner oval is not true. What is in this oval, the assertion that something is praised to Y by X. Thus, the whole graph means ‘there is something X of which it is not true that there is something Y of which it is not true that to Y X praises something’. [...] Compare this with the final statement of the meaning of the first graph, and you will see that the only difference is in the relative order of settling upon the persons taken as Y and Z. The effect is, that the former asserts that somebody praises to each person some one thing or another, while the latter asserts that he praises the same thing to all persons. A graph with heavy lines is an assertion about a set of individual objects as many in number as the separate heavy lines (each of which may branch to any extent). Each individual of the set which forms the subject of the assertion is either to be suitably chosen by the speaker or by an omniscient being desirous of supporting his assertion or else is to be taken at pleasure by the auditor or by the most knowing antagonist of the assertion. It is of the set so made up that the predication is made. When the proponent or opponent has to designate an individual object as a member of the set, he is entitled to know what are the objects so far selected, so that he may shape his choice accordingly. Now every heavy line is regarded as enclosed only in those ovals which entirely enclose it. All these heavy lines which are evenly enclosed (that is, are wholly within an even number of envelopes) represent members of the set of subjects to be selected by the proponent in a suitable manner. All the heavy lines oddly enclosed represent individuals which the opponent is free to select as he likes. Individuals represented by less enclosed lines are to be selected before individuals represented by less [sic, more] enclosed lines. In all cases in which this rule leaves the order of choice indeterminate it makes no difference which order is pursued. The reason this convention is sufficient is that if either proponent or opponent has, at any stage, several individuals to select successively, it is obvious that the order in which he names them will be indifferent, since he will decide upon them in his own mind simultaneously. (MS 430, p. 53-64).

The fifth office, which Peirce either forgets to mention here or this MS was discontinued for any reason, is for the ovals to show the non-identity between the extremities of the identifications (lines of identities or ligatures).

I will in a moment comment on the fourth role, the indication of the order of successions. But first, we note that Peirce grasps the fact that the analogy of logical with chemical graphs can no longer be maintained. He observes that there is nothing corresponding to cuts in chemistry (and we can add that ditto for diagrams that are not visual). He argues that it is nevertheless possible, in the sense of “more than just logically possible”, that such elements may be found in the
rapidly evolving field of chemistry: “I only affirm that the constitution of thought is like that of a chemical molecule in being composed of atoms each with a definite valency”.

Incidentally, in this same MS S 499 from 1907 he presents lots of analogies from chemistry to logic. For instance, the classification of “indecomposable elements of thought” must be done according to their valencies. These “simple ideas” are represented in EGs by the spots (so there is a connection between phaneroscopy and logic, a highly relevant topic which I nevertheless have to forego here).

We must therefore deny isomorphism between logical and chemical graphs and leave it open what in each case is the real correlate to separation. To find the right kinds of correlates has to do with non-material and imaginary considerations rather than visual observations. The objects of negation are not physically concrete objects although they can well be manipulated in the imagination and although they have manifest and observable consequences.

Be this as it may, Peirce had by the mid-nineties succeeded in identifying the crucial pieces needed for the development of the full theory of logical graphs. What remained to be done was to conjugate these ingredients into one system of logic. This he was able to carry out rapidly, within a couple of weeks or at most months during early 1897. The crucial innovation here was that of the cut, which at the same time plays the roles of isolating the areas of juxtaposition, showing where and what the main connectives of the assertion are, giving the order of quantifiers, and effectuating the non-identity by cutting off continuous connections.

Peirce reflects on these events in 1906 when he mentioned the lost possibility of coming up with the full system of logical graphs already in 1882, in connection with Oscar Howard Mitchell’s proposal for a logic of algebra spreading in two dimensions. Nevertheless, given the idea of the separation, the Beta part was ready for presentation even before anything was published on their precursor of Entitative Graphs in 1897.

In general, it is worth remarking that Peirce is keen on emphasizing the importance of the imaginary in logic and mathematics. See MS 430, 1902, p. 7:

As a general rule the diagram will be so complicated that ordinary language is put to a severe strain to express it at all, even though its facile perspicuity be not attempted; and naturally the clear mental representation of the problem, and then the invention of the proper alteration of the diagram, call for the closest of thought. Thus all the marked general characteristics of mathematics are consequences of its being a scientific study of imaginary states of things.

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6 The development was so quick that Peirce speculated on the possibility that the reason for Paul Carus’s decision to refuse publishing more of Peirce’s work had to do with the pace of the developments.

7 “The system of expressing propositions which is called Existential Graphs was invented by me late in the year 1896, as an improvement upon another system published in the Monist for January 1897. But it is curious that 14 years previously, I had, but for one easy step, entered upon the system of Existential Graphs, reaching its threshold by a more direct way. The current of my investigations at that time swept me past the portal of this rich treasury of ideas. I must have seen that such a system of expression was possible, but I failed to appreciate its merits” (MS 498, 1-2). He also credits Mitchell for the discovery of the quantifiers Σ and Π in being “probably” the first to use the existential (sum) and universal (product) quantifiers as strings of dependent quantifiers and thus being able to give the system “considerable power”, in his “epoch-making paper” of the 1883 Studies in Logic (Dictionary of Philosophy and Psychology, Vol. II, entry on “Symbolic Logic”, p. 650, 1902).
But what is imaginary is not non-real of fictional. Einstein, in The Herbert Spencer Lecture, “On the Method of Theoretical Physics” delivered at Oxford, June 10, 1933 (1974, pp. 5-6) importantly remarked: “If you wish to learn from the theoretical physicist anything about the methods which he uses, I would give you the following piece of advice: Don’t listen to his words, examine his achievements. For to the discoverer in that field, the constructions of his imagination appear so necessary and so natural that he is apt to treat them not as the creations of his thoughts but as given realities”.

In conclusion, I would like to point out that Peirce’s focus on the actual practice of mathematics and its autonomous nature with respect to logic shows in not being limited by any particular logical ‘formalism’ when it comes to the meaning of the central notions such as quantifiers and their behaviour. He had a good grasp of how quantifiers work in the context of mathematics similar to that of Weierstrass, who also would not and could not have been bound by any formal rules of logic when defining the crucial mathematical concepts.

Some glimpses of this fact are contained in the following passages of Peirce’s manuscript draft, where he contemplates the tasks of cuts (separations/negations) in logic, bringing us back to the five roles the closed line of separation (“ovals”) is supposed to play. Here he comments on the fourth role of the ovals, that of showing the order of succession of selections (MS 430, earlier draft version, pp. 52-53):

This method fails when we attempt to apply it to such a case as [Ex.1, figure omitted]. The reason it fails is that in [Ex.2, figure omitted] the line, in order to connect ‘is born of’ to ‘woman’ has to pass outside the ovals; so that what is asserted is that there is some individual of whom any man there may be is born, and this individual, in case there exists any man, is a woman. Thus, the difficulty is the purely geometrical one that our graphs are written in such a kind of space, that it is impossible to pass from one oval to another without traversing a region outside them both. The ovals were chosen for negation because they would at the same time show the order of connection. They answer the two purposes well in most all cases, but it would occasionally where there is no identification. But when there is identification, a third office is imposed upon the ovals, that of determining the order of the lines of identity. Even this they will always do fairly well; but there are a few cases in which they do not give us the freedom of manipulation which is desirable in mathematics. Three dyadic relatives may be combined in four general ways in a proposition. Two of these ways, which are very common, are illustrated by these examples:

(1) There is a benefactress of everybody whom anybody she has not rejected must love.
(2) Everybody has some benefactress who rejects everybody that does not love her.

The other two modes of combination are uncommon. They are, for example,

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8 From another, intuitionistic point of view, we note the following remark by Brouwer (1907) about the possibility of mathematical misconceptions: “Those misconceptions did not arise through insufficient mathematical insight, but because mathematics, lacking a pure language, has to make do with the language of logical reasonings; whereas the thoughts of mathematics don’t reason logically, but mathematically, which is something completely different.” “But the mathematician who, because of the poverty of language, phrases the above mentioned theorem as a logical theorem, thinks something different from the logical interpretation just mentioned. He imagines that he starts to construct an isosceles triangle, and then that after the construction either the angles will turn out to be acute, or the construction doesn’t succeed if a right or obtuse angle is postulated. In other words, he gives the theorem in his mind a mathematical, not a logical interpretation. It is precisely the main content of the 3rd chapter to show that the naïve use of a logical language rather than a mathematical one has led parts of mathematics astray.”
After these sentences, Peirce presents four beta diagrams corresponding to these four sentences. Notable is that the numbers 2 and 3 of them are actually equivalent, although the sentences 2 and 3 in the above quotation are obviously different in meaning. Peirce might have well felt that this difference cannot be brought out in the Beta part as its expressive capacity does not go beyond what a normal predicate logic can capture (though see Pietarinen 2005 for the qualifications needed to this statement). Note also the abundant anaphoric pronouns and complex quantifier structure in the sentences (2), (3) and (4). Their logic is even more complex than ordinary donkey sentences, which cannot be captured by the resources of ordinary first-order logic. So how one is to express these Peircean donkey sentences (‘benefactress’ sentences)? We leave this as a future problem to be explored, suggesting merely that some of the Peircean sentences, presumably at least the sentence (2), appear to need the resources provided by independent quantifiers, and independent connectives, that is to say, IF logic (independence-friendly logic, see Hintikka 1996) to be adequately captured.

But could Peirce really have had in mind the notion of an independent quantifier, or perhaps that of an independent connective, to get around the fact that the sentences 2 and 3, though different in meaning, have exactly the same graph representations if all we have is the notion of a dependent quantifier or a connective? It is possible. Crucial passage in the previous quotation is really a note about quantifier ordering which is restrictive in the sense that there are “cases in which they do not give us the freedom of manipulation which is desirable in mathematics.” Peirce does not specify what those cases are, but we must remember that he was working on many mathematical topics and followed closely Weierstrass’s groundbreaking work in the analysis. In Weierstrass’s work, in particular in his epsilon-delta definition for uniform continuity or uniform differentiability, what is intended is not generally captured in ordinary first-order notation for quantifiers. There is no general method of expressing uniform continuity in linearly ordered quantifier structures that would show only the dependencies between quantifiers. Peirce seems to have been aware of this fact by 1894 when writing a review of Thomas Forsyth’s Theory of Functions of a Complex Variable (1893) to the Nation. In that review he corrected some oversights in Forsyth’s book, such as the author ignoring the importance of the differences between uniform and standard definitions.\(^9\) It has turned out much later on that only the latter, standard definition is first-order representable. It is also to be noted that Peirce wrote this review very soon after having invented the iconic notation for negation to express, among others, the succession ordering for identity lines, and in the preceding year of Weierstrass’s lectures and notes being published as a posthumous book (Weierstrass 1895).\(^10\)

So how to express the independencies between identity lines in Beta graphs, in addition to the orders of succession and the function dependencies? Because of parity considerations and the use of separation of the space of assertions, the lines of identities have a direction within the

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\(^9\) Peirce (1894. p. 47): “It would be unfair to convey the idea that Forsyth is quite impeccable in his expressions. This is far from being true. Thus, at the beginning of chapter iii., in enunciating Cauchy’s fundamental theorem on the expansion of a holomorphic function, the important words ‘unconditionally and uniformly,’ as describing the mode of convergence, are omitted, as they are overlooked in the proof given.”

\(^10\) Forsyth’s book refers to Weierstrass’s mimeograph of 1886 which has not been preserved.
areas (this is part of the important idea which Peirce terms the “endoporeutic interpretation” of EGs). If the interpretation of ligatures, which are complex lines that can branch or cross the cuts, can be reversed, we get a system of logic more expressive than either the Beta (which is really in its usual definitions a fragment of first-order logic) or the full first-order logic. The question of the reversibility, or the polarity change, was never taken up by Peirce, but it amounts to a very curious logical phenomenon where the idea of the dependent quantifier gets considerably extended. Such graphs are not first-orderizable anymore.

Pietarinen (2004, 2006) proposes an alternative way for representing sentences that may require independent quantification, by taking such extensions to be scribed in three-dimensional spaces instead of two-dimensional sheets of assertion. The same idea works equally well for independent connectives. For instance, the Beta graph for the infamous branching quantifier sentence introduced in Hintikka (1976), “Some relative of each villager and some friend of each townsman hate each other”, is the following:

All these marvelous possibilities to develop the crucial concepts of logic and quantification that would have been based on the 19th century concept of mathematics – a concept not constrained by formal logic or attempted to be based on logical foundations, remained unexplored for a century. No systematic study on Beta graphs was published during Peirce’s lifetime and the only printed appearance was in the 1906 Monist paper Prolegomena to an Apology for Pragmaticism. Alas, that exposition was already slated for another big fish, the proof of pragmaticism, rather than being a systematic presentation of the logic of EGs (Pietarinen 2011b). The Monist Prolegomena was for Peirce the last opportunity to record his endless ideas about EGs. He availed of this opportunity just in order to demonstrate a particular development of thought and to prevent it falling into oblivion. He predicted that it is something “not likely to be reproduced in a century” (MS 3XX). My brief account here of some of the key factors that led to this development is intended to show not only the lasting value of his logical innovations concerning Beta EGs but also that his prediction turned out to be largely true.

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11 A possible hint from Peirce occurs in MS 430 (see also MS 553): “The ovals were chosen for negation because they would at the same time show the order of connection. They answer to the purposes well in most all cases; but it would occasionally where there is no identification. But when there is identification, a third office is imposed upon the ovals, that of determining the order of the lines of identity. Even this they will always do fairly well; but there are a few cases in which they do not give us the freedom of manipulation which is desirable in mathematics.” A number of natural-language sentences follow the form of which appears not to be captured only by having dependent quantifiers at our disposal.

12 A Syllabus for Certain Topics of Logic was a small circulation booklet that contained some basic theory of Beta slightly before, in 1902.
References


