Peirce’s Logic and Philosophy

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Hangzhou, May 2011
Synopsis:
Charles S. Peirce (pron. ‘purse’), a scientist and logician, is known of his seminal contributions to logic and philosophy (semiotic) and their application to the sciences.

We look at selected topics on Peirce’s overall thoughts about semiotics, language, logic and cognition.
Brief Chronology

1839  b. 10 September, Cambridge, Mass., † 19 April 1914, Milford, Penn.

1861  Worked for the U.S. Coastal and Geodetic Survey until 1891 (astronomy, gravity, spectroscopy, measurement, the economics of research, metrology, geodesy . . . ).

1877  Elected to the National Academy of Sciences.

1879-84 Lecturer in Logic, the Johns Hopkins University, Baltimore.

1889  Retires to Arisbe, Milford as independent researcher.
  ▶ Published ~800 pieces, almost 400 reviews.
  ▶ 90,000 manuscript pages in microfilm, The Writings of Charles S. Peirce: A Chronological Edition, 1980–)

Peirce's Logic and Philosophy
Philosophy and Language

Peirce on language:

- Language cannot express thought in full.
- Exact words corresponding to thought are hard to find.
- Thinking in terms of symbolic systems (such as natural languages) will in the end be flawed.

*People who cannot reason exactly simply cannot understand my philosophy,—neither the process, methods, nor results.... My philosophy and all philosophy worth attention, repose entirely upon the theory of logic.*

(1898, Letter to William James)
Peirce’s Scientific Works

- **Logic**: algebra, quantifiers, graphs...
- **Mathematics**: algebra, topology, lattices, continuum...
- **Physics**: gravitation, geodesy, astronomy...
- **Economics**: mathematical economics, economy of research...
- **Computer Science**: switching circuits, mind & machines...
- **Linguistics**: speech acts/pragmatics, rhetoric, semantics, grammar, typology, pronunciations, dictionaries...
- **Psychology**: experimental psychology
- **History**: ancient, history of science...

He is very interdisciplinary and relevant to contemporary science.
My view is that there are three modes of being. I hold that we can directly observe them in elements of whatever is at any time before the mind in any way. They are the being of positive qualitative possibility, the being of actual fact, and the being of law that will govern facts in the future. (CP 1.23)

Or: Firstness, Secondness and Thirdness. Peirce conceives everything through these three categories.
1. **Firstness** is defined by *one-place (monadic) predicates*: $P(x)$ describes $x$ by its inherent qualities.

2. **Secondness** is defined by *two-place (dyadic) relations*: $R(x, y)$ describes a reaction between $x$ and $y$.

3. **Thirdness** is defined by *three-place (triadic) relations*: $Q(x, y, z)$ describes that $x$ mediates $y$ and $z$.

4. No higher categories are needed, since all $n$-place ($n > 3$) relations can be defined in terms of three-place relations (Peirce’s ‘Irreducibility Thesis’, proved in Burch 1991).

At the bottom of Peirce’s categories thus lies a mathematical fact.

*The Dao gave birth to the Great One;*
*The Great One gave birth to the Two;*
*The Two gave birth to the Three;*
*And the Three gave birth to the ten thousand things.*

— *Dao De Jing*
Semeiotic: Signs, Objects, Interpretants

1. **Sign**: That which *conveys or mediates* something to the mind. That “which stands to somebody for something in some respect or capacity” (CP 2.228)

2. **Object**: That which *determines* the sign (a thing, fact, real, being, thinkable, speakable).

3. **Interpretant**: The effect which a sign *creates* in its interpreter (idea, image, mental effect, conclusion, affection, emotion).

*I define a Sign as anything which on the one hand is so determined by an Object and on the other hand so determines an idea in a person’s mind, that this latter determination, which I term the Interpretant of the sign, is thereby mediately determined by that Object. A sign, therefore, has a *triadic* relation to its Object and to its Interpretant* (CP 8.343, 1908).
There are three kinds of signs which are all indispensable in all reasoning:

- **the first** is the diagrammatic sign or icon, which exhibits a similarity or analogy;
- **the second** is the index, which like a pronoun demonstrative or relative, forces the attention to the particular object intended without describing it;
- **the third [symbol]** is the general name or description which signifies its object by means of an association of ideas or habitual connection between the name and the character signified. (CP 1.369, c.1885).
Science of Discovery

1. Mathematics
   1.1 Formal (symbolic) logic
   1.2 Discrete mathematics (algebra, computation, ...)
   1.3 Continuous mathematics (topical geometry, continuity)

2. Philosophy
   2.1 Phenomenology (phaneroscopy)
   2.2 Normative sciences
      2.2.1 Esthetics
      2.2.2 Ethics
      2.2.3 Logic/Semiotic: Grammar, Logic Proper, Rhetoric
   2.3 Metaphysics

3. The Special Sciences (idiosity)
   3.1 Physical sciences
   3.2 Psychical sciences
We form in the imagination some sort of diagrammatic, that is, iconic, representation of the facts, as skeletonized as possible. The impression of the present writer is that with ordinary persons this is always a visual image, or mixed visual and muscular; but this is an opinion not founded on any systematic examination. (CP 2.778, 1901)

The words or the language, written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which serve as elements in thought are certain signs and more or less clear images... There is, of course, a certain connection between those elements and relevant logical concepts... The elements are, in my case, of visual and some of muscular type. Conventional words or other signs have to be sought for laboriously only in a secondary stage, when the associative play is sufficiently established and can be reproduced at will... The play with the mentioned elements is aimed to be analogous to certain logical conceptions one is searching for. (A. Einstein, quoted in Hadamard 1949: 142-3).
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Existential Graphs: “The Moving Pictures of Thought”

- *I do not think I ever reflect in words: I employ visual diagrams, firstly, because this way of thinking is my natural language of self-communion, and secondly, because I am convinced that it is the best system for the purpose.* (Peirce, MS 619, 1909).

His goal was a logical analysis of thought and reasoning even when symbolic expressions fall short of fulfilling that purpose:

- *There are countless Objects of consciousness that words cannot express; such as the feelings a symphony inspires or that which is in the soul of a furiously angry man in [the] presence of his enemy.* (Peirce, MS 499, 1906).
According to Peirce, graphical representation of natural language puts before us

- “moving pictures of thought” (CP 4.11)
- “a moving picture of the action of the mind in thought” (MS 298: 1)

The precise vehicle is the iconic logic of diagrams, which will

- “furnish a moving picture of the intellect” (MS 298: 10) and provides a “system for diagrammatizing intellectual cognition” (MS 292: 41).
A **Diagram** is a [sign] which is predominantly an icon of relations and is aided to be so by conventions. Indices are also more or less used. (MS 492: 22).

A diagram should be “as iconic as possible” in order to represent “visible relations” (MS 492: 22).

There are three kinds of iconic signs (**hypoicons**):

*Those which partake of simple qualities, or First Firstnesses, are **images**; those which represent the relations, mainly dyadic, or so regarded, of the parts of one thing by analogous relations in their own parts, are **diagrams**; those which represent the representative character of a representamen by representing a parallelism in something else, are **metaphors**. (EP 2:273, 1903).*

So iconic logic should also cover images and metaphors.
I extend logic to embrace all the necessary principles of semiotic, and I recognize a logic of icons, and a logic of indices, as well as a logic of symbols. (CP 4.9, 1906).
EGs: Alpha, Beta, Gamma

1. Alpha Graphs \(\sim\) propositional logic
   *Cat and dog are on a mat.*

2. Beta Graphs \(\sim\) predicate logic
   *Every man is mortal.*

3. Gamma Graphs \(\sim\)
   3.1 Modal logic;
   *It is possible that it rains.*
   3.2 Higher-order logic;
   *Aristotle has all the virtues of a philosopher.*
   3.3 Metagraphs;
   *‘You are a good cook’ is well said.*
   3.4 Non-declaratives;
   interrogatives, imperatives, metaphors, emotions, interpretation of music...
Definition

The set of Alpha Graphs \( \mathcal{G}_\alpha \) is the smallest set of satisfying:

1. Sheet of Assertion (SA): 
   
   \[ \in \mathcal{G}_\alpha. \]

2. Closure under \textit{juxtaposition}:
   
   If \( P_1 \in \mathcal{G}_\alpha, P_2 \in \mathcal{G}_\alpha \ldots P_n \in \mathcal{G}_\alpha \), then \( P_1 \ldots P_n \in \mathcal{G}_\alpha. \)

3. Closure under \textit{cuts}:
   
   If \( P_1 \in \mathcal{G}_\alpha \), then \( P_1 \in \mathcal{G}_\alpha. \)

Remark

- The SA doesn’t really have \textit{boundaries}.
- Cuts may not overlap.
- Juxtaposition is \textit{commutative} and \textit{associative} (homology).
EGs are scribed on a surface.

1. **Sheet of Assertion**
   \[ \top \text{(verum)} \]

2. **Juxtaposition** \sim
   conjunction

3. **Cut** \sim negation
EGs are scribed on a surface.

1. **Sheet of Assertion**  
   \[ \top \text{ (verum)} \]

2. **Juxtaposition**  
   conjunction

*Pear is ripe*
EGs are scribed on a surface.

1. **Sheet of Assertion**
   \[ \top \text{ (verum)} \]

2. **Juxtaposition** \(\sim\)
   conjunction
   
   *Pear is ripe*
   
   *A dog stumbles over a quick fox*
   
   \[ p \land q \]

3. **Cut** \(\sim\) negation
EGs are scribed on a surface.

1. **Sheet of Assertion**
   \[ \top \text{ (verum)} \]

2. **Juxtaposition** \sim 
   conjunction
   
   *Pear is ripe*
   
   *A dog stumbles over a quick fox*
   
   \[ p \land q \]

3. **Cut** \sim 
   negation
   
   *You are cautious* 
   
   \[ p \]
EGs are scribed on a surface.

1. **Sheet of Assertion**
   $\top$ (verum)

2. **Juxtaposition** $\sim$
   conjunction
   *Pear is ripe*
   *A dog stumbles over a quick fox*
   $p \land q$

3. **Cut** $\sim$ negation
   *You are cautious* $\neg p$
EGs are scribed on a surface.

1. **Sheet of Assertion**
   
   $\top$ (verum)

2. **Juxtaposition**
   
   $\sim$ conjunction
   
   *Pear is ripe*
   
   *A dog stumbles over a quick fox*
   
   $p \land q$

3. **Cut**
   
   $\sim$ negation
   
   *You are cautious* $\neg p$
   
   *Pear is ripe*
   
   *A dog stumbles over a quick fox*
Alpha Graphs

EGs are scribed on a surface.

1. **Sheet of Assertion**
   \( \top \) (verum)

2. **Juxtaposition** \( \sim \)
   conjunction
   - **Pear is ripe**
   - **A dog stumbles over a quick fox**
   \( p \land q \)

3. **Cut** \( \sim \) negation
   - **You are cautious** \( \neg p \)
   - **Pear is ripe**
   - **A dog stumbles over a quick fox**
   \( \neg (p \land q) \)
EGs are scribed on a surface.

1. **Sheet of Assertion**

   \[ \top \text{ (verum)} \]

2. **Juxtaposition**

   \( \sim \text{ conjunction} \)

   *Pear is ripe*

   *A dog stumbles over a quick fox*

   \( p \land q \)

3. **Cut** \( \sim \text{ negation} \)

   \( \neg p \)

   *You are cautious*

   *Pear is ripe*

   *A dog stumbles over a quick fox*

   \( \neg (\neg \neg p \land \neg q) = p \lor q \)
EGs are scribed on a surface.

1. **Sheet of Assertion**
   \( \top \) (verum)

2. **Juxtaposition**
   conjunction
   - **Pear is ripe**
   - **A dog stumbles over a quick fox**
   \( p \land q \)

3. **Cut**
   \( \sim \) negation
   - **You are cautious** \( \neg p \)
   - **Pear is ripe**
   - **A dog stumbles over a quick fox**
   \( \neg (\neg p \land \neg q) = p \lor q \)

   (the scroll)
   \( \neg (p \land \neg q) = \neg p \lor q = p \to q \)
EGs are scribed on a surface.

1. **Sheet of Assertion**
   \[ \top \text{ (verum)} \]

2. **Juxtaposition**
   \[ \sim \text{ conjunction} \]
   \[ \text{Pear is ripe} \]
   \[ \text{A dog stumbles over a quick fox} \]
   \[ p \land q \]

3. **Cut**
   \[ \sim \text{ negation} \]
   \[ \text{You are cautious} \quad \neg p \]
   \[ \text{Pear is ripe} \]
   \[ \text{A dog stumbles over a quick fox} \]
   \[ \neg (\neg p \land \neg q) = p \lor q \]
   \[ \neg (p \land \neg q) = \neg p \lor q = p \rightarrow q \]
   \[ \neg \top = \bot \text{ (falsum)} \]
Definition (Area)
Space within the cut without the cut is the \textit{area of the cut}.

Definition (Enclosure)
A cut, its area and everything in that area comprise the \textit{enclosure of the cut}.

Remark
1. Area is not part of the SA.
2. Area is not a graph.
3. Cut is not a graph.
4. Enclosure is a graph.
Alpha Graphs

Definition (Positive and negative areas)

1.a Any graph $P \in G_\alpha$ not enclosed by any cut or enclosed by an even number of cuts is evenly enclosed.

b Any graph $P \in G_\alpha$ enclosed by an odd number of cuts is oddly enclosed.

2.a Area on which an evenly enclosed graph rests is positive.

b Area on which an oddly enclosed graph rests is negative.

Remark
The union of evenly and oddly enclosed graphs of any $P \in G_\alpha$ comprise the set of all subgraphs of $P$. 
Definition (Nest)

- A linearly ordered finite sequence of areas from the SA to the areas of cuts of increasing depth makes a *nest*.
- A nest terminating on a cut-free area is a *maximal nest*.

1. One of 5 areas, or 4 cuts A-B-C-E-F
   Three of 4 areas or 3 cuts each:
   2. A-B-C-D
   3. A-B-H-I
   4. A-B-H-J
   5. One of 3 areas, or 2 cuts, A-B-G.
Five rules of transformation:

1. **Add/remove double cuts:**

   \[
   \ldots \quad P \quad Q \quad \leftrightarrow \quad \ldots \quad P \quad Q
   \]

2. **Insertion:** Any \( P \in G_\alpha \) may be added on negative area:

   \[
   \ldots \quad 2k+1 \quad k \quad \ldots \quad \rightarrow \quad \ldots \quad P \quad 2k+1 \quad k \quad \ldots
   \]

3. **Erasure:** Any \( P \in G_\alpha \) may be erased from positive area:

   \[
   \ldots \quad P \quad 2k \quad k \quad \ldots \quad \rightarrow \quad \ldots \quad 2k \quad k \quad \ldots
   \]
4. **Iteration**: Any copy of $P$ may be scribed on the same area or on the area in its nest (not part of $P$):

\[
\ldots P \ldots \quad \Rightarrow \quad \ldots PP \ldots
\]

5. **Deiteration**: Any copy of $P$ may be removed from the same area or from the area in its nest (not part of $P$):

\[
\ldots PP \ldots \quad \Rightarrow \quad \ldots P \ldots
\]
“Moving pictures of thought”: 

Example
Definition (Beta Graphs)

The set of beta graphs $G_\beta$ is the smallest set satisfying:

1. **SA**: $\HRule \in G_\beta$.

2. **Dot**, Line of Identity (LI) $\begin{array}{c} 1 \\ \hline n \end{array}$ and finitely branching $\begin{array}{c} \bullet \\ \hline n \end{array} \in G_\beta$.

3. Closure under spots:

   If $\begin{array}{c} \bullet \\ \hline n \end{array}$, $\begin{array}{c} \hline n \\ \bullet \end{array}$ and $\begin{array}{c} \bullet \hline n \end{array} \in G_\beta$, then $\begin{array}{c} \bullet P_1 \end{array}$, $\begin{array}{c} \hline P_1 \end{array}$ and $\begin{array}{c} \bullet P_1 \end{array} \in G_\beta$.

4. Closure under juxtaposition:

   If $\varphi_1 \in G_\beta \ldots \varphi_n \in G_\beta$, then $\Box \varphi_1 \ldots \varphi_n \in G_\beta$. 

---

Peirce’s Logic and Philosophy
Definition (cont.)

5. Closure under *cuts*:

If \[ \begin{array}{c}
\bullet \\
\hline
\end{array},
\begin{array}{c}
\hline
\hline
\end{array}
\] and \[ \begin{array}{c}
\hline
\hline
\end{array} \in G_\beta, \text{ then }
\begin{array}{c}
\bullet \\
\hline
\end{array},
\begin{array}{c}
\hline
\hline
\end{array},
\begin{array}{c}
\hline
\hline
\end{array}
\] and \[ \begin{array}{c}
\hline
\hline
\end{array} \in G_\beta. \]

Remark

- SA represents the *universe of discourse*.
- LIs are finitely long, finitely branching.

Definition (Outermost end)

The least enclosed portion of an LI is its *outermost end*. 
4. $\text{LI} \sim \text{quantification}$

- $\exists x \ (x = x)$
4. LI \sim quantification

\exists x \exists y \ (x = y)
4. LI $\sim$ quantification

$\exists x \, R(x)$
4. \( \text{LI} \sim \text{quantification} \)

\[
\exists x \exists y \ R(x, y)
\]

*Someone loves someone.*
4. $L_1 \sim$ quantification

\begin{align*}
\exists x \exists y \ R(x, y)
\end{align*}

Someone loves someone.

\begin{align*}
\exists x \exists y \ (P(x) \land Q(y) \land (x = y))
\end{align*}

Someone is identical to someone.
4. $\mathbf{L I} \sim$ quantification

\[ \exists x \exists y \, R(x, y) \]

*Someone loves someone.*

\[ \exists x \exists y \, (P(x) \land Q(y) \land (x \neq y)) \]

*Someone is not identical to someone.*
4. LI $\sim$ quantification

Someone loves someone.

$\exists x \exists y \, R(x, y)$

Someone is not identical to someone.

$\exists x \exists y \, (P(x) \wedge Q(y) \wedge (x \neq y))$

Pain does not hurt.

$\exists x \, (P(x) \wedge \neg H(x))$
4. $\text{LI} \sim \text{quantification}$

- $\exists x \exists y \, R(x, y)$
  
  Someone loves someone.

- $\exists x \exists y \, (P(x) \land Q(y) \land (x \neq y))$
  
  Someone is not identical to someone.

$\exists x \, (P(x) \land \neg H(x))$

Pain does not hurt.

$\neg (\exists x \, (P(x) \land \neg H(x))) = \forall x \, (P(x) \rightarrow H(x))$

All pain hurts.
4. LI ∼ quantification

\[ \exists x \exists y \ R(x, y) \]

Someone loves someone.

\[ \exists x \exists y \ (P(x) \land Q(y) \land (x \neq y)) \]

Someone is not identical to someone.

▶ **Existential assertion**: The outermost end of an LI rests on a positive area.

▶ **Universal assertion**: The outermost end of an LI rests on a negative area.
Beta Graphs

Definition (Loose End)
Unattached end of an LI is a *loose end*.
- Any hook may be attached by an LI.
- At most one LI may be attached to any one hook.
- Attachment corresponds to a bound variable (no free variables).

Definition (Ligature)
A line that branches or crosses a cut is a *ligature*.
- LIs may not cross cuts, ligatures may.
- Ligatures that cross cuts $\notin G_\beta$ (graphs may not overlap).
- The outermost loose ends of LIs of a ligature that lie within the same area form an *equivalence class*.

Definition (Cycle, Bridge)
- A self-returning ligature is a *cycle*.
- LIs that cross one another but do not join form a *bridge*.
Examples

The following beta graphs (below left) are the diagrammatic counterparts of the sentences to their right:

- \( \exists x (x = x) \) or \( P \) • \( \forall x \neg P(x, x) \)
- \( P \) or \( P \) • \( \exists x P(x) \) • \( \forall x \exists y (P(x, y) \land (x = y)) \)
- \( P \) • \( \exists x P(x, x) \) • \( \forall x (P_1(x) \land \exists y P_2(y)) \)
- \( P \) or \( P \) • \( \exists x \exists y P(x, y) \) • \( \exists x \neg P(x) \)
Examples

There is a woman who loves (and is loved by) every man:

Every man loves (and is loved by) a woman:

Remark
For simplicity, we assume that all relations are symmetric.
Examples: Dynamic Logic & Anaphora

Equivalent graphs:

However: $\exists x (S_1(x) \land S_2(x)) \not\Leftrightarrow \exists x S_1(x) \land S_2(x) \not\Leftrightarrow \exists x S_2(x) \land S_1(x)$.
(cf. A man walks in the park. He whistles.)

$\forall x \exists y (S_1(x, y) \rightarrow S_2(x, y)) \not\Leftrightarrow \forall x \exists y S_1(x, y) \rightarrow S_2(x, y) \not\Leftrightarrow \forall x (\exists y S_1(x, y) \rightarrow S_2(x, y)) \not\Leftrightarrow [\forall x \exists y (S_1(x, y)) \rightarrow S_2(x, y))$.
(cf. Every player picks a card. He puts it in his hand.)
Examples: Dynamic Logic & Anaphora

Equivalent graphs:

However: \( \exists x (S_1(x) \land S_2(x)) \not\leftrightarrow \exists x S_1(x) \land S_2(x) \not\leftrightarrow \exists x S_2(x) \land S_1(x). \)

(cf. *A man walks in the park. He whistles.*)

\( \forall x \exists y (S_1(x, y) \rightarrow S_2(x, y)) \not\leftrightarrow \forall x \exists y S_1(x, y) \rightarrow S_2(x, y) \not\leftrightarrow \forall x ([\exists y S_1(x, y)] \rightarrow S_2(x, y)) \not\leftrightarrow [\forall x \exists y (S_1(x, y)) \rightarrow S_2(x, y)]. \)

(cf. *Every player picks a card. He puts it in his hand.*
Shin’s Account


1. Shin’s interpretation uses predicate terms, so in fact amounts to heterogeneous logics.

2. Is what the “visually clear” and “intuitive” ways of how one could “read off” EGs the bottom question?
   - There might always be multiple ways – this is not so much of an issue in EGs, and Diagrams need not be visual in the first place.

For example:

\[
\begin{align*}
(P \land \neg Q) \lor (R \land \neg S), & \quad (P \land \neg Q) \lor (\neg S \land R), \\
(\neg Q \land P) \lor (R \land \neg S), & \quad (\neg Q \land P) \lor (\neg S \land R), \\
(R \land \neg S) \lor (P \land \neg Q), & \quad (\neg S \land R) \lor (P \land \neg Q), \\
(R \land \neg S) \lor (\neg Q \land P), & \quad (\neg S \land R) \lor (\neg Q \land P), \\
\neg((\neg(P \land Q) \land \neg(R \land S))), & \quad \neg((\neg(R \land S) \land \neg(P \land Q))), \\
\neg((\neg(Q \land P) \land \neg\neg(S \land R))), & \quad \neg((\neg(R \land S) \land \neg\neg(Q \land P))), \\
\neg((\neg(P \land Q) \land \neg\neg(S \land R))), & \quad \neg((\neg(S \land R) \land \neg(P \land Q))), \\
\neg((\neg(Q \land P) \land \neg\neg(R \land S))), & \quad \neg((\neg(S \land R) \land \neg(Q \land P))), \\
(P \rightarrow Q) \rightarrow (R \land \neg S), & \quad (\neg(R \land \neg S) \rightarrow (P \land \neg Q)), \\
(P \rightarrow Q) \rightarrow (\neg S \land R), & \quad (\neg(R \land \neg S) \rightarrow (\neg Q \land P)), \\
(R \rightarrow S) \rightarrow (P \land \neg Q), & \quad (\neg(S \land R) \rightarrow (P \land \neg Q)) \\
(R \rightarrow S) \rightarrow (\neg Q \land P), & \quad (\neg(S \land R) \rightarrow (\neg Q \land P)).
\end{align*}
\]
Peirce created surprisingly expressive diagrammatic logics, backed by was his conviction that,

*Three dimensions are necessary and sufficient for the expression of all assertions; so that, if man’s reason was originally limited to the line of speech (which I do not affirm), it has now outgrown the limitation.* (MS 654: 6, 1910).

Still more to come? A year later he writes:

> At great pains, I learned to think in diagrams, which is a much superior method [to algebraic symbols]. I am convinced that there is a far better one, capable of wonders; but the great cost of the apparatus forbids my learning it. It consists in thinking in stereoscopic moving pictures. (MS L 231, 1911)
The **DiaMind**

1. Doesn’t have the shortcomings of theories of Mental Models, Conceptual Spaces, Image Schemas,…

2. Breaks free from the idea of Symbolic Systems

3. Related to the Evolution of Communication

We don’t strive for the unique, correct interpretation of expressions. We strive for guides, plans, maps, precepts, observations, behaviours, actions… according to which we understand what is asserted.
**DiaMind: The Diagrammatic Mind**

- Representation of mind’s thought processes need logic
- Thinking and reasoning is diagrammatic in nature
- Diagrams are iconic: their parts show their own meaning
- Diagrams are the “moving pictures of our thought” (Peirce).

The **DiaMind**

1. Doesn’t have the shortcomings of theories of Mental Models, Conceptual Spaces, Image Schemas,…
2. Breaks free from the idea of Symbolic Systems
3. Related to the Evolution of Communication

We don’t strive for the unique, correct interpretation of expressions. We strive for guides, plans, maps, precepts, observations, behaviours, actions… according to which we understand what is asserted.