Expressibility and Higher-Order Logic of Existential Graphs

Ahti-Veikko Pietarinen, University of Helsinki

According to Peirce, “ideas too lofty to be expressed in diagrams are mere rubbish for the purposes of philosophy” (W8: 24, 1890, [On Framing Philosophical Theories]). Now what can be expressed in diagrams? What can be expressed in logic? Peirce’s diagrammatic logic of potentials has remained seriously understudied. Besides expressing abstraction, this system of diagrammatic logic is the true logic of generalities: its second-order part can accommodate all mathematics.

But what are we quantifying with second-order diagram logic? The lines of identities that attach to the hooks on the peripheries of spots do not express the existence of the values attached to them but “qualitative possibilities”. Moreover, the potentials assert “substantive” and “objective possibilities” and need to be taken to precede existence (MS 508: Syllabus B.6).

These modal features of second-order graphs are remarkable. But how can higher-order quantification be about such qualitative possibilities? Normally, second-order variables range over properties, classes, relations or functions of what there are in the domain of discourse. The language that has first and second-order variables and a well-defined language and logic (that is, a logical consequence relation) is commonly taken to give birth to the second-order logic. But properties and attributes are intensional entities. Quine famously asked which properties are there. The rub for Quine and other die-hard extensionalists was that there is no determinate criterion of identity by which elements in the domain of second-order languages could be determined. Second-order logic devotees typically sweep this problem under the carpet of set theory, making the whole issue seem part of mathematics proper.

Since Peirce is an extreme anti-nominalist and since he disapproved Cantor’s set theory in which members of sets are first-order elements, this answer is unsatisfactory. Higher-order variables must have the range that is distinctive from individuals referred to by propositions de inesse. So what can he do? The answer is surprising. By having the quantification in the higher-order graphs of logic to consist in assertions of substantive, objective possibilities Peirce can keep with the genuinely second-order range of the values of quantification while incorporating the criterion of identity into the notation of identity lines which quantify qualitative possibilities. This criterion guarantees the feasibility of asserting not the identities of existing individuals but those of substantive possibilities.

By the iconicity of second-order diagrams we avoid the worry of the meaning of the existence of higher-order elements. No separate sign for identity is needed to supply the identity conditions: identities are denoted by exactly the same signs that denote quantification, predication and substantive possibilities of second-order entities. As a consequence, the empirical distinction – the fourth dogma of empiricism – of what is a logical and what an extra-logical diagram cannot be maintained.