CLMPS 2015 affiliated meeting: Proof theory of modal and non-classical logics

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1 Aims and description

In recent years, alongside with the publication of pessimistic views on the possibility of developing satisfactory proof systems for modal logic, there has been an impressive burst of new ideas, methods, and results for the proof theory of modal and non-classical logics. All such endeavours converge to the creation of novel inferential methods that cover a wide variety of logics for which no analytic proof systems were previously known; they extend the methods of structural proof theory from pure logic to philosophical logics and axiomatic theories, and use a well developed semantic apparatus as a ground for the generation of proof systems.

The purpose of this affiliated meeting is to bring together experts who are contributing to this growing field, to present their recent work and share ideas with a more generous time frame for talks and discussion and a specialized audience.

The following specific topics will be treated by the talks to be presented at the meeting:

- Gentzen's systems and contraction-free sequent systems
- The widening scope of inferentialism
- Beyond Gentzen's systems: labelled, hypersequent, display, and nested sequent calculi

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- Metatheorems: cut elimination, completeness, correspondence, interpolation, decidability
- Applications: Euclid's geometry, bi-connexive logic, counterfactuals, conditional logics, deontic logic, relevance (or relevant) logic, social choice theory
- Comparisons between proof systems for modal and non-classical logics

2 Contributors and abstracts

Arnon Avron (Tel-Aviv University, Israel, `http://www.cs.tau.ac.il/~aa/`, e-mail: aa@math.tau.ac.il):

Using Assumptions in Gentzen-type Systems

The consequence relation between formulas which is induced by a fully structural Gentzen-type system G is usually taken to be: $\Gamma \vdash_G \psi$ iff the sequent $\Gamma \Rightarrow \psi$ is provable in G. However, no less useful is the consequence relation $\vdash^v_G$ defined by: $\Gamma \vdash^v_G \psi$ iff the sequent $\Rightarrow \psi$ is derivable in G from the set of sequents $\{\Rightarrow \varphi \mid \varphi \in \Gamma\}$. This is one particular case in which it is useful to infer a sequent from a set of assumptions where these assumptions are again sequents. We present several other examples of the usefulness of such inferences, like coherence of canonical and quasi-canonical systems (which determines whether such a system is analytic or not), and the problem of processing information from different sources, where the use of sequents is not only useful, but really essential for the expressive power of the logic.

The main technical tool used in the various applications we present is a generalization of the usual cut-elimination theorem (which treats only assumptions-free derivations) to what we call the *strong cut-elimination theorem* (which applies also to derivations of sequents from other sequents). We describe (with examples) several methods for proving strong cut-elimination in systems:

- Prove ordinary cut-elimination. Then prove the strong cut-elimination by induction on the number of premises. (This works fine if the system is *pure* and closed under weakening).
- Use some version of Gentzen's syntactic proof for LK and LJ.
- Use semantic methods.

Our examples of the use of these methods include the propositional provability logic GL, hypersequential systems for Gödel-Dummet logic and some paraconsistent extensions of it, and classical first-order logic.

Agata Ciabattoni, **Elisa Freschi**, **Francesco A. Genco**, and **Bjorn Lellmann** (Vienna University of Technology, Austria, `http://www.logic.at/staff/agata/`, e-mail:agata@logic.at):

Mimamsa deontic logic: proof theory and applications

We define a new deontic logic justifying its components with principles contained in texts of the Mimamsa school of Indian Philosophy. We use general proof-theoretic methods to obtain a cut-free sequent calculus for this logic, resulting in decidability, complexity results and neighbourhood semantics. The latter is used to analyse a well known example of seemingly conflicting obligations contained in the Vedas which proved to be a stumbling block for a number of interpretations of Mimamsa scholars.

Pierluigi Graziani (University of Chieti-Pescara, Italy, `https://sites.google.com/site/grazianipierluigi/`, e-mail: pierluigigraziani@yahoo.it):

Proof theory for non-classical Euclid's geometrical logic

The talk focuses on whether formal logic constitutes a valuable instrument for analyzing ancient mathematics. Starting from the mid-seventies*, this question has been the subject of a new wave of interest. Yet it can be traced back to previous formal renditions of syllogistic logic**.
I will first analyze different contemporary foundations of Euclid's geometry from a logical point of view*, then look at them against the backdrop of current philological studies. In particular, these proposals will be analyzed with respect to:

- the role of geometrical construction procedures;
- the role of diagrams;
- their answers to the generalization problem;

*Hintikka and Remes [1974]; Hintikka and Remes [1976]; Mueller [1981]; Mäenpää and von Plato [1990]; Mäenpää [1993; 1997]; von Plato [1995; 1998]; Mumma [2006]; Graziani [2007; 2014]; Miller [2008]; Mumma and Avigad and Dean [2009]; Beeson [2009; 2012; 2014]

**Notably Lukasiewicz [1957].

*For example: Mueller [1981]; Mäenpää and von Plato [1990]; Mäenpää [1993; 1997]; von Plato [1995; 1998]; Mumma [2006]; Graziani [2007; 2014]; Miller [2008]; Mumma and Avigad and Dean [2009]; Beeson [2014].

Focusing on these aspects will show which amongst classical vs. non-classical, and proof theoretical vs. model theoretical approaches are best suited to formally capture Euclid's reasonings. In particular I will argue that:

1. Contemporary proposals often rest on different ideas of 'formal'[‡].

2. Many of them prove inadequate to formalize Euclid's geometrical thinking.

3. In the light of new philological evidence, the approaches that seem most promising are the non-classical (constructive) and proof theoretical ones, in that they both seem indispensable to capture the dynamics of Euclid's geometrical reasoning.

4. Ancient mathematics can offer a very interesting context of application and further development for contemporary research in proof theory and constructive mathematics.

Rosalie Iemhoff (Utrecht University, The Netherlands, `http://www.phil.uu.nl/~iemhoff/`, e-mail: R.Iemhoff@uu.nl):

Uniform interpolation and proof systems

In 1992 a paper by Andrew Pitts appeared in which a syntactic method to construct what later became known as uniform interpolants was introduced, for intuitionistic propositional logic. The existence of such uniform interpolants imply the existence of interpolants, but not necessarily vice versa. An example of a modal logic with uniform interpolation was first given by Volodya Shavrukov who showed, by semantic means, that K has this property. Later Silvio Ghilardi proved the same for GL. Intriguingly, $\mathsf{S4}$ has interpolation but not uniform interpolation, also proved by Ghilardi, and the same holds for $\mathsf{K4}$, as shown by Marta Bílková. The latter also showed that Pitts' technique to prove uniform interpolation can be applied to several modal logics as well.

Here we generalise Pitts' method in such a way that having uniform interpolation becomes a property of proof systems rather than of logics. Some general conditions on axioms and rules are formulated so that any proof system satisfying these conditions has uniform interpolation. This has the advantage that many proof systems, and whence logics, can be treated at once. Moreover, from the fact that a logic does not have uniform interpolation it follows that it cannot have a proof system of the above kind. These insights can be applied to several modal and intermediate logics.

[‡]See Panbuccian [2000]; Dutilh Novaes [2013]

Hidenori Kurokawa (University of Helsinki, `https://helsinki.academia.edu/HidenoriKurokawa`, e-mail: hidenori.kurokawa@gmail.com) and **Sara Negri**: (University of Helsinki, `http://www.helsinki.fi/~negri/`, e-mail: sara.negri@helsinki.fi):

Labelled sequent calculi for substructural logics I: Relevant logics

Substructural logics have been identified as logics obtained by dropping the structural rules of weakening and/or contraction from the sequent calculus for classical logic formulated in a traditional style. Although traditional relevant logics have slightly different formulations from these cases due to the presence of distributive laws, these logics have also been broadly considered to be in the same family of substructural logics as other typical substructural logics (such as linear logic and BCK logic). This is because relevant logics have the common feature that the monotonicity principle $A \to (B \to A)$, an axiomatic counterpart of weakening rule, fails to hold. The family of relevant logics has also been semantically characterized by Routley-Meyer semantics, a relational semantics based on ternary accessibility relations. In this talk, we formulate these traditional relevant logics by using labelled sequent calculi with a ternary relational symbol, analogously to the binary labelled sequent calculi for modal logics in (Negri, 2005). In particular, we develop those calculi for relevant logics by adopting G3-style sequent calculi, which are formulated in such a way that all the rules are invertible and all the structural rules (including cut) are admissible. We highlight the fact that, although relevant logics are usually formulated by omitting the structural rules of weakening and contraction, in the labelled sequent calculi presented in this talk, we can show that all the rules are invertible and the structural rules of weakening and contraction are admissible in a height-preserving manner.

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Proof theory for first-order logic of social choice

In social choice theory, order-theoretic notions have always played an important role in providing a formal representation of individual and collective preferences. Properties such as transitiviy and connectedness as well as majority voting and Pareto optimality can be easily expressed using order relations. Traditionally, such properties are formulated in the language of predicate logic (with or without identity) and presented as axioms of Hilbert-style calculi. Since axiomatic theories make it difficult to analyze the structure of formal derivations and consequently to regiment the proofs in ordinary mathematics, we present a first-order theory of social choice as a calculus based on rules of inference, following the tradition of sequent calculi originated with Genzten. In this way, the standard axioms of social choice are formulated as a rules of inference and it is shown how the structural properties of the system, in particular the admissibility of the rules of cut and contraction, can be preserved. Secondly, such structural properties are used to provide a fully formal reconstruction of well-known results in social choice theory like Arrow's impossibility theorem and Sen's paradox of Paretian liberal.

Sara Negri (University of Helsinki, Finland, `http://www.helsinki.fi/~negri/`, e-mail: sara.negri@helsinki.fi):

Proof theory for neighborhood semantics

The internalization of possible worlds semantics in labelled sequent calculi provides a versatile formalism for the proof-theoretical investigation of large families of philosophical logics. In recent work (Dyckhoff and Negri, *Geometrization of first-order logic*, BSL, in press) it was shown that the method indeed encompasses logics characterized by arbitrary first-order conditions in their Kripke frames. The semantics of important intensional connectives such as Lewis' counterfactual conditionals, as well as the modalities of non-normal systems, however, cannot be captured by standard Kripke semantics and requires the more general neighbourhood semantics, a topological semantics which has had an intensive development since the 1970's. The question arises as to whether the successes of the semantic methods can be matched by equally powerful and general syntactic theories of modal and conditional concepts and reasoning.

In perfect analogy to the method of proof analysis in modal logic based on relational semantics, systems of proof for modal and philosophical logics based on neighborhood semantics are introduced. The procedure follows the standard path of inferentialism, suitably widened to accommodate the topological meaning explanation of the logical constants. In particular, the nesting of quantifiers in the truth conditions for the modalities and other intensional connectives makes the determination of the rules of the calculus an interesting and challenging task. The rules are obtained directly through a conservative extension of the modal language, without exploiting the known translations of the neighborhood semantics into the relational one, and without using non-local rules. For the calculi obtained, admissibility of the structural rules can be established either syntactically, through suitable inductions on the structure of derivations. or semantically, through completeness. The completeness proof, in turn, gives a construction of formal proofs for derivable sequents and countermodels for underivable ones and can be turned into a proof of decidability through saturation and filtration. Case studies include standard non-normal modal logics, deontic logics, and conditionals.

Nicola Olivetti (Aix-Marseille University, France, `http://www.lsis.org/olivetti/`, e-mail: nicola.olivetti@univ-amu.fr):

Internal and External Calculi for conditional logics

The recent history of conditional logics begins with the pioneering works by Lewis, Stalnaker, Nute, Chellas and Burgess, among others, who aimed to formalize a kind of hypothetical reasoning that cannot be captured by material implication of classical logic. Conditional logics have found an interest in epistemology, artificial intelligence and knowledge representation to formalise counterfactual reasoning, this was the original motivation, but also to model belief change (if the agent learns A (s)he will believe/know B), to represent plausible inferences (in normal circumstances if A then B") and to handle rules with exceptions (nonmomonotonic reasoning).

Semantically, all conditional logics enjoy a possible world semantics, with the intuition that a conditional $A > B$ is true in a world x if B is true in the set of worlds where A is true and that are most similar to/closest to/"as normal as" x. Since there are different ways of formalizing "the set of worlds similar/closest/..." to a given world, there are expectedly different semantics for conditional logics, from the most general selection function semantics to the stronger sphere semantics.

The proof theory of conditional logics is not as developed as the one of other extensions of classical logics, first of all modal logics of which they might be considered a generalisation. We shall present several calculi for conditional logics, following the traditional distinction between external proof systems, which extend the object language by partially importing the semantics, and internal proof systems, where any proof configuration may be directly read as a formula of the object language. In particular we shall present recently introduced nested sequent calculi, a generalisation of Gentzen systems, which seem particularly natural for conditional logics, at least for the basic ones characterised by the selection function semantics. We shall finally discuss some open problems, in particular the challenge of obtaining natural internal calculi for stronger conditional logics, such as Lewis' logics of counterfactuals.

Eugenio Orlandelli (University of Bologna, Italy,
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Proof theory of non-normal modal logics

In the context of deontic and epistemic logics it is widely recognized that a logic weaker than a normal modal logic should be employed. Non-normal logics are quite well understood from a semantic point of view, where they are studied by means of neighborhood semantics. Their proof theory, nevertheless, is rather limited since it is confined to Hilbert-style axiomatic systems. There have been some work in the area of sequent systems for non-normal modal logics, however the existing sequent calculi for non-normal logics either consider only some limited class of non-normal logics or do not allow to eliminate all the structural rules of inference.

We fill this gap by introducing **G3**-style sequent calculi for the minimal non-normal modal logics **E** and for all its extensions obtained by some combination of the axioms $M, C, N, D, D^{\star}$. For all these calculi we prove that weakening and contraction are height-preserving admissible, and we give a syntactic proof of the admissibility of cut. This yields that the subformula property holds for them and that they are decidable. Then we show that our calculi are equivalent to the axiomatic ones, and therefore that they are sound and complete w.r.t. neighborhood semantics. Finally, we use the well-known Maehara-Takeuti technique to prove Craig's interpolation theorem for non-normal modal logics. In this way we obtain not only a proof of the interpolation theorem, but also an explicit procedure to construct interpolants. Thus we show that in the context of non-normal logics the **G3**-style calculi are extremely well-behaved and allow to give constructive proofs of many deep logical results.

Alessandra Palmigiano (Delft University of Technology, The Netherlands; joint work with Giuseppe Greco, Minghui Ma , Apostolos Tzimoulis, Zhiguang Zhao, `http://www.appliedlogictudelft.nl/giuseppe-greco/`,
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Unified Correspondence as a Proof-Theoretic Tool

This talk focuses on the formal connections which have recently been highlighted between correspondence phenomena, well known from the area of modal logic, and the theory of display calculi originated by Belnap.

These connections have been seminally observed and exploited by Marcus Kracht, in the context of his characterisation of the modal axioms (which he calls primitive formulas) which can be effectively transformed into 'good' structural rules of display calculi. In this context, a rule is 'good' if adding it to a display calculus preserves Belnap's cut-elimination theorem.

In recent years, correspondence theory has been uniformly extended from classical modal logic to diverse families of nonclassical logics, ranging from (bi-)intuitionistic (modal) logics, linear, relevant and other substructural logics, to hybrid logics and mu-calculi. This generalisation has given rise to a theory called unified correspondence, the most important technical tool of which is the algorithm ALBA.

We put ALBA to work to obtain a generalisation of Kracht's transformation procedure from axioms into 'good' rules. This generalisation concerns more than one aspect. Firstly, we define primitive formulas/inequalities in any logic the algebraic semantics of which is based on distributive lattices with operators. Secondly, in the context of each such logic, we significantly generalise the class of primitive formulas/inequalities, and we apply ALBA to obtain an effective transformation procedure for each member of this class.

Heinrich Wansing (Ruhr-University Bochum Germany,
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Natural deduction for bi-connexive logic

Both bi-intuitionistic logic and connexive logic have received considerable attention recently, see, for example, (Goré 2000), (McCall 2012), (Wansing 2014). A bi-intuitionistic system, 2Int, different form the bi-intuitionistic logic BiInt that is also know as Heyting-Brouwer logic, has been introduced in (Wansing 2013). In this talk I will present a natural deduction proof system for a connexive version of 2Int. It combines the use of proofs as well as dual roofs with a connexive interpretation of the implication and co-implication connectives of 2Int. Moreover, a formulas-as- types notion of construction is presented for the co-negation, implication, and co-implication fragment of 2Int. This construction makes use of a two-sorted typed lambda calculus.

References

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3 Schedule

9.00–10.30 CLMPS plenary lecture

10.30–11.00 break

11.00–11.30 Avron: *Using Assumptions in Gentzen-type Systems*

11.30–12.00 Iemhoff: *Uniform interpolation and proof systems*

12.00–12.30 Wansing: *Natural deduction for bi-connexive logic*

12.30–13.00 Genco: *Mimamsa deontic logic: proof theory and applications*

13.00–14.30 lunch

14.30–15.00 Palmigiano: *Unified Correspondence as a Proof-Theoretic Tool*

15.00–15.30 Graziani: *Proof theory for non-classical Euclid's geometrical logic* (cancelled)

15.30–16.00 Maffezioli: *Proof theory for first-order logic of social choice*

16.00–16.30 Orlandelli: *Proof theory of non-normal modal logics*

16.30–17.00 break

17.00–17.30 Negri: *Proof theory for neighborhood semantics*

17.30–18.00 Kurokawa: *Labelled sequent calculi for substructural logics I: Relevant logics*

18.00–18.30 Olivetti: *Internal and External Calculi for conditional logics*