

# Epistemic possibilities for constructive reasoning with open assumptions

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## Abstract

For the verificationist principle of truth underlying intuitionistic logic and constructivism, semantic truth conditions for propositions are substituted by syntactically defined assertion conditions for judgments ([2]). Where the standard interpretation of such notion of truth is given in a language for arithmetic, far less clear is how to interpret it for the representation of rational knowledge processes. In this talk, I will show philosophical and logical issues involved by such task, and in particular for the required interpretation of open assumptions in Constructive Type Theory (CTT, [1]).

In the first part of the talk, the crucial problem of assertion conditions for judgments will be illustrated, introducing epistemic modalities for formulae of the form  $\Box(A \text{ true})$  and  $\Diamond(A \text{ true})$  (see [7] and [6]). Such judgmental modal operators express epistemic attitudes for assertion conditions and significantly extend the quantificational fragment of CTT.

In the second part, some existent modal contextual type-theoretical calculi based on *propositional* modal operators are surveyed ([3], [4]) and then the alternative version based on their judgmental translation is introduced ([5]). Their interpretation requires a non-standard extension of the constructive definition of type to introduce separate truth constructors from verified and open assumptions. Judgmental modal operators are then used to internalize modes of correctness on truth expressions: necessity for standard constructive verification, possibility for provability up to refutation of contextual conditions.

## References

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