Mesonic correlation lengths in high-temperature QCD*

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Finite T field theory

$$Z = \mathrm{Tr} e^{-\beta H} \Rightarrow \int \mathcal{D}\phi e^{-\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3 x \mathcal{L}(\phi, \partial\phi)}$$

- "imaginary time", $\tau = it$
- τ direction periodic, $\tau + \beta \leftrightarrow \tau$
- Matsubara modes: $\phi(\tau, \boldsymbol{x}) = \sum_{n=-\infty}^{\infty} \phi_n(\boldsymbol{x}) e^{i\omega_n \tau}$,

with $\omega_n = \begin{cases} 2n\pi T & \text{bosons} \\ (2n+1)\pi T & \text{fermions} \end{cases}$

Correlators

We want to calculate the τ-averaged correlators of various quark bilinears:

$$\begin{array}{rclcrcl} S^a & \equiv & \bar{\psi} F^a \psi & & V^a_\mu & \equiv & \bar{\psi}\gamma_\mu F^a \psi \\ P^a & \equiv & \bar{\psi}\gamma_5 F^a \psi & & A^a_\mu & \equiv & \bar{\psi}\gamma_\mu\gamma_5 F^a \psi \end{array},$$

where $F^s \equiv I_{N_f}$ and $F^n, \quad n = 1, \ldots, N_f^2 - 1$ generate the flavor $U(N_f)$ group.

$$C_{\boldsymbol{q}}[O^a, O^b] \equiv \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^3 x e^{i\boldsymbol{q}\cdot\boldsymbol{x}} \langle O^a(\tau, \boldsymbol{x}) O^b(0, \mathbf{0}) \rangle$$

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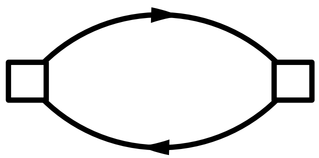
Leading order



$$\sim \mathrm{Tr}[F^a F^b] N_c \frac{i}{8\pi q} \ln \frac{2p_0 - iq}{2p_0 + iq} + \mathrm{constants}$$

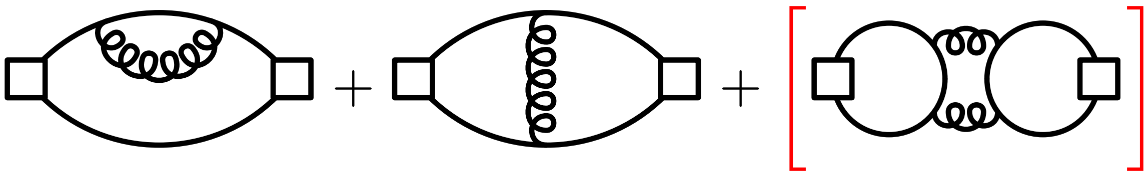
or, in the configuration space,

$$C_{\boldsymbol{x}} \propto \mathrm{Tr}[F^a F^b] N_c \frac{1}{x^2} \exp(-2p_0 x) + \delta' \mathrm{s},$$

which gives the screening mass $2p_0$ as expected. Two exceptions: $\langle V_0^a V_0^b \rangle$ and $\langle A_0^a A_0^b \rangle$ are further suppressed by $1/p_0 x$.

Next to leading order

In NLO we need the following graphs:



This is not enough, however: the screening state consists of two nearly on-shell quarks, which may well have $|1/\not{p}| \sim \mathcal{O}(1/g^2T)$ after emitting a soft gluon, compensating for g^2T in the vertices.

$\Rightarrow$ Need to resum a large collection of diagrams. An effective theory is a nice way to formulate this.

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Effective theory (NRQCD_3)

In high temperature gluons undergo the usual dimensional reduction,

$$\mathcal{L}_\mathrm{b} = \frac{1}{2}\mathrm{Tr}F_{ij}^2 + \mathrm{Tr}[D_i, A_0]^2 + m_\mathrm{E}^2 \mathrm{Tr}A_0^2 + \ldots,$$

with $D_i = \partial_i - i g_\mathrm{E} A_i$. The parameters that need to be matched with QCD are

$$m_\mathrm{E}^2 = \left(\frac{N_c}{3} + \frac{N_f}{6}\right) g^2 T^2, \qquad g_\mathrm{E}^2 = g^2 T.$$

What about fermions? Integrating out all the fermionic modes except $\pm\pi T$ does not seem justified. However, for almost on-shell quarks $|p_3 \pm i p_0| \lesssim gT$. The relevant expansion parameter is the "off-shellness" $\left[(ip_3)^2 - \boldsymbol{p}_\perp^2 - p_0^2\right]/T^2$.

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Lagrangian, matching

In the reduced theory the Matsubara frequency $\pi T \equiv M$ acts as a mass term,

$$\mathcal{L}_{\mathrm{f}} = i\chi^{\dagger}\left(M - g_{\mathrm{E}}A_0 + D_t - \frac{\nabla_{\perp}^2}{2p_0}\right)\chi + i\phi^{\dagger}\left(M - g_{\mathrm{E}}A_0 - D_t - \frac{\nabla_{\perp}^2}{2p_0}\right)\phi$$

with ϕ and χ given by $\psi^T = (\chi^T \quad \phi^T)$. NLO the mass M becomes a parameter that needs to be matched. We did this by comparing the locations of poles in fermion propagators, which is both a gauge invariant and an IR safe method, and found

$$M = \pi T + g^2 T \frac{C_{\mathrm{F}}}{8\pi}$$

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Potential, results

The static potential computed in the 2+1 -d theory will be of the form

$$V(r) \sim g_{\rm E}^2 \ln(r/?) + g_{\rm E}^4 r + \mathcal{O}(g_{\rm E}^6 r^2)$$

The Schrödinger equation gives $\frac{1}{p_0}\frac{\partial^2}{\partial r^2} \sim V(r) \sim g_{\rm E}^2 \ln r$, which leads to $|\boldsymbol{p}_\perp| \sim \partial/\partial r \sim 1/r \sim \sqrt{g_{\rm E}^2 p_0} \sim gT$.

$\Rightarrow$ potential is expanded in $\mathcal{O}(g_{\rm E}^2 r) \sim \mathcal{O}(g)$

$\Rightarrow$ To get the leading correction $E \sim \boldsymbol{p}_\perp^2/p_0 \sim g^2 T$ it is enough to compute the $g_{\rm E}^2 \ln r$ term in the potential, provided it is IR safe.

$$V(\boldsymbol{r}) = g^2 T \frac{C_\mathrm{F}}{2\pi} \left[\ln \frac{m_\mathrm{E} r}{2} + \gamma_\mathrm{E} - K_0(m_\mathrm{E} r) \right],$$

which is both gauge independent and IR finite. Solving the corresponding Schrödinger equation numerically gives

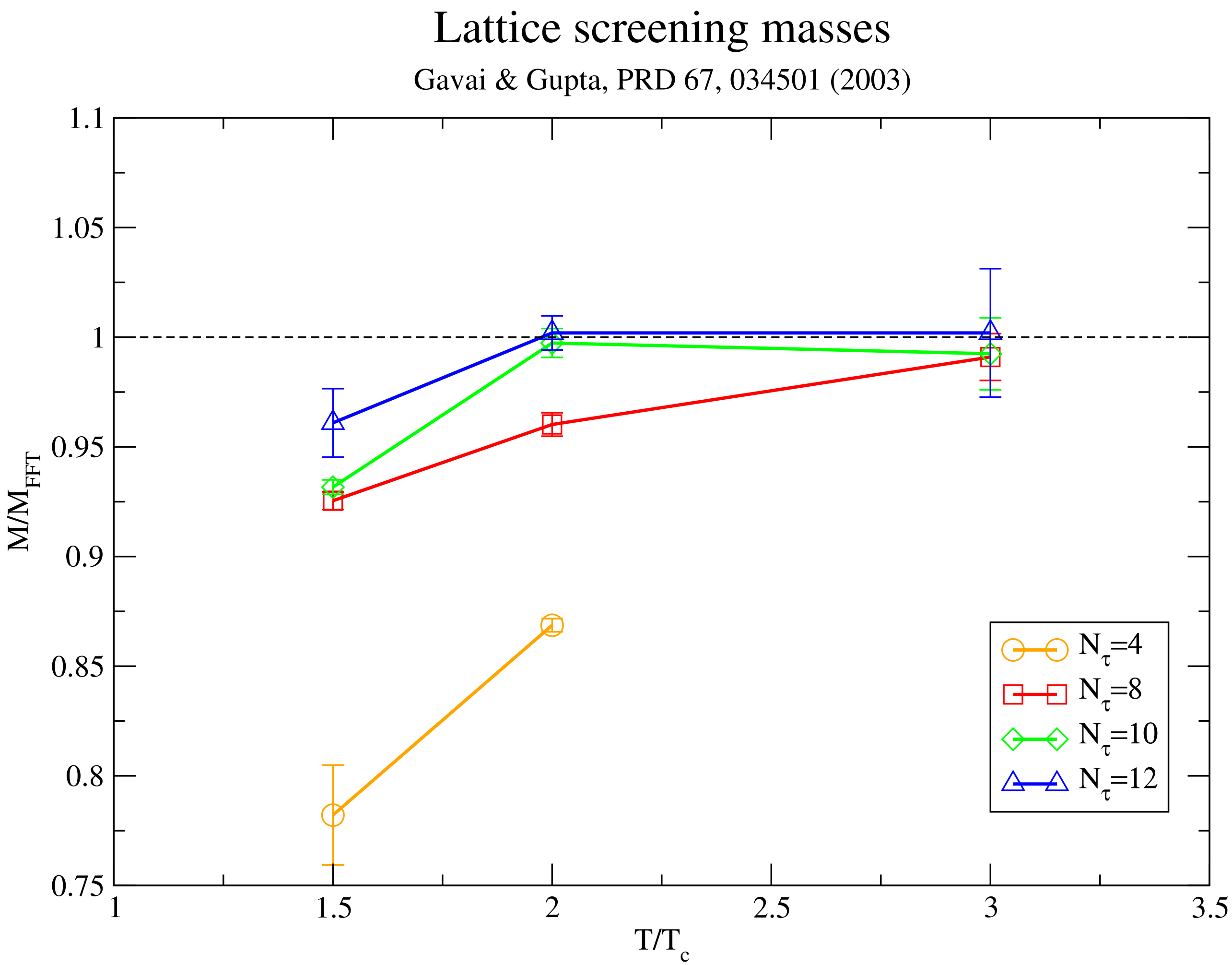
$$\frac{E - 2M}{g_\mathrm{E}^2 \frac{C_\mathrm{F}}{2\pi}} = \begin{cases} 0.16368014 & (N_f = 0) \\ 0.38237416 & (N_f = 2) \\ 0.46939139 & (N_f = 3) \end{cases}$$

$$\Rightarrow \quad E \approx 2\pi T + 0.14083730\, g^2 T \qquad (N_f = 0, \text{quenched})$$

At $T \sim 2T_c$ and $N_f = 0$ $g_\mathrm{E}^2/T \approx 2.7$,[1] only a 5% correction.

[1]Kajantie, Laine, Rummukainen and Shaposhnikov, Nucl.Phys. B503 (1997) 357

Lattice



Few years ago, PS/S used to have an exceptionally low screening mass, while all the other bilinears were only slightly below the free theory value. This has turned out to be a discretization artifact.

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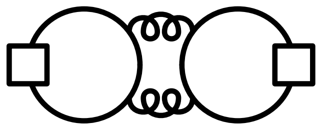
Flavour singlets

These couple to glueballs via the disconnected contribution: 

$$
\begin{aligned}
\frac{1}{N_f}\bar{\psi}\psi &\quad\leftrightarrow\quad N_c\frac{mT^2}{6} - \frac{m}{2\pi^2}g^2\mathrm{Tr}A_0^2 \\
\frac{1}{N_f}\bar{\psi}\gamma_5\psi &\quad\leftrightarrow\quad \frac{7\zeta(3)m}{8\pi^2T^2}\frac{g^2}{32\pi^2}\epsilon_{\alpha\beta\mu\nu}F^a_{\alpha\beta}F^a_{\mu\nu} \\
\frac{1}{N_f}\bar{\psi}\gamma_0\psi &\quad\leftrightarrow\quad -\frac{i}{3\pi^2}g^3\mathrm{Tr}A_0^3
\end{aligned}
$$

The glueball screening masses have been measured in DR on the lattice,[2] and are in general lower than $2\pi T$.

[2] Hart, Laine and Philipsen, Nucl.Phys. B586 (2000) 443

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Conclusions, outlook

- Consistent method for calculating spatial correlators of fermionic operators
- Systematics clear
- Sign differs from that of 4d lattice QCD
- NRQCD_3 should be easy to implement on lattice
- The overall scaling of correlators needs to be calculated

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