Mesonic correlation lengths in high-temperature QCD*

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Motivation

With RHIC already producing data at high rate, we need quantitative predictions from high-T QCD to compare the data with.

- Because of IR problems, large coupling and nonperturbative contributions the lattice computations are usually the most reliable theoretical tool.

- Fermions are hard to put on a lattice, and the systematic errors of different methods are difficult to estimate.

- Some of the infrared problems can be dealt with using an effective 3d theory, where one integrates out all the heavy modes, including fermions.

- Fermionic operators should behave nicely in perturbation theory, but the typical dimesional reduction approach needs to be modified.

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Finite T field theory

$$Z = \mathrm{Tr} e^{-\beta H} \Rightarrow \int \mathcal{D}\phi e^{-\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3 x \mathcal{L}(\phi, \partial\phi)}$$

- "imaginary time", $\tau = it$
- τ direction periodic, $\tau + \beta \leftrightarrow \tau$
- Matsubara modes: $\phi(\tau, \boldsymbol{x}) = \sum_{n=-\infty}^{\infty} \phi_n(\boldsymbol{x}) e^{i\omega_n \tau}$,

$$\text{with } \omega_n = \begin{cases} 2n\pi T & \text{bosons} \\ (2n+1)\pi T & \text{fermions} \end{cases}$$

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In terms of Matsubara modes the QCD Lagrangian reads

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\mathrm{Tr}\, F_n^{ij} F_{-n}^{ij} + \mathrm{Tr}\,\omega_n^2 A_n^2 + \mathrm{Tr}[D_i, A_0^n]^2 + \frac{1}{\xi}\mathrm{Tr}(\partial_i A_i^n)^2 \\ &\quad - \left(i\omega_n \mathrm{Tr} A_i^n \, [D_i, A_0]^{-n} + \mathrm{h.c.}\right) + \bar{\psi}_n \left(\vec{\not{D}} + (i\omega_n - igA_4)\gamma_4\right)\psi_n\end{aligned}$$

and the corresponding propagators are

$$\begin{aligned}D_{\mu\nu}(p) &= \frac{1}{\omega_n^2 + \boldsymbol{p}^2}\left(\delta_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{\omega_n^2 + \boldsymbol{p}^2}\right) \\ S(p) &= \frac{\not{p}}{\omega_n^2 + \boldsymbol{p}^2}\end{aligned}$$

Masses and correlators

The mass of an operator $\mathcal{O}$ is measured on the lattice by determining the (plane averaged) temporal correlator:

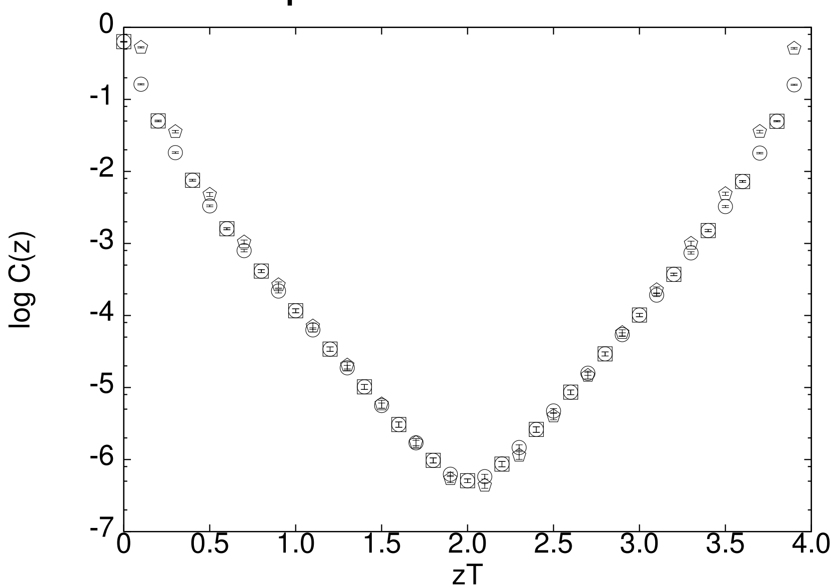
$$C(t) \equiv \langle \int \mathrm{d}^3 x \mathcal{O}(t, \boldsymbol{x}) \mathcal{O}(0, \mathbf{0})^\dagger \rangle = \langle \int_x e^{iHt} \mathcal{O}(0, \boldsymbol{x}) e^{-iHt} \mathcal{O}(0, \mathbf{0})^\dagger \rangle$$

$$= \sum_{n=0} \int_x e^{-iE_n t} \langle \mathcal{O}(0, \boldsymbol{x}) | n \rangle \langle n | \mathcal{O}(0, \mathbf{0})^\dagger \rangle$$

In Euclidean space (lattice) the time coordinate is imaginary, $t = -i\tau$, giving an exponential decay.

In finite temperature the rotational $O(4)$ symmetry of the Euclidean space is broken down, and the correlators in spatial directions (the screening correlators) are independent of the temporal real-time correlators.

Figure 1: Some screening correlators in finite temperature.



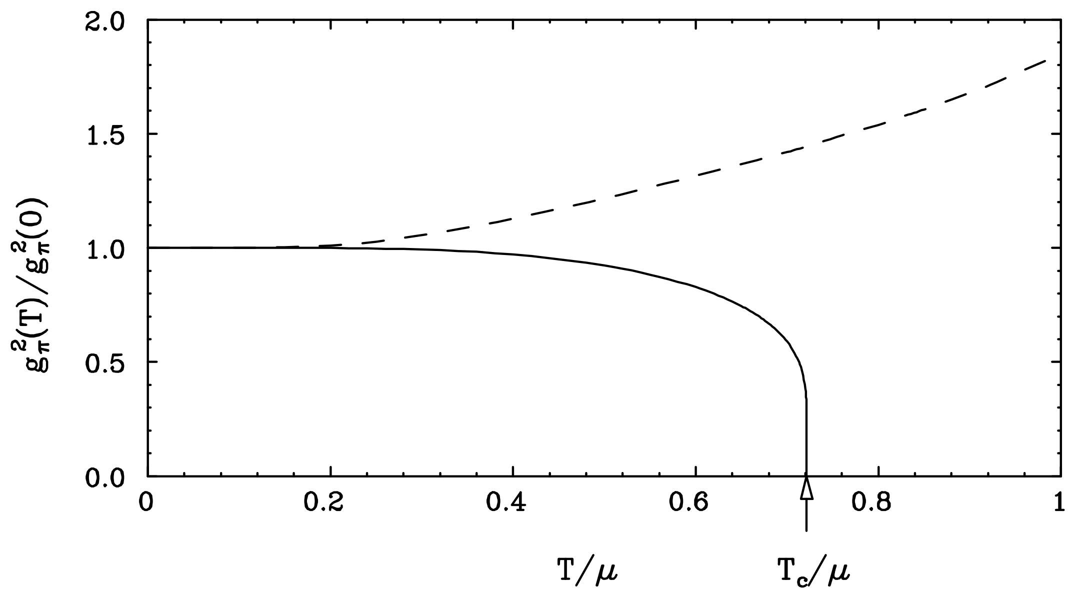


figure 2

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To compare with the recent lattice data, we want to calculate the τ-averaged correlators of various quark bilinears:

$$\begin{array}{llllll} S^a & \equiv & \bar{\psi} F^a \psi & V^a_\mu & \equiv & \bar{\psi}\gamma_\mu F^a \psi \\ P^a & \equiv & \bar{\psi}\gamma_5 F^a \psi & A^a_\mu & \equiv & \bar{\psi}\gamma_\mu\gamma_5 F^a \psi \end{array} ,$$

where $F^s \equiv I_{N_f}$ and $F^n, \quad n = 1, \ldots, N_f^2 - 1$ generate the flavor $U(N_f)$ group.

$$C_{\boldsymbol{q}}[O^a, O^b] \equiv \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^3 x e^{i\boldsymbol{q}\cdot\boldsymbol{x}} \langle O^a(\tau, \boldsymbol{x}) O^b(0, \mathbf{0}) \rangle$$

Expect:

$$C(r) \equiv \int \frac{\mathrm{d}^3}{(2\pi)^3} C_{\boldsymbol{q}}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \propto e^{-M(T)r},$$

where $M(T) = 2\pi T + \# g^2 T$.

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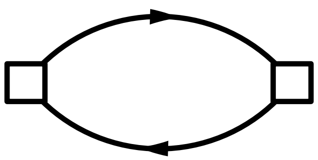
Leading order



$$\sim \mathrm{Tr}[F^a F^b] N_c \frac{i}{8\pi q} \ln \frac{2p_0 - iq}{2p_0 + iq} + \mathrm{constants}$$

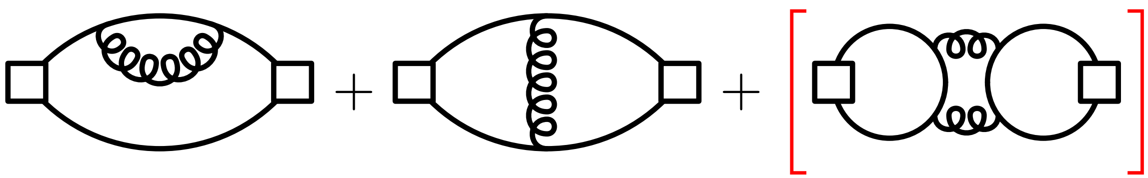
or, in the configuration space,

$$C_{\boldsymbol{x}} \propto \mathrm{Tr}[F^a F^b] N_c \frac{1}{x^2} \exp(-2p_0 x) + \delta'\mathrm{s},$$

which gives the screening mass $2p_0$ as expected. Two exceptions: $\langle V_0^a V_0^b \rangle$ and $\langle A_0^a A_0^b \rangle$ are further suppressed by $1/p_0 x$.

Next to leading order

In NLO we need the following graphs:



This is not enough, however: the screening state consists of two nearly on-shell quarks, which may well have $|1/\not{p}| \sim \mathcal{O}(1/g^2T)$ after emitting a soft gluon, compensating for g^2T in the vertices.

$\Rightarrow$ Need to resum a large collection of diagrams. An effective theory is a nice way to formulate this.

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IR problems in general

There are infrared singularities related to soft and/or collinear momenta at $T = 0$, but those are well understood and under control. In finite temperature the situation is more severe because of the Bose-Einstein factors in the propagators:

$$n(k_0) = \frac{1}{e^{\beta|k_0|} - 1} \rightarrow \frac{1}{\beta k}$$

Part of these additional IR divergences can be cured by resumming gluon self-energy insertions into an effective mass for the temporal gluon. The remaining IR problems are poorly understood, but contribute only at higher order than our calculation.

3D theory

In high temperature gluons undergo a dimensional reduction,

$$\mathcal{L}_{\mathrm{b}} = \frac{1}{2}\mathrm{Tr}F_{ij}^2 + \mathrm{Tr}[D_i, A_0]^2 + m_{\mathrm{E}}^2 \mathrm{Tr}A_0^2 + \ldots,$$

with $D_i = \partial_i - i g_{\mathrm{E}} A_i$. The parameters that need to be matched with QCD are

$$m_{\mathrm{E}}^2 = \left(\frac{N_c}{3} + \frac{N_f}{6}\right) g^2 T^2, \qquad g_{\mathrm{E}}^2 = g^2 T.$$

What about fermions? Integrating out all the fermionic modes except $\pm\pi T$ does not seem justified. However, for almost on-shell quarks $|p_3 \pm i p_0| \lesssim gT$. The relevant expansion parameter is the "off-shellness" $\left[(ip_3)^2 - \boldsymbol{p}_\perp^2 - p_0^2\right]/T^2$.

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Fermionic Lagrangian

Write $\psi = \begin{pmatrix} \chi \\ \phi \end{pmatrix}$ and solve the EOM for heavy components near $p_3 = \pm i p_0$:

$$\Rightarrow \mathcal{L}_{\rm f} = i\chi^\dagger \left(M - g_{\rm E} A_0 + D_t - \frac{\nabla_\perp^2}{2p_0} \right) \chi + i\phi^\dagger \left(M - g_{\rm E} A_0 - D_t - \frac{\nabla_\perp^2}{2p_0} \right) \phi.$$

The particle (χ) and antiparticle (ϕ) sectors are clearly separated and propagate only forward/backward in the x_3 direction:

$$\langle \chi_u(x)^* \chi_v(y) \rangle = -i\delta_{uv}\theta(x_3 - y_3) \frac{p_0}{2\pi|x_3 - y_3|} e^{-M|x_3 - y_3| - \frac{p_0(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)^2}{2|x_3 - y_3|}}$$

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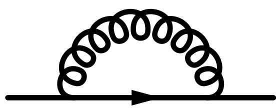
In the reduced theory the Matsubara frequency $\pi T \equiv M$ acts as a mass term. NLO M becomes a parameter that needs matching. We did this by comparing the locations of poles in fermion propagators, which is both a gauge invariant and an IR safe method.

To preserve the correct power counting $\pi T \gg \boldsymbol{p}_\perp$ while integrating over $\boldsymbol{p}_\perp$, we expand the ET quark propagators in $\boldsymbol{p}_\perp / p_0$.



$$= g_{\mathrm{E}}^2 C_{\mathrm{F}} \int \frac{\mathrm{d}^{3-2\epsilon} q}{(2\pi)^{3-2\epsilon}} \frac{1}{M + ip_3 - iq_3} \frac{1}{q^2 + \lambda^2}$$

Matching the QCD and NRQCD results gives the mass parameter in effective theory:

$$M = \pi T + g^2 T \frac{C_{\mathrm{F}}}{8\pi}$$

Potential

The static potential computed in the 2+1 -d theory will be of the form

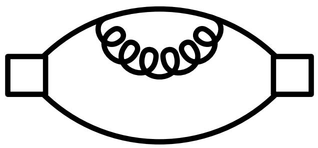
$$V(r) \sim g_{\rm E}^2 \ln(r/?) + g_{\rm E}^4 r + \mathcal{O}(g_{\rm E}^6 r^2)$$

The Schrödinger equation gives $\frac{1}{p_0}\frac{\partial^2}{\partial r^2} \sim V(r) \sim g_{\rm E}^2 \ln r$, which leads to $|\boldsymbol{p}_\perp| \sim \partial/\partial r \sim 1/r \sim \sqrt{g_{\rm E}^2 p_0} \sim gT$.

$\Rightarrow$ potential is expanded in $\mathcal{O}(g_{\rm E}^2 r) \sim \mathcal{O}(g)$

$\Rightarrow$ To get the leading correction $E \sim \boldsymbol{p}_\perp^2/p_0 \sim g^2 T$ it is enough to compute the $g_{\rm E}^2 \ln r$ term in the potential, provided it is IR safe.

$V(\boldsymbol{r}) =$



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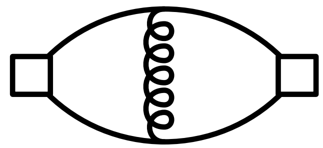

$$= g_{\mathrm{E}}^2 C_{\mathrm{F}} \int \frac{\mathrm{d}^{2-2\epsilon}}{(2\pi)^{2-2\epsilon}} \left[\frac{1}{q^2+\lambda^2}\left(1-e^{iq\cdot r}\right) - \frac{1}{q^2+m_{\mathrm{E}}^2}\left(1+e^{iq\cdot r}\right) \right]$$

With this specific combination of signs, the potential is both IR and UV finite. Taking the limit $\epsilon \to 0$, $\lambda \to 0$ gives

$$V(\boldsymbol{r}) = g^2 T \frac{C_{\mathrm{F}}}{2\pi} \left[\ln \frac{m_{\mathrm{E}} r}{2} + \gamma_{\mathrm{E}} - K_0(m_{\mathrm{E}} r) \right].$$

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Results

Solving the corresponding Schrödinger equation numerically gives

$$\frac{E-2M}{g_{\mathrm{E}}^2\frac{C_{\mathrm{F}}}{2\pi}} = \begin{cases} 0.16368014 & (N_f=0) \\ 0.38237416 & (N_f=2) \\ 0.46939139 & (N_f=3) \end{cases}$$

$$\Rightarrow \quad E \approx 2\pi T + 0.14083730\, g^2 T \qquad (N_f = 0, \text{quenched})$$

At $T \sim 2T_c$ and $N_f = 0$ $g_{\mathrm{E}}^2/T \approx 2.7$,[1] only a 5% correction.

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[1] Kajantie, Laine, Rummukainen and Shaposhnikov, Nucl.Phys. B503 (1997) 357

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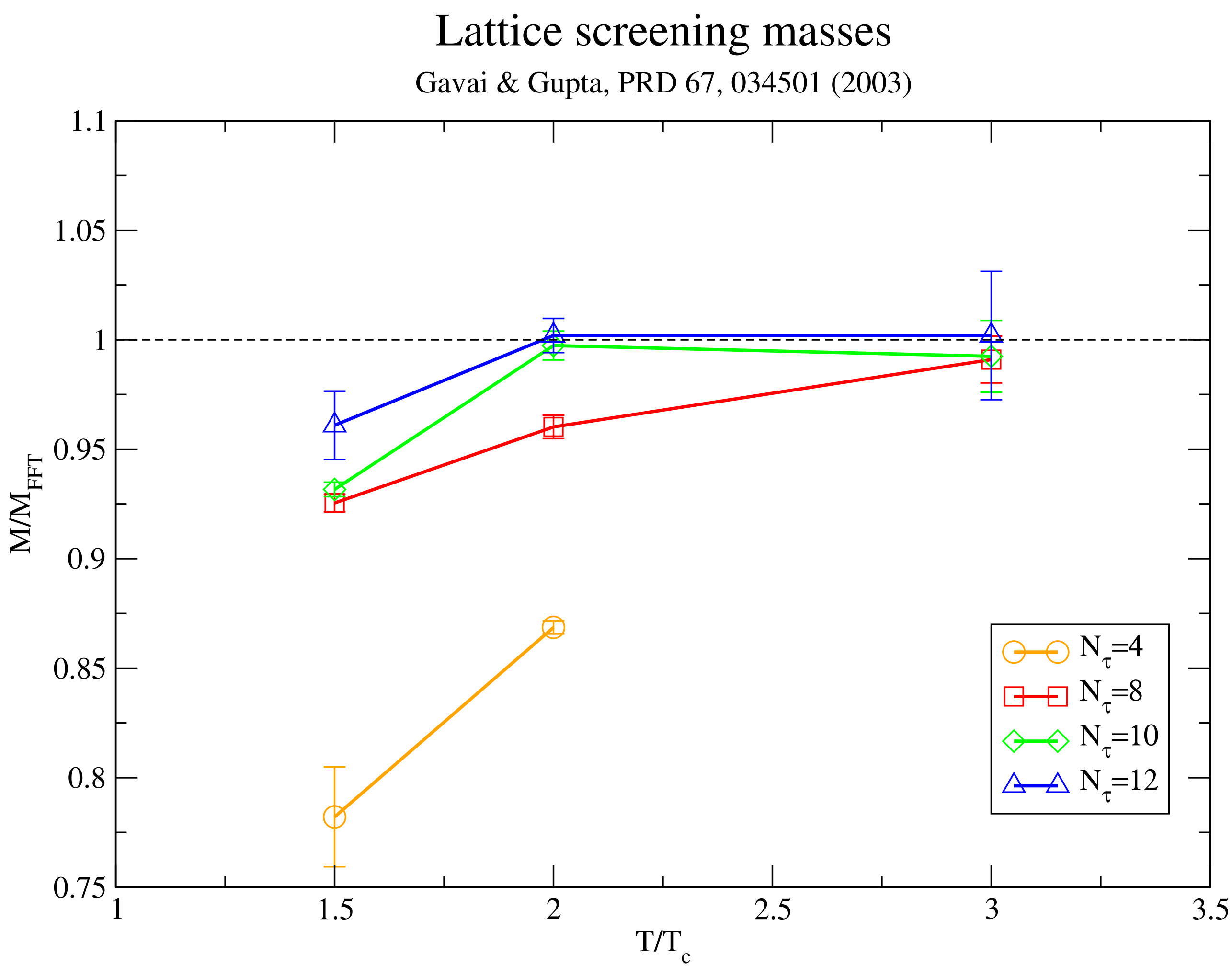
Lattice



Few years ago, PS/S used to have an exceptionally low screening mass, while all the other bilinears were only slightly below the free theory value. This has turned out to be a discretization artifact.

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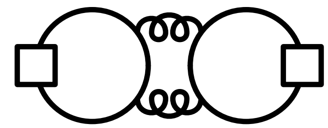
Flavour singlets

These couple to glueballs via the disconnected contribution: 

$$
\begin{aligned}
\frac{1}{N_f}\bar\psi\psi &\leftrightarrow N_c\frac{mT^2}{6} - \frac{m}{2\pi^2}g^2\mathrm{Tr}A_0^2 \\
\frac{1}{N_f}\bar\psi\gamma_5\psi &\leftrightarrow \frac{7\zeta(3)m}{8\pi^2T^2}\frac{g^2}{32\pi^2}\epsilon_{\alpha\beta\mu\nu}F^a_{\alpha\beta}F^a_{\mu\nu} \\
\frac{1}{N_f}\bar\psi\gamma_0\psi &\leftrightarrow -\frac{i}{3\pi^2}g^3\mathrm{Tr}A_0^3
\end{aligned}
$$

The glueball screening masses have been measured in DR on the lattice,[2] and are in general lower than $2\pi T$.

[2] Hart, Laine and Philipsen, Nucl.Phys. B586 (2000) 443

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Conclusions, outlook

- Consistent method for calculating spatial correlators of fermionic operators
- Systematics clear
- Sign differs from that of 4d lattice QCD
- NRQCD_3 should be easy to implement on lattice
- The overall scaling of correlators needs to be calculated
- Flavour singlets still problematic