Mesonic screening masses at high temperature*

Mikko Vepsäläinen

Lattice QCD at finite temperature and density
BNL 12.2.2004

*M. Laine and M. Vepsäläinen, hep-ph/0311268

UNIVERSITY OF HELSINKI

Correlators

$$\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(\gamma_\mu D_\mu + M)\psi$$

To compare with the recent lattice data, we want to calculate the τ-averaged spatial correlators of various quark bilinears:

$$\begin{array}{lclclcl} S^a & \equiv & \bar{\psi} F^a \psi & V^a_\mu & \equiv & \bar{\psi}\gamma_\mu F^a \psi \\ P^a & \equiv & \bar{\psi}\gamma_5 F^a \psi & A^a_\mu & \equiv & \bar{\psi}\gamma_\mu\gamma_5 F^a \psi \end{array},$$

where $F^s \equiv I_{N_f}$ and $F^n, \quad n = 1, \ldots, N_f^2 - 1$ generate the flavor $U(N_f)$ group.

$$C_{\boldsymbol{q}}[O^a, O^b] \equiv \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}^3 x e^{i\boldsymbol{q}\cdot\boldsymbol{x}} \langle O^a(\tau, \boldsymbol{x}) O^b(0, \boldsymbol{0}) \rangle$$

UNIVERSITY OF HELSINKI

The screening mass is defined by the exponential decay of the correlator:

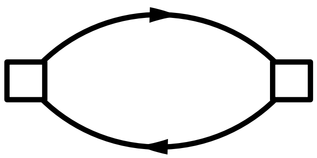
$$C(r) \equiv \int \frac{\mathrm{d}^3 q}{(2\pi)^3} C_{\boldsymbol{q}}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \propto e^{-M(T)r},$$

$$\Rightarrow\ M(T) = -\lim_{r\to\infty} \frac{1}{r} \ln C(r).$$

Since the lowest fermionic Matsubara mode is $p_0 = \pi T$, we expect the screening mass of the two-fermion operator to be

$$M(T) = 2\pi T + \# g^2 T.$$

Leading order



$$= \mathrm{Tr}[F^a F^b] N_c T \sum_{n=-\infty}^{\infty} \int \frac{\mathrm{d}^{3-2\epsilon} p}{(2\pi)^{3-2\epsilon}} \frac{\mathrm{Tr}(\not{p} + \not{q})\Gamma^a \not{p} \Gamma^b}{[p_n^2 + \boldsymbol{p}^2][p_n^2 + (\boldsymbol{p} + \boldsymbol{q})^2]}$$

Besides terms constant in q, the correlator is given by

$$C_q \sim B_{3d}(2p_0) \equiv \frac{i}{8\pi q} \ln \frac{2p_0 - iq}{2p_0 + iq}$$

or, in the configuration space,

$$C_{\boldsymbol{x}} \propto \mathrm{Tr}[F^a F^b] N_c \frac{1}{x^2} \exp(-2p_0 x) + \delta(\boldsymbol{x})'\mathrm{s},$$

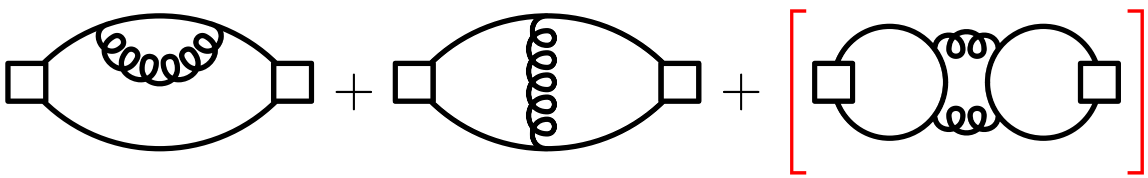
which gives the screening mass $2p_0$ as expected.

Operator	Behavior
S	$q^2 B_{3d}(2p_0)$
P	$q^2 B_{3d}(2p_0)$
V_0	$(q^2 + 4p_0^2) B_{3d}(2p_0)$
$V_\perp$	$(q^2 - 4p_0^2) B_{3d}(2p_0)$
A_0	$(q^2 + 4p_0^2) B_{3d}(2p_0)$
$A_\perp$	$(q^2 - 4p_0^2) B_{3d}(2p_0)$

Note that the singularity at $q = \pm i p_0$ is suppressed by a factor $q^2 + 4p_0^2$ in V_0 and A_0 channels. Working out the same correlators in configuration space shows that they are actually suppressed by $1/p_0 x$ relative to other channels.

Next to leading order

NLO we need the following graphs:



This is not enough, however: the screening state consists of two nearly on-shell quarks, which may well have $|1/\not{p}| \sim \mathcal{O}(1/g^2T)$ after emitting a soft gluon, compensating for g^2T in the vertices.

$\Rightarrow$ Need to resum a large collection of diagrams. An effective theory is a nice way to formulate this.

3D theory

In high temperature gluons undergo a dimensional reduction,

$$\mathcal{L}_{\rm b} = \frac{1}{2}{\rm Tr}F_{ij}^2 + {\rm Tr}[D_i, A_0]^2 + m_{\rm E}^2{\rm Tr}A_0^2 + \ldots,$$

with $D_i = \partial_i - ig_{\rm E}A_i$. The parameters that need to be matched with QCD are

$$m_{\rm E}^2 = \left(\frac{N_c}{3} + \frac{N_f}{6}\right) g^2T^2, \qquad g_{\rm E}^2 = g^2T.$$

What about fermions? Integrating out all the fermionic modes except $\pm\pi T$ does not seem justified. However, for almost on-shell quarks $|p_3 \pm ip_0| \lesssim gT$. The relevant expansion parameter is the "off-shellness" $\left[(ip_3)^2 - \boldsymbol{p}_\perp^2 - p_0^2\right]/T^2$.

UNIVERSITY OF HELSINKI

Fermionic Lagrangian

Write $\psi = \begin{pmatrix} \chi \\ \phi \end{pmatrix}$ and solve the EOM for heavy components near $p_3 = \pm i p_0$:

$$\Rightarrow \mathcal{L}_{\mathrm{f}} = i\chi^\dagger \left(M - g_{\mathrm{E}} A_0 + D_t - \frac{\nabla_\perp^2}{2p_0} \right) \chi + i\phi^\dagger \left(M - g_{\mathrm{E}} A_0 - D_t - \frac{\nabla_\perp^2}{2p_0} \right) \phi .$$

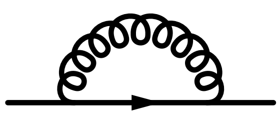
The particle (χ) and antiparticle (ϕ) sectors are clearly separated and propagate only forward/backward in the x_3 direction:

$$\langle \chi_u(x)^* \chi_v(y) \rangle = -i\delta_{uv}\theta(x_3 - y_3) \frac{p_0}{2\pi|x_3 - y_3|} e^{-M|x_3 - y_3| - \frac{p_0(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)^2}{2|x_3 - y_3|}}$$

UNIVERSITY OF HELSINKI

In the reduced theory the Matsubara frequency $\pi T \equiv M$ acts as a mass term. NLO M becomes a parameter that needs matching. We did this by comparing the locations of poles in fermion propagators, which is both a gauge invariant and an IR safe method.

To preserve the correct power counting $\pi T \gg \boldsymbol{p}_\perp$ while integrating over $\boldsymbol{p}_\perp$, we expand the ET quark propagators in $\boldsymbol{p}_\perp / p_0$.



$$= g_{\rm E}^2 C_{\rm F} \int \frac{{\rm d}^{3-2\epsilon} q}{(2\pi)^{3-2\epsilon}} \frac{1}{M + ip_3 - iq_3} \frac{1}{q^2 + \lambda^2}$$

Matching the QCD and NRQCD results gives the mass parameter in effective theory:

$$M = \pi T + g^2 T \frac{C_{\rm F}}{8\pi}$$

Potential

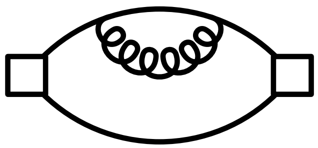
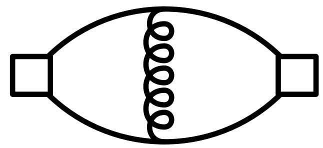
The static potential computed in the 2+1 -d theory will be of the form

$$V(r) \sim g_{\rm E}^2 \ln(r/?) + g_{\rm E}^4 r + \mathcal{O}(g_{\rm E}^6 r^2)$$

The Schrödinger equation gives $\frac{1}{p_0}\frac{\partial^2}{\partial r^2} \sim V(r) \sim g_{\rm E}^2 \ln r$, which leads to $|\boldsymbol{p}_\perp| \sim \partial/\partial r \sim 1/r \sim \sqrt{g_{\rm E}^2 p_0} \sim gT$.

$\Rightarrow$ potential is expanded in $\mathcal{O}(g_{\rm E}^2 r) \sim \mathcal{O}(g)$

$\Rightarrow$ To get the leading correction $E \sim \boldsymbol{p}_\perp^2/p_0 \sim g^2 T$ it is enough to compute the $g_{\rm E}^2 \ln r$ term in the potential, provided it is IR safe.

$V(\boldsymbol{r}) =$  $+$ 

$$= g_{\rm E}^2 C_{\rm F} \int \frac{{\rm d}^{2-2\epsilon}}{(2\pi)^{2-2\epsilon}} \left[\frac{1}{q^2+\lambda^2}\left(1-e^{iq\cdot r}\right) - \frac{1}{q^2+m_{\rm E}^2}\left(1+e^{iq\cdot r}\right) \right]$$

With this specific combination of signs, the potential is both IR and UV finite. Taking the limit $\epsilon \to 0$, $\lambda \to 0$ gives

$$V(\boldsymbol{r}) = g^2 T \frac{C_{\rm F}}{2\pi} \left[\ln \frac{m_{\rm E} r}{2} + \gamma_{\rm E} - K_0(m_{\rm E} r) \right].$$

UNIVERSITY OF HELSINKI

Results

Solving the corresponding Schrödinger equation numerically gives

$$\frac{E-2M}{g_{\rm E}^2\frac{C_{\rm F}}{2\pi}} = \begin{cases} 0.16368014 & (N_f=0) \\ 0.38237416 & (N_f=2) \\ 0.46939139 & (N_f=3) \end{cases}$$

$$\Rightarrow \quad E \approx 2\pi T + 0.14083730\, g^2 T \qquad (N_f=0, \text{quenched})$$

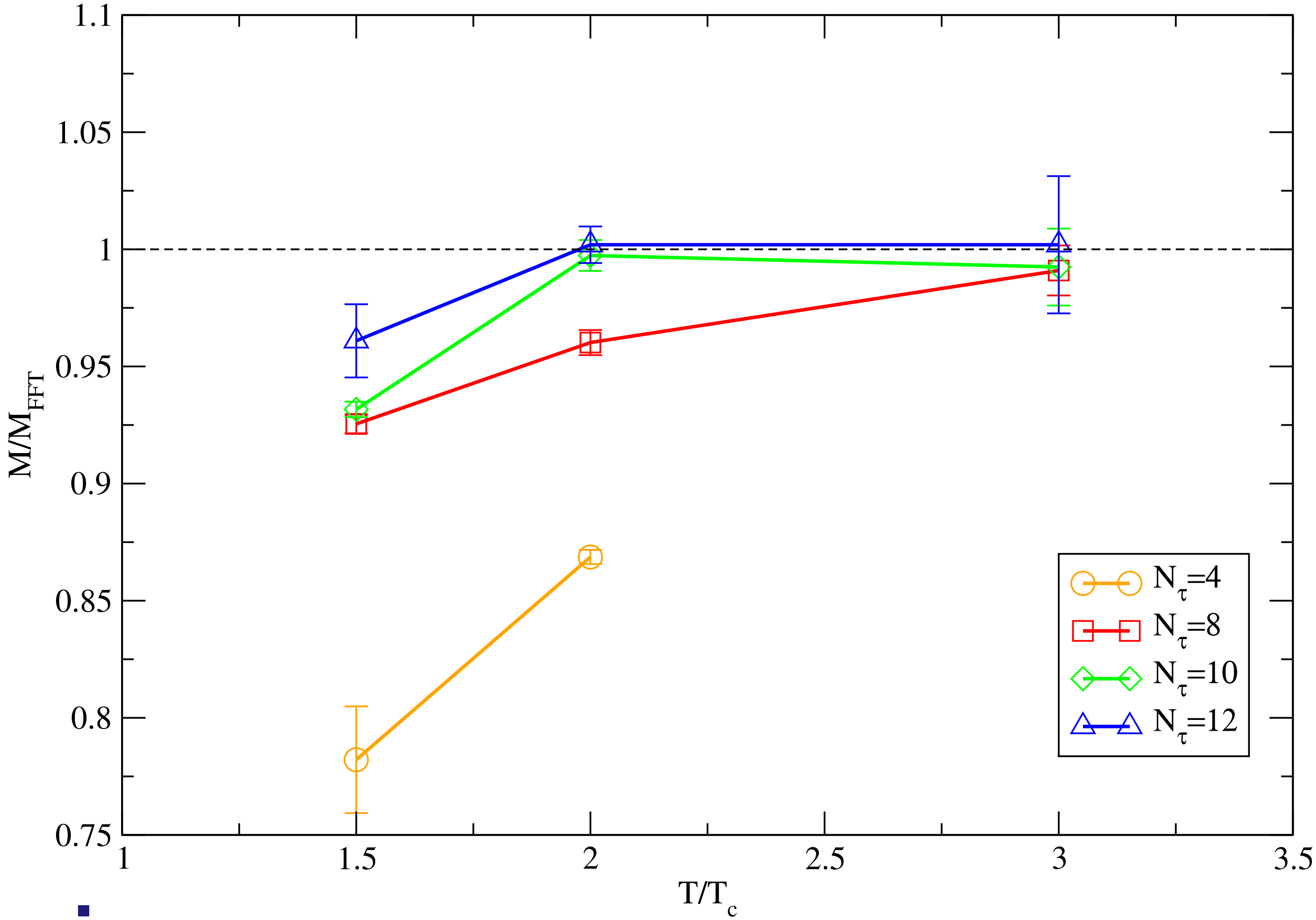
At $T \sim 2T_c$ and $N_f = 0$ $g_{\rm E}^2/T \approx 2.7$,[1] only a 5% correction.

[1] Kajantie, Laine, Rummukainen and Shaposhnikov, Nucl.Phys. B503 (1997) 357

UNIVERSITY OF HELSINKI

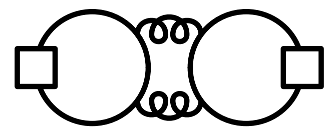
Lattice screening masses

Gavai & Gupta, PRD 67, 034501 (2003)



UNIVERSITY OF HELSINKI

Flavour singlets

These couple to glueballs via the disconnected contribution: 

$$\frac{1}{N_f}\bar{\psi}\psi \quad \leftrightarrow \quad N_c\frac{mT^2}{6} - \frac{m}{2\pi^2}g^2\mathrm{Tr}A_0^2$$

$$\frac{1}{N_f}\bar{\psi}\gamma_5\psi \quad \leftrightarrow \quad \frac{7\zeta(3)m}{8\pi^2T^2}\frac{g^2}{32\pi^2}\epsilon_{\alpha\beta\mu\nu}F^a_{\alpha\beta}F^a_{\mu\nu}$$

$$\frac{1}{N_f}\bar{\psi}\gamma_0\psi \quad \leftrightarrow \quad -\frac{i}{3\pi^2}g^3\mathrm{Tr}A_0^3$$

The glueball screening masses have been measured in DR on the lattice,[2] and are in general lower than $2\pi T$.

[2]Hart, Laine and Philipsen, Nucl.Phys. B586 (2000) 443

Axial symmetry (non-)restoration

The anomaly equation

$$\partial_\mu \left[\bar{\psi}\gamma_\mu\gamma_5\psi\right] = 2m\bar{\psi}\gamma_5\psi + N_f \frac{g^2}{32\pi^2}\epsilon\alpha\beta\gamma\delta F^a_{\alpha\beta}F^a_{\gamma\delta}$$

gives, when integrated over $\int \mathrm{d}\tau \mathrm{d}^2\boldsymbol{x}_\perp$ in the chiral $(m = 0)$ limit,

$$A^s_3 \equiv \bar{\psi}\gamma_3\gamma_5\psi \sim K_3 \equiv g^2\epsilon_{ij3}A^a_0 F^a_{ij},$$

whose correlation length has been determined to be

$$m[K_3] \approx m_\mathrm{E} + \frac{g^2_\mathrm{E} N_c}{4\pi}\left(\ln\frac{m_\mathrm{E}}{g^2_\mathrm{E}} + 7.0\right) + \mathcal{O}(g^3 T).$$

On the other hand, the vector current related to $\bar{\psi}\gamma_3\psi$ is conserved and the correlator vanishes in infinite volume.

Conclusions, outlook

- Consistent method for calculating spatial correlators of fermionic operators
- Systematics clear
- Sign differs from that of 4d lattice QCD
- NRQCD_3 should be easy to implement on lattice
- Flavour singlets still problematic