

## 8.0 Interpreting Chi Square Output

### 8.1 the Meaning of a Significant Chi-Square

Sometimes the purpose of Crosstabulation is wrongly seen as being merely about computing and interpreting the Chi-Square Statistic

As we have seen above - there is a lot of interpretative effort that can be put into uncovering the meaning of a crosstab before any reference to Chi-Square is made.

Chi-Square is a tool in the armoury of inferential statistics or significance testing. In this context it is a test of the Significance of Association shown in Contingency Tables (Crosstabulations).

Any pattern of association we find in a Crosstab is usually a pattern in Sample Data. The issue that Chi Square addresses is whether any relationship in the sample data is strong enough to justify our making inferences about the larger population from which the sample has been drawn.

Chi-Square sets about addressing this issue essentially by comparing the Observed / Data Frequencies of each cell with their corresponding Expected / Fit Values computed from the data on the basis of the assumption (null hypothesis) that there is no association between the two variables in question.

If we get big disparities between Data and Fit then we will get a bigger Chi-Square value and be more likely to detect a “significant” association.

The precise calculation of Chi-Square ( $\chi^2$ ) is given by:

$$\chi^2 = \sum \frac{(\text{Observed Frequency} - \text{Expected Frequency})^2}{\text{Expected Frequency}}$$

Though since we would normally get a computer to calculate this for us you may not wish to know that! However, you do require to know how to interpret Chi Square. Consider the following Chi-Square Output corresponding to the Vote by Social Class crosstab which we have referred to several times previously:

Fig 8.1

Chi-Square	Value	DF	Significance
Pearson	125.07354	4	.00000
Likelihood Ratio	130.62201	4	.00000
Mantel-Haenszel	93.37928	1	.00000
Minimum Expected Frequency -	20.970		

Here we see that the SPSS package generates for us 3 alternative Chi-Square values - we do not have any control over this. However the only Chi-Square value we are interested in for our present purposes is the Pearson Chi-Square\*\*

The actual interpretation of Chi-Square computer output involves us looking at the significance probability quoted - if this is *less than* 0.05 then we have a Chi-Square value which is significant at the conventional cut off point of 5% - ie the association found in the sample data is significant and would be regarded as evidence that there is an association between the variables in question in the Population from which the sample has been drawn.

We might report our reading and interpretation of this output as follows:

“The Chi-Square value for the association between Voting Preference and Social Class was obtained as 125.1 with 4 degrees of freedom and a Significance Probability less than 0.00001 - ie. a very highly significant result. On the evidence of this data there would appear to be no doubt that there is an association between Voting Preference and Social Class in the population from which this sample of 695 respondents was drawn.”

Some caveats in interpreting Chi-Square:

Before interpreting Chi-Square we should really have looked at the bottom line of this output which reports the “Minimum Expected Frequency”. It is a requirement for the application of the Chi-Square test that the value for the Expected Frequency should not fall below 5 in more than 25% of the cells. Were this to happen then we would not be strictly justified in carrying on with the interpretation of the Chi-Square statistic.

## **8.2 Delving Further into the Chi-Square Analysis.**

One of the limitations of the Chi-Square analysis is that what it really tells us is only whether or not there is a significant association *overall* between the two variables being investigated, or more specifically whether the *overall* pattern of Observed Cell Frequencies is significantly different from the pattern of Expected Cell Frequencies had there been no relation between the variables in question. What it doesn't tell us is whether the Observed Frequency *in any particular cell* is significantly different from the Expected Frequency.

Consider the following Crosstabulation of Housing Tenure (OWN\_RENT) by Class -

Fig 8.2

OWN\_RENT WHETHER HOUSE OWNER OR RENTER by CLASS OCCUPATIONAL CLASS

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		Count Col Pct	CLASS					Row Total
			Upper Mi ddle 1.00	Lower Mi ddle 2.00	Skilled Manual 3.00	SemiSkil led 4.00	Unskille d 5.00	
OWN_RENT	1.00							
OWNS/BUYING	1.00	117 74.5	60 53.6	89 35.0	46 35.4	8 19.0	320 46.0	
RENTING	2.00	40 25.5	52 46.4	165 65.0	84 64.6	34 81.0	375 54.0	
	Column Total	157 22.6	112 16.1	254 36.5	130 18.7	42 6.0	695 100.0	

  

Chi-Square	Value	DF	Significance
Pearson	84.45524	4	.00000

Inspection of the Column Percentages in this Crosstab shows a clear Class gradient in the percentage of home owners. The top Occupational Class - Upper Middle Class - has almost three quarters of respondents (74.5%) as house owners compared with less than one in five (19%) of Unskilled Manual respondents. Also over the intervening Class categories we have pretty clear evidence of a consistent gradient with a fall from about 75% House Owners in the Upper Middle Class through about 54% in Lower Middle Class and a further fall to around 35% for both Skilled Manual and Semi Skilled Class categories to the 19% House Owners in the Unskilled manual category. We recognise also a “discontinuity” in this gradient with effectively a “plateau” in the % of House Ownership between the Skilled Manual and the Semi-Skilled categories at 35.0% and 35.4% respectively.

The Pearson Chi-Square of 84.5 with 4 degrees of freedom is very highly significant at conventional levels (significance probability is less than 0.00001) which confirms the interpretation of the column percentages and allows us to conclude that the association between Tenure and Class in this sample of 695 respondents is strong enough to be generalised to the population from which the sample was drawn.

However, whilst the significant Chi-Square allows us to conclude that the pattern of Observed Frequencies is overall significantly different from what we would have expected with no relation between Tenure and Class, we are not able to conclude that each and every Observed Frequency is different from its Expected Frequency.

In particular consider the Lower Middle Class category. The percentage of House Owners here at 53.6% is only 7.6 percentage points greater than

the overall average of House Owners - given by the Row Marginal Percentage of 46%. This is a smaller difference than for the other Class Categories - Upper Middle Class at 74.5% is almost 30 percentage points higher than expected, Unskilled Manual at 19% is somewhat similar (27 percentage points lower than what expected), and even Skilled Manual and Semiskilled at around 35% is about 11 percentage points below the expected percentage of 46%.

The question therefore is how we can test particular cells within a Crosstab as to whether they are significantly different from what would have been expected had there been no relation between the variables in question.

### 8.3 The Significance of a Sample Percentage

One answer to the previous question might be to apply the test for the significance of a single sample percentage. Specifically in the context of the current problem we would test the % of House Owners in the Lower Middle Class category (53.6%) against the overall % of 46%.

The Test Statistic used is the Z-Test for a Sample Percentage which is given by the following expression:

$$Z = \frac{(\text{Sample \%} - \text{Population \%})}{(\text{Population \%} * (100 - \text{Population \%}) / \text{Sample Size})}$$

Substituting in this expression we have -

$$Z = \frac{(53.6 - 46)}{(46 * 54) / 112}$$

This evaluates to  $Z = 1.61378921$  or approx 1.6

The Critical Value for Z for a two tailed significance test is 1.96. Since our calculated value (1.6) is not greater than the critical value (1.96) we cannot reject the Null Hypothesis that there is no significant difference between this Cell frequency and what would have been expected had there been no association between Tenure and Class.

## 8.4 Use of Residuals, & Standardised & Adjusted Residuals

The previous approach of comparing the percentage having the indicated characteristic (eg. who Vote Conservative) for a particular category of the Independent Variable (eg. Class) against the overall population percentage is limited in that it requires a separate manual calculation for each Cell of interest (– SPSS doesn't provide a procedure to simplify this).

However, SPSS does provide an alternative approach which enables us to analyse the Residuals in each cell of the crosstabulation.

Consider the following Crosstab:

Fig 8.3

		CLASS					Page 1 of 1
OWN_RENT	Count	Upper	Lower	Skilled	SemiSkil	Unskille	Row Total
	Exp Val	ddle	ddle	Manual	led	d	
	Residual						
	Std Res						
	Adj Res	1.00	2.00	3.00	4.00	5.00	
OWNS/BUYING	1.00	117	60	89	46	8	320
		72.3	51.6	116.9	59.9	19.3	46.0%
		44.7	8.4	-27.9	-13.9	-11.3	
		5.3	1.2	-2.6	-1.8	-2.6	
		8.1	1.7	-4.4	-2.7	-3.6	
RENTING	2.00	40	52	165	84	34	375
		84.7	60.4	137.1	70.1	22.7	54.0%
		-44.7	-8.4	27.9	13.9	11.3	
		-4.9	-1.1	2.4	1.7	2.4	
		-8.1	-1.7	4.4	2.7	3.6	
Column		157	112	254	130	42	695
Total		22.6%	16.1%	36.5%	18.7%	6.0%	100.0%

The above crosstab has more than two numbers in each cell as compared with the more familiar crosstab format we have previously recommended. The key to the contents of each cell is provided at the top left of the Crosstab. The numbers in each cell can be identified in vertical order as:

- the Cell Count - this is the actual frequency observed for that combination of the variable codes that identify the cell;

- the Expected Value - this is the frequency count we would have expected in the situation where there was no relationship between the two variables being investigated;
- the Residual - this is the difference between the Cell Count and the Expected Value
- the Standardised Residual - see below
- the Adjusted Standardised Residual - see below

*Clarification of the different variants of Residuals -*

the Residual = Cell Count - Expected Value

eg. for the topmost left cell concerning Upper Middle Class respondents who Own their Houses Outright the Residual is  $117 - 72.3 = +44.7$ . This means that the Observed Frequency of Upper Middle Class Outright House Owners (117) is 44.7 greater than the value we would expect (72.3) had there been no relationship between Tenure and Class. Clearly the bigger the Residual, other things being equal, the less likely the Null Hypothesis of No Association between the two variables is to hold and the bigger would appear to be the strength of association between the two variables. However the magnitude of the Residual is not determined only by the strength of association but also by the size of the sub categories relating to the cells. To compensate for this the Standardised Residual - the 4th number down in each Cell is calculated -

the Standardised Residual is obtained by dividing the Residual by the Square Root of the Expected Value for each cell, ie.

$$\text{Standardised Residual} = \frac{(\text{Observed Cell Count} - \text{Expected Value})}{\sqrt{\text{Expected Value}}}$$

eg. for the top left cell -

$$\text{Standardised Residual} = (117 - 72.3) / \sqrt{72.3} = 5.2570048 = 5.3 \text{ approx}$$

as shown in the above crosstab.

The Standardised Residual should ring a bell for those familiar with the formula for Chi Square -

$$Z = \frac{(\text{Observed Frequency} - \text{Expected Frequency})}{\sqrt{\text{Expected Frequency}}}$$

If we compare the expression for the Standardised Residual with that for Chi Square then we can see that Chi-Square is nothing other than the Sum of the Squared Standardised Residuals over all Cells in the Crosstab.

The Standardised Residuals are said to be approximately distributed as a Normal Distribution with Mean of 0 and Standard Deviation of 1. This would enable us to compare individual Standardised Residuals with the conventional cut off point of approximately 2 for the Standard Normal Distribution. Thus Standardised Residuals greater than 2 would be deemed to be significantly large, for those less than 2 we would be unable to say with any confidence that the Observed Frequency Count in the Cell in question was significantly different from that which would have been expected had there been no association between the two variables in question.

However, although the Standardised Residuals are an approximate fit to the Standard Normal Distribution, in some cases this fit can be poor. In the latter case cells which are significantly different from what would be expected given the null hypothesis of no association might not be identified. A better test for the significance of individual cells is given by inspection of the Adjusted Residuals.

$$\text{the Adjusted Residual} = \frac{\text{the Standardised Residual}}{\text{estimated Standard Error}}$$

(- we shall not delve further into how the Standard Error is estimated!). The Adjusted Residual also is distributed according to the Standard Normal Distribution with Mean of 0 and Standard Deviation of 1. Once again the cut off point would be the value 2 - ie any Adjusted Residual which had an absolute value greater than 2 would be deemed significant, less than 2 would be insignificant.

If we opt for the Adjusted Residual as the way in which we shall identify cells which do not seem to fit in with the pattern of overall association suggested by a significant Chi-Square then we can simplify the previous Crosstab by merely adding the Adjusted Standardised Residual to the list of Cell contents requested when specifying the Crosstab. This would then have given us the following output:

Fig 8.4

OWN\_RENT WHETHER HOUSE OWNER OR RENTER by CLASS OCCUPATIONAL CLASS

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		CLASS					Row
		Upper Mi	Lower Mi	Skilled	SemiSkil	Unskille	Total
Count	Col Pct	ddle	ddle	Manual	led	d	
Adj Res							
OWN_RENT		1.00	2.00	3.00	4.00	5.00	
1.00		117	60	89	46	8	320
OWNS/BUYING		74.5%	53.6%	35.0%	35.4%	19.0%	46.0%
		8.1	1.7	-4.4	-2.7	-3.6	
2.00		40	52	165	84	34	375
RENTING		25.5%	46.4%	65.0%	64.6%	81.0%	54.0%
		-8.1	-1.7	4.4	2.7	3.6	
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Chi-Square	Value	DF	Significance
Pearson	84.45524	4	.00000

We should remind ourselves that the Pearson Chi-Square is highly significant indicating that overall there is a relationship between Tenure and Class - or alternatively put - that the pattern of Tenure differs across the different Class Groupings.

Inspecting the individual cells we see for example that Upper Middle Class House Ownership is substantially higher at 74.5% than the overall average (46%), whilst that for Unskilled Manual Respondents (19%) is substantially below average. Turning now to the Adjusted Standardised Residuals and comparing them to the cut-off of 2, we see that these residuals are clearly above the value 2 in all but 2 of the cells - those corresponding to the Lower Middle Class category of respondents. The Adjusted Standardised Residual in both of these Cells (1.7 and -1.7) is below the cut-off point. We are led to conclude therefore that despite the overall association between Tenure and Class, for this particular sub category of class (Lower Middle Class) we could not say with confidence that the percentage of Home Owners (53.6%) is significantly higher than the overall average percentage of 46% for the sample of 695 respondents as a whole.

**A warning** - Residual Analysis here is presented as a way of delving deeper into a Crosstabulation. It makes no sense to carry out a Residual Analysis on Two by Two Crosstabs because if Chi-Squared is significant so also will each of the Residuals within the Crosstab. However, with Crosstabs of larger dimension, Residual Analysis will sometimes throw up interesting results along the lines of particular sub-categories that “buck the trend” of the overall association between the variables. Alternatively, much higher values for residuals - whether +ve or -ve - may be taken as indicating those cells which make a particularly strong contribution to the relationship depicted in the table.