Basics of Monte Carlo simulations 2005. Exercise 2

To be handed in Mon 21.2, exercise session Thu 24.2 10:15.

1. (12 p) Physical tests of random-number generators. A random walk is an important concept in both statistics and physics. The simplest possible random walk is the one-dimensional one, where a walker starts from 0 and takes exactly N steps of length 1 either to the left or right, with equal probability. It can be analytically shown that the average displacement of the walker after N steps $< x(N) >= 0$ and the average of the square of the displacement $< x^2(N) >= N$. These are exact results.

Write a program which simulates this 1D random walk to test random number generators. Perform walks to 100 steps, and repeat enough times to get a statistically good average $< x(N) >$ and $< x^2(N) >$. Consider the following random number generators: a) the Mersenne twister, b) the Park-Miller generator, c) the lowest-order $(ia = 106, ic = 1283, im = 6075)$ quick-and-dirty generator, and d) the higher-order $(ia = 259200, ic = 7141, im = 54773)$ quick-and-dirty generator.

Calculate $< x(N) >$ and $< x^2(N) >$ after 300 000 random walks of each generator. Repeat a few times with different seeds to get an idea of the statistical fluctuations.

Report $< x(N) >$ and $< x^2(N) >$. Which generators fail the test, and which one is the worst? Return the code.

2. (6 p) In nuclear physics one of the most important distributions is given by the Breit-Wigner formula

$$\sigma(E) = a\frac{\Gamma^2}{(E - E_R)^2 + \Gamma^2/4}$$

which for instance gives the shape of single, isolated nuclear resonances at an energy E_R. Typically $E_R >> \Gamma$. Write a program which generates random numbers in this distribution using the analytical inversion method. Verify the correctness by plotting the distribution and an analytical prediction with the same parameter values on top of each other (use proper normalization). Hand in the code and plot.

3. (6 p) Write a program which generates random numbers distributed as

$$f(t) = \cos^2(t)e^{-t}$$

in the interval $[0, \infty]$ by a hit-and-miss method of your choice. Let the code also report the number of hits and misses. Verify the correctness by plotting the distribution and an analytical prediction with the same parameter values Hand in code, plot, and report of number of hits and misses in a long run.

4. (12 p) Write a program which reads in an arbitrary discrete distribution from a file with data in x_i, p_i format, with the x data evenly distributed, and generates random numbers in this distribution. The p_i data does not need to be normalized. The program should output the random distribution as well as the ratio between the random and original data.

Apply the program to generate 1000000 points in the distribution given in the file 'bimodal.dat' on the course web page, and return two plots: the original vs. the random data, and the fraction between the two data sets. Also return the source code.