After these excursions to extra dim'l spaces let us return to AdS/CFT prescription:

p.46: $$ds^2 = \frac{R^2}{\rho^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + d\rho^2\right) + R^2 d\Omega_5^2 = AdS_5 \times S_5$$

$$\underset{\text{Mink}}{-dt^2 + d\vec{x}^2} \underset{\tau = it}{=} \underset{\text{Eucl}}{d\tau^2 + d\vec{x}^2}$$

What is the meaning of "4d QFT lives on boundary at $\rho = 0$"? And how does one get nonperturbative exact info on the 4d QFT (which is encoded in gauge inv. exp. values $\langle O(x)\rangle$, $\langle O(x_1) O(x_2)\rangle, \dots$) from solutions of bulk Einstein gravity?

Example $$\eta = \lim_{\omega \to 0} \int_{-\infty}^{\infty} dt\, e^{i\omega t} \underset{(\vec{k}=0)}{\int d^3x} \langle [T_{xy}(t,\vec{x}), T_{xy}(0,\vec{0})]\rangle_T$$ Minkowski space correlator!!

$$= s T_H \frac{\sqrt{-G(r_0)}}{\sqrt{-G_{00}(r_0) G_{rr}(r_0)}} \int_{r_0}^{\infty} dr \frac{-G_{00}(r) G_{rr}(r)}{G_{xx}(r)\sqrt{-G(r)}}$$

for a near'y extremal black 3-brane

$$ds^2 = +G_{00}\, dt^2 + G_{xx}(dx_1^2 + \dots + dx_3^2) + G_{rr}\, dr^2 + Z(r) \overbrace{\sum_1^d K_{mn}\, dy^m dy^n}^{d_r \Omega_d^2}$$

$$1 \quad + p \quad + 1 \quad + d \quad = 10$$

p.47:

$$r \ll R \quad G_{00} = -\frac{r^2}{R^2}\left(1 - \frac{r_0^4}{r^4}\right) \quad G_{xx} = \frac{r^2}{R^2} \quad G_{rr} \approx \frac{R^2}{4r_0}\frac{1}{(r - r_0)} \quad Z(r) = R^2$$

$$\approx \frac{4r_0}{R^2}(r - r_0) \quad T_H = \frac{r_0}{\pi R^2} \quad G(r) = \underbrace{G_{00}(r)\, G_{rr}(r)}_{-1} G_{xx}^3(r)\, Z^5 = -\frac{r^6}{R^6} \cdot Z^5$$

$$\frac{\eta}{s} = T_H \cdot \frac{r_0^3}{R^3} \int_{r_0}^{\infty} dr \frac{1}{\frac{r^2}{R^2}\frac{r^3}{R^3}} = T_H\, r_0^3 R^2 \left[-\frac{1}{4} r^{-4}\right]_{r_0}^{\infty} = \frac{1}{4} T_H\, r_0^3 R^2 \frac{1}{r_0^4} = \frac{1}{4\pi}$$

so here we have the metric, gravity giving a field theory result

Gauge-gravity duality concretely:

Gauge theory: Physics is in correlators $\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle$

some gauge inv. op.

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}\varphi \, \mathcal{O} \, e^{-S[\varphi]}}{\int \mathcal{D}\varphi \, e^{-S[\varphi]}}$$

all fields of gauge theory $S[\varphi]$

$$Z[J] = e^{-\Gamma(\bar\varphi) - J\bar\varphi} = e^{-W(J)} = \int \mathcal{D}\varphi \, e^{-S[\varphi] - J\varphi} \qquad \bar\varphi(J) = W'(J)$$

$$\frac{d\Gamma}{d\bar\varphi} = -J \qquad \frac{d^2\Gamma}{d\bar\varphi^2} = -\frac{1}{W''(J)} \qquad \langle \varphi^2(J) \rangle - \langle \varphi(J) \rangle^2 = W''(J)$$

For any local operator $\mathcal{O}$ we may construct the generating functional

$$e^{-W(\varphi_0)} \equiv Z(\varphi_0) = \int \mathcal{D}\varphi \, e^{-S[\varphi] - \int \varphi_0(x) \mathcal{O}(x)} \equiv \left\langle e^{-\int d^4x \, \varphi_0(x) \mathcal{O}(x)} \right\rangle \cdot Z(0)$$

"external classical current"

AdS/CFT now is

$\begin{cases} x \to \lambda x \\ r \to \frac{1}{\lambda} r \end{cases}$

!! $$\left[\frac{1}{Z(0)} Z(\varphi_0(x)) = \left\langle e^{-\int d^4x \, \varphi_0(x) \mathcal{O}(x)} \right\rangle = e^{-S_{SUGRA}[\varphi(x,r),\ \varphi(x, r\to\infty) = \varphi_0(x)]} \right.$$

$$ds^2 = \frac{r^2}{R^2}(d\tau^2 + d\vec{x}^2) + \frac{R^2}{r^2} dr^2$$

Practical applications thus boil down to solving classical SUGRA EOM's of type

$$\begin{cases} R_{\mu\nu} = \frac{1}{2} \partial_\mu \phi \, \partial_\nu \phi + \cdots e^{a\phi} (F_{\mu \cdots} F_\nu{}^{\cdots} - g_{\mu\nu} F^2) \\ \nabla_\mu (e^{a\phi} F^{\mu\nu}) = 0 \\ \nabla_\mu \nabla^\mu \phi = \cdots e^{a\phi} F^2 \end{cases}$$

$$\begin{cases} \nabla_\mu \nabla^\mu \phi = \frac{1}{\sqrt{g}} \partial_\mu \sqrt{-g} \, g^{\mu\nu} \partial_\nu \phi \\ \nabla_\mu F^{\mu\nu} = \frac{1}{\sqrt{g}} \partial_\mu \big(\underbrace{\sqrt{-g} \, g^{\mu\alpha} g^{\nu\beta}}_{\text{put here some relevant background metric}} F_{\alpha\beta} \big) = 0 \end{cases}$$

with suitable BC's.

For transport coefficients ($D, \eta, \cdots$) one needs two-point correlators (Kubo formulas), i.e., $S''_{SUGRA}[\phi_0]$. See

S-S + Policastro hep-th/0104066 η for SYM graviton absorption
Son-Starinets "/0205051 General formulas, CS diffusion rate

—"— + Policastro	—"— 052	Diffusion, η
—"—	/0210220	Sound
—"— + Kovtun	/0309213	D, η for many metrics, universal $\frac{\eta}{s}$

<u>Example</u> The simplest correlator is that of 1, the free energy!

$$p = \underbrace{15}_{8+\frac{7}{8}8} \cdot \overset{d_A}{8} \frac{\pi^2}{90} T^4 = d_A \frac{\pi^2}{6} T^4 = \frac{\pi^2}{6} \overset{N_c^2-1}{N_c^2} T^4$$

$$s = \frac{2}{3} \pi^2 N_c^2 T^3 \xrightarrow[\frac{3}{4}]{} \frac{1}{2} \pi^2 N_c^2 T^3$$

See pp. 69-70 of 2004 lectures