Thermodynamics

Non-extremal $r_0 \neq 0$ but $r \ll R$ ("throat appro"):

$$ds^2 = \frac{r^2}{R^2}\left[-\left(1-\frac{r_0^4}{r^4}\right)dt^2 + d\vec{x}^2\right] + \frac{R^2}{r^2}\frac{1}{1-\frac{r_0^4}{r^4}}dr^2 + R^2 d\Omega_5^2$$

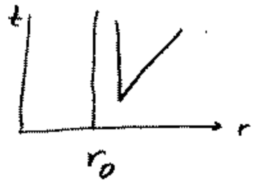
$$\xrightarrow{r \gg r_0} \frac{r^2}{R^2}(\eta_{\mu\nu}dx^\mu dx^\nu) + \frac{R^2}{r^2}dr^2$$

$\begin{cases} x^\mu \to \lambda x^\mu \\ r \to \frac{1}{\lambda} r \end{cases}$

$$ds^2 = -g(r)dt^2 + \frac{1}{f(r)}dr^2 \approx -g'(r_0)(r-r_0)dt^2 + \frac{1}{f'(r_0)(r-r_0)}dr^2$$

light $\frac{dt}{dr} = \frac{1}{\pm\sqrt{g_0' f_0'}}\frac{1}{r-r_0}$

$r \approx r_0$



write $\tau = it$ and transform to the form

$$ds^2 = \rho^2 d\varphi^2 + d\rho^2 = g_0'(r-r_0)\,d\tau^2 + \frac{1}{f_0'(r-r_0)}dr^2$$

periodic imag time $\equiv \tau$ $\quad \left(\rho = 2\sqrt{\frac{1}{f_0'}(r-r_0)} \Rightarrow dr^2 = f_0'(r-r_0)\,d\rho^2\right)$

$$= \rho^2\,\frac{g_0' f_0'}{4}d\tau^2 + d\rho^2$$

Above $\begin{cases} r - r_0 = \frac{\rho^2}{R^2} \\ g_0' = f_0' = \frac{4r_0}{\sqrt{R^4+r_0^4}} \end{cases}$ ← without throat appro!

$$= \rho^2\, d\left(\frac{\tau}{2\sqrt{f_0' g_0'}}\right)^2 + d\rho^2$$

Black non-extr 3-brane: $\boxed{T_H = \frac{\sqrt{g_0' f_0'}}{4\pi} = \frac{r_0}{\pi\sqrt{R^4+r_0^4}}}$ ⇐ periodic under $\tau \to \tau + \frac{4\pi}{\sqrt{g_0' f_0'}} \equiv \tau + \frac{1}{T}$

$\left(\text{Schw}: T_H = \frac{1}{4\pi r_s} = \frac{1}{8\pi G M}\right)$

Another metric: AdS_5 black hole = Schw. generalised to AdS_5: Witten 1998

$$ds^2 = -\left(\frac{r^2}{b^2} + 1 - \underbrace{\frac{8G_5}{3\pi}\frac{M}{r^2}}\right)dt^2 + \frac{dr^2}{(\text{same})} + r^2 d\Omega_3^2, \quad \boxed{R_{\mu\nu} = -\frac{4}{b^2}g_{\mu\nu}}$$

$[G_d] = \frac{1}{M^{d-2}}$

AdS_{m+1}: $\frac{16\pi G_{m+1}}{(m-1)\Omega_{m-1}}\cdot\frac{M}{r^{m-2}}$ $\quad$ Note: G_{n+1} is not in, it comes like G_N to Schw. by physically fixing one const of int. $\quad$ m everywhere, not only for $r \gg r_0$

$$\xrightarrow{r\to\infty} -\frac{r^2}{b^2}dt^2 + r^2 d\Omega_3^2 + b^2\frac{dr^2}{r^2} = \frac{r^2}{b^2}\left(-dt^2 + b^2 d\Omega_3^2\right) + \frac{b^2}{r^2}dr^2$$

$= AdS_5$ (p. 40)

AdS_5 BH has a horizon at some r_+:

$$r_+^2 = \frac{b^2}{2}\sqrt{1+\frac{4}{b^2}\frac{8G_5}{3\pi}M} - \frac{b^2}{2} \xrightarrow[\begin{cases} M \text{ large} \\ r_+ \text{ -''-} \end{cases}]{} b\sqrt{\frac{8G_5}{3\pi}M}$$

$$T_H \cdot b = \frac{1}{\pi}\left(\frac{r_+}{b} + \frac{b}{2r_+}\right) \xrightarrow[r_+ \gg b]{} \frac{r_+}{\pi b}$$

For $\begin{cases} G_5 M \gg b^2 \\ r_+ \gg b \end{cases}$ non-extremal D3-brane = AdS_5 black hole

$$r_+^2 \to b\sqrt{\frac{r_0^4}{R^2}} \to r_0^2 \qquad \begin{cases} R = b \\ \dfrac{r_0^4}{R^2} = \dfrac{8}{3\pi}G_5 M \\ r_0 = r_+ \gg b \end{cases}$$

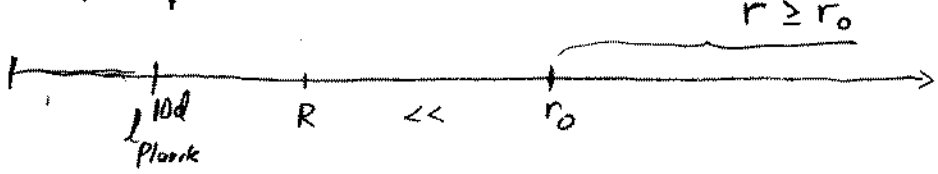
So this limit is also $r_0 \gg R$

$$T_H = \frac{r_0}{\pi R^2} = \frac{1}{\pi R}\cdot\frac{r_0}{R} \gg \frac{1}{R}$$

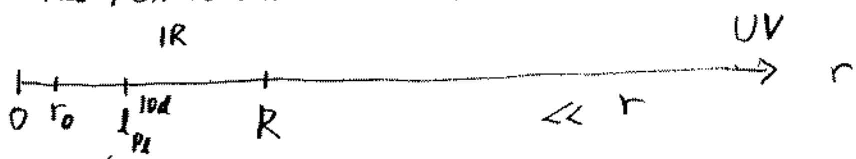
Altogether:

$$\frac{1}{\sqrt{1+\frac{R^4}{r^4}}}\left[-\left(1-\frac{r_0^4}{r^4}\right)dt^2 + \dots\right.$$

BIG BH



For the nearly extremal sol'n we had



$g_s^{\frac{1}{4}}\sqrt{\alpha'}$

$= (4\pi g_s N \alpha'^2)^{\frac{1}{4}}$

where is $\Lambda_{QCD} R^2$?

and Gubser-Kl-Peet hep-th/9805156, eq. (2) claim this is the relevant limit for

SMALL BH

$$\text{Entropy} = \frac{\text{Area of horizon in } n-1 \text{ dim}}{4 G_{n+1}} \overset{AdS_{n+1}}{=} \frac{\Omega_{n-1}\, r_+^{\,n-1}}{4 G_{n+1}}$$

$$\left(\text{Schw: } S = \frac{\Omega_2\, r_s^2}{4G} = \frac{\pi 4 G^2 M^2}{G} = 4\pi G M^2 \cdot \frac{1}{\hbar c} = \frac{1}{2}\frac{M}{T_H} \right)$$

$\Omega_2 = 4\pi$ $\qquad T_H = \frac{1}{8\pi G M}$

$r_s = \frac{2GM}{c^2}$

$$AdS_5 : \quad S = \frac{2\pi^2}{4G_5} r_+^3 = \frac{\pi^2}{2}\,\frac{8}{3\pi}\,\frac{MR^2}{r_0^2} = \frac{4}{3}\,\frac{M}{T_H}$$

$\Omega_3 = 2\pi^2$ $\qquad \frac{1}{G_5} = \frac{8}{3\pi}\frac{MR^2}{r_+^4} \qquad T_H = \frac{r_0}{\pi R^2}$

Where is this Hawking radiation as it appears as hot $N=4$ susy gas?

Somewhat more conventional $d>4$ theories are obtained by just adding:

$$S = \int d^d x \sqrt{|g|}\left[\frac{1}{2\kappa^2} R - \frac{1}{4} F^a_{MN} F^{MNa} + [(\partial_M + ig A_M)\phi]^\dagger (\partial^M + ig A^M)\phi + \dots\right]$$

and looking for soln's of type

$$ds^2 = g_{MN}\, dx^M dx^N = \sigma(y)\, g_{\mu\nu}(x)\, dx^\mu dx^\nu + \gamma_{mn}(y)\, dx^m dx^n,$$