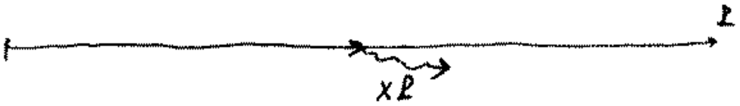
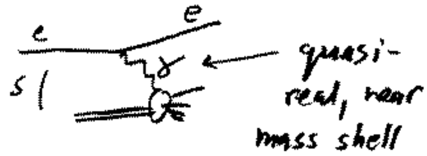
Collinear factorisation of gluonic MHV amps

- Weizsäcker-Williams photons





$$f_{\gamma/p}(x) = \frac{\alpha}{2\pi} \log \frac{s}{4m^2} \frac{1+(1-x)^2}{x}$$

- AP splitting functions:

$$x = \frac{p^+}{P^+}$$

$$f_{g/g}(x) = \frac{\alpha_s}{2\pi} \log Q^2 \underbrace{P_{gg}(x)}_{6\left[\frac{1-x}{x} + \frac{x}{1-x} + x(1-x) + \cdots\right]}$$

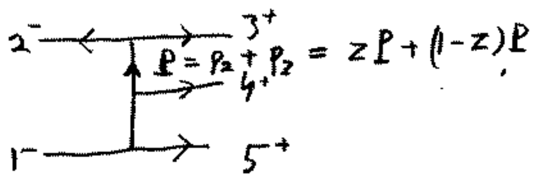
evol. eqs $\Rightarrow$

$$\frac{d g(x,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dz}{z} P_{gg}(z)\, g(\tfrac{x}{z}, Q^2) + \cdots$$

$$= \int_0^1\int_0^1 dy\, dz\, P_{gg}(z)\, g(y, Q^2)\, \delta(x - zy)$$

- How is this seen in MHV amps? Take

$$A_5(1^- 2^- 3^+ 4^+ 5^+) = i g^3 \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle} \quad \text{when } \langle 23\rangle \to 0:$$



$$\langle 34\rangle = \lambda_3^a \lambda_{4a}$$

$$p_{3a\dot a} = \lambda_{3a}\tilde\lambda_{3\dot a} \qquad \tilde\lambda = \lambda^*$$

$$= (1-z)P_{a\dot a} = \lambda_{Pa}\tilde\lambda_{P\dot a}$$

$$\Rightarrow \langle 34\rangle = \sqrt{1-z}\,\langle P4\rangle$$

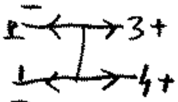
$$\langle 12\rangle = \sqrt{z}\,\langle 1P\rangle$$

$$\approx \underbrace{\frac{1}{\langle 23\rangle}\, g\sqrt{\frac{z^4}{z(1-z)}}}_{\text{splitting fn}}\;\underbrace{i g^2 \frac{\langle 1P\rangle^4}{\langle 1P\rangle\langle P4\rangle\langle 45\rangle\langle 51\rangle}}_{\text{quasireal}}$$

$$P^2 = 2p_2\cdot p_3 = \langle 23\rangle[23] \to 0$$

splitting fn = (polarised A-P)$^{\frac{1}{2}}$

quasireal



Gluons → Gravitons; weak field gravity

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B. DeWitt PR 162 (1967)

Donoghue gr-qc/9512024

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$$p_{a\dot a} = \lambda_a \tilde\lambda_{\dot a}$$

$$\varepsilon^{(+)}_{\mu\nu} \to \varepsilon^{(+)}_{a\dot a\, b\dot b} = \varepsilon^{(+)}_{a\dot a}\,\varepsilon^{(+)}_{b\dot b}$$

$$\frac{\mu_a \tilde\lambda_{\dot a}}{[\lambda\mu]}$$

$$\boxed{\begin{gathered} A\left(\;\right) = \frac{i}{M_{Pl}^2} T^{\mu\nu} \frac{iP_{\mu\nu\alpha\beta}}{q^2+i\varepsilon} T^{\alpha\beta} \\ P_{\mu\nu\alpha\beta} \hat= \frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}\right] \\ q^\mu T_{\mu\nu} = 0 \quad \text{harmonic gauge} \end{gathered}}$$

Free gravitons

$$S = \frac{1}{2\kappa_d^2}\int d^dx \sqrt{g}\, R \overset{\text{Linearize,}}{=} \frac{1}{2}\int d^4x\; h^{\mu\nu}\left(R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\,\eta^{\alpha\beta}R_{\alpha\beta}\right)_{\text{Linear}}$$

$$2\kappa_d^2 = 16\pi G_d \qquad d=4 \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \qquad g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\alpha}h_\alpha{}^\nu + \cdots$$

$$\kappa_4 = \sqrt{8\pi G_N}$$

$$R^{Lin}_{\mu\nu} = \left[-\partial^2\eta_{\mu\alpha}\eta_{\nu\beta} + \partial_\mu\partial_\alpha\eta_{\nu\beta} + \partial_\nu\partial_\alpha\eta_{\mu\beta} - \partial_\mu\partial_\nu\eta_{\alpha\beta}\right]h^{\alpha\beta}$$

$$R^\mu{}_\mu{}^L = \left(-\partial^2\eta_{\alpha\beta} + 2\partial_\alpha\partial_\beta - \partial^2\eta_{\alpha\beta}\right)h^{\alpha\beta} = 2\left(\partial_\alpha\partial_\beta - \partial^2\eta_{\alpha\beta}\right)h^{\alpha\beta}$$

$$S_{Lin} = \int d^4x\; h^{\mu\nu}\left[-\partial^2\eta_{\mu\alpha}\eta_{\nu\beta} + \partial_\mu\partial_\alpha\eta_{\nu\beta} + \partial_\nu\partial_\alpha\eta_{\mu\beta} - \partial_\mu\partial_\nu\eta_{\alpha\beta} + \eta_{\mu\nu}\left(\partial^2\eta_{\alpha\beta} - \partial_\alpha\partial_\beta\right)\right]h^{\alpha\beta}$$

$$D^{(0)-1}_{\mu\nu,\alpha\beta} \quad \text{inverse graviton propagator}$$

Invariant under $\boxed{h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\varepsilon_\nu(x) + \partial_\nu\varepsilon_\mu(x)}$ $\qquad (A_\mu \to A_\mu + \partial_\mu\varepsilon)$

Reminder:

$$R_{\mu\nu\lambda\rho} = \frac{1}{2}\left(\partial_\rho\partial_\lambda g_{\mu\nu} - \partial_\rho\partial_\mu g_{\nu\lambda} - \partial_\lambda\partial_\nu g_{\mu\nu} + \partial_\lambda\partial_\mu g_{\nu\rho}\right) + g_{\alpha\beta}\left(\Gamma^\alpha_{\lambda\mu}\Gamma^\beta_{\nu\rho} - \Gamma^\alpha_{\rho\mu}\Gamma^\beta_{\nu\lambda}\right)$$

$$\nabla_\nu V_\mu = \partial_\nu V_\mu - \Gamma^\alpha_{\mu\nu}V_\alpha \qquad \left(D_\mu = \partial_\mu + igA_\mu\right)$$

$$\left(\nabla_\lambda\nabla_\nu - \nabla_\nu\nabla_\lambda\right)V_\mu = -V_\alpha R^\alpha{}_{\mu\nu\lambda} \qquad \left([D_\mu, D_\nu] = \frac{i}{g}F_{\mu\nu}\right)$$

$$\Gamma^\sigma_{\mu\nu} = \frac{1}{2}g^{\alpha\sigma}\left(\partial_\mu g_{\nu\alpha} + \partial_\nu g_{\mu\alpha} - \partial_\alpha g_{\mu\nu}\right)$$

Contractions $\quad R_{\mu\nu\lambda\rho} = 0 \quad \varepsilon^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} = 0$

∃ unique scalar! $\quad R_{\nu\rho} = R_{\mu\nu\lambda\rho} \equiv g^{\mu\lambda}R_{\mu\nu\lambda\rho} \qquad R = R_{\mu\nu\lambda\rho} = -R_{\mu\nu\lambda\rho}$

Field eq. $\quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \quad$ or $\quad R_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^\alpha{}_\alpha\right)$

Due to inv. under $\underbrace{h_{\mu\nu}}_{\substack{6 \text{ off diag} \\ + 4 \text{ diag comps}}} \to h_{\mu\nu} + \partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu$ lots of extra dofs

$$\partial^\mu F_{\mu\nu} = \partial^\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) = \partial^2 A_\nu - \partial_\nu \partial^\mu A_\mu = 0 \Rightarrow \partial^2 A_\nu = 0 \text{ if } \partial_\mu A^\mu = 0$$

Gauge fixing:

4 relations $\partial_\alpha h^\alpha{}_\mu - \frac{1}{2} \partial_\mu h^\alpha{}_\alpha = 0 \to \partial_\mu \partial_\alpha h^{\alpha\mu} = \frac{1}{2}\partial^2 h^\alpha{}_\alpha$

$$-\partial^2 h_{\mu\nu} + \underbrace{\partial_\mu \partial_\alpha h^\alpha{}_\nu + \partial_\nu \partial_\alpha h_\mu{}^\alpha - \partial_\mu \partial_\nu h^\alpha{}_\alpha}_{\text{removes these terms}} + \eta_{\mu\nu}(\partial^2 h^\alpha{}_\alpha - \underbrace{\partial_\alpha \partial_\beta h^{\alpha\beta}}_{= \frac{1}{2}\partial^2 h^\alpha{}_\alpha})$$

$$\Rightarrow \boxed{\partial^2 h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} \partial^2 h^\alpha{}_\alpha = 16\pi G T_{\mu\nu}} \quad + \frac{1}{2}\eta_{\mu\nu}\partial^2 h^\alpha{}_\alpha$$

$$\eta^{\mu\nu} \times : \quad \partial^2 h - \frac{1}{2} \underbrace{\eta^\mu{}_\mu}_{4} \partial^2 h = -\partial^2 h$$

$$\tilde{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h \qquad \partial^2 \tilde{h}_{\mu\nu} = 16\pi G T_{\mu\nu}$$

$$\tilde{h} \equiv \tilde{h}^\mu{}_\mu = h - \frac{1}{2}\cdot 4 h = -h$$

<u>Plane waves</u> $\quad h_{\mu\nu}(x) = \mathrm{Re}\ \varepsilon_{\mu\nu} e^{-ip\cdot x}$

$T_{\mu\nu} = 0 \quad \partial^2 h = 0$

$\partial^2 h_{\mu\nu} = 0 \Rightarrow p^2 = 0$ — 10 dofs

gauge fixing $\quad p_\alpha \varepsilon^\alpha{}_\mu = \frac{1}{2} p_\mu \varepsilon^\alpha{}_\alpha$ — 6 dofs

inv. under $\varepsilon'_{\mu\nu} \to \varepsilon_{\mu\nu} + \eta_\mu p_\nu + \eta_\nu p_\mu$

$A_\mu(x) = \mathrm{Re}\ \varepsilon_\mu e^{-ipx}$

$\Box A_\mu = 0 \Rightarrow p^2 = 0$ — 4 dofs

$\partial^\mu A_\mu = 0 \quad p\cdot\varepsilon = 0$ — 3 dofs

$A_\mu \to A_\mu + \partial_\mu \varepsilon \quad \varepsilon^\mu \to \varepsilon^\mu + \eta p^\mu$

2 dofs

Choose $p^\mu = (E, 0, 0, E)$ and work out explictly what the above conditions imply for $\varepsilon_{\mu\nu}$ (Weinberg sect 10.2). See that by proper choice of η_μ can choose

$$\varepsilon_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \varepsilon_{11} & \varepsilon_{12} & 0 \\ 0 & \varepsilon_{12} & -\varepsilon_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Now rotate around z axis, by angle θ leaving p^μ inv. Find

$$e_{11} \mp i e_{12} \longrightarrow \underset{\substack{\uparrow \\ \text{helicity } 2}}{e^{\pm 2 i\theta}} (e_{11} \mp i e_{12}) \quad !!$$

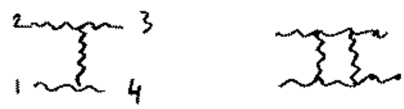
We have redone the little group analysis of reps of Poincaré!

$\Rightarrow$ gravitational waves

[Here pp. 33–36]

<u>Back to MHV of gravitons</u> Higher orders give couplings 

and one can evaluate diags like

 nonrenormalisable! coupling const $\sqrt{G} \sim \frac{1}{M_{Pl}}$

But tree diags can be evaluated! $\dim A_4 = \frac{1}{M_{Pl}^2}$ $\dim \langle 12\rangle^2 = 1$

$$A_4(1^- 2^- 3^+ 4^+) = 8\pi G \frac{\langle 12\rangle^8 \, [12]}{\underbrace{\langle 12\rangle\langle 13\rangle\langle 14\rangle \, \langle 23\rangle\langle 24\rangle \, \langle 34\rangle}_{\equiv N(4)} \cdot \langle 34\rangle}$$ ↙ note both ⟨⟩ and [ ]

$$A_5(1^- 2^- 3^+ 4^+ 5^+) = -4i\,(8\pi G)^{3/2} \frac{\langle 12\rangle^8}{N(5)} \underbrace{\varepsilon_{\mu\nu\alpha\beta}\, p_1^\mu p_2^\nu p_3^\alpha p_4^\beta}_{\frac{1}{4i}\left[[12]\langle 23\rangle[34]\langle 41\rangle - h.c.\right]}$$

$$\dim A_5 = \frac{1}{M_{Pl}^3} \cdot \frac{\langle 12\rangle^8}{\langle 12\rangle^9}\, p^4 = \frac{p}{\langle 12\rangle} = \frac{1}{p} \qquad \dim A_n = \mathrm{GeV}^{4-n}$$

Much is happening here, too!

Work out $\varepsilon_{\mu\nu}$ for $p^\mu = (E\ 0\ 0\ E)$:

Conditions: $p^\alpha \varepsilon_{\alpha\mu} = \frac{1}{2} p_\mu \varepsilon^\alpha{}_\alpha$

$$\Rightarrow E(\varepsilon_{0\mu} + \varepsilon_{3\mu}) = \frac{1}{2} p_\mu (\varepsilon_{00} - \varepsilon_{11} - \varepsilon_{22} - \varepsilon_{33})$$

$$\begin{array}{ll} \mu=0 & \varepsilon_{00} + \varepsilon_{30} = \frac{1}{2}(\varepsilon_{00} - \varepsilon_{ii}) \\ \mu = 1,2 & \varepsilon_{01} + \varepsilon_{31} = \varepsilon_{02} + \varepsilon_{32} = 0 \\ \mu = 3 & \varepsilon_{03} + \varepsilon_{33} = -\frac{1}{2}(\varepsilon_{00} - \varepsilon_{ii}) \end{array}$$

$$\varepsilon_{00} + \varepsilon_{03} = -\varepsilon_{03} - \varepsilon_{33}$$
$$\downarrow$$
$$\varepsilon_{00} + \varepsilon_{33} = -2\varepsilon_{03}$$

1st eq: $\varepsilon_{00} - \frac{1}{2}(\varepsilon_{00} + \varepsilon_{33}) = \frac{1}{2}(\varepsilon_{00} - \varepsilon_{33} - \varepsilon_{11} - \varepsilon_{22})$

$$\Rightarrow \varepsilon_{11} + \varepsilon_{22} = 0$$

4 eqs are, e.g. $\begin{cases} \varepsilon_{01} = -\varepsilon_{31} \quad \varepsilon_{02} = -\varepsilon_{32} \quad \varepsilon_{03} = -\frac{1}{2}(\varepsilon_{00} + \varepsilon_{33}) \\ \varepsilon_{22} = -\varepsilon_{11} \end{cases}$

But now do $\varepsilon_{\mu\nu} \to \varepsilon_{\mu\nu} + \eta_\mu p_\nu + \eta_\nu p_\mu$ to leave only ε_{11}, ε_{12}:

$$\begin{pmatrix} \varepsilon_{00} & -\varepsilon_{13} & -\varepsilon_{23} & -\frac{1}{2}(\varepsilon_{00} + \varepsilon_{33}) \\ & \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \text{symm} & & -\varepsilon_{11} & \varepsilon_{23} \\ & & & \varepsilon_{33} \end{pmatrix} \Longrightarrow \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \varepsilon_{11} & \varepsilon_{12} & 0 \\ 0 & \varepsilon_{12} & -\varepsilon_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{array}{lll} \varepsilon_{00} \to \varepsilon_{00} + 2\eta_0 E = 0 & \eta_0 = -\frac{\varepsilon_{00}}{2E} & \\ \varepsilon_{13} \to \varepsilon_{13} - \eta_1 E = 0 & \eta_1 = \frac{\varepsilon_{13}}{E} & \eta^\mu = -\frac{1}{E}\left(\frac{\varepsilon_{00}}{2}, \varepsilon_{13}, \varepsilon_{23}, \frac{\varepsilon_{33}}{2}\right) \\ \varepsilon_{23} \to \varepsilon_{23} - \eta_2 E = 0 & \eta_2 = \frac{\varepsilon_{23}}{E} & \\ \varepsilon_{33} \to \varepsilon_{33} - 2\eta_3 E = 0 & \eta_3 = \frac{\varepsilon_{33}}{2E}. & \end{array}$$

Two polarisation states

$$\varepsilon^{(1)}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \varepsilon^{(2)}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \sum_{r=1}^{2} \varepsilon^{(r)}_{\mu\nu} \varepsilon^{(r)}_{\alpha\beta} = \frac{1}{2}\left(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}\right)$$

Now rotate around 3-axis $\quad x^\mu \to x^{\mu\prime} = \Lambda^\mu{}_\nu x^\nu = R^\mu{}_\nu x^\nu$

$$\varepsilon_{\mu\nu} \to R_\mu{}^\alpha R_\nu{}^\beta \varepsilon_{\alpha\beta}$$

$$R_\mu{}^\alpha = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c\theta & s\theta & 0 \\ 0 & -s\theta & c\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\varepsilon'_{11} = R_1{}^\alpha R_1{}^\beta \varepsilon_{\alpha\beta} = R_1{}^1 R_1{}^1 \varepsilon_{11} + (R_1{}^1 R_1{}^2 + R_1{}^2 R_1{}^1)\varepsilon_{12} + R_1{}^2 R_1{}^2 \varepsilon_{22}$$

$$= (c^2\theta - s^2\theta)\,\varepsilon_{11} + \sin 2\theta\ \varepsilon_{12}$$

$$\varepsilon'_{12} = (R_1{}^1 R_2{}^1 - R_1{}^2 R_2{}^2)\,\varepsilon_{11} + (R_1{}^1 R_2{}^2 + R_1{}^2 R_2{}^1)\,\varepsilon_{12}$$

$$= -2c\theta\, s\theta\ \varepsilon_{11} + (c^2\theta - s^2\theta)\,\varepsilon_{12}$$

$$\begin{cases} \varepsilon'_{11} = \cos 2\theta\ \varepsilon_{11} + \sin 2\theta\ \varepsilon_{12} \\ \varepsilon'_{12} = -\sin 2\theta\ \varepsilon_{11} + \cos 2\theta\ \varepsilon_{12} \end{cases}$$

$$\varepsilon'_{11} - i\varepsilon'_{12} = e^{i2\theta}\,\varepsilon_{11} + \underbrace{(-i\cos 2\theta + \sin 2\theta)}_{-i e^{i2\theta}}\,\varepsilon_{12}$$

$$\varepsilon^{(+)} \quad \boxed{\varepsilon'_{11} - i\varepsilon'_{12} = e^{i2\theta}(\varepsilon_{11} - i\varepsilon_{12})}$$

The state $\quad (\varepsilon_{11} \mp i\varepsilon_{12})\, e^{-ipx}$ has helicity ± 2!

(there are also two $h=0$ comps $\varepsilon_{00}, \varepsilon_{33}$ and $h=\pm 1$ comps $\varepsilon_{31} \mp i\varepsilon_{23}$ which were gauge transformed away)

Graviton scattering

in quantised weak-field gravity

$$\text{Full:}\quad S = \int d^{d}x \left\{ -\frac{1}{16\pi G}\sqrt{|g|}\,\big(R - \underbrace{2\Lambda}_{\rho}\big) + \sqrt{g}\,\mathcal{L}_m(g_{\mu\nu}) \right\} \qquad d=4,\quad \tfrac{1}{2} g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi$$

related to

$$\delta\sqrt{g} = -\tfrac{1}{2}\sqrt{g}\, g_{\mu\nu}\,\delta g^{\mu\nu}$$

$$T_{\mu\nu} = \varepsilon_{vac}\, g_{\mu\nu}$$

$$\Rightarrow \frac{\delta S}{\delta g^{\mu\nu}} = -\int d^4x\, \frac{1}{16\pi G}\sqrt{g}\left\{ R_{\mu\nu} - \tfrac{1}{2}(R - 2\Lambda)g_{\mu\nu} + 8\pi G\left(g_{\mu\nu}\mathcal{L}_m - 2\frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}}\right)\right\}$$

$$\text{EOM:}\quad R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \underbrace{T_{\mu\nu}}_{2\frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}} - g_{\mu\nu}\mathcal{L}_m = -\frac{2}{\sqrt{g}}\frac{\delta S}{\delta g^{\mu\nu}}}$$

$$\left[\begin{array}{l} \mathcal{L}_m = 0 \qquad R_{\mu\nu} - \tfrac{1}{2}R g_{\mu\nu} = -\Lambda g_{\mu\nu} \equiv 8\pi G\,\varepsilon_{vac}\, g_{\mu\nu} \\ R - 2R + 4\Lambda = 0 \\ \Rightarrow R = 4\Lambda \Rightarrow R_{\mu\nu} = \Lambda g_{\mu\nu} \end{array}\right.$$

Weak field $\quad R = g^{\mu\nu}R_{\mu\nu}$

$$\left\{\begin{array}{l} g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \\ g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\alpha}h_\alpha{}^{\nu} + \dots \end{array}\right.$$

like $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\sqrt{-g} = e^{\frac{1}{2}\operatorname{Tr}\log g} = 1 + \kappa h - \tfrac{1}{4}\kappa^2 h_{\alpha\beta}h^{\alpha\beta} + \tfrac{\kappa^2}{8}h^2 + \dots$$

$$R_{\mu\alpha\nu\beta} = \underbrace{R_{[\mu\alpha][\nu\beta]}}_{\text{antis within pairs, symm in pairs}} = -\frac{\kappa}{2}\left[\partial_\mu\partial_\nu h_{\alpha\beta} - \partial_\mu\partial_\beta h_{\alpha\nu} - \partial_\alpha\partial_\nu h_{\mu\beta} + \partial_\alpha\partial_\beta h_{\mu\nu}\right] + \dots$$

$$R_{\mu\nu} = \eta^{\alpha\beta}R_{\mu\alpha\nu\beta} = \frac{\kappa}{2}\left[-\partial^2 h_{\mu\nu} + \partial_\mu\partial_\alpha h^\alpha{}_\nu + \partial_\nu\partial_\alpha h^\alpha{}_\mu - \partial_\mu\partial_\nu h\right] + \dots$$

$$R = \eta^{\mu\nu}R_{\mu\nu} = \kappa\left[-\partial^2 h + \partial_\mu\partial_\nu h^{\mu\nu}\right] + \dots$$

$$S = \int d^4x \left[\frac{1}{2} h^{\mu\nu}\, D^{-1}_{\mu\nu\alpha\beta}\, h^{\alpha\beta} + \kappa\,\mathcal{O}(h^3) + \kappa^2\mathcal{O}(h^4)\right]$$

$$\frac{1}{\mathrm{GeV}^4}\left[\mathrm{GeV}\quad \mathrm{GeV}^2\quad \mathrm{GeV} + \mathrm{GeV}^2\frac{1}{\mathrm{GeV}}\mathrm{GeV}^3 + \mathrm{GeV}^2\frac{1}{\mathrm{GeV}^2}\mathrm{GeV}^4\right.$$

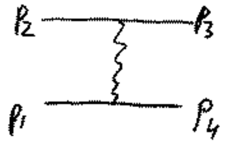
two momenta

For  and  see DeWitt PR 162 (1967) 1239
eqs (2.6) & (2.7)

$PP\eta\eta\eta$ $\quad$ $PP\eta\eta\eta\eta$ $\quad$ or Bjerrum-Bohr hep-th/0410097

171 terms $\quad$ 2850 terms $\quad$ eqs (4.10), (4.11)

Example (from DeWitt): scalar scattering

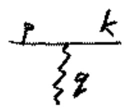


$$\cong i\delta^4(p)\left\{\frac{1}{2\sqrt{E_2E_3}}\left[p_2^\mu p_3^\nu + p_3^\mu p_2^\nu - \eta_{\mu\nu}(p_2\cdot p_3 + m^2)\right]\right.$$

$$\frac{1}{q^2}\left(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}\right)$$

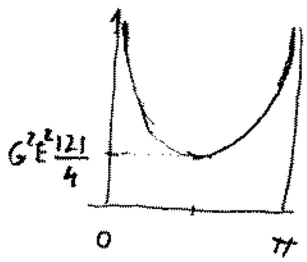
$$\left.\frac{1}{2\sqrt{E_1E_4}}\left[p_1^\alpha p_4^\beta + \cdots - \eta^{\alpha\beta}(\qquad)\right]\right\}$$

$$\int d^4x\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2\right)$$



$$T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}\mathcal{L} \Rightarrow p_\mu k_\nu + p_\nu k_\mu - \eta_{\mu\nu}(p\cdot k + m^2)$$

$$\frac{d\sigma}{d\Omega} \underset{\substack{E\gg m\\ \theta\approx 0}}{\cong} G^2E^2\,\frac{4}{\sin^4\frac{\theta}{2}} + \cdots$$



overall magnitude $\left(\frac{E}{M_{Pl}}\right)^2 \cdot \underbrace{\frac{1}{M_{Pl}^2}}$

$10^{-38}\,0.2\,\text{fm}^2$ = infinitesimal, of course

Gluonic processes $\left|\right.$  $+ \cdots$  $\left.\right|^2$ clearly exceedingly complicated, but the MHV amp on p. 32 is again very simple!

$\Rightarrow \frac{d\sigma}{d\Omega}$ as above