Exercise 10

General instructions: Follow these instructions, as they facilitate the revision of the exercises. The review takes into account that you always use the requested file names. Send only the files requested in the exercise. Return your answers to your assistant as an e–mail entitled TilaI,2017. If you have not programmed before, choose only one of the programming languages (**octave**/**python**), and do not change it during the course. If you are sure that you want to try both languages, you can of course do the exercises of both languages. However return the exercises to your assistant in one language only.

- **Exercise 10a:** The instructions apply for both **python** and **octave**.

  There are two images on the course homepage. The image `H10aPmalli.jpg` is made with **python**. The image `H10aOmalli.jpg` is made with **octave**. In both images is the periodogram $z(f_j)$ of the power spectrum described in the LaTeX document `H9bmalli.pdf` of the previous Exercise 9b.

  The periodogram $z(f_j)$ is computed for the time points t_i and observations $y_i = y(t_i)$ in the file `H7ainput.dat`. The first column of the file contains the values t_i and the second column the y_i values. The period range to be tested is $P_{min} = 1.0$ and $P_{max} = 10$.

  In the upper part of the images are plotted the observations y_i as a function of the time t_i.

  In the lower part of the images is presented the power spectrum of the observations. Calculate first the mean of the observations $m_y = [\sum y_i]/n$. Subtract the mean from the observations, and you will get $y'_i = y_i - m_y$. Compute the value of the power spectrum with the test frequency f_j using the formula

$$z(f_j) = \frac{\{\sum_{i=1}^{n} y'_i \cos[2\pi f_j(t_i - \tau)]\}^2}{2\sum_{i=1}^{n}\{\cos[2\pi f_j(t_i-\tau)]\}^2} + \frac{\{\sum_{i=1}^{n} y'_i \sin[2\pi f_j(t_i - \tau)]\}^2}{2\sum_{i=1}^{n}\{\sin[2\pi f_j(t_i-\tau)]\}^2},$$

  where τ fulfills

$$\tan(4\pi f_j \tau) = \left[\sum_{i=1}^{n} \sin(4\pi f_j t_i)\right] \left[\sum_{i=1}^{n} \cos(4\pi f_j t_i)\right]^{-1}$$

  The frequency range to be tested is between $f_{min} = 1/P_{max}$ and $f_{max} = 1/P_{min}$. The distance between two undependent frequencies is $f_0 = 1/\Delta T$, where $\Delta T = t_n - t_1$ i.e. the length of the whole observation range. Make the distance between the frequencies to be tested ten times denser than f_0 i.e. $f_{step} = f_0/\mathrm{OFAC}$, where $\mathrm{OFAC} = 10$ is constant. The number of frequencies that fit to the tested frequency range is

$$M = \mathrm{INT}[(f_{max} - f_{min})/f_{step}],$$

  where INT removes the decimal part of the argument (e.g.: $\mathrm{INT}[1.23] = 1$). Compute the value of the power spectrum $z(f_j)$ for all the following frequency values

$$f_j = f_{min} + j f_{step},$$

  where $j = 0, 1, 2, 3, ..., M$.

  Plot the power spectrum in the lower part of the image $z_j = z(f_j)$ as a function of the frequencies f_j. Mark in the plot the highest peak of the power spectrum $z(f_j)$ at the location $1/f_{best} = P_{best}$. The best achieved value P_{best} is the best period for these observations. Due to computational accuracy and rounding **python** ja **octave** programs can give slightly different values, e.g. $P_{best} = 1.91$ or 1.92.
  Hint: In Exercise **H6b** the values τ, $z_1(f_j)$ and $z_2(f_j)$ were computed for one frequency.

  Requirements for Exercise 10a
  Write a **python** program `H10avalmis.py`, that produces the image `H10aPvalmis.jpg`. The **content** of the image `H10aPvalmis.jpg` has to match as accurately as possible the image `H10aPmalli.jpg` on the course homepage. The program must not crash with the the command `python H10avalmis.py`.

  or

  Write an **octave** program `H10avalmis.m`, that produces the image `H10aOvalmis.jpg`. The **content** of the image `H10aOvalmis.jpg` has to match as accurately as possible the image `H10aOmalli.jpg` on the course homepage. The program must not crash with the the command `octave H10avalmis.m`.

- **Exercise 10b:** The instructions apply for both **python** and **octave**.

  There are two images on the course homepage. The image `H10bPmalli.jpg` is made with **python**. The image `H10bOmalli.jpg` is made with **octave**. In both images is the least squares fit of the observations described in the LaTeX document `H9bmalli.pdf` in the previous Exercise 9b.

  The time points t_i and observations $y_\mathrm{i} = y(t_\mathrm{i})$ are read from the file `H7ainput.dat`. The first column of the file contains the values t_i and the second column the values y_i.

  The period $P_\mathrm{best} = 1.91$ that best fits the observations corresponds to the frequency $f_\mathrm{best} = 1/P_\mathrm{best}$. The phases of the observations are with this period

  $$\phi_\mathrm{i} = \mathrm{FRAC}[(t_\mathrm{i} - t_0) f_\mathrm{best}],$$

  where $t_0 = 0$ and $\mathrm{FRAC}[x]$ removes the integer part of the argument x (e.g.: $\mathrm{FRAC}[21.34] = 0.34$). In the image are plotted the observations $y_\mathrm{i} = y(t_\mathrm{i}) = y(\phi_\mathrm{i})$ as a function of the phases ϕ_i.

  These observations are modeled with the least squares fit

  $$g(t, \bar{\beta}) = M + A\cos(2\pi\phi_\mathrm{i}) + B\sin(2\pi\phi_\mathrm{i}),$$

  where the free parameters are $\bar{\beta} = [M, A, B]$. The model given by the fit is presented in the image as a continuous curve. In the same image are given also the values for the free parameters M, A ja B.

  ### Requirements for Exercise 10b

  Write a **python** program `H10bvalmis.py`, that produces the image `H10bPvalmis.jpg`. The **content** of the image `H10bPvalmis.jpg` has to match as accurately as possible the image `H10bPmalli.jpg` on the course homepage. The program must not crash with the the command `python H10bvalmis.py`.

  or

  Write an **octave** program `H10bvalmis.m`, that produces the image `H10bOvalmis.jpg`. The **content** of the image `H10bOvalmis.jpg` has to match as accurately as possible the image `H10bOmalli.jpg` on the course homepage. The program must not crash with the the command `octave H10bvalmis.m`.

Turning in the exercises

Send to the course assistant an e–mail with the following attachments:
H10a: `H10avalmis.py` & `H10aPvalmis.jpg` or `H10avalmis.m` & `H10aOvalmis.jpg`
H10b: `H10bvalmis.py` & `H10bPvalmis.jpg` or `H10bvalmis.m` & `H10bOvalmis.jpg`