ExerciseCosineOne: Introduction

If the **linear** model $g(t)$ has the free parameters $\bar{\beta} = [A, B, C, ...]$, the error for $g(t)$ is

$$\sigma_{g(t)}^2 = \left[\frac{\partial g(t)}{\partial A}\right]^2 \sigma_A^2 + \left[\frac{\partial g(t)}{\partial B}\right]^2 \sigma_B^2 + \left[\frac{\partial g(t)}{\partial C}\right]^2 \sigma_C^2 + ... \qquad (1)$$

For example, the error of

$$g(t) = A + Bt$$

is obtained from

$$\frac{\partial g(t)}{\partial A} = 1, \qquad \frac{\partial g(t)}{\partial B} = t$$

$$\begin{aligned} \sigma_{g(t)} &= \sqrt{\left(\frac{\partial g(t)}{\partial A}\right)^2 \sigma_A^2 + \left(\frac{\partial g(t)}{\partial B}\right)^2 t^2 \sigma_B^2} \\ &= \sqrt{1^2\sigma_A^2 + t^2\sigma_B^2} \\ &= \sqrt{\sigma_A^2 + t^2\sigma_B^2} \end{aligned}$$

Note that this $\sigma_{g(t)}$ error depends on the argument t.

ExerciseCosineOne: Problem

The model is

$$g(t) = M + B\cos t + C\sin t,$$

where $\bar{\beta} = [M, B, C]$ are the free parameters having errors $\sigma_{\bar{\beta}} = [\sigma_M, \sigma_B, \sigma_C]$. The argument units are $[t] =$ radians.

(a) The peak to peak amplitude A of this model is the difference between the maximum value $g_{\max}$ of $g(t)$ and the minimum value $g_{\min}$ of $g(t)$.
Solve $A \pm \sigma_A$ from the given known $B \pm \sigma_B$ and $C \pm \sigma_C$ values.

(b) The primary minimum $t_{\min}$ of this model fulfils $g(t_{\min}) = g_{\min}$.
Solve $t_{\min} \pm \sigma_{t_{\min}}$ from the given known $B \pm \sigma_B$ and $C \pm \sigma_C$ values.

Your A, σ_A, $t_{\min}$ and $\sigma_{t_{\min}}$ solutions can contain **only** parameters B, σ_B, C and σ_C.

Send your full solution to the assistant via email. Your solution can be a scanned handwritten pdf-file, or even better, a pdf-file compiled from a latex-file.