Binaries: Exercise 1.

A third body causes periodic changes in the observed (O) minus the computed (C) epochs of an eclipsing binary. The mass function of this light-time effect fulfills

$$\frac{(m_3 \sin i)^3}{[m_1(1+q)+m_3]^2} = \frac{(173.15\ a)^3}{p_3^2}, \tag{1}$$

where the parameters are

$i =$ inclination of the orbital plane of the third body

$m_1 =$ mass of primary of eclipsing binary (units of solar mass $[m_\odot]$)

$q = m_2/m_1 =$ dimensionless mass ratio of secondary and primary of eclipsing binary

$m_3 =$ mass of third body (units of solar mass $[m_\odot]$)

$a = A/2$ is half of the peak the peak amplitude A of O-C modulation caused by the third body (units of days [d])

$p_3 =$ period of O-C modulations caused by the third body (units of years [y])

The period $p_3 = 19180$ days was detected from the O-C changes of Algol ($m_1 = 3.7m_\odot$ and $m_2 = 0.8m_\odot$). The peak to peak amplitude of these regular O-C changes was $A = 0.035$ days. Compute the lower limit for the mass m_3 of the third body by assuming that the inclination of orbital plane of this third body is $i = 90^{\rm o}$. Give the result using an accuracy of two decimals, like $m_3 = 0.12m_\odot$.

The easiest solution is iterative. It proceeds through the following stages.

1. Compute the numerical value on the right side of Eq. 1.
2. Insert the known values of m_1, q and i into the left side of Eq. 1.
3. Test all m_3 values between $0.10m_\odot$ and $2.00m_\odot$. Use a step of $0.01m_\odot$ to achieve the desired accuracy. For the correct m_3 value, the difference between the left and right side of Eq. 1 is closest to zero.
4. A short computer code is the fastest way to do this.