Univalency and Integral Transforms

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Abstract

Let $\mathcal{U}(\lambda)$ denote the class of all analytic functions f in the unit disk Δ of the form $f(z) = z + a_2 z^2 + \cdots$ satisfying the condition

$$\left| f'(z) \left(\frac{z}{f(z)} \right)^2 - 1 \right| \leq \lambda, \quad z \in \Delta.$$

We begin our discussion in finding conditions on λ and on $c \in \mathbb{C}$, with $\operatorname{Re} c \geq 0 \neq c$, such that for each $f \in \mathcal{U}(\lambda)$ satisfying

$$(z/f(z)) * F(1, c; c+1; z) \neq 0 \quad \text{for all } z \in \Delta,$$

the transform

$$G(z) = G_f^c(z) = \frac{z}{(z/f(z)) * F(1, c; c+1; z)}, \quad z \in \Delta,$$

is univalent or starlike. Here $F(a, b; c; z)$ denotes the Gauss hypergeometric function and $*$ denotes the convolution (or Hadamard product) of analytic functions on Δ. Also, we provide a more general extension of this result along with a number of recent development in the class $\mathcal{U}(\lambda)$.