

Postcritically-finite maps on the sphere

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Abstract

A continuous map $f: S^2 \rightarrow S^2$ on the 2-sphere S^2 is called topologically holomorphic if near each point it can be written as $z \mapsto z^n$ for some $n \in \mathbb{N}$ in suitable local coordinates in domain and image. The critical set of f is the set of all points where $n \geq 2$. The postcritical set is the forward orbit of the critical set under iteration of f . A topologically holomorphic map $f: S^2 \rightarrow S^2$ is called postcritically finite if it has a finite postcritical set.

Thurston studied postcritically-finite maps and gave a necessary and sufficient condition for such a map to be equivalent (in a suitable sense) to a rational map. There is a close connection of this equivalence problem to the question when a metric 2-sphere is quasimetrically equivalent to the standard 2-sphere.

In my talk I will give a survey on this subject and discuss some recent joint work with D. Meyer.