Generalized Differentiation and Bi-Lipschitz Non-embedding

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Abstract

This talk will concern joint work with Bruce Kleiner on generalized differentiation for Lipschitz maps, $f : X \to L^1$, where (X, μ) is a metric measure space satisfying a doubling condition and a Poincaré inequality. For such L^1 targets, differentiability need not hold in the classical sense. We establish a new connection with sets of finite perimeter, which we use to show that for certain domains such as the Heisenberg group, $\mathbb{H}$, with its Carnot-Caratheodory metric, differentiability can be restored if understood in a generalized sense. This leads to a bi-Lipschitz non-embedding theorem for $\mathbb{H}$ in L^1. There is a second proof of this non-embedding result which depends on metric differentiation and the classification of what we call monotone subsets. In joint work with Kleiner and Assaf Naor, the non-embedding result is made quantitative. Thus, we show that a 1-Lipschitz map, $f : \mathbb{H} \to L^1$, must exhibit a definite amount of compression above an explicitly estimable scale. The argument involves combining both of the approaches mentioned above.