# Curvature and Dynamics

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**Abstract**

Most of us have seen a bit of the Fatou/Julia theory of iteration of rational mappings of the Riemann sphere and pictures of associated parameter spaces such as the Mandelbrot set. A natural question is "are there such rational (conformal) dynamical systems on manifolds in higher dimensions". Classical rigidity theorems (eg the Liouville theorem from 1860) suggests strongly that there are not. Surprisingly there are, but we must give up smooth Riemannian structures to allow singular structures where branching of mappings may occur and quite fascinating examples can be found. The Lichnerowicz problem asks us to classify those (closed) manifolds which admit a rational (conformal away from singular set) endomorphism. For injective mappings this problem was solved in the 70's : only the sphere admits a noncompact family of conformal self maps. For rational mappings the situation is more complicated. Using ideas from Sela's proof of the Hopf property for Gromov hyperbolic groups and old results of Walsh and Smale on open mappings we can prove quite strong rigidity theorems (preventing branching) for open self mappings of negatively curved spaces since we prove the fundamental groups of such spaces are virtually Hopf (self homomorphism with image of finite index is an isomorphism). Recent work also identifies those knot groups which are virtually Hopf and therefore identifies those knot complements which admit a proper open self map which is not homotopic to a homeomorphism. This is a general talk aimed at a broad audience and represents joint work largely with Martin Bridson (Imperial), Jonathan Hillman (Sydney), Volker Mayer (Lille) and Kirsi Peltonen (Helsinki).