A pattern in algebra and geometry illustrated by all the moduli spaces of finite type Riemann surfaces taken together

Dennis Sullivan

Abstract

All of the moduli spaces of Riemann surfaces of finite type when taken together form a single algebraic structure. This is so because the frontier at infinity of any one moduli space can be described in terms of gluing operations applied to earlier moduli spaces in a natural partial order on the system of all moduli spaces. This pattern appears in other contexts like configuration spaces of distinct points in a fixed 3-manifold or the moduli spaces of connections on a bundle over a Riemannian four manifold with the minimal possible energy (square integral of curvature). Another rich supply of examples comes from the moduli spaces of pseudo holomorphic curves in a given homology class of a symplectic manifold provided with a compatible almost complex structure. Our work is motivated by the synthesis understood by physicists who see all of the above examples as unified by the mathematically undefined concepts of quantum field theory and string theory. Just for the above examples which the physicists call topological theories because there are interesting correlations which remain unchanged when the background structures used in the definitions are continuously varied, we try to derive a description of the rigid information in these geometric examples in terms of algebra based on algebraic topology.