Euclidean Quasiconvexity

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Abstract

A metric space is quasiconvex provided it is bilipschitz equivalent to a length space: each pair of points can be joined by a rectifiable path whose length is comparable to the distance between its endpoints.

We consider a closed set in Euclidean n-space and ask when is its complement quasiconvex. In dimension $n = 2$, a complete description is available, at least for closed sets with finitely many components. In general, there are sufficient conditions that such a complement be quasiconvex; one such condition is that the set have zero $(n-1)$-dimensional Hausdorff measure.

For each $d \in [n-1, n]$, there is a compact totally disconnected set with positive finite Hausdorff d-measure whose complement is quasiconvex. On the other hand, we construct a compact totally disconnected set with non-zero Hausdorff $(n-1)$-measure whose complement fails to be quasiconvex. This is joint work with Hrant Hakobyan.