

Positivity of the universal pairing in 3 dimensions, and the topological Cauchy-Schwarz inequality

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Abstract

Fix a topological surface S , and let V be the complex vector space spanned by all 3-manifolds which bound S . There is a Hermitian pairing on V , with values in the complex vector space spanned by all closed 3-manifolds. The main result is that this pairing is nondegenerate: if $\langle v, v \rangle = 0$ then $v = 0$. The proof involves the construction of a suitable complexity function c on all closed 3-manifolds so that if A and B are two 3-manifolds which bound S , there is an inequality

$$c(AB) \leq \max(c(AA), c(BB))$$

with equality if and only if $A = B$. We discuss some details of the construction of the function c , which involves input ranging from finite group TQFT's to Perelman's recent proof of the geometrization conjecture.

This is joint work with Mike Freedman and Kevin Walker.