Quantum Correlations, Information and Entropy

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And

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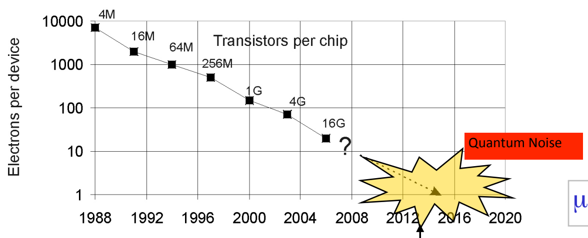
emergence of quantum information technology



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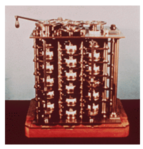

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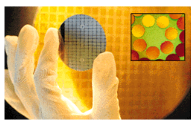

Quantum technology

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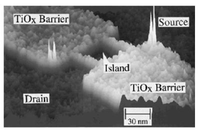

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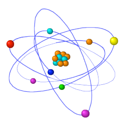

*quantum registers
*new atomic clocks
*quantum communications

EPSRC

QIRC

Computation = physical process

Hardware obeys the laws of physics-
but nature is quantum mechanical

So what would a quantum computer look like?

"Computers of the future may weigh no more than 1.5 tons"

Popular mechanics, 1949!

EPSRC
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The qubit.

- A single two-state system can store a single bit in computational basis.

$$\frac{1}{2}(|1\rangle\langle 1| + |0\rangle\langle 0|)$$

- Superpositions are allowed
  - the qubit.

$$\frac{1}{\sqrt{2}}(|1\rangle \pm |0\rangle)$$

Entanglement

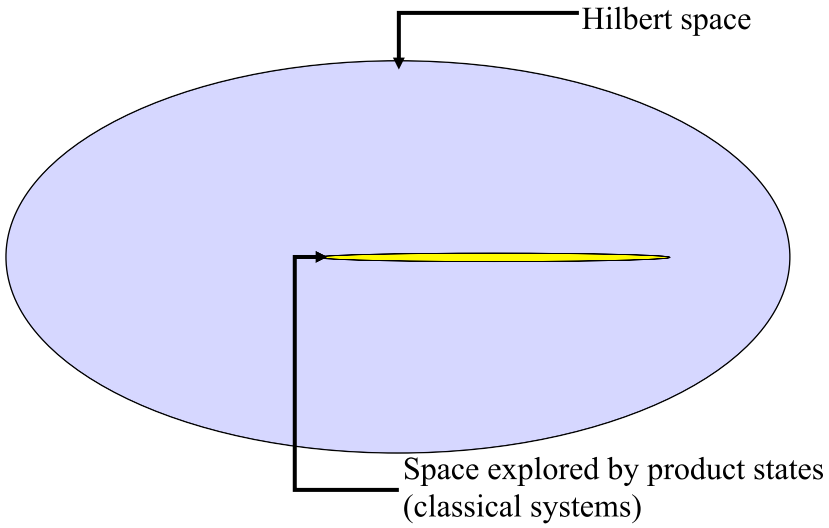
- Superpositions:

$$|\psi\rangle = |\uparrow\rangle + |\downarrow\rangle$$

- Superposed correlations:

$$|\psi\rangle = |\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2$$

- Entanglement
- (pure state) $|\psi\rangle \neq |\psi\rangle_1 \otimes |\psi\rangle_2$



Quantum system explores whole Hilbert space ⟶ more parameters.

Opportunity ←→ Challenge

Quantum parallelism.

- Code binary string for input as an integer.

$$k = S_1 2^0 + S_2 2^1 + \ldots + S_N 2^{N-1} \; : \; (k = 0,1 \ldots 2^{N-1})$$

- Quantum Turing Machine.

$$f : \; |k\rangle_{input} \otimes |0\rangle_{output} \rightarrow \; |k\rangle_{input} \otimes |f(k)\rangle_{output}$$

- Quantum parallelism

$$f : \sum_{k=0}^{2^{N-1}} |k\rangle_{input} \otimes |0\rangle_{output} \rightarrow \sum_{k=0}^{2^{N-1}} |k\rangle_{input} \otimes |f(k)\rangle_{output}$$

INITIAL IDEAS - QUANTUM MORE POWERFUL THAN CLASSICAL
BENIOFF (82), FEYNMAN (84), DEUTSCH (85)

QUANTUM PARALLELISM - ORACLES, HADAMARDS...
DEUTSCH-JOZSA (92)/ BERNSTEIN-VAZIRANI (93) / SIMON (93), EKERT

QUANTUM FACTORING AND SEARCHING- EXPLOSION OF INTEREST
SHOR (94), GROVER (95) - AND RECENTLY ON QUANTUM WALKS

PHYSICAL IMPLEMENTATIONS- HARDWARE, GATES, DECOHERENCE
CIRAC-ZOLLER (94), WINELAND, KIMBLE, HAROCHE, BLATT, STEANE, HINDS, RARITY, O' BRIEN....

ERROR CORRECTION- THE CONQUEST OF DECOHERENCE
SHOR (95), STEANE (96)

EPSRC Engineering and Physical Sciences Research Council

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Separability

Separable states (with respect to the subsystems A, B, C, D, …)

$$\rho = \sum_i p_i \, \rho_A^i \otimes \rho_B^i \otimes \rho_C^i \otimes \rho_D^i \otimes \ldots$$

Everything else is entangled

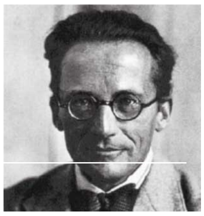
e.g. $$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Entangle and Imperial?

- “Is it local trouble?
- Lets just say we’d like to avoid any Imperial entanglements.“
- *Dialogue between Han Solo and Ben (Obi-Wan) Kenobi, Star Wars Episode IV-A New Hope*

History

Schrödinger coined the term "entanglement" in 1935



- "When two systems, …… enter into temporary physical interaction due to known forces between them, and …… separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [the quantum states] have become entangled."

- Schrödinger (Cambridge Philosophical Society)

- for bipartite systems there are four important basis states
  - The Bell states

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right)$$

$$|\Psi_+\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2\right)$$

$$|\Phi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\uparrow\rangle_2 - |\downarrow\rangle_1|\downarrow\rangle_2\right)$$

$$|\Phi_+\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\uparrow\rangle_2 + |\downarrow\rangle_1|\downarrow\rangle_2\right)$$

  - Braunstein et al. PRL 68, 3259 (1992)
    - States maximally violating Bell's inequality

Bell

12

Entanglement for pure states

- For "Quantum businesses" such as
  - — Quantum Computation
  - — Quantum Teleportation
  - — Quantum Cryptography ....

  quantum mechanical entanglement is a key issue.

- Entangled states are the states that can not be written in the product of states

$$|\phi_A\rangle \otimes |\phi_B\rangle \otimes \cdots \otimes |\phi_Z\rangle$$

i.e. inseparable

- Maximally Entangled States (MES):

  For two spin 1/2 particles (Bell states)

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle \pm |1\rangle|1\rangle),$$
$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle \pm |1\rangle|0\rangle).$$

For many spin 1/2 particles, for example,

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle|0\rangle\cdots|0\rangle \pm |1\rangle|1\rangle|1\rangle\cdots|1\rangle)$$

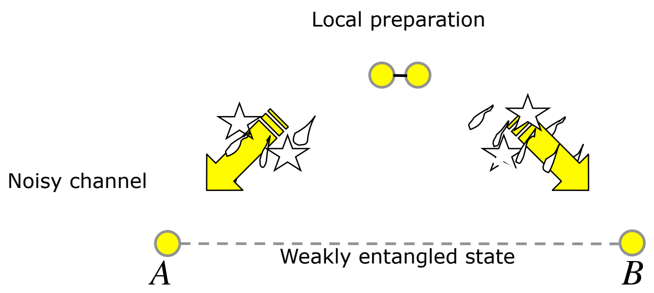
(GHZ state for three particles)

13

Imperial College London

purity. . . and the reality

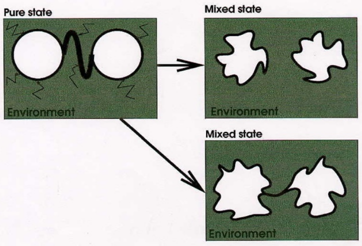
We need a pretty pure nonclassical states as a resource for quantum information processing. Decoherence will degrade entanglement......



Can we quantify the degree of entanglement?
Can Alice and Bob 'repair' the damaged entanglement?
Can we purify/distil impure states to improve the resource?

Entanglement of Mixed States

- Effects of the environment cause decoherence...



- Our definition of entanglement of mixed States:

  The density matrix of a mixed state ρ is disentangled

  — if it can be written as $\rho = \sum p_i \rho_i^A \otimes \rho_i^B \otimes \cdots \otimes \rho_i^B$

  — If it can not be purified by local operations and classical communications

  $\Rightarrow$ Otherwise, ρ contained some entanglement.

- Entanglement measures for mixed states:

  — Entanglement of Formation

  — Horodecki's measure for two particle entanglement

  — Relative entropy of entanglement

15

Entanglement

$$\rho = \sum_{ijkl} p_{kl}^{ij} |i\rangle\langle j| \otimes |k\rangle\langle l|$$

- If a system of two (or more) particles is not represented by a weighted sum of product states, the particles are said to be entangled: $\rho \neq \sum_i p_i \rho_a(i) \otimes \rho_b(i)$
- Peres criterion [A. Peres, Phys.Rev.Lett. 77, 1413 (1996)] If the partial transposition of its density matrix

$$\rho^{T_B} := I \otimes T(\rho) = \sum_{ijkl} p_{kl}^{ij} |i\rangle\langle j| \otimes (|k\rangle\langle l|)^T = \sum_{ijkl} p_{kl}^{ij} |i\rangle\langle j| \otimes |l\rangle\langle k|$$

- has a negative eigenvalue, the state is said to be entangled.
- For example, $\frac{1}{\sqrt{2}}\left(|1\rangle|0\rangle + |0\rangle|1\rangle\right)$
- correlation and nonlocality?

A state ρ is disentangled (separable) if it is of the form

$$\rho = \sum p_i \rho_A^i \otimes \rho_B^i$$

where $\sum p_i = 1$ and $p_i \geq 0$.

Example of maximally entangled states are Bell states

$$|\Psi^\pm\rangle = |01\rangle \pm |10\rangle$$
$$|\Phi^\pm\rangle = |00\rangle \pm |11\rangle$$

which all violate the standard Bell inequalities.

But what about the Werner States?

$$\rho = F|\Psi^-\rangle\langle\Psi^-| + \frac{1-F}{3}(|\Psi^+\rangle\langle\Psi^+| + |\Phi^-\rangle\langle\Phi^-| + |\Phi^-\rangle\langle\Phi^-|)$$

- $F > 1/2$ states violate Bell's inequality.
- $F \leq 1/2$ states don't violate Bell's inequality, **but** can be purified to a state that does.

Central Question:

Which states contain entanglement and how much?

17

- **Conditions for a Measure of Entanglement**

E1. $E(\sigma) = 0$ iff σ is separable.

E2. Invariance under local unitary operations , i.e.

$$E(\sigma) = E(U_A \otimes U_B \, \sigma \, U_A^\dagger \otimes U_B^\dagger).$$

E3. The measure of entanglement $E(\sigma)$ cannot increase under LGM+CC given by $\sum V_i^\dagger V_i = I$, i.e.

$$\sum tr(\sigma_i) \; E(\sigma_i/tr(\sigma_i)) \leq E(\sigma),$$

where $\sigma_i = V_i \sigma V_i^\dagger$.

The origin of the conditions:

1) Separable states are disentangled,

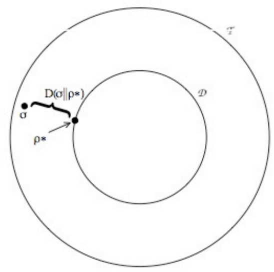
2) Local unitary transformations represent a local change of basis only and leave quantum correlations unchanged.

3) Any increase in correlations due to a purification protocol should be classical in nature and therefore entanglement should not be increased.

18

Entanglement measures

Vedral, Plenio, Rippin, Knight, Phys. Rev. Lett. **78**, 2275 (1997)
Vedral, Plenio, Phys. Rev. A **57**, February (1998)



Define measure of entanglement as:

$$E(\sigma) := \min_{\rho \in \mathcal{D}} \; D(\sigma || \rho)$$

Use the relative entropy as a measure of distance

$$D(\sigma||\rho) = tr\{\sigma \ln \sigma - \sigma \ln \rho\}$$

⇒ Relative entropy of entanglement.

19

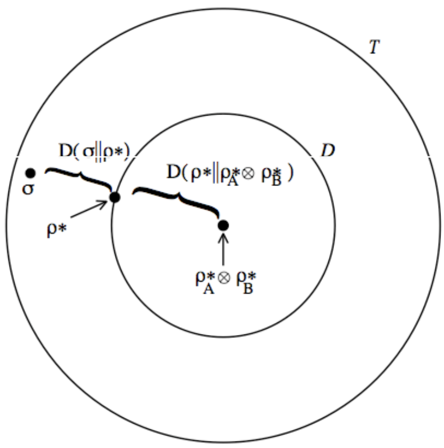


Figure 2.1: The set of all density matrices, $\mathcal{T}$ is represented by the outer circle. Its subset, a set of disentangled states $\mathcal{D}$ is represented by the inner circle. A state σ belongs to the entangled states, and ρ^* is the disentangled state that minimizes the distance $D(\sigma||\rho)$, thus representing the amount of quantum correlations in σ. State $\rho^*_A \otimes \rho^*_B$ is obtained by tracing ρ^* over A and B. $D(\rho^*||\rho^*_A \otimes \rho^*_B)$ represent the classical part of the correlations in the state σ.

20

Measures of entanglement: Schmidt decomposition- Ekert & PLK Amer J Phys 63, 415 (1995)

Bipartite pure states:

$$|\psi_{AB}\rangle = \sum_i \alpha_i |\psi_{A,i}\rangle |\phi_{B,i}\rangle$$

Schmidt decomposition

Positive, real coefficients

$$\sum_i \alpha_i^2 = 1$$

Entangled quantum systems and

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(Received 20 June 1994; accepted 31 Octob

Quantum systems comprised of interactin individual identities become entangled. Thi decomposition, in which a pair of preferred o tight correlations between two quantum subs can be exploited to shed new light on enta attention to two-mode squeezed states and to

[3]The original reference is E.Schmidt, "Zur Theorie der linearen und nichtlinearen Integralgleichungen," Math. Annalen **63**, 433–476 (1906), in the context of quantum theory see H. Everett III, "The theory of the universal wave function," in *The Many-World Interpretation of Quantum Mechanics*, edited by B. S. DeWitt and N. Graham (Princeton University, Princeton, 1973), pp. 3–140, and H.Everett III, " 'Relative state' formulation of quantum mechanics," Rev. Mod. Phys. **29**, 454–462 (1957). A graduate level textbook by A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer, Dordrecht, 1993), Chap. 5 includes a brief description of the Schmidt decomposition. Modern application of the Schmidt decomposition can be found in H. Huang and J. H. Eberly, "Correlations and one-quantum pulse shapes in photon pair generation," J. Mod. Opt. **40**, 915–930 (1993); S. M. Barnett and S. J. D. Phoenix, "Information theory, squeezing and quantum correlations," Phys. Rev. A **44**, 535–545 (1991); S. M. Barnett and S. J. D. Phoenix, "Bell's inequality and the Schmidt decomposition," Phys. Lett. A **167**, 233–237 (1992), and in Ref. 7.

Measures of entanglement

Bipartite pure states:

$$|\psi_{AB}\rangle = \sum_i \alpha_i |\psi_{A,i}\rangle |\phi_{B,i}\rangle$$

Schmidt decomposition

Positive, real coefficients

Reduced density operators

$$\rho_A = \mathrm{tr}_B(\rho_{AB}) = \sum_i \alpha_i^2 |\psi_{A,i}\rangle\langle\psi_{A,i}|$$

$$\rho_B = \mathrm{tr}_A(\rho_{AB}) = \sum_i \alpha_i^2 |\phi_{B,i}\rangle\langle\phi_{B,i}|$$

Same coefficients

Measure of mixedness

Unique measure of entanglement (Entropy)

$$S(\rho_A) = S(\rho_B) = -\sum_i \alpha_i^2 \log(\alpha_i^2)$$

Phoenix & Knight, Annals of Physics (N.Y.) 186, 381 (1988)

Example

- Consider the Bell state: $|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

This can be written as:

$$
\begin{aligned}
|\psi\rangle &= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right] \\
&+ \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\frac{1}{\sqrt{2}}(-|0\rangle + |1\rangle)\right]
\end{aligned}
$$

So $\alpha_1 = \alpha_2 = \frac{1}{\sqrt{2}}$ and $S = -\frac{1}{2}\log\left(\frac{1}{2}\right) - \frac{1}{2}\log\left(\frac{1}{2}\right) = \log(2)$

Maximally entangled (S is maximised for two qubits)

“Monogamy of entanglement”

Measures of entanglement

Bipartite mixed states:

- Average over pure state entanglement that makes up the mixture
- Problem: infinitely many decompositions and each leads to a different entanglement
- Solution: Must take minimum over all decompositions (e.g. if a decomposition gives zero, it can be created locally and so is not entangled)

Entanglement of formation

$$E_F(\rho) = \min \sum_i p_i \, S(\rho_{A,i})$$

$S(\rho_A)$ von Neumann entropy

Minimum over all realisations of: $\rho_{AB} = \sum_j p_j \, |\psi_j\rangle\langle\psi_j|$

measures

Conditions for Measures of Entanglement [5]

E1. $E(\sigma) = 0$ iff σ is disentangled.

E2. Invariance under local unitary operations, i.e.

$$E(\sigma) = E(U_A \otimes U_B \, \sigma \, U_A^\dagger \otimes U_B^\dagger).$$

E3. The expected entanglement can not be increased with LGM+CC+SS. That is

$$\sum tr(\sigma_i) \; E(\frac{\sigma_i}{tr(\sigma_i)}) \leq E(\sigma),$$

where $\sigma_i = A_i \otimes B_i \sigma A_i^\dagger \otimes B_i^\dagger$.

The origin of the conditions:

2) Local unitary transformations represent a local change of basis only and leave quantum correlations unchanged.

3) Any increase in correlations due to a purification protocol should be classical in nature and therefore entanglement should not be increased.

[5] V. Vedral, M.B. Plenio, M.A. Rippin, and P.L. Knight, Phys. Rev. Lett. **78**, 2275 (1997)

25

entropy

- Boltzmann generalization

$$S = -\mathrm{Tr}\{\rho \ln \rho\}$$

- density operator ρ analogous to classical density of points in phase space

$$\rho = \sum_k p_k |k\rangle\langle k|$$

p_k: probability of $|k\rangle$ in the 'mixture'
  - ensemble of similar systems
  - additional uncertainty over and above those required by q.m.

- pure state $\rho = |k\rangle\langle k|$ — only uncertainties from q.m.
  - pure state $S = 0$
  - mixed state $S > 0$

. measure disorder above that inherent in q.m.

- dynamics?

entropy and dynamics

- $\dot{\rho}$ governed by unitary time evolution, so eigenvalues of ρ are constant; also true for any $f(\rho)$

  → S = constant

- but subsystems: reduced density operator, trace over parts to be ignored

$$\rho_{A[B]} = \mathrm{Tr}_{B[A]}\{\rho\}$$

no longer governed by unitary time evolution

$$S(\rho_{A[B]}) = -\mathrm{Tr}_{A[B]}\{\rho_{A[B]} \ln \rho_{A[B]}\} \quad \text{t-dependent}$$

- non-additive

$$S \neq S_A + S_B$$

— trace has thrown away A, B correlations

Araki-Lieb theorem

$$|S(\rho_A) - S(\rho_B)| \leq S \leq S(\rho_A) + S(\rho_B)$$

- if whole system prepared in a pure state, $S = 0$

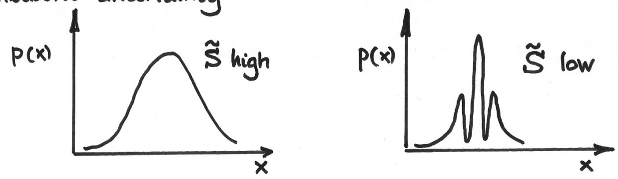
  → $S(\rho_A) = S(\rho_B)$

- consequence of Schmidt...

Shannon entropy

$$\tilde{S} \equiv -\sum_i p_i \ln p_i$$

- measures uncertainty in a distribution



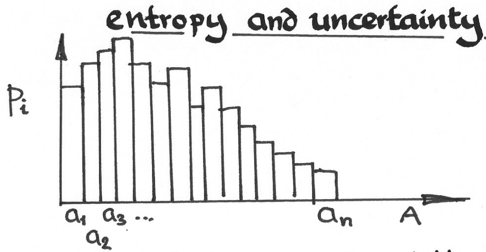
- defined with respect to an observable
- measure of fluctuations in that observable: basis dependent

$$\tilde{S}\left(\rho_{A[B]} ; \hat{O}_{A[B]}\right) = -\sum_{\alpha[\beta]} \left\{ \left(\rho_{A[B]}\right)_{\alpha\alpha[\beta\beta]} \times \ln \left(\rho_{A[B]}\right)_{\alpha\alpha[\beta\beta]} \right\}$$

$$\hat{O}_{A[B]} \, |\alpha[\beta]\rangle = \alpha[\beta] \, |\alpha[\beta]\rangle$$

$$\left(\rho_{A[B]}\right)_{\alpha\alpha[\beta\beta]} = \langle \alpha[\beta] | \rho_{A[B]} | \alpha[\beta] \rangle$$

entropy and uncertainty



probability distribution of variable A

$$S = -k \sum_i p_i \log p_i$$

$S_{max} = \log N$ for $p_i = 1/N$

$S_{min} = 0$ for $p_{i=k} = 1, p_{i \neq k} = 0$

$S_{initial} - S_{final}$ = information gained

entropy and quantum mechanics

- von Neumann

$$S(\hat{\rho}) = -k \, \mathrm{Tr} \, \hat{\rho} \log \hat{\rho}$$

depends only on quantum state $\hat{\rho}$

$$\underset{\rho}{[\text{full matrix}]} \rightarrow \underset{\rho}{[\text{diagonal}]} \rightarrow -k \sum_i p_i \log p_i$$

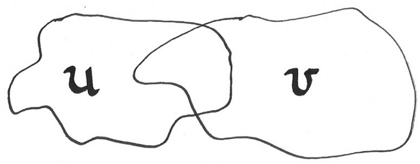
- Shannon

density operator in the basis of eigenvectors of a given observable $\hat{A}$

$$\underset{|u_i\rangle}{\overset{\rho}{[\;\cdot\;]}} \rightarrow \overset{\rho'}{[\text{diagonal}]} \rightarrow -k \sum_i q_i \log q_i$$

$$S(A_\mu) = -k \, \mathrm{Tr} \, \rho \log \rho'$$

Entanglement

$|u_i\rangle$  $|v_j\rangle$

$$|\psi\rangle = \sum_{ij} c_{ij} \, |u_i\rangle \otimes |v_j\rangle$$

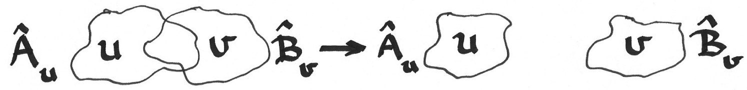
- subsystem density operator

$$|\psi\rangle = c_1 |u_1\rangle \otimes |v_1\rangle + c_2 |u_2\rangle \otimes |v_2\rangle$$
$$\neq |\psi_u\rangle \otimes |\psi_v\rangle$$

- in general $\hat{\rho} \neq \rho_u \otimes \rho_v$: u and v are entangled

Correlations

past connectivity



- two observables $\hat{A}_u$ and $\hat{B}_v$ are correlated for a given quantum state $\hat{\rho}$ if

$$\langle \hat{A}_u \otimes \hat{B}_v \rangle \neq \langle \hat{A}_u \rangle \langle \hat{B}_v \rangle$$

$$\mathrm{Tr}[\hat{\rho}\,\hat{A}_u \otimes \hat{B}_v] \neq (\mathrm{Tr}\,\hat{\rho}_u \hat{A}_u)(\mathrm{Tr}\,\hat{\rho}_v B_v)$$

Entanglement or Correlation?

- no entanglement, no correlation

$$\rho = \rho_u \otimes \rho_v \rightarrow \mathrm{Tr}[\rho\,\hat{A}_u \otimes \hat{B}_v] = \mathrm{Tr}[\rho_u \otimes \rho_v \cdot \hat{A}_u \otimes \hat{A}_v]$$
$$= \mathrm{Tr}[\rho_u \cdot \hat{A}_u \otimes \rho_v \cdot \hat{B}_v] = (\mathrm{Tr}_u\,\rho_u \hat{A}_u).(\mathrm{Tr}_v\,\rho_v \hat{B}_v$$

Entanglement
↙ no correlation ↘ correlation

eg: the EPR state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle)$$

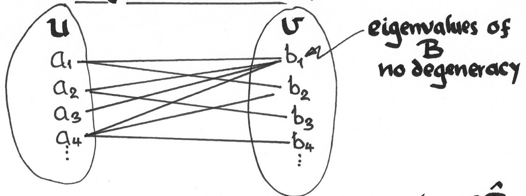
observables σ_u^x and σ_v^x <u>are</u> correlated

but σ_u^x and σ_v^y are <u>not</u>

correlations → entanglement

entanglement ↛ correlations

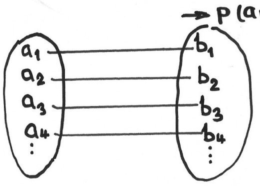
Perfect Correlation



Can you predict value of $\hat{A}_u$ given value of $\hat{B}_v$?

✓ w. probability $p(a_i|b_j) = \dfrac{p(a_i, b_j)}{p(b_j)}$

$$p(a_i|b_j) = 1 \rightarrow p(a_i, b_j) = p(b_j) = \sum_k p(a_k, p_j)$$
$$\rightarrow p(a_k, b_j) = \delta_{ik}$$



States for Perfect Correlations

- to have perfect correlations between A_u and B_v total system should be in state

$$\hat{\rho} = \sum_{ik} \rho_{ik} |u_i\rangle\langle u_k| \otimes |v_i\rangle\langle v_k|$$

$$A_u|u_i\rangle = a_i|u_i\rangle \qquad B_v|v_k\rangle = b_v|v_k\rangle$$

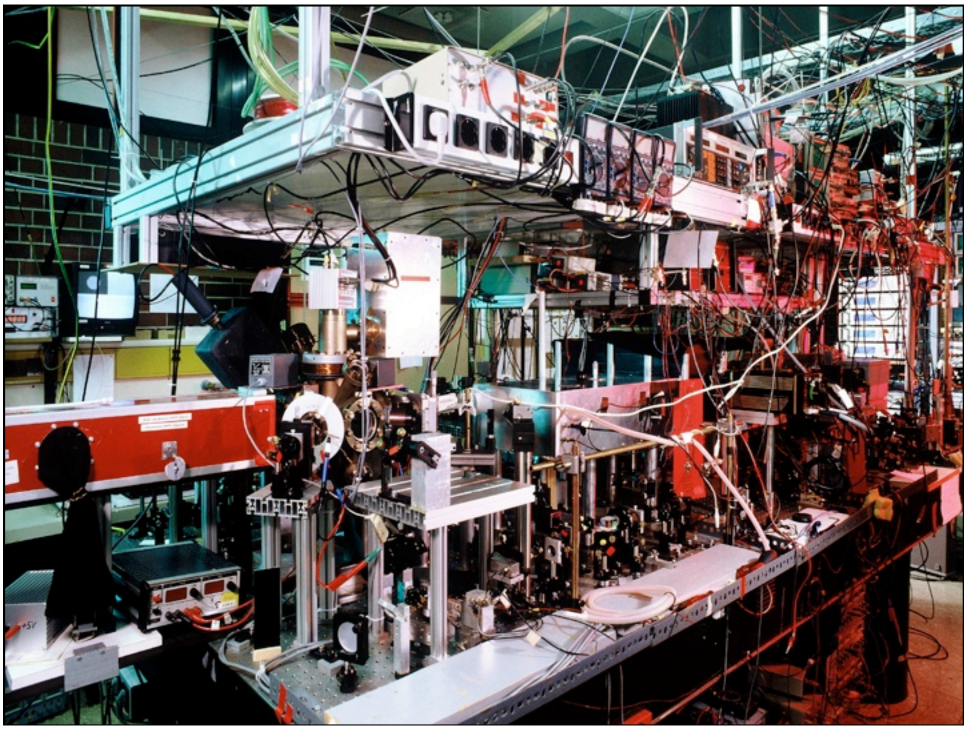
- given $\hat{\rho}$, can we always decompose it like this?

  not always, but when total system is in a pure state $\hat{\rho}^2 = \hat{\rho}$, then its possible, and there exist perfectly correlated observables

- Schmidt:

$$|\psi\rangle = \sum_{ij} \bar{c}_{ij} |\bar{u}_i\rangle \otimes |\bar{v}_j\rangle = \sum_k c_k |u_k\rangle \otimes |v_k\rangle$$

$$\hat{\rho} = |\psi\rangle\langle\psi| = \sum_{ik} c_i c_k^* |u_i\rangle\langle u_k| \otimes |v_i\rangle\langle v_k|$$



Qubits & Quantum Registers

Classical Bit

0 or 1 →

Quantum Bit

0 or 1 or 0 1

0

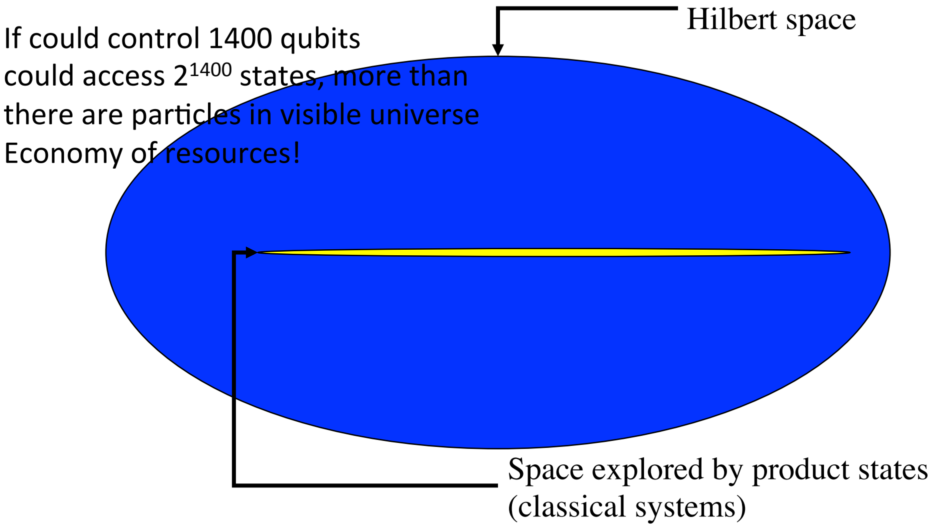
Classical register

101 →

Quantum register

000 001 010 011
100 101 110 111

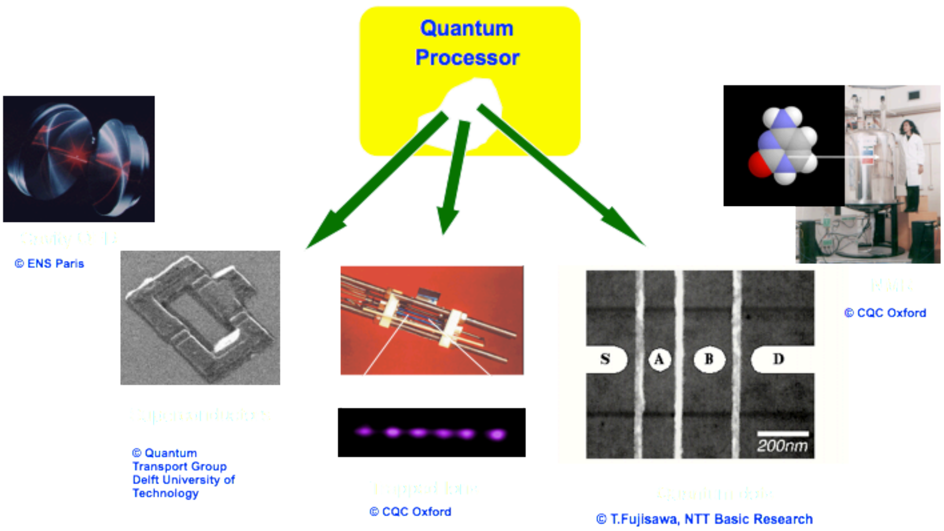
10110101



Quantum system explores whole Hilbert space → more parameters.

Situation for mixed states less well understood (Entangled and disentangled systems have same number of parameters).

Which quantum technology ?

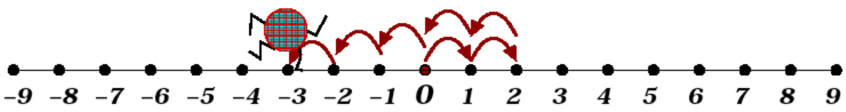


Classical random walk on a line

1. Start at the origin. *(discrete space and time)*
2. Toss a coin, move one unit right for heads, left for tails.
3. Repeat step 2. T times.
4. Record current position, $-T \leq x \leq T$.

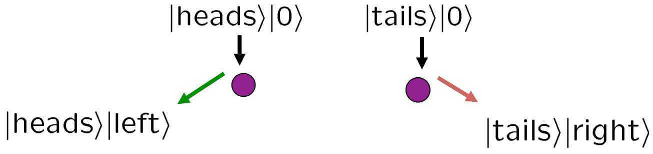
Repeat steps 1. to 4. many times $\longrightarrow$ prob. dist. $P(x,T)$, binomial

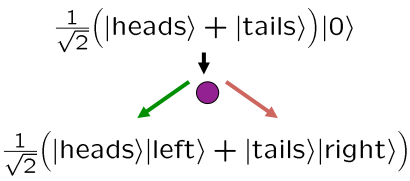
standard deviation $\langle x^2 \rangle^{1/2} = \sqrt{T}$



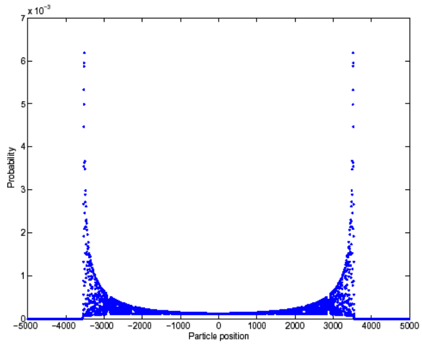
Quantum Quincunx

- Head for L, Tail for R:



- 

Symmetric walk



$$\mathbf{C}_2^{(\mathrm{Had})} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

FIG. 1: Probability distribution for a walk on a line after 5000 steps. Only even positions are shown since odd positions are unoccupied. Hadamard coin, symmetric $|R,0\rangle + i|L,0\rangle$ initial state.

Decoherence: walk on line (Kendon & Tregenna

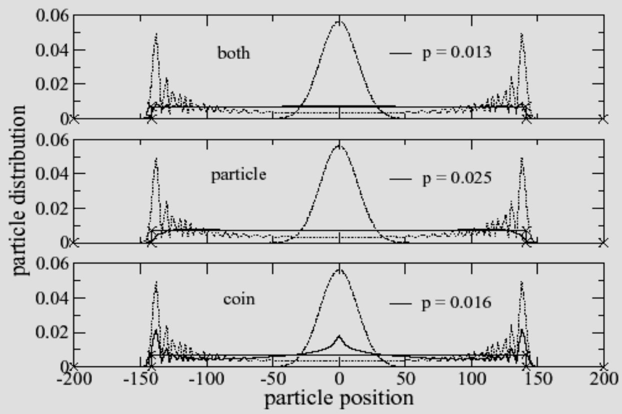


FIG. 1: Distribution of the particle position for a quantum walk on a line after $T = 200$ time steps. Pure quantum (dotted), fully classical (dashed), and decoherence at rate shown on part of system indicated by key (solid). Uniform distribution between $-T/\sqrt{2} \leq x \leq T/\sqrt{2}$ (crosses) also shown.

Beating Nature?

nature Vol 446|12 April 2007|doi:10.1038/nature05678

LETTERS

Evidence for wavelike energy transfer through quantum coherence in photosynthetic systems

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quantum coherence in photosynthetic complexes have been predicted[12,13] and indirectly observed[14]. Here we extend previous two-dimensional electronic spectroscopy investigations of the FMO bacteriochlorophyll complex, and obtain direct evidence for remarkably long-lived electronic quantum coherence playing an important part in energy transfer processes within this system. The quantum coherence manifests itself in characteristic, directly observable quantum beating signals among the excitons within the *Chlorobium tepidum* FMO complex at 77 K. This wavelike characteristic of the energy transfer within the photosynthetic complex can explain its extreme efficiency, in that it allows the complexes to sample vast areas of phase space to find the most efficient path.

Conclusions and acknowledgments

- Many years of discussions with Stig
- My former students: Steve Barnett, Artur Ekert, Myungshik Kim, Simon Phoenix, Barry Sanders, Joerg Steinbach, Vlatko Vedral
- My former postdocs: Almut Beige, Vlado Buzek, Hugo Cable, Barry Garraway, Philip Gou, Jaewoo Joo, Christoph Keitel, Viv Kendon, Gerard Milburn, Mio Murao, Jiannis Pachos, Martin Plenio, Stefan Scheel, Ben Tregenna, Jason Twamley