Perturbative aspects of soft QCD dynamics

ECT* Seminar, 12 September 2019

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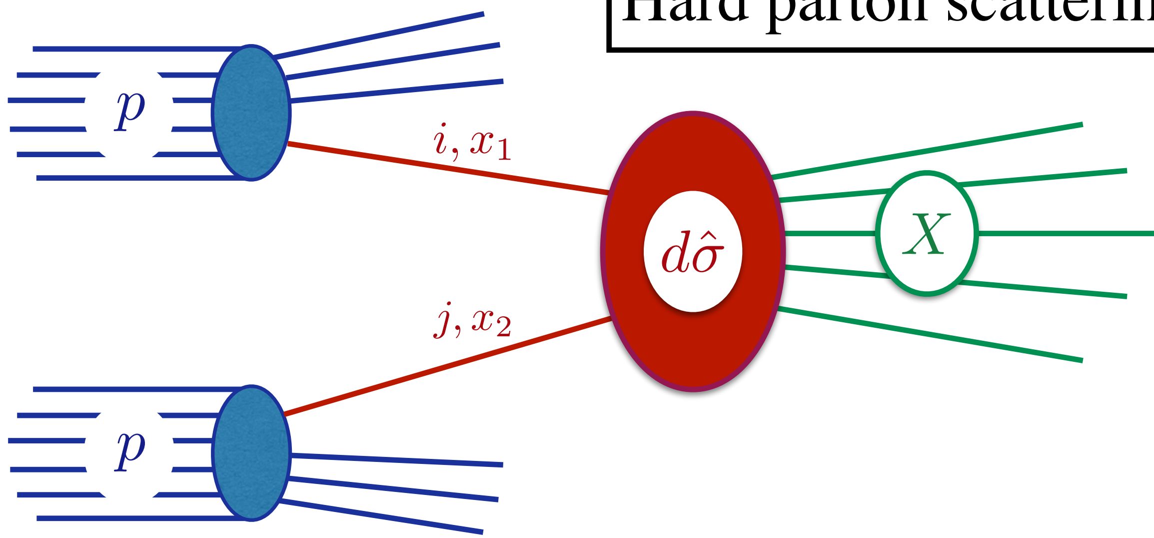
Hard parton scattering $\Longrightarrow$ PQCD



Soft parton distributions $\Longrightarrow$ Universality, Lattice QCD

$\Longrightarrow$ PQCD (bound state)

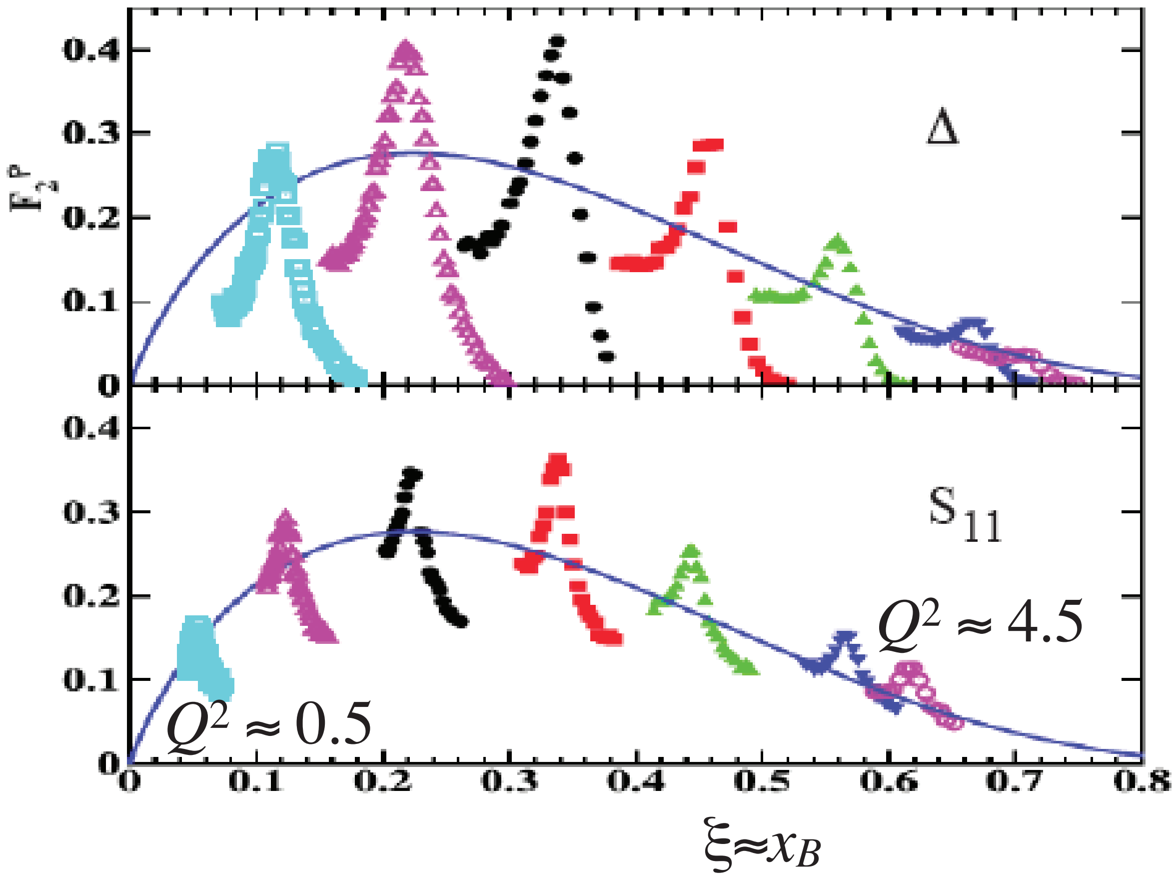
PH 1807.05598v2 and refs. therein

Resonances build the pdf's

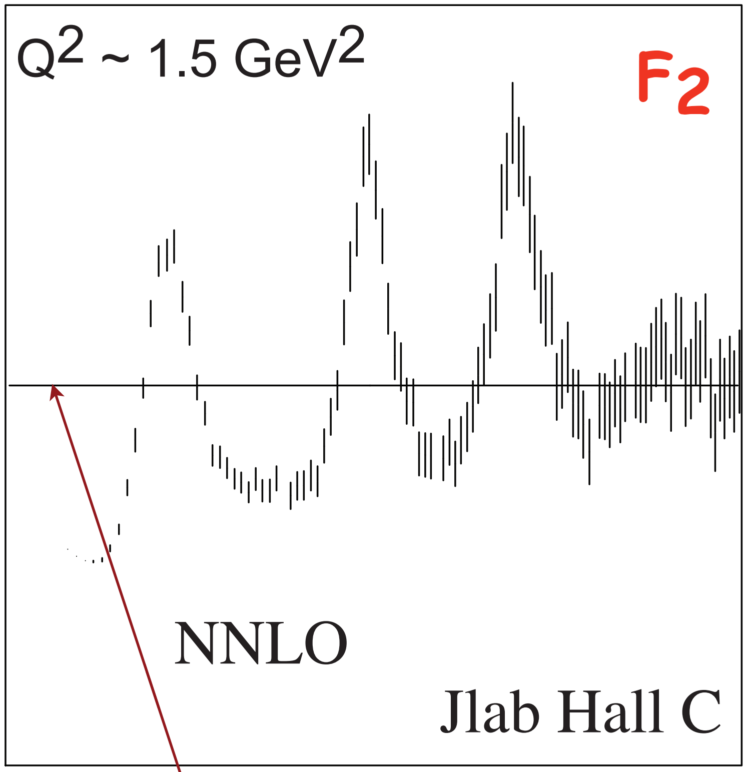
Duality is a pervasive and surprising feature of hadron dynamics:
Bound states form the dynamics.

Bloom-Gilman duality (1970): Resonances build the pdf's

Jlab Hall C

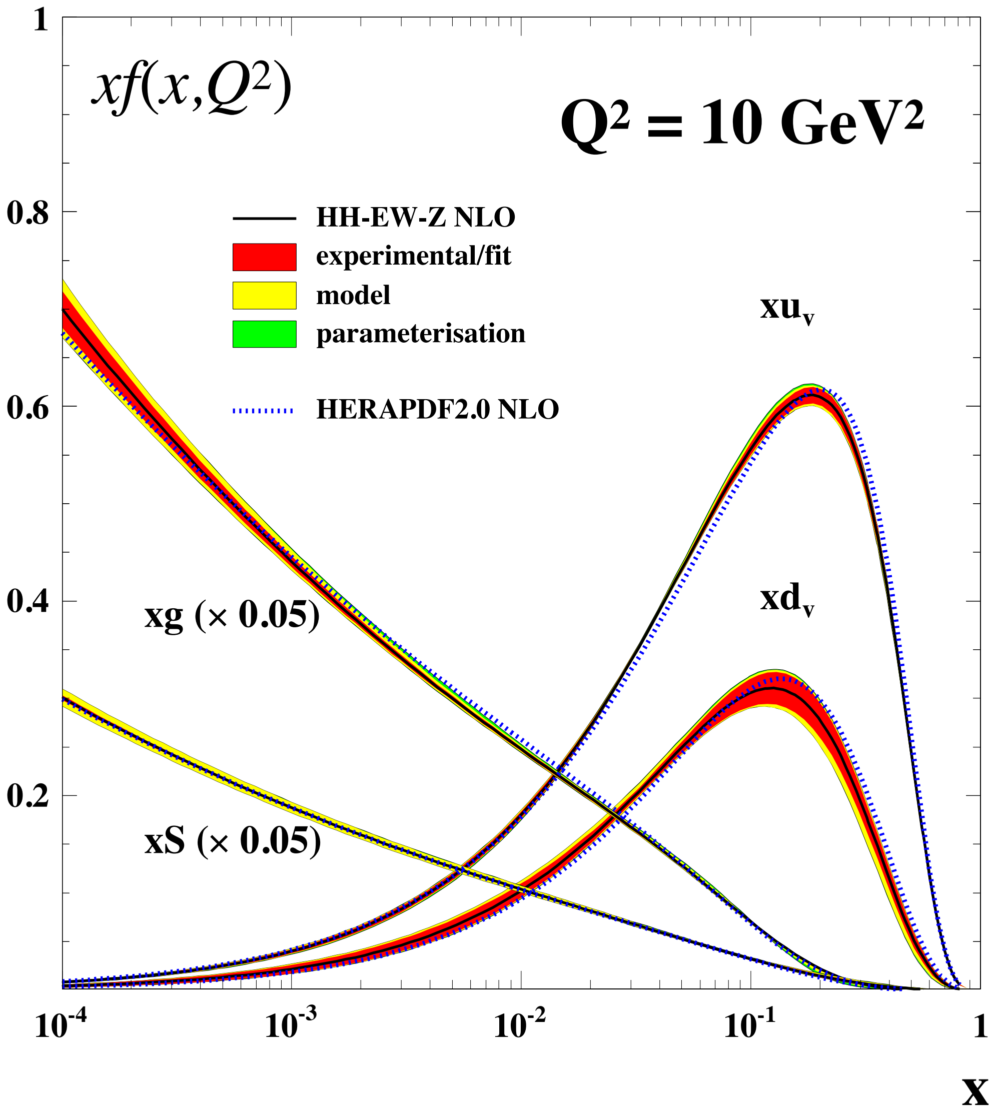


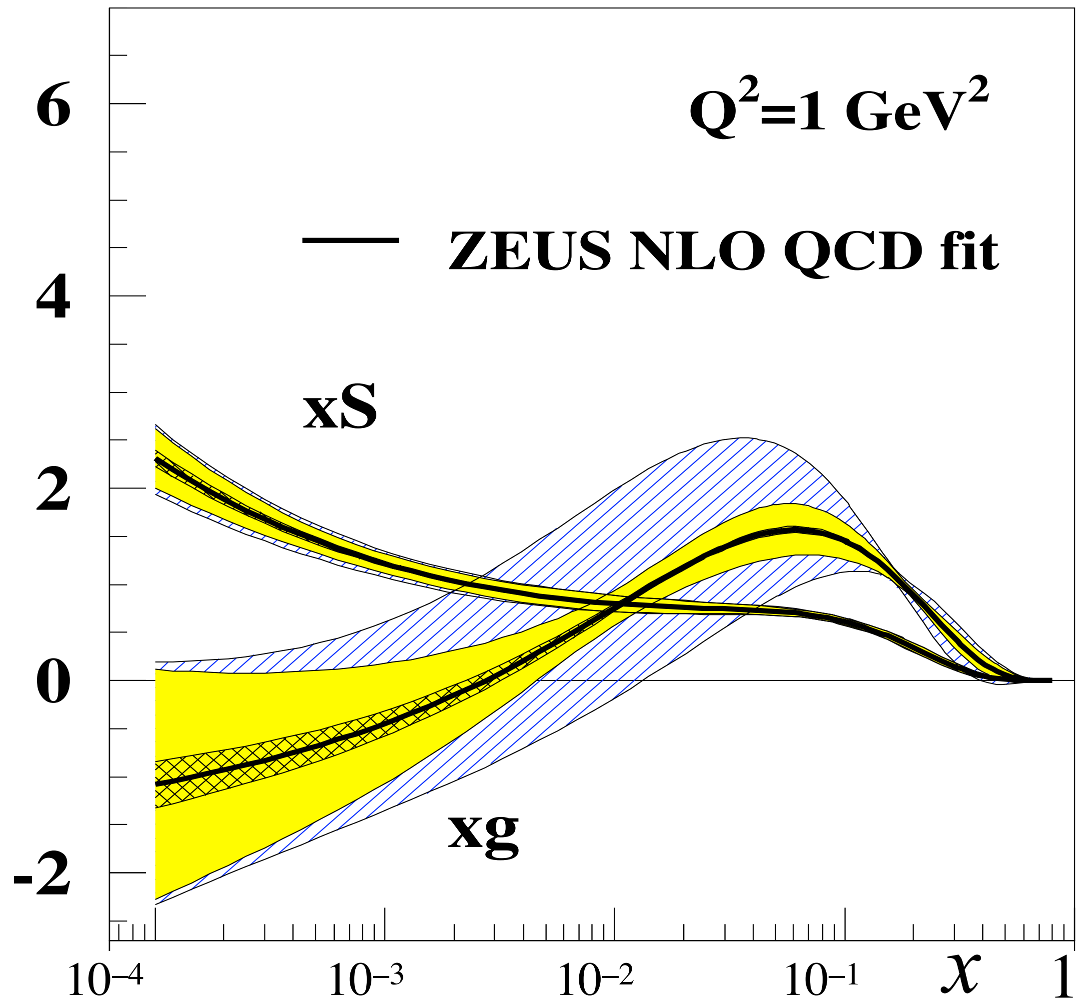
E=4 GeV, Θ=24 Deg



S. Alekhin, PRD 68 (2003) 014002

Gluons evolve away with decreasing Q^2





Resonances are not gluon dominated.

But the low x sea quarks remain.

A. M. Cooper-Sarkar et al, 1206.0894, 1604.05083

The meaning of “non-perturbative”

Perturbative expansion diverges

Feynman diagrams lack essential features

Common view for soft QCD: $\alpha_s \gg 1$ $\Rightarrow$ Use lattice QCD (or models)

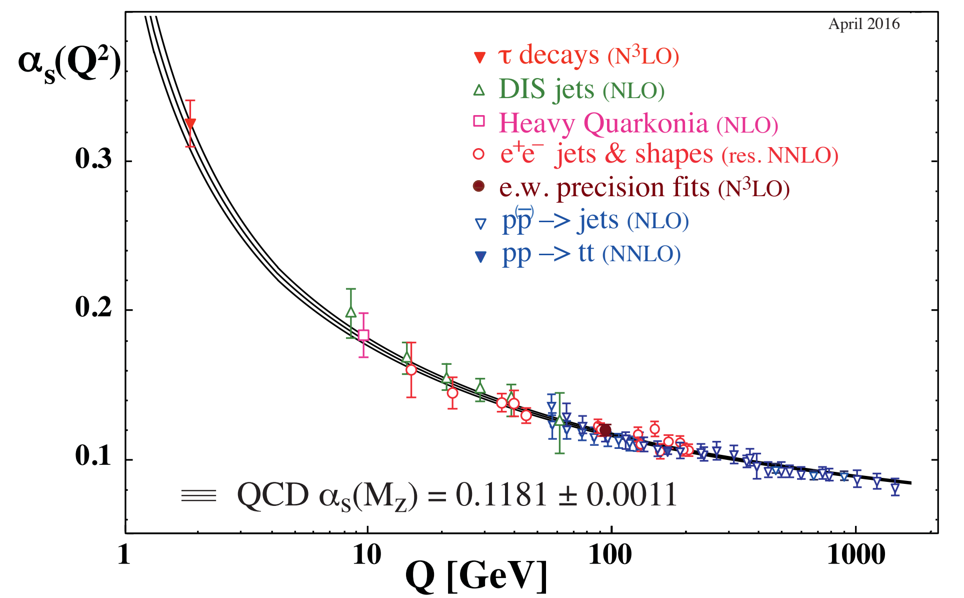
Alternative possibility: Coupling freezes, remains perturbative $\alpha_s(0)/\pi \approx 0.14$

★ ⟵ $\alpha_s{}^{crit} \approx 0.43$ Gribov

Divergence of perturbative expansion is due to low momentum transfers

This is the case for classical fields in QED

and for QED bound states $\alpha(0) \simeq 1/137$



Yuri Dokshitzer (2011):

infrared coupling

Theory + Phenomenology of 1/Q effects in event shape observables, both in e^+e^- annihilation and **DIS** systematically pointed at the ***average value*** *of the* ***infrared coupling***

$$\alpha_0 \equiv \frac{1}{2\text{ GeV}} \int_0^{2\text{ GeV}} dk\, \alpha_s(k^2) \sim 0.5$$

$\alpha_s = 0.1153 \pm 0.0017(\text{exp}) \pm 0.0023(\text{th})$

$\alpha_0 = 0.5132 \pm 0.0115(\text{exp}) \pm 0.0381(\text{th})$

T.Ghermann, M.Jaquier, G.Luisoni

The main features of this result are as follows : the average IR coupling is

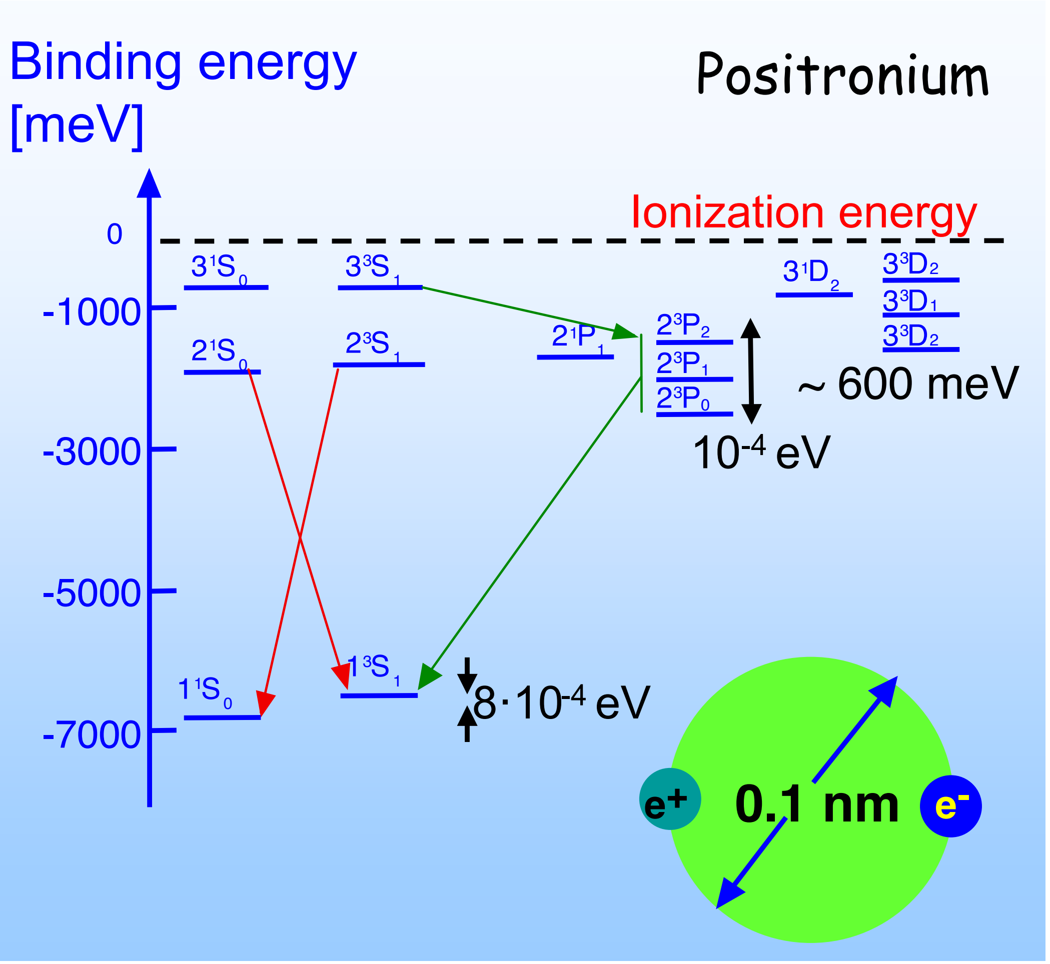
- Universal — holds to within $\pm 15\%$

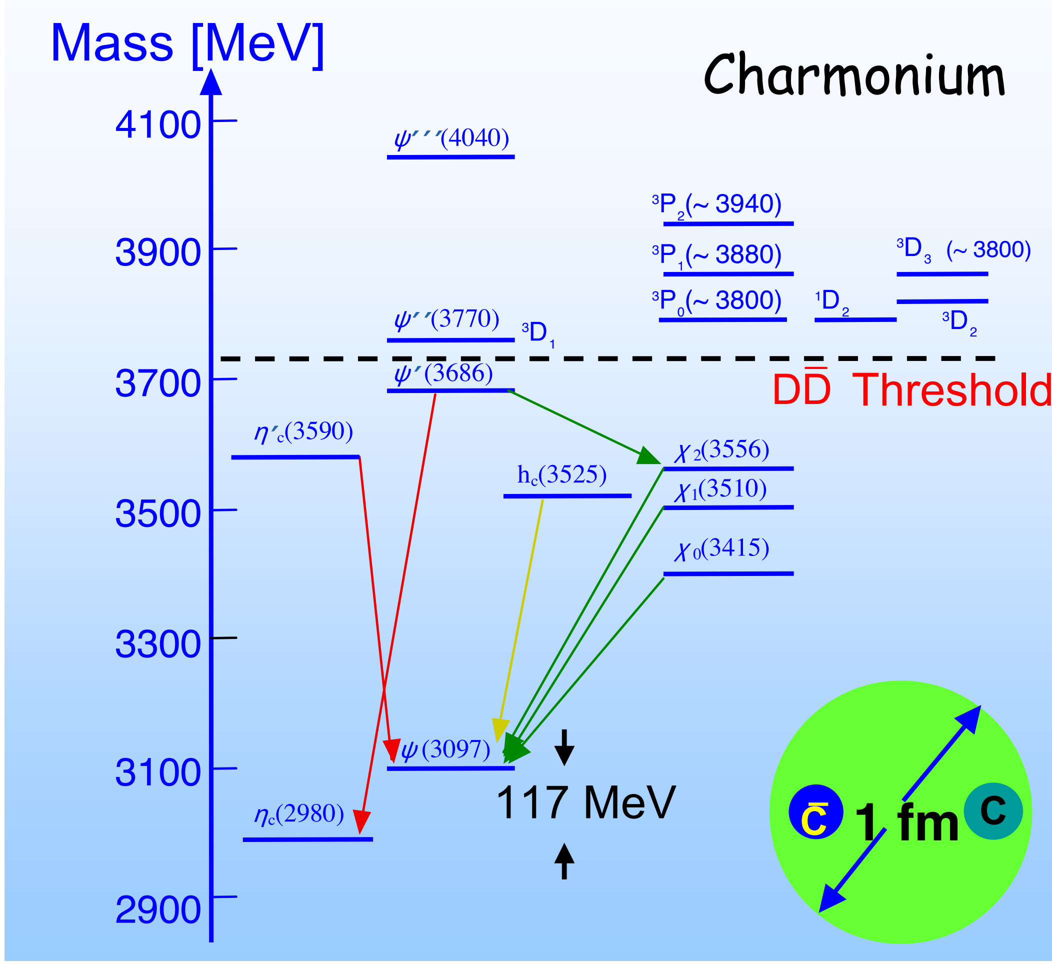
  If not for the ***universality***,
  the whole game would made no sense : it would have meant just trading ***one unknown*** - non-perturbative "smearing" effects in a given observable (like in MC event generators) - for ***another unknown*** function - the shape of the coupling in the infrared...

- Reasonably small — (which opens intriguing possibilities . . .)

- Comfortably above the Gribov's critical value ($\pi \cdot 0.137 \simeq 0.4$)

Similarity of quarkonia and atoms



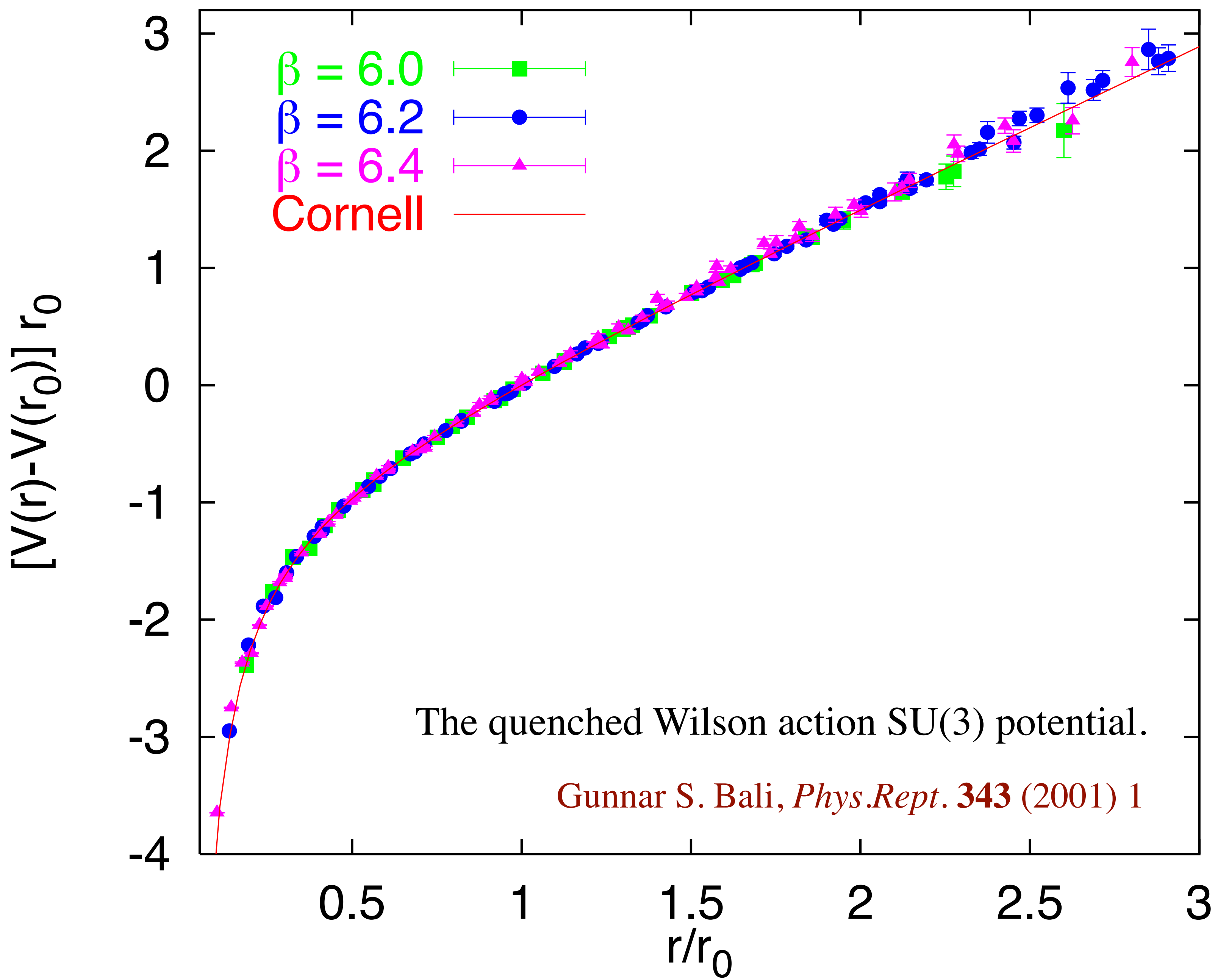


$$V(r) = -\frac{\alpha}{r}$$

$$V(r) = c\,r - \frac{4}{3}\frac{\alpha_s}{r}$$

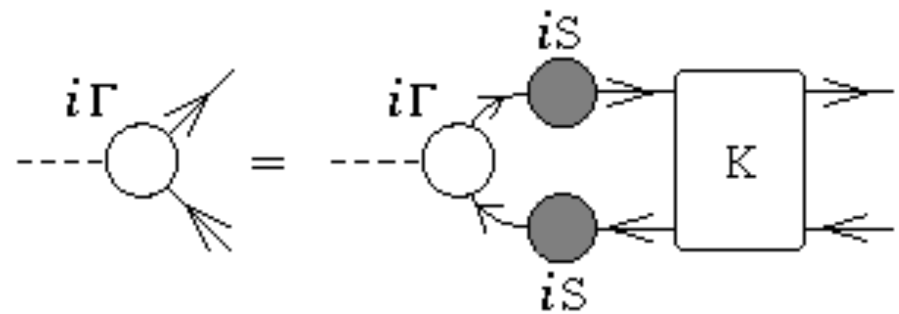
"The J/ψ is the Hydrogen atom of QCD"

Linear Cornell potential agrees with Lattice QCD



Three developments in the theory of atoms

- 1951: Salpeter & Bethe



Expand propagators S and kernel K in powers of α
Explicit Lorentz covariance (frame dependent time separations)
No analytic solution even at lowest order in S and K

- 1975: Caswell & Lepage: BS is not unique: ∞ # of equivalent equations, $S \leftrightarrow K$

We may choose to expand around Schrödinger atoms
Give up explicit boost invariance

- 1986: Caswell & Lepage NRQED: Effective NR field theory

Expand QED action in powers of ∇/m_e
Choose to start from Schrödinger atoms (at rest)

$\Rightarrow$ Need a physical principle for the choice of initial wave function.

Perturbative expansions for atoms

PT for atoms start with an initial approximation, *e.g.*, the Schrödinger eq.

Atomic wave functions are of $\mathcal{O}(\alpha^\infty)$: $\Psi(\boldsymbol{x}) \sim \exp(-\alpha m r/2)$

The wave function is not an observable (gauge dependent).

Binding energies are physical and they can be expanded in α and logα.

Example: Hyperfine splitting in Positronium

G. S. Adkins, Hyperfine Interact. **233** (2015) 59

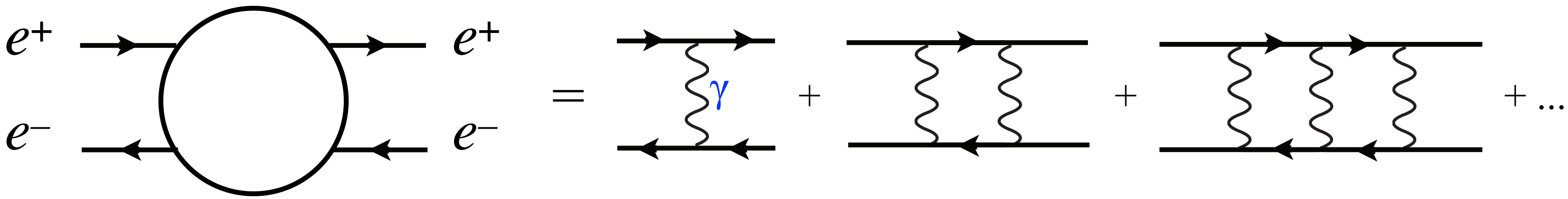
$$\begin{aligned}\Delta\nu_{QED} &= m_e\alpha^4\left\{\frac{7}{12} - \frac{\alpha}{\pi}\left(\frac{8}{9}+\frac{\ln 2}{2}\right)\right. \\ &+\frac{\alpha^2}{\pi^2}\left[-\frac{5}{24}\pi^2\ln\alpha + \frac{1367}{648} - \frac{5197}{3456}\pi^2 + \left(\frac{221}{144}\pi^2+\frac{1}{2}\right)\ln 2 - \frac{53}{32}\zeta(3)\right] \\ &\left. -\frac{7\alpha^3}{8\pi}\ln^2\alpha + \frac{\alpha^3}{\pi}\ln\alpha\left(\frac{17}{3}\ln 2 - \frac{217}{90}\right) + \mathcal{O}\left(\alpha^3\right)\right\} = 203.39169(41)\ \text{GHz}\end{aligned}$$

$\Delta\nu_{\text{EXP}} = 203.394 \pm .002$ GHz

Principles of bound state perturbation theory?

A generalization to QCD requires a derivation of the Schrödinger eq. from L_{QED}.

Summing ladder diagrams is not the answer: *E.g.*, for $e^+ e^- \rightarrow e^+ e^-$



The divergence of the ladder sum gives rise to Positronium poles.

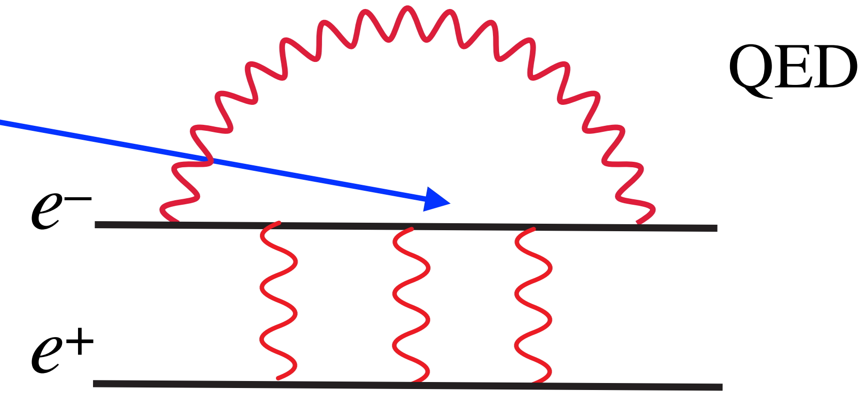
But: The free *in* and *out* states of PQED lack overlap with Positronia.

Free quark states at $t = \pm\infty$ are incompatible with confinement in QCD.

Beware of using Feynman diagrams, based on free propagation, for bound states!

Bound state constituents propagate in a field

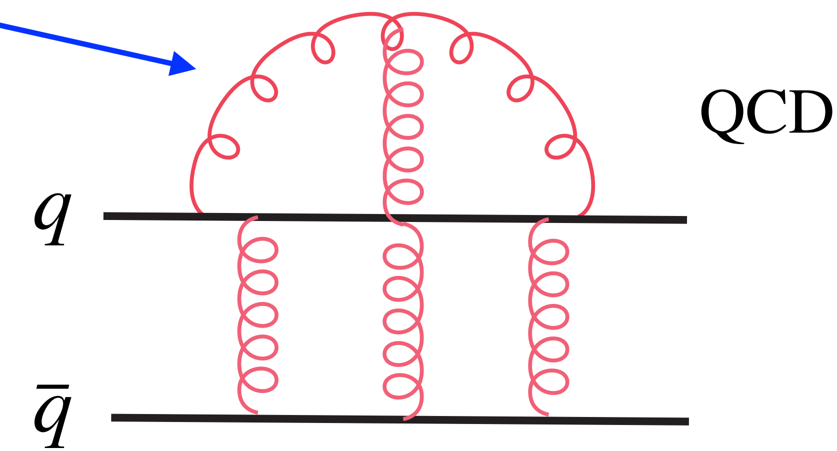
For QED lamb shift, need to calculate e^- propagator in the field of e^+



In an NR approximation, this can be described by a fixed $-\alpha/r$ potential.

Lamb shift

In QCD, relativistic gluons interact with colored quarks



Gluon and quark propagators depend on the state in which they propagate.

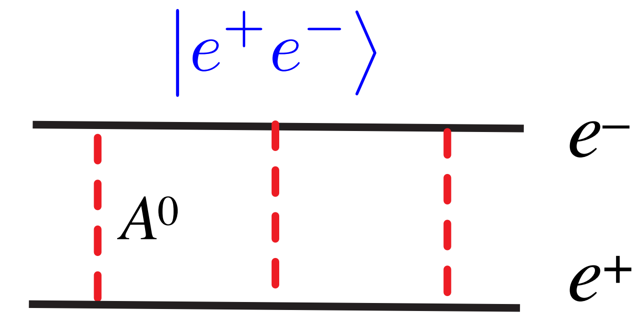
Cannot build bound states with constituents that have predetermined propagators.

$\Rightarrow$

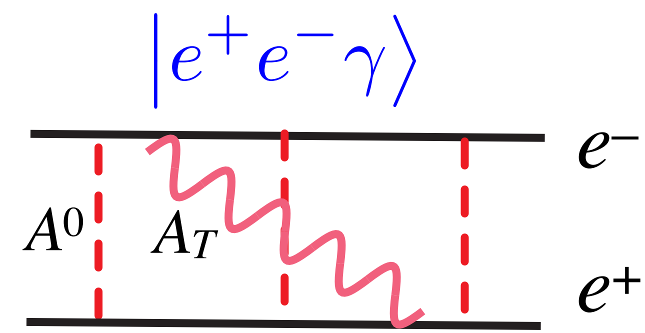
Constituents propagate in their instantaneous field.
Bound states are eigenstates of H.

Fock state expansion for Positronium (at rest)

The $|e^+e^-\rangle$ Fock state determines the binding energy at lowest order, $\mathcal{O}(\alpha^2)$.
Binding is due to instantaneous A^0 photons.
The classical field is not suppressed by α.

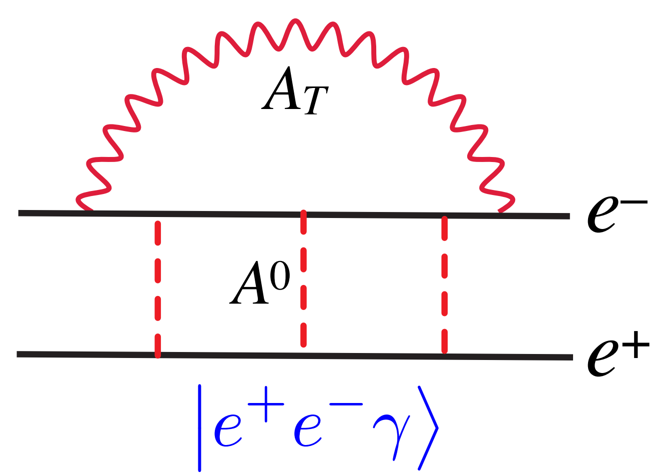


Spin dependence arises at $\mathcal{O}(\alpha^4)$ from states with a transverse photon, $|e^+e^-\gamma\rangle$.
A_T exchange is suppressed by powers of α.



The Lamb shift also arises from $|e^+e^-\gamma\rangle$.

Perturbative theory is equivalent to a Fock expansion in the classical field.



How can this be implemented in a Hamiltonian approach?

Canonical quantisation in temporal gauge: $A^0 = 0$

Avoids problem due to the missing conjugate field of A^0. No ghosts.

$$E^i = F^{i0} = -\partial_0 A^i \quad \text{conjugate to } -A^i \quad (i = 1,2,3)$$

$$\left[E^i(t,\boldsymbol{x}), A^j(t,\boldsymbol{y})\right] = i\delta^{ij}\delta(\boldsymbol{x}-\boldsymbol{y}) \qquad \left\{\psi_\alpha^\dagger(t,\boldsymbol{x}), \psi_\beta(t,\boldsymbol{y})\right\} = \delta_{\alpha\beta}\,\delta(\boldsymbol{x}-\boldsymbol{y})$$

$$H = \int d\boldsymbol{x}\left[\tfrac{1}{2}\boldsymbol{E}_L^2 + \tfrac{1}{2}\boldsymbol{E}_T^2 + \tfrac{1}{4}F^{ij}F^{ij} + \psi^\dagger(-i\alpha^i\partial_i - e\alpha^i A^i + m\gamma^0)\psi\right]$$

Gauss' operator does not vanish: $G(x) \equiv \dfrac{\delta \mathcal{S}}{\delta A^0(x)} = \partial_i E_L^i(x) - e\psi^\dagger\psi(x)$

$G(x)$ generates *time*-independent gauge transformations, consistent with $A^0 = 0$

Fix the gauge by constraining physical states: $G(x)\,|phys\rangle = 0$

This determines $E_L(x)$ for each state, imposing Gauss' law.

J. D. Bjorken, SLAC Summer Institute (1979)
G. Leibbrandt, Rev. Mod. Phys. 59, 1067 (1987)

Schrödinger equation for Positronium

$$G(x)\,|phys\rangle = 0 \quad \Longrightarrow \quad \begin{aligned} &\partial_i E_L^i(t,\boldsymbol{x})\,|phys\rangle = e\psi^\dagger\psi(t,\boldsymbol{x})\,|phys\rangle \\ &E_L^i(t,\boldsymbol{x})\,|phys\rangle = -\partial_i^x \int d\boldsymbol{y}\,\frac{e}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\psi^\dagger\psi(t,\boldsymbol{y})\,|phys\rangle \end{aligned}$$

For the component of Positronium with an electron at $\boldsymbol{x}_1$ and a positron at $\boldsymbol{x}_2$:

$$\big|e^-(\boldsymbol{x}_1)e^+(\boldsymbol{x}_2)\big\rangle = \bar{\psi}_\alpha(\boldsymbol{x}_1)\psi_\beta(\boldsymbol{x}_2)\,|0\rangle$$

$$E_L^i\,\big|e^-(\boldsymbol{x}_1)e^+(\boldsymbol{x}_2)\big\rangle = -\partial_i^x\,\frac{e}{4\pi}\Big(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|}-\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|}\Big)\big|e^-(\boldsymbol{x}_1)e^+(\boldsymbol{x}_2)\big\rangle$$

The instantaneous Hamiltonian $H_V \equiv \frac{1}{2}\int d\boldsymbol{x} E_L^i E_L^i(\boldsymbol{x})$ gives the classical potential:

$$H_V\,\big|e^-(\boldsymbol{x}_1)e^+(\boldsymbol{x}_2)\big\rangle = -\frac{\alpha}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}\big|e^-(\boldsymbol{x}_1)e^+(\boldsymbol{x}_2)\big\rangle$$

The Schrödinger equation follows from

$$H\,\big|e^+e^-\big\rangle = (2m+E_b)\,\big|e^+e^-\big\rangle$$

Science, not art

A Fock state expansion for QCD

The Fock expansion is compatible with the quark model of hadrons:

- Valence quantum numbers of mesons and baryons (lowest Fock state)
- Physical (transverse) gluon constituents contribute at $O(\alpha_s)$
- The E_L field is instantaneous also for relativistic constituents

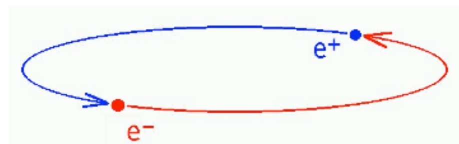
How can color confinement arise?

Gauss' law has no Λ_{QCD} scale

A crucial difference between QED and QCD

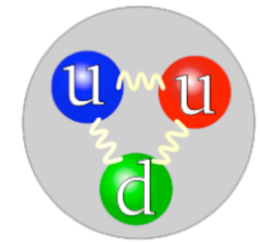
Global gauge invariance allows a classical gauge field for neutral atoms, but not a color octet gluon field for color singlet hadrons.

Positronium (QED)



$$E^i_L(\boldsymbol{x}) = -\frac{e}{4\pi}\,\partial^x_i\left(\frac{1}{\boldsymbol{x}-\boldsymbol{x}_1} - \frac{1}{\boldsymbol{x}-\boldsymbol{x}_2}\right)$$

Proton (QCD)



$$E^i_{L,a}(\boldsymbol{x}) = 0$$

However:

The classical gluon field is non-vanishing for each color component C of the state

$$E^i_{L,a}(\boldsymbol{x}, C) \neq 0$$

The blue quark feels the color field generated by the red and green quarks.

An external observer sees no field: The gluon field generated by a color singlet state vanishes

$$\sum_C E^i_{L,a}(\boldsymbol{x}, C) = 0$$

Temporal gauge in QCD: $A_a^0 = 0$

Gauss' operator $G_a(x) \equiv \dfrac{\delta\mathcal{S}}{\delta A_a^0(x)} = \partial_i E_a^i(x) + g f_{abc} A_b^i E_c^i - g\psi^\dagger T^a \psi(x)$

generates time-independent gauge transformations, which keep $A_a^0 = 0$

The gauge is fully defined (in PT) by the constraint $G_a(x)\,|phys\rangle = 0$

$$\Rightarrow \qquad \partial_i E_{L,a}^i(\boldsymbol{x})\,|phys\rangle = g\big[-f_{abc}A_b^i E_c^i + \psi^\dagger T^a \psi(\boldsymbol{x})\big]\,|phys\rangle$$

In QED one solves for $\boldsymbol{E}_L$ requiring $\boldsymbol{E}_L(\boldsymbol{x}) \to 0$ for $|\boldsymbol{x}| \to \infty$

In QCD, for (globally) color singlet bound states: $\sum_C E_{L,a}^i(\boldsymbol{x}, C) = 0$

For each color component C there are homogeneous solutions of Gauss' law for E_L, which do not vanish at spatial infinity.

Translation invariance requires a constant field energy density (Λ_{QCD}) .

The solution is unique, up to the magnitude of the energy density.

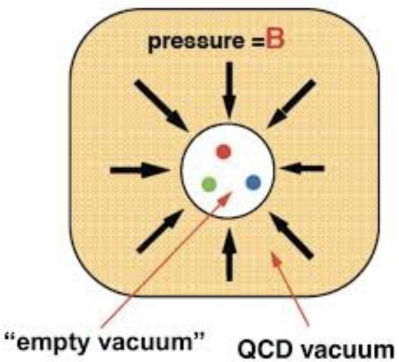
Including a homogeneous solution for $E^i_{L,a}$

$$E^i_{L,a}(\boldsymbol{x})\,|phys\rangle = -\partial_i^x \int d\boldsymbol{y}\Big[\kappa\,\boldsymbol{x}\cdot\boldsymbol{y} + \frac{g}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\Big]\mathcal{E}_a(\boldsymbol{y})\,|phys\rangle$$

where $\mathcal{E}_a(\boldsymbol{y}) = -f_{abc}A^i_b E^i_c(\boldsymbol{y}) + \psi^\dagger T^a \psi(\boldsymbol{y})$

$\kappa \neq \kappa(\boldsymbol{x},\boldsymbol{y})$ ensures $\partial_i \boldsymbol{E}^i(\boldsymbol{x}) = 0$ (a homogeneous solution)

The linear dependence on $\boldsymbol{x}$ makes $\boldsymbol{E}_L$ independent of $\boldsymbol{x}$, as required by translation invariance:
The field energy density is spatially constant. *Cf*. bag model:



The $\boldsymbol{E}_L$ contribution to the QCD Hamiltonian is

$$H_V = \int d\boldsymbol{y}d\boldsymbol{z}\Big\{\boldsymbol{y}\cdot\boldsymbol{z}\Big[\tfrac{1}{2}\kappa^2\int d\boldsymbol{x} + g\kappa\Big] + \tfrac{1}{2}\frac{\alpha_s}{|\boldsymbol{y}-\boldsymbol{z}|}\Big\}\mathcal{E}_a(\boldsymbol{y})\mathcal{E}_a(\boldsymbol{z})$$

The field energy $\propto$ volume of space is irrelevant only if it is universal.
This relates the normalisation $\varkappa$ of all Fock components, leaving an overall scale Λ_{QCD} as the single parameter.

Examples: Fock state potentials (I)

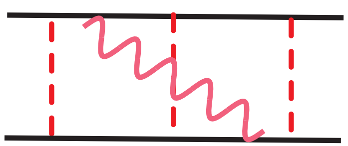
$q\bar{q}$: $$H_V\,|q(\boldsymbol{x}_1)\bar{q}(\boldsymbol{x}_2)\rangle = V_{q\bar{q}}\,|q(\boldsymbol{x}_1)\bar{q}(\boldsymbol{x}_2)\rangle$$

$$V_{q\bar{q}} = \Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2| - C_F\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$$

"Cornell potential" also for relativistic quarks

$qg\bar{q}$: $$V^{(0)}_{qgq}(\boldsymbol{x}_1,\boldsymbol{x}_g,\boldsymbol{x}_2) = \frac{\Lambda^2}{\sqrt{C_F}}\,d_{qgq}(\boldsymbol{x}_1,\boldsymbol{x}_g,\boldsymbol{x}_2)$$

(universal Λ)



$$d_{qgq}(\boldsymbol{x}_1,\boldsymbol{x}_g,\boldsymbol{x}_2) \equiv \sqrt{\tfrac{1}{4}(N-2/N)(\boldsymbol{x}_1-\boldsymbol{x}_2)^2 + N(\boldsymbol{x}_g-\tfrac{1}{2}\boldsymbol{x}_1-\tfrac{1}{2}\boldsymbol{x}_2)^2}$$

$$V^{(1)}_{qgq}(\boldsymbol{x}_1,\boldsymbol{x}_g,\boldsymbol{x}_2) = \tfrac{1}{2}\,\alpha_s\Big[\frac{1}{N}\,\frac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_2|} - N\Big(\frac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_g|}+\frac{1}{|\boldsymbol{x}_2-\boldsymbol{x}_g|}\Big)\Big]$$

When q and g coincide:

$$V^{(0)}_{qgq}(\boldsymbol{x}_1=\boldsymbol{x}_g,\boldsymbol{x}_2) = \Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2| = V^{(0)}_{q\bar{q}}$$

$$V^{(1)}_{qgq}(\boldsymbol{x}_1=\boldsymbol{x}_g,\boldsymbol{x}_2) = V^{(1)}_{q\bar{q}}$$

Fock state potentials (II)

qqq :

$$V_{qqq} = \Lambda^2 d_{qqq}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) - \frac{2}{3}\alpha_s\Big(\frac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_2|} + \frac{1}{|\boldsymbol{x}_2-\boldsymbol{x}_3|} + \frac{1}{|\boldsymbol{x}_3-\boldsymbol{x}_1|}\Big)$$

$$d_{qqq}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) \equiv \frac{1}{\sqrt{2}}\sqrt{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2 + (\boldsymbol{x}_2-\boldsymbol{x}_3)^2 + (\boldsymbol{x}_3-\boldsymbol{x}_1)^2}$$

gg : $$V_{gg} = \sqrt{\frac{N}{C_F}}\,\Lambda^2\,|\boldsymbol{x}_1-\boldsymbol{x}_2| - N\,\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$$

This agrees with the $qg\bar{q}$ potential where the quarks coincide:

$$V_{gg}(\boldsymbol{x}, \boldsymbol{x}_g) = V_{qg\bar{q}}(\boldsymbol{x}, \boldsymbol{x}_g, \boldsymbol{x})$$

It is straightforward to work out the instantaneous potential for any Fock state.

$\mathcal{O}\left(\alpha_s^0\right)$ $q\bar{q}$ bound states

The $\mathcal{O}\left(\alpha_s^0\right)$ meson is a superposition of $q\bar{q}$ Fock states with wave function Φ,

$$|M\rangle = \sum_{A,B;\alpha,\beta} \int d\boldsymbol{x}_1 d\boldsymbol{x}_2\, \bar{\psi}_\alpha^A(t=0,\boldsymbol{x}_1)\delta^{AB}\Phi_{\alpha\beta}(\boldsymbol{x}_1-\boldsymbol{x}_2)\psi_\beta^B(t=0,\boldsymbol{x}_2)\,|0\rangle$$

The bound state condition $H\,|M\rangle = M\,|M\rangle$ gives

$$\left[i\gamma^0\boldsymbol{\gamma}\cdot\overrightarrow{\boldsymbol{\nabla}}+m\gamma^0\right]\Phi(\boldsymbol{x})+\Phi(\boldsymbol{x})\left[i\gamma^0\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}}-m\gamma^0\right]=\left[M-V(|\boldsymbol{x}|)\right]\Phi(\boldsymbol{x})$$

where $\boldsymbol{x} \equiv \boldsymbol{x}_1 - \boldsymbol{x}_2$ and $V(|\,\boldsymbol{x}\,|) = V'|\,\boldsymbol{x}\,| = \Lambda^2|\,\boldsymbol{x}\,|$.

In the non-relativistic limit ($m \gg \Lambda$) this reduces to the Schrödinger equation, and we may add the instantaneous gluon exchange potential.

$\Longrightarrow$ The successful quarkonium phenomenology with the Cornell potential.

Relativistic $q\bar{q}$ bound states

$$i\boldsymbol{\nabla}\cdot\left\{\gamma^0\boldsymbol{\gamma},\Phi(\boldsymbol{x})\right\}+m\left[\gamma^0,\Phi(\boldsymbol{x})\right]=\left[M-V(\boldsymbol{x})\right]\Phi(\boldsymbol{x})$$

Expanding the 4×4 wave function in a basis of 16 Dirac structures $\Gamma_i(\boldsymbol{x})$

$$\Phi(\boldsymbol{x})=\sum_i \Gamma_i(\boldsymbol{x})F_i(r)Y_{j\lambda}(\hat{\boldsymbol{x}})$$

we may use rotational, parity and charge conjugation invariance to determine which $\Gamma_i(\boldsymbol{x})$ may occur for a state of given j^{PC}:

0^{-+} trajectory $[s=0,\ \ell=j]:$ $-\eta_P=\eta_C=(-1)^j$ $\gamma_5,\ \gamma^0\gamma_5,\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L}$

0^{--} trajectory $[s=1,\ \ell=j]:$ $\eta_P=\eta_C=-(-1)^j$ $\gamma^0\gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma^0\gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \boldsymbol{\alpha}\cdot\boldsymbol{L},\ \gamma^0\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$

0^{++} trajectory $[s=1,\ \ell=j\pm 1]:$ $\eta_P=\eta_C=+(-1)^j$ $1,\ \boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma^0\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \gamma^0\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \gamma^0\gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$

0^{+-} trajectory [exotic] $:$ $\eta_P=-\eta_C=(-1)^j$ $\gamma^0,\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$

$\Rightarrow$ There are no solutions for quantum numbers that would be exotic in the quark model (despite the relativistic dynamics)

Example: 0⁻⁺ trajectory wf's

$$\Phi_{-+}(\boldsymbol{x}) = \Big[\frac{2}{M-V}(i\boldsymbol{\alpha}\cdot\overrightarrow{\boldsymbol{\nabla}} + m\gamma^0) + 1\Big]\gamma_5\, F_1(r) Y_{j\lambda}(\hat{\boldsymbol{x}})$$

$$\eta_P = (-1)^{j+1}$$

$$\eta_C = (-1)^j$$

Radial equation: $F_1'' + \Big(\frac{2}{r} + \frac{V'}{M-V}\Big)F_1' + \Big[\tfrac{1}{4}(M-V)^2 - m^2 - \frac{j(j+1)}{r^2}\Big]F_1 = 0$

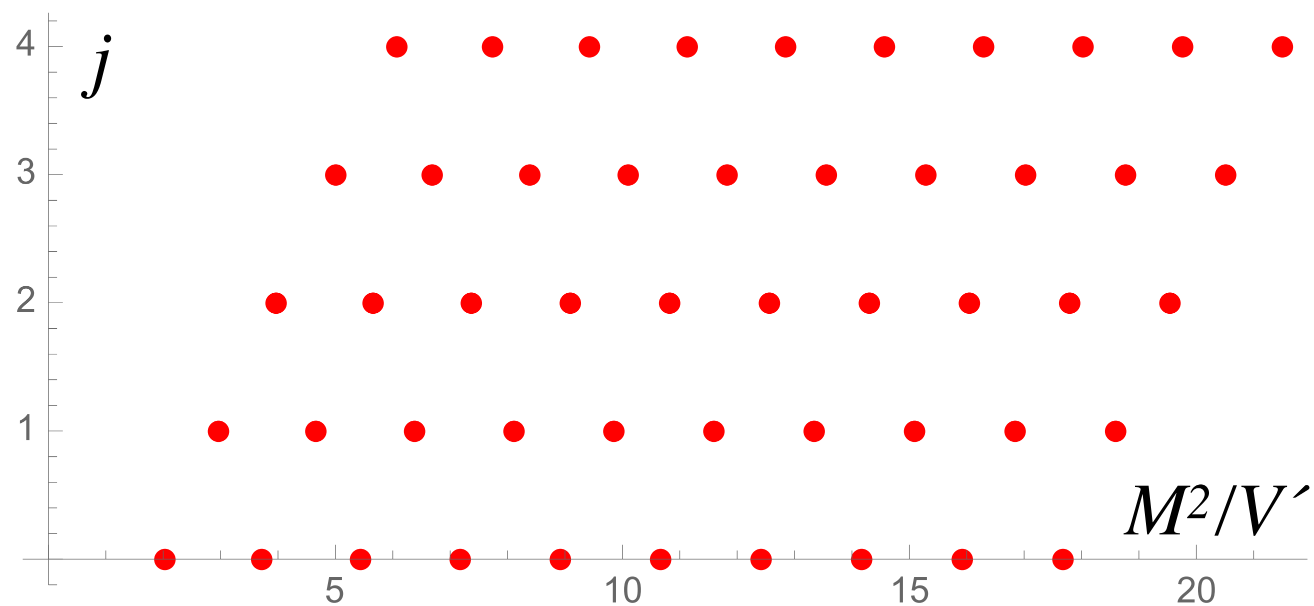
Local normalizability at $r = 0$ and at $V(r) = M$ determines the discrete M

Mass spectrum:

Linear Regge trajectories with daughters

Spectrum similar to dual models

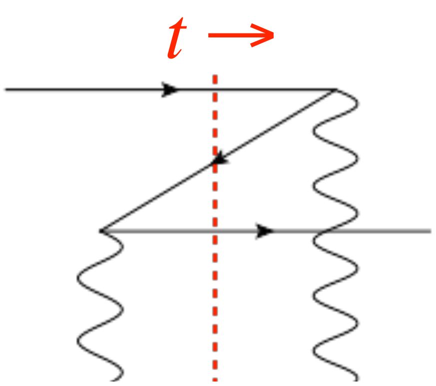
$m = 0$



Sea quark contributions

Quark states in a strong field have E<0 components

Bogoliubov transformation, cf. Dirac states.



In time-ordered PT, these correspond to Z-diagrams, and interpreted as contributions from $q\bar{q}$ pairs.

This effect is manifest in the behavior of the wave function Φ for large V = V´|$\boldsymbol{x}$| :

$$\lim_{\boldsymbol{x}\to\infty} |\Phi(\boldsymbol{x})|^2 = const.$$

The asymptotically constant norm reflects, via duality, pair production as the linear potential $V(|\,\boldsymbol{x}\,|)$ increases.

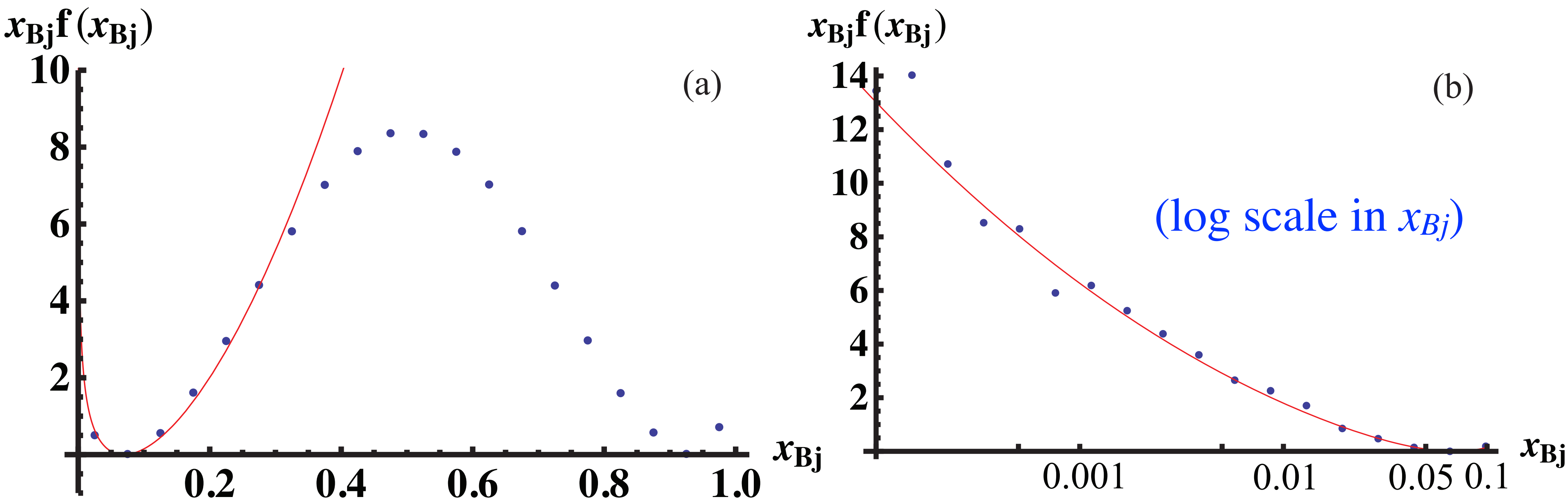
These sea quarks show up in the parton distribution measured in DIS.

Parton distributions have a sea component

In D=1+1 dimensions the sea component is prominent at low m/e :

$m/e = 0.1$

D. D. Dietrich, PH, M. Järvinen
arXiv 1212.4747



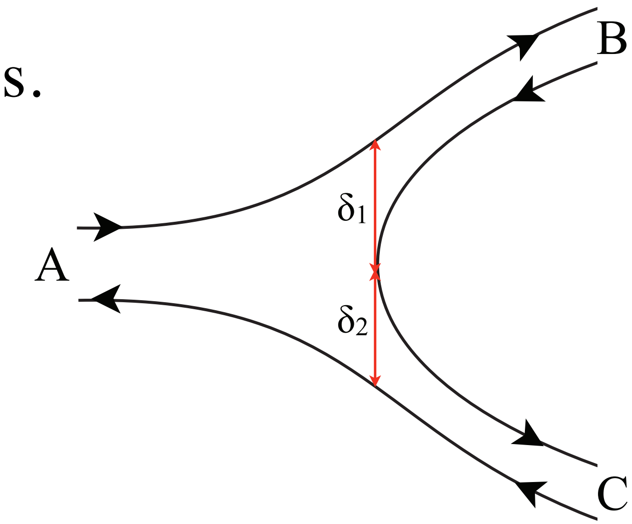
The red curve is an analytic approximation, valid in the $x_{Bj} \to 0$ limit.

Note: Enhancement at low x is due to bd (sea), not to $b^\dagger d^\dagger$ (valence) component.

To be calculated in D=3+1 (and in various frames!)

Decays and hadron loops

The bound state equation determines zero-width states.

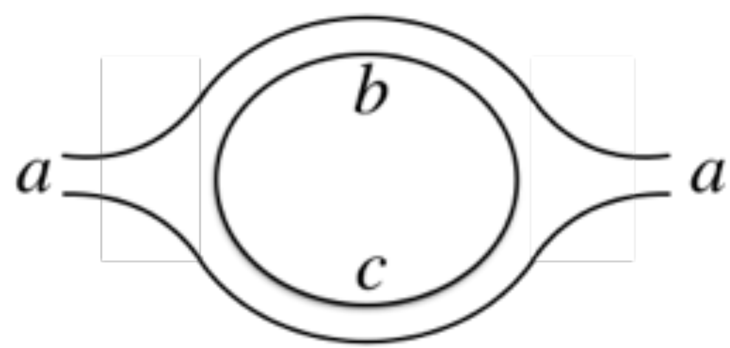


There is an $\mathcal{O}\left(1/\sqrt{N_C}\right)$ coupling between the states: string breaking

$$\langle B, C|A\rangle =$$

$$-\frac{(2\pi)^3}{\sqrt{N_C}}\delta^3(\boldsymbol{P}_A-\boldsymbol{P}_B-\boldsymbol{P}_C)\int d\boldsymbol{\delta}_1 d\boldsymbol{\delta}_2\, e^{i\boldsymbol{\delta}_1\cdot\boldsymbol{P}_C/2-i\boldsymbol{\delta}_2\cdot\boldsymbol{P}_B/2}\mathrm{Tr}\left[\gamma^0\Phi_B^\dagger(\boldsymbol{\delta}_1)\Phi_A(\boldsymbol{\delta}_1+\boldsymbol{\delta}_2)\Phi_C^\dagger(\boldsymbol{\delta}_2)\right]$$

When squared, this gives a $1/N_C$ hadron loop unitarity correction:



Unitarity should be satisfied at hadron level at each order of $1/N_C$.

Bound states in motion

An $\mathcal{O}\left(\alpha_s^0\right)$ $q\bar{q}$ bound state with CM momentum $\boldsymbol{P}$ may be expressed as

$$|M,\boldsymbol{P}\rangle = \int dx_1\, dx_2\, \bar\psi(t=0,x_1)\, e^{i\boldsymbol{P}\cdot(\boldsymbol{x}_1+\boldsymbol{x}_2)/2}\, \Phi^{(\boldsymbol{P})}(x_1-x_2)\, \psi(t=0,x_2)\, |0\rangle$$

The instantaneous potential is $\boldsymbol{P}$-independent, $V(\boldsymbol{x}) = V'|\boldsymbol{x}|$, hence the BSE:

$$i\boldsymbol{\nabla}\cdot\left\{\boldsymbol{\alpha},\Phi^{(\boldsymbol{P})}(\boldsymbol{x})\right\} - \tfrac{1}{2}\boldsymbol{P}\cdot\left[\boldsymbol{\alpha},\Phi^{(\boldsymbol{P})}(\boldsymbol{x})\right] + m\left[\gamma^0,\Phi^{(\boldsymbol{P})}(\boldsymbol{x})\right] = \left[E - V(\boldsymbol{x})\right]\Phi^{(\boldsymbol{P})}(\boldsymbol{x})$$

The solution for $\Phi^{(P)}(\boldsymbol{x})$ is not simply Lorentz contracting in $\boldsymbol{x}$.

States with general $\boldsymbol{P}$ are needed for:

- $\boldsymbol{P}$-dependence of angular momentum ($P \to \infty$ frame).
- EM form factors (gauge invariance has been verified)
- Parton distributions
- Hadron scattering
- . . .

"Perturbative expansion of non-perturbative states"

A perturbative approach to soft QCD:

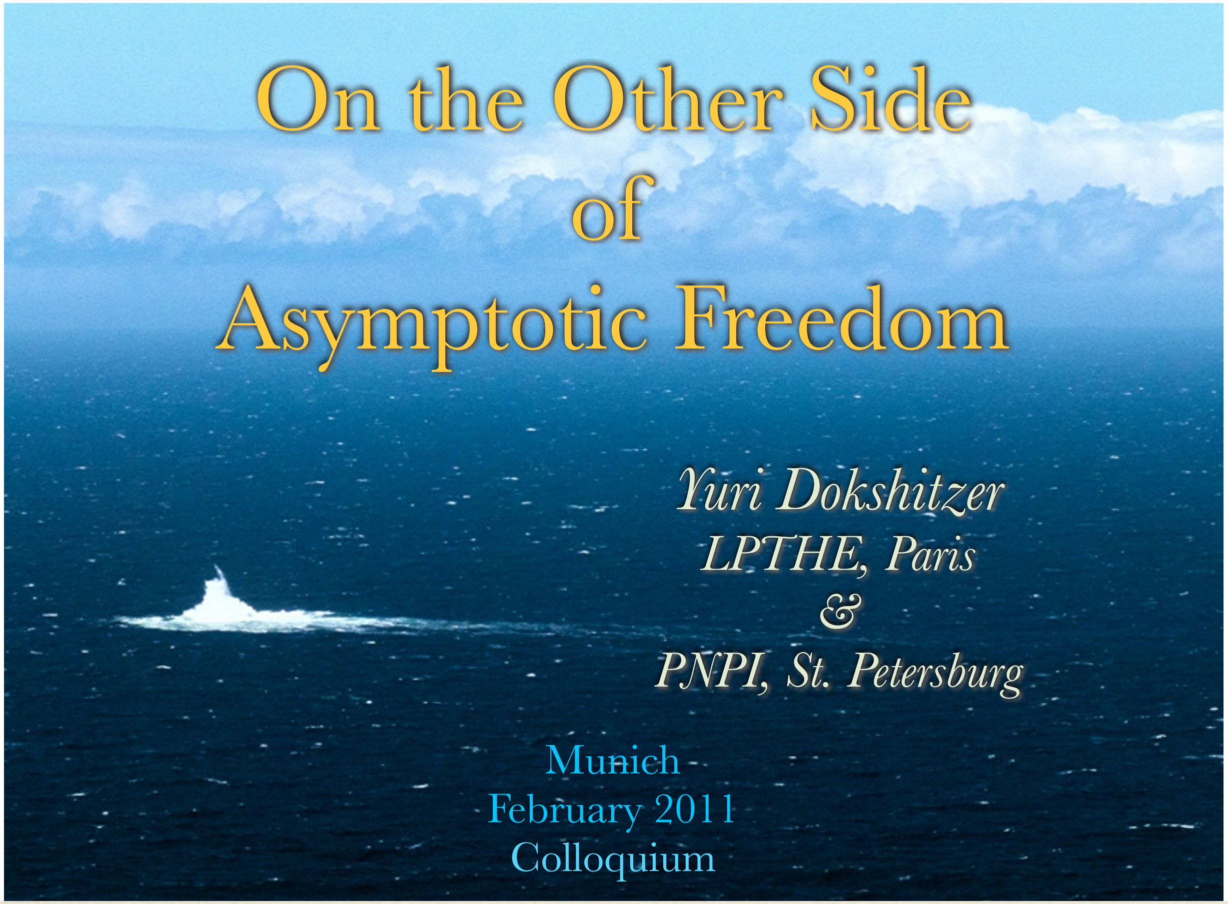
- The instantaneous $\mathcal{O}\left(\alpha_s^0\right)$ field binds the lowest Fock states
- The higher Fock states given by the Hamiltonian H_{QCD} are of $\mathcal{O}\left(\alpha_s\right)$
- Makes bound state calculations less of an art

For the approach to be viable the $\mathcal{O}\left(\alpha_s^0\right)$ dynamics must have:

Poincaré symmetry
Unitarity
Confinement
Chiral Symmetry Breaking (CSB)
Reasonable mass spectrum
. . .

Not all of these have been demonstrated, but the outlook is promising.

A new appearance of PQCD



PQCD can be relevant also for soft interactions.

$\alpha_s/\pi \sim 0.14$

QCD *is about to undergo a* **faith transition**

QCD practitioners prepare themselves - slowly but steadily - to start using, in earnest, the language of ***quarks*** and ***gluons*** down into the region of small characteristic momenta - "**large distances**"

Extra slides

States with M = 0

PRELIMINARY

We required the wave function to be normalizable at $r = 0$ and $V'r = M$

For $M = 0$ the two points coincide. Regular, massless solutions are found.

The massless 0^{++} meson "σ"

$$|\sigma\rangle = \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, \bar{\psi}(\boldsymbol{x}_1)\, \Phi_\sigma(\boldsymbol{x}_1 - \boldsymbol{x}_2)\, \psi(\boldsymbol{x}_2)\, |0\rangle \equiv \hat{\sigma}\, |0\rangle$$

For $m = 0$ and $V' = 1$:

$$\Phi_\sigma(\boldsymbol{x}) = N_\sigma \Big[J_0(\tfrac{1}{4}r^2) + \boldsymbol{\alpha}\cdot\boldsymbol{x}\frac{i}{r}\, J_1(\tfrac{1}{4}r^2) \Big]$$

J_0 and J_1 are Bessel functions.

$\hat{P}^\mu\, |\sigma\rangle = 0$ State has *vanishing four-momentum* in any frame.
It may mix with the perturbative vacuum.
This spontaneously breaks chiral invariance.

A chiral condensate (m = 0)

PRELIMINARY

Since | σ ⟩ has vacuum quantum numbers and zero momentum it can mix with the perturbative vacuum without violating Poincaré invariance

Consider: $|\chi\rangle = \exp(\hat{\sigma})\,|0\rangle$ for which $\langle\chi|\bar{\psi}\psi\,|\chi\rangle = 4N_\sigma$

An infinitesimal chiral rotation of the condensate generates a pion

$$U_\chi(\beta) = \exp\left[i\beta\int d\boldsymbol{x}\,\psi^\dagger(\boldsymbol{x})\gamma_5\psi(\boldsymbol{x})\right] \qquad U_\chi(\beta)\,|\chi\rangle = (1-2i\beta\,\hat{\pi}\,|\chi\rangle$$

where $\hat{\pi}$ is the massless 0^{-+} state with wave function $\Phi_\pi = \gamma_5\Phi_\sigma$

This may provide an explicit example of chiral condensate.

Small quark mass: m > 0

When $m \neq 0$ the massless ($M_\sigma = 0$) sigma 0^{++} state has wave function

$$\Phi_\sigma(\boldsymbol{x}) = f_1(r) + i\,\boldsymbol{\alpha}\cdot\boldsymbol{x}\,f_2(r) + i\,\boldsymbol{\gamma}\cdot\boldsymbol{x}\,g_2(r)$$

Radial functions are Laguerre fn's

An $M_\pi > 0$ pion 0^{-+} state has rest frame wave function

$$\Phi_\pi(\boldsymbol{x}) = \big[F_1(r) + i\,\boldsymbol{\alpha}\cdot\boldsymbol{x}\,F_2(r) + \gamma^0\,F_4(r)\big]\gamma_5 \qquad F_4(0) = \frac{2m}{M}F_1(0)$$

$$F_1'' + \Big(\frac{2}{r} + \frac{1}{M-r}\Big)F_1' + \big[\tfrac{1}{4}(M-r)^2 - m^2\big]F_1 = 0$$

$$\langle\chi|j_5^\mu(x)\hat{\pi}\,|\chi\rangle = iP^\mu f_\pi\,e^{-iP\cdot x} \quad\Longrightarrow\quad F_4(0) = \tfrac{1}{4}iM_\pi f_\pi$$

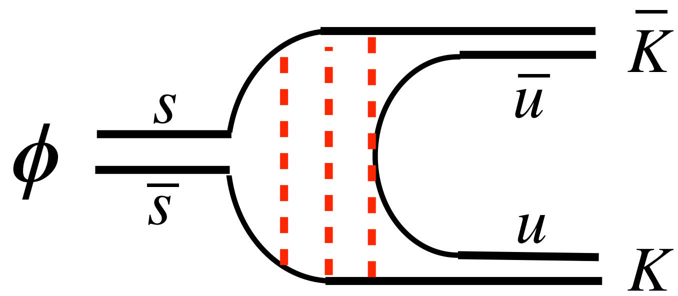
$$\langle\chi|\bar{\psi}(x)\gamma_5\psi(x)\,\hat{\pi}\,|\chi\rangle = -i\,\frac{M^2}{2m}\,f_\pi\,e^{-iP\cdot x} \quad\Longrightarrow\quad F_1(0) = i\,\frac{M^2}{8m}\,f_\pi$$

CSB relations are satisfied for any *P*.

The OZI rule

Connected diagrams: Unsuppressed, string breaking from confining potential

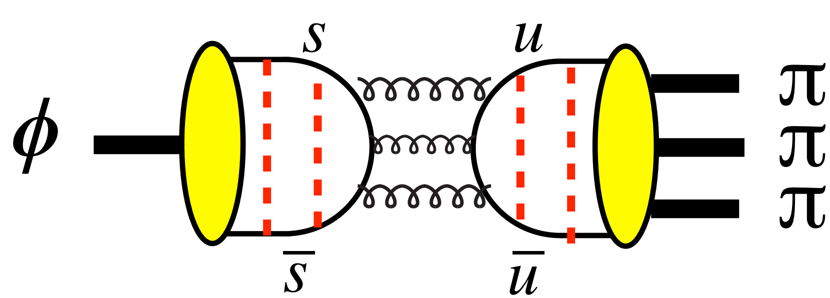
$\phi(1020) \to K\bar{K}$



	ΔE	Br
$\phi(1020) \to K\bar{K}$	26 MeV	83.1 %
$\phi(1020) \not\to \pi\pi\pi$	610 MeV	15.3 %

Disconnected, perturbative diagrams are suppressed

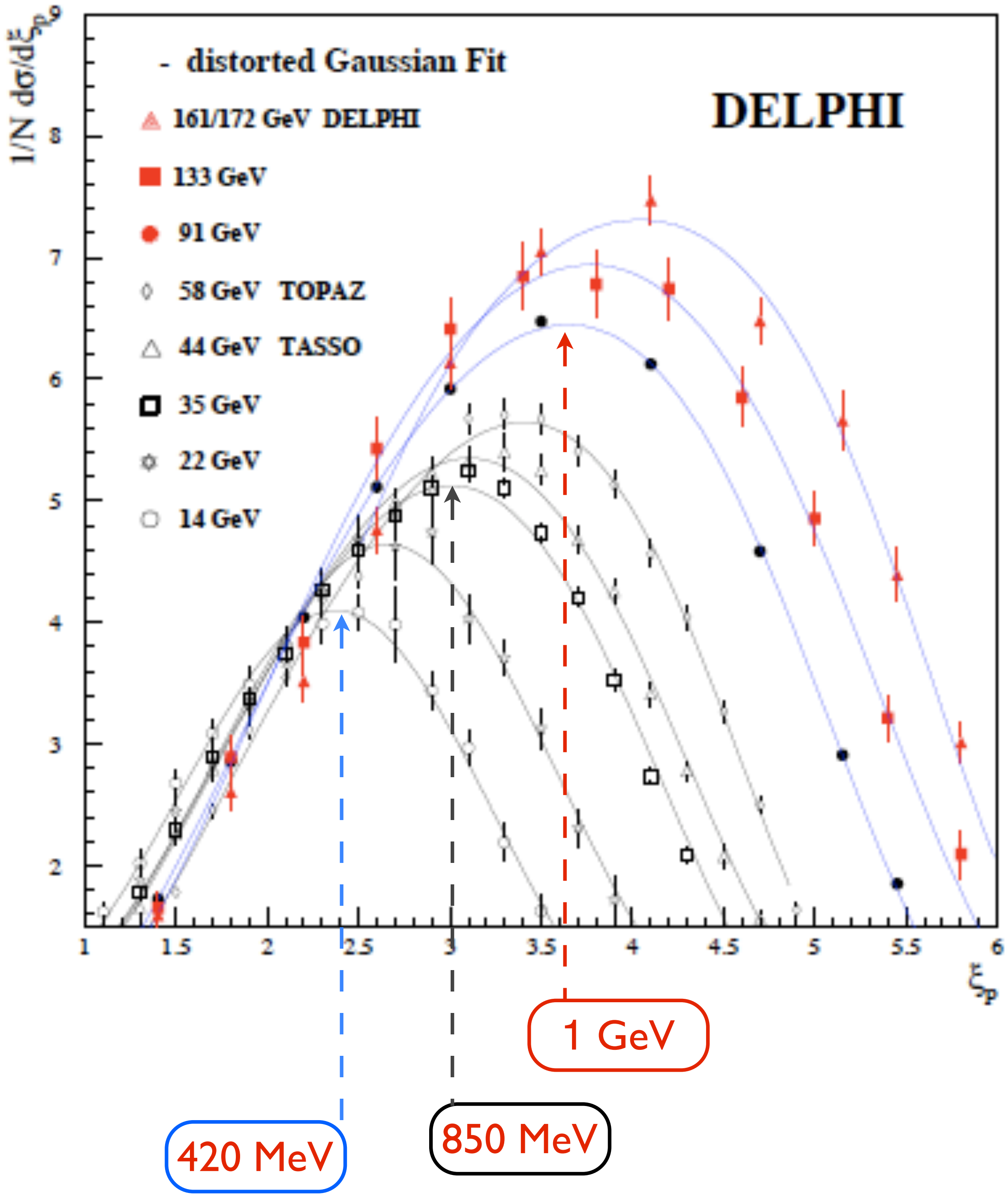
$\phi(1020) \not\to \pi\pi\pi$



This suggests that perturbative corrections are small even in the soft regime.

Soft Physics: hadron production *inside* jets

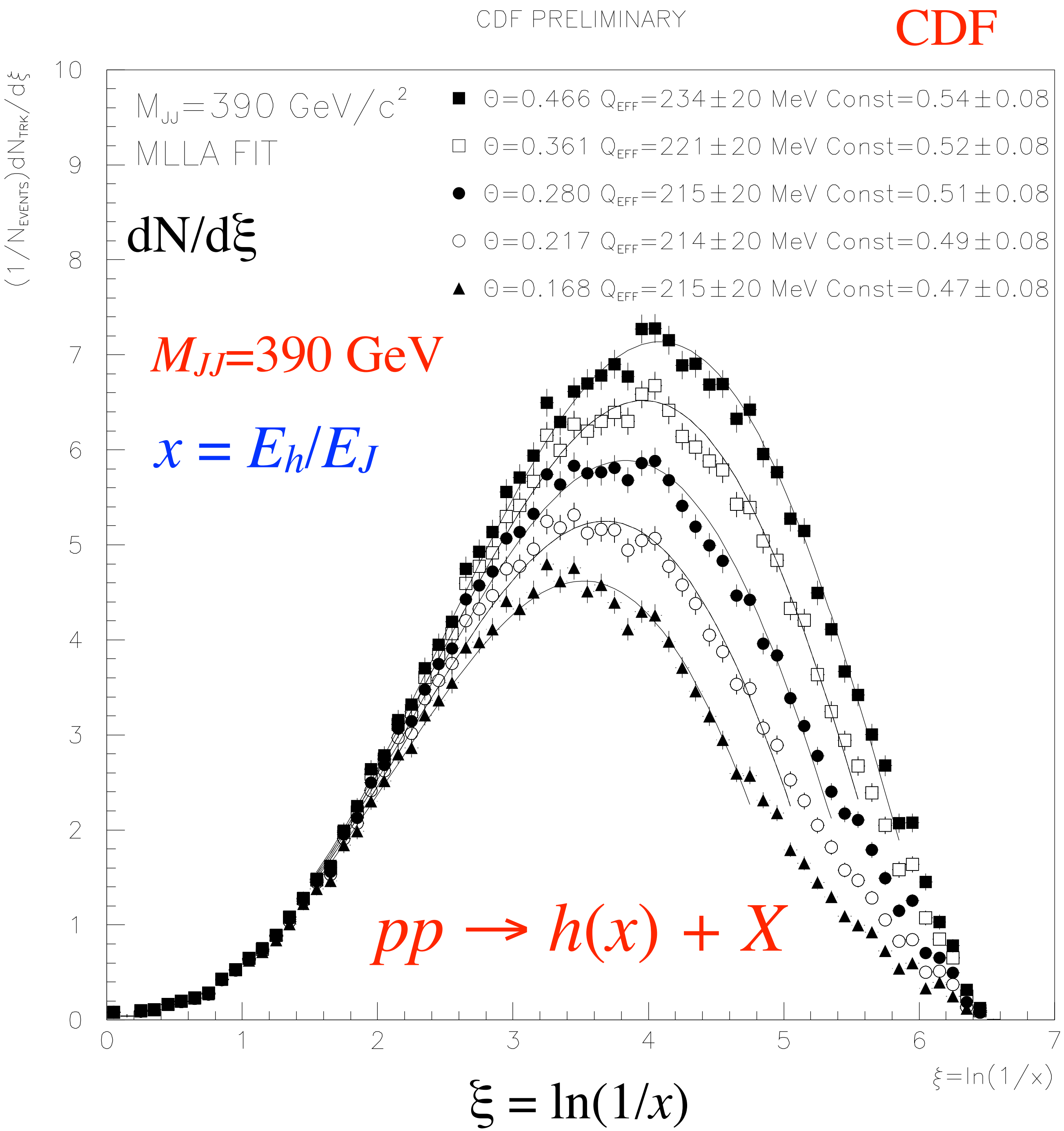
Dokshitzer 2011



$$\ln \frac{1}{x}$$

Dokshitzer (Les Houches 2008)

Hump-backed plateau



First confronted with theory in $e^+e^- \to h+X$.

CDF (Tevatron)

$pp \to 2$ jets

Charged hadron yield as a function of $\ln(1/x)$ for different values of jet hardness, versus (MLLA) QCD prediction.

One free parameter – overall normalization (the number of final π's per extra gluon)