

## QCD Bound States

Hadrons are the only truly relativistic bound states found in Nature. They are unique also in that their quark and gluon constituents do not appear as free particles. Besides confinement, QCD must have spontaneous chiral symmetry breaking in order to describe observed phenomena. Despite these novel features the non-relativistic quark model successfully classifies the hadron spectrum analogously to QED atoms. This makes the study of QCD bound states both tantalizing and challenging [1].

The low-lying states of charmonia and bottomonia consist dominantly of a heavy quark-antiquark pair. Light hadrons have many sea quark and gluon constituents, and their valence quarks contribute only  $\mathcal{O}(1\%)$  to the bound state mass. Data on light hadrons nevertheless has many simple features. Hadron spectra reflect only the valence quark degrees of freedom. There is an early onset of dimensional scaling in fixed angle exclusive scattering. Exclusive form factors are related to inclusive parton distributions via Bloom-Gilman duality [2].

There are interesting and solvable issues related to relativistic bound states that are largely ignored. The wave function of an ordinary Hydrogen atom in relativistic CM motion was calculated only recently [3]. This appears to be one of the first times that the Lorentz contraction of a physical state was studied in quantum field theory. A relativistic electron bound in an external potential is accompanied by virtual pairs (due to “Z-diagrams”). Why can such a state nevertheless be described by the single-particle Dirac wave function? Bound states are treated briefly, if at all, in modern field theory courses.

Tree diagrams generally provide a good first approximation of scattering amplitudes. Bound state poles on the other hand arise from the *divergence* of the perturbative expansion. The “lowest order” approximation then depends on the subset of diagrams one chooses to sum. For non-relativistic bound states the Schrödinger equation is a natural starting point, whereas there are numerous proposals for relativistic bound state equations. An objective choice would be the Born approximation, *i.e.*, the lowest order term in an  $\hbar$  expansion [4, 5]. The  $\hbar$  expansion has surprisingly not been much discussed in a field theory context [6]. There are interesting issues related to the classical limit as well as to the relation between the power of  $\hbar$  and the number of loops, not least in a bound state context.

Atoms are dominantly bound by the Coulomb interaction. The  $A^0$  field does not propagate in time and is instantaneously determined by the charged fields through Gauss’ law – when the boundary condition at spatial infinity is specified. Imposing a non-vanishing asymptotic energy density is interesting for the study of relativistic bound states, since it gives rise to a linear potential which is of lower order in the coupling than gauge boson exchange. Each order of an expansion in  $\alpha$  and  $\hbar$  must be fully Poincaré invariant. Hence Born-level states, bound by a purely linear potential, have equal time wave functions which transform dynamically under boosts such that the bound state energy correctly depends on its CM momentum.

## References

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