7 Big Bang Nucleosynthesis

One quarter (by mass) of the baryonic matter in the universe is helium. Heavier elements make up a few per cent. The rest, i.e., the major part, is hydrogen.

The building blocks of atomic nuclei, the nucleons, or protons and neutrons, formed in the QCD phase transition at $T \sim 150$ MeV and $t \sim 20$ μs. Elements heavier than lithium, up to iron, cobalt, and nickel, have been made from lighter elements by fusion reactions in stars. These reactions provide the energy source for the stars. Elements heavier than these have been formed in supernova explosions. However, the amount of helium and some other light isotopes in the universe cannot be understood by these mechanisms. It turns out that ^{2}H, ^{3}He, ^{4}He, and ^{7}Li were mainly produced already in the big bang, in a process we call *Big Bang Nucleosynthesis* (BBN).

The nucleons and antinucleons annihilated each other soon after the QCD phase transition, and the small excess of nucleons left over from annihilation did not have a significant effect on the expansion and thermodynamics of the universe until much later ($t \sim t_{\mathrm{eq}} = \Omega_m^{-2} h^{-4} 1000$ a), when the universe became matter-dominated. The ordinary matter in the present universe comes from this small excess of nucleons. Let us now consider what happened to it in the early universe. We shall focus on the period when the temperature fell from $T \sim 10$ MeV to $T \sim 10$ keV ($t \sim 10$ ms – few h).
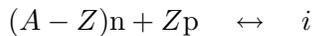
7.1 Equilibrium

The total number of nucleons stays constant due to baryon number conservation. This baryon number can be in the form of protons and neutrons or atomic nuclei. Weak nuclear reactions may convert neutrons and protons into each other and strong nuclear reactions may build nuclei from them.

During the period of interest the nucleons and nuclei are nonrelativistic ($T \ll m_p$). Assuming thermal equilibrium we have

$$n_i = g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{\frac{\mu_i - m_i}{T}} \tag{1}$$

for the number density of nucleus type i. If the nuclear reactions needed to build nucleus i (with mass number A and charge Z) from the nucleons,

$$(A - Z)\mathrm{n} + Z\mathrm{p} \quad \leftrightarrow \quad i$$

occur at sufficiently high rate to maintain chemical equilibrium, we have

$$\mu_i = (A - Z)\mu_{\mathrm{n}} + Z\mu_{\mathrm{p}} \tag{2}$$

for the chemical potentials. Since for free nucleons

$$
\begin{aligned}
n_{\mathrm{p}} &= 2 \left(\frac{m_{\mathrm{p}} T}{2\pi} \right)^{3/2} e^{\frac{\mu_{\mathrm{p}} - m_{\mathrm{p}}}{T}} \\
n_{\mathrm{n}} &= 2 \left(\frac{m_{\mathrm{n}} T}{2\pi} \right)^{3/2} e^{\frac{\mu_{\mathrm{n}} - m_{\mathrm{n}}}{T}},
\end{aligned}
\tag{3}
$$

we can express n_i in terms of the neutron and proton densities,

$$n_i = g_i A^{\frac{3}{2}} 2^{-A} \left(\frac{2\pi}{m_N T} \right)^{\frac{3}{2}(A-1)} n_p^Z n_n^{A-Z} e^{B_i/T}, \tag{4}$$

where

$$B_i \equiv Z m_p + (A - Z) m_n - m_i \tag{5}$$

is the binding energy of the nucleus. Here we have approximated $m_p \approx m_n \approx m_i/A$ outside the exponent, and denoted it by m_N ("nucleon mass").

$^A Z$	B	g
^{2}H	2.22 MeV	3
^{3}H	8.48 MeV	2
^{3}He	7.72 MeV	2
^{4}He	28.3 MeV	1
^{12}C	92.2 MeV	1

Table 1. Some of the lightest nuclei and their binding energies.

The different number densities add up to the total baryon number density

$$\sum A_i n_i = n_B. \tag{6}$$

The baryon number density n_B we get from the photon density

$$n_\gamma = \frac{2}{\pi^2} \zeta(3) T^3 \tag{7}$$

and the baryon/photon -ratio

$$\frac{n_B}{n_\gamma} = \frac{g_{*s}(T)}{g_{*s}(T_0)} \eta \tag{8}$$

as

$$n_B = \frac{g_{*s}(T)}{g_{*s}(T_0)} \eta \frac{2}{\pi^2} \zeta(3) T^3. \tag{9}$$

After electron-positron annihilation $g_{*s}(T) = g_{*s}(T_0)$ and $n_B = \eta n_\gamma$. Here η is the *present baryon/photon ratio.* It can be estimated from various observations in a number of ways. It's order of magnitude is 10^{-9}.

For temperatures $m_N \gg T \gtrsim B_i$, we have that
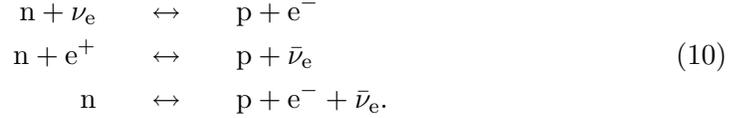
$$(m_N T)^{3/2} \gg T^3 \gg n_B > n_p, n_n$$

and thus Eq. (4) implies that

$$n_i \ll n_p, n_n$$

for $A > 1$. Thus we have initially only free neutrons and protons in large numbers.

7.2   Neutron-proton ratio

What can we say about n_{p} and n_{n}? Protons and neutrons are converted into each other in the weak reactions

$$
\begin{aligned}
\mathrm{n} + \nu_{\mathrm{e}} &\leftrightarrow \mathrm{p} + \mathrm{e}^{-} \\
\mathrm{n} + \mathrm{e}^{+} &\leftrightarrow \mathrm{p} + \bar{\nu}_{\mathrm{e}} \\
\mathrm{n} &\leftrightarrow \mathrm{p} + \mathrm{e}^{-} + \bar{\nu}_{\mathrm{e}}.
\end{aligned}
\tag{10}
$$

If these reactions are in equilibrium, $\mu_{\mathrm{n}} + \mu_{\nu_{\mathrm{e}}} = \mu_{\mathrm{p}} + \mu_{\mathrm{e}}$, and the neutron/proton ratio is

$$
\frac{n_n}{n_p} \equiv \frac{n}{p} = e^{-Q/T + (\mu_{\mathrm{e}} - \mu_{\nu_{\mathrm{e}}})/T},
\tag{11}
$$

where $Q \equiv m_{\mathrm{n}} - m_{\mathrm{p}} = 1.293$ MeV.

   We need now some estimate for the chemical potentials of electrons and electron neutrinos. The universe is electrically neutral, so the number of electrons (or $n_{\mathrm{e}^-} - n_{\mathrm{e}^+}$) equals the number of protons, and μ_e can be calculated exactly in terms of η and T. We leave the exact calculation as an exercise, but give below a rough estimate for the ultrarelativistic limit ($T > m_{\mathrm{e}}$):

   In the ultrarelativistic limit

$$
n_{\mathrm{e}^-} - n_{\mathrm{e}^+} = \frac{2T^3}{6\pi^2}\left(\pi^2\left(\frac{\mu_{\mathrm{e}}}{T}\right) + \left(\frac{\mu_{\mathrm{e}}}{T}\right)^3\right) = n_{\mathrm{p}}^* \approx n_{\mathrm{B}} \approx \eta n_\gamma = \eta\frac{2}{\pi^2}\zeta(3)T^3.
\tag{12}
$$

Here n_p^* includes the protons inside nuclei. Since η is small, $\mu \ll T$, and we can drop the $(\mu/T)^3$ term to get

$$
\frac{\mu_{\mathrm{e}}}{T} \approx \frac{6}{\pi^2}\zeta(3)\eta.
\tag{13}
$$

Thus $\mu_{\mathrm{e}}/T \sim \eta \sim 10^{-9}$. The nonrelativistic limit can be done in a similar manner. It turns out that μ_e rises as T falls, and somewhere between $T = 30$ keV and $T = 10$ keV μ_e becomes larger than T, and, in fact, comparable to m_e.

   For $T \gtrsim 30$ keV, $\mu_e \ll T$, and we can drop the μ_e in Eq. (11).

   Since we cannot detect the cosmic neutrino background, we don't know the neutrino chemical potentials. Usually it is *assumed* that also $\mu_\nu \ll T$, so that the difference in the number of neutrinos and antineutrinos is small. Thus we ignore both μ_e and μ_ν and get that the equilibrium neutron/proton ratio is

$$
\frac{n}{p} = e^{-Q/T}.
\tag{14}
$$

(This is not valid for $T \lesssim 30$ keV, since μ_e is no longer small, but we shall use this formula only at higher temperatures as will be seen below.)

   Thus we can express the number densities of all nuclei in terms of the free proton number density n_p, as long as chemical equilibrium holds.

7.3   Bottlenecks

We define the *mass fraction* of nucleus i as

$$
X_i \equiv \frac{A_i n_i}{n_{\mathrm{B}}}.
\tag{15}
$$

Since

$$n_{\mathrm{B}} = \sum_i A_i n_i, \tag{16}$$

(where the sum includes protons and neutrons), we have $\sum X_i = 1$.

   Using also the normalization condition,[1] Eq. (16), or $\sum_i X_i = 1$, we get all equilibrium abundances as a function of T (they also depend on the value of the parameter η). There are two items to note, however:

1. The normalization condition, Eq. (16), includes all nuclei up to uranium etc. Thus we would get a huge polynomial equation from which to solve X_p.

2. In practice we don't have to care about the first item, since as the temperature falls the nuclei no longer follow their equilibrium abundances. The reactions are in equilibrium only at high temperatures, when the other equilibrium abundances except X_{p} and X_{n} are small, and we can use the approximation $X_{\mathrm{n}} + X_{\mathrm{p}} = 1$.

   In the early universe the baryon density is too low and the time available is too short for reactions involving three or more incoming nuclei to occur in any appreciable rate. The heavier nuclei have to be built sequentially from lighter nuclei in two-particle reactions, so that deuterium is formed first in the reaction

$$\mathrm{n} + \mathrm{p} \quad \rightarrow \quad \mathrm{d} + \gamma.$$
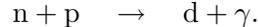
Only when deuterons are available can helium nuclei be formed, and so on. This process has "bottlenecks": the lack of sufficient densities of lighter nuclei hinders the production of heavier nuclei, and prevents them from following their equilibrium abundances.

   As the temperature falls, the equilibrium abundances rise fast. They become large later for nuclei with small binding energies. Since deuterium is formed directly from neutrons and protons it can follow its equilibrium abundance as long as there are large numbers of free neutrons available. Since the deuterium binding energy is rather small, the deuterium abundance becomes large rather late (at $T < 100$ keV). Therefore heavier nuclei with larger binding energies, whose equilibrium abundances would become large earlier, cannot be formed. This is the *deuterium bottleneck*. Only when there is lots of deuterium ($X_{\mathrm{d}} \sim 10^{-3}$), can helium be produced in large numbers.

   The nuclei are positively charged and there is thus an electromagnetic repulsion between them. The nuclei need thus large kinetic energies to overcome this *Coulomb barrier* and get within the range of the strong interaction. Thus the cross sections for these fusion reactions fall rapidly with energy and the nuclear reactions are "shut off" when the temperature falls below $T \sim 30$ keV. Thus there is less than one hour available for nucleosynthesis. Because of additional bottlenecks (e.g., there are no stable nuclei with $A = 8$) and the short time available, only very small amounts of elements heavier than helium are formed.

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[1] For n_{p} and n_{n} we know just their ratio, since we do not know μ_{p} and μ_{n}, only that $\mu_{\mathrm{p}} = \mu_{\mathrm{n}}$. Therefore this extra equation is needed to solve all n_i.

7.4   Calculation of the helium abundance

Let us now calculate the numbers. For $T > 0.1$ MeV, we still have $X_n + X_p \approx 1$, so the equilibrium abundances are

$$X_n = \frac{e^{-Q/T}}{1 + e^{-Q/T}} \qquad \text{and} \qquad X_p = \frac{1}{1 + e^{-Q/T}}. \tag{17}$$

Nucleons follow these equilibrium abundances until neutrinos decouple at $T \sim 0.8$ MeV, shutting off the weak $n \leftrightarrow p$ reactions. After this the neutrons decay, so that

$$X_n(t) = X_n(t_1)e^{-(t-t_1)/\tau_n}, \tag{18}$$

where $\tau_n = 887 \pm 2$ s is the mean lifetime of a free neutron. (The half-life is $\tau_{1/2} = (\ln 2)\tau_n$.) In reality, the decoupling and thus the shift from behavior (17) to behavior (18) is not instantaneous, but an approximation where one takes it to be instantaneous at time t_1 when $T = 0.8$ MeV, so that $X_n(t_1) = 0.1657$, gives a fairly accurate final result.

The equilibrium mass fractions are, from Eq. (4),

$$X_i = \frac{1}{2}X_p^Z X_n^{A-Z} g_i A^{\frac{5}{2}} \epsilon^{A-1} e^{B_i/T} \tag{19}$$

where

$$\epsilon \equiv \frac{1}{2}\left(\frac{2\pi}{m_N T}\right)^{3/2} n_B = \frac{1}{\pi^2}\zeta(3)\left(\frac{2\pi T}{m_N}\right)^{3/2}\frac{g_{*s}(T)}{g_{*s}(T_0)}\eta \sim \left(\frac{T}{m_N}\right)^{3/2}\eta.$$

The factors which change rapidly with T are $\epsilon^{A-1}e^{B_i/T}$. For temperatures $m_N \gg T \gg B_i$ we have $e^{B_i/T} \sim 1$ and $\epsilon \ll 1$. Thus $X_i \ll 1$ for others ($A > 1$) than protons and neutrons. As temperature falls, ϵ becomes even smaller and at $T \sim B_i$ we have $X_i \ll 1$ still. The temperature has to fall below B_i by a large factor before the factor $e^{B_i/T}$ wins and the equilibrium abundance becomes large.

Deuterium has $B_d = 2.22$ MeV, and we get $\epsilon e^{B_d/T} = 1$ at $T_d = 0.06$ MeV–0.07 MeV (assuming $\eta = 10^{-10} - 10^{-8}$), so the deuterium abundance becomes large near this temperature. Since ^{4}He has a much higher binding energy, $B_4 = 28.3$ MeV, the corresponding situation $\epsilon^3 e^{B_4/T} = 1$ occurs at a higher temperature $T_4 \sim 0.3$ MeV. But we noted earlier that only deuterium stays close to its equilibrium abundance once it gets large. Helium begins to form only when there is sufficient deuterium available, in practice slightly above T_d. Helium forms then rapidly. The available number of neutrons sets an upper limit to ^{4}He production. Since helium has the highest binding energy per nucleon (of all isotopes below A=12), almost all neutrons end up in ^{4}He, and only small amounts of the other light isotopes, ^{2}H, ^{3}H, ^{3}He, ^{7}Li, and ^{7}Be, are produced.

The Coulomb barrier shuts off the nuclear reactions before there is time for heavier nuclei ($A > 8$) to form. One gets a fairly good approximation for the ^{4}He production by assuming instantaneous nucleosynthesis at $T = T_{ns} \sim 1.1 T_d \sim 70$ keV, with all neutrons ending up in ^{4}He, so that

$$X_4 \approx 2X_n(T_{ns}). \tag{20}$$

After electron annihilation ($T \ll m_\mathrm{e} = 0.511$ MeV) the time-temperature relation is

$$t = 0.301 g_*^{-1/2} \frac{m_\mathrm{Pl}}{T^2}, \tag{21}$$

where $g_* = 3.363$. Since most of the time in $T = 0.8$ MeV–0.07 MeV is spent at the lower part of this temperature range, this formula gives a good approximation for the time

$$t_\mathrm{ns} - t_1 = 266.5 \text{ s} \qquad \text{(in reality 264.3 s).}$$

Thus we get for the final ^{4}He abundance

$$X_4 = 2X_\mathrm{n}(t_1)e^{-(t_\mathrm{ns}-t_1)/\tau_\mathrm{n}} = 24.5 \text{ \%.} \tag{22}$$

Accurate numerical calculations, using the reaction rates of the relevant weak and strong reaction rates give $X_4 = 21$–26 % (for $\eta = 10^{-10} - 10^{-9}$).

As a calculation of the helium abundance X_4 the preceding calculation is of course a cheat, since we have used the results of those accurate numerical calculations to infer that we need to use $T = 0.8$ MeV as the neutrino decoupling temperature, and $T_\mathrm{ns} = 1.1T_\mathrm{d}$ as the "instantaneous nucleosynthesis" temperature, to best approximate the correct behavior. However, it gives us a quantitative description of what is going on, and an understanding of how the helium yield depends on various things.

Exercise: Using the preceding calculation, find the dependence of X_4 on η, i.e., calculate $dX_4/d\eta$.
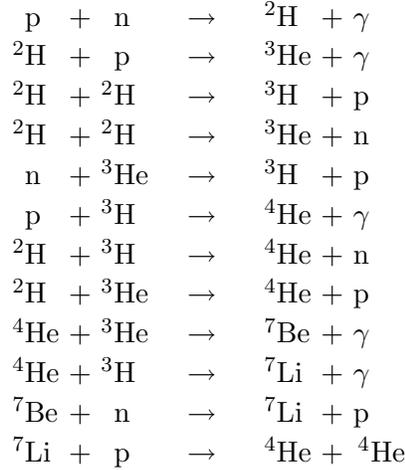
7.5   Why so late?

Let us return to the question, why the temperature has to fall so much below the binding energy before the equilibrium abundances become large. From the energetics one might conclude that when typical kinetic energies, $\langle E_k \rangle \approx \frac{3}{2}T$, are smaller than the binding energy, it would be easy to form nuclei but difficult to break them. Above we saw that the smallness of the factor $\epsilon \sim (T/m_\mathrm{N})^{3/2}\eta$ is the reason why this is not so. Here $\eta \sim 10^{-9}$ and $(T/m_\mathrm{N})^{3/2} \sim 10^{-6}$ (for $T \sim 0.1$ MeV). The main culprit is thus the small baryon/photon ratio. Since there are 10^9 photons for each baryon, there is a sufficient amount of photons who can disintegrate a nucleus in the high-energy tail of the photon distribution, even at rather low temperatures. One can also express this result in terms of entropy. A high photon/baryon ratio corresponds to a high entropy per baryon. High entropy favors free nucleons.

7.6   The most important reactions

In reality, neither neutrino decoupling, nor nucleosynthesis, are instantaneous processes. Accurate results require a rather large numerical computation where one uses the cross sections of all the relevant weak and strong interactions. These cross sections are energy-dependent. Integrating them over the energy and velocity distributions and multiplying with the relevant number densities leads to temperature-dependent reaction rates. The most important reactions are the weak $n \leftrightarrow p$ reactions (10) and the following strong reactions[2](see also Fig. 1):

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[2]The reaction chain which produces helium from hydrogen in BBN is not the same that occurs in stars. The conditions is stars are different: there are no free neutrons and the temperatures are

$$
\begin{array}{rcl}
\text{p} + \text{n} & \rightarrow & {}^2\text{H} + \gamma \\
{}^2\text{H} + \text{p} & \rightarrow & {}^3\text{He} + \gamma \\
{}^2\text{H} + {}^2\text{H} & \rightarrow & {}^3\text{H} + \text{p} \\
{}^2\text{H} + {}^2\text{H} & \rightarrow & {}^3\text{He} + \text{n} \\
\text{n} + {}^3\text{He} & \rightarrow & {}^3\text{H} + \text{p} \\
\text{p} + {}^3\text{H} & \rightarrow & {}^4\text{He} + \gamma \\
{}^2\text{H} + {}^3\text{H} & \rightarrow & {}^4\text{He} + \text{n} \\
{}^2\text{H} + {}^3\text{He} & \rightarrow & {}^4\text{He} + \text{p} \\
{}^4\text{He} + {}^3\text{He} & \rightarrow & {}^7\text{Be} + \gamma \\
{}^4\text{He} + {}^3\text{H} & \rightarrow & {}^7\text{Li} + \gamma \\
{}^7\text{Be} + \text{n} & \rightarrow & {}^7\text{Li} + \text{p} \\
{}^7\text{Li} + \text{p} & \rightarrow & {}^4\text{He} + {}^4\text{He}
\end{array}
$$

The cross sections of these strong reactions can't be calculated from first principles, i.e., from QCD, since QCD is too difficult. Instead one uses cross sections measured in laboratory. The cross sections of the weak reactions (10) are known theoretically (there is one parameter describing the strength of the weak interaction, which is determined experimentally, in practice by measuring the lifetime τ_n of free neutrons). The relevant reaction rates are now known sufficiently accurately, so that the nuclear abundances produced in BBN (for a given value of η) can be calculated with better accuracy than the present abundances can be measured from astronomical observations.

The reaction chain proceeds along stable and long-lived (compared to the nucleosynthesis timescale—minutes) isotopes towards larger mass numbers. At least
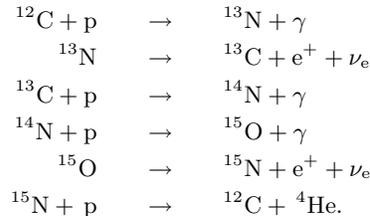
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lower, but the densities are higher and there is more time available. In addition, second generation stars contain heavier nuclei (C,N,O) which can act as catalysts in helium production. Some of the most important reaction chains in stars are [Karttunen et al: Fundamental Astronomy, s. 251] :

1. The proton-proton chain

$$
\begin{array}{rcl}
\text{p} + \text{p} & \rightarrow & {}^2\text{H} + \text{e}^+ + \nu_\text{e} \\
{}^2\text{H} + \text{p} & \rightarrow & {}^3\text{He} + \gamma \\
{}^3\text{He} + {}^3\text{He} & \rightarrow & {}^4\text{He} + \text{p} + \text{p,}
\end{array}
$$

2. and the CNO-chain

$$
\begin{array}{rcl}
{}^{12}\text{C} + \text{p} & \rightarrow & {}^{13}\text{N} + \gamma \\
{}^{13}\text{N} & \rightarrow & {}^{13}\text{C} + \text{e}^+ + \nu_\text{e} \\
{}^{13}\text{C} + \text{p} & \rightarrow & {}^{14}\text{N} + \gamma \\
{}^{14}\text{N} + \text{p} & \rightarrow & {}^{15}\text{O} + \gamma \\
{}^{15}\text{O} & \rightarrow & {}^{15}\text{N} + \text{e}^+ + \nu_\text{e} \\
{}^{15}\text{N} + \text{p} & \rightarrow & {}^{12}\text{C} + {}^4\text{He.}
\end{array}
$$

The cross section of the direct reaction d+d $\rightarrow$ ${}^4\text{He} + \gamma$ is small (i.e., the ${}^3\text{H} + \text{p}$ and ${}^3\text{He} + \text{n}$ channels dominate d+d $\rightarrow$), and it is not important in either context.

The triple-α reaction ${}^4\text{He} + {}^4\text{He} + {}^4\text{He} \rightarrow {}^{12}\text{C}$, responsible for carbon production in stars, is also not important during big bang, since the density is not sufficiently high for three-particle reactions to occur (the three ${}^4\text{He}$ nuclei would need to come within the range of the strong interaction within the lifetime of the intermediate state, ${}^8\text{Be}$, 2.6×10^{-16} s). (Exercise: calculate the number and mass density of nucleons at $T = 1$ MeV.)
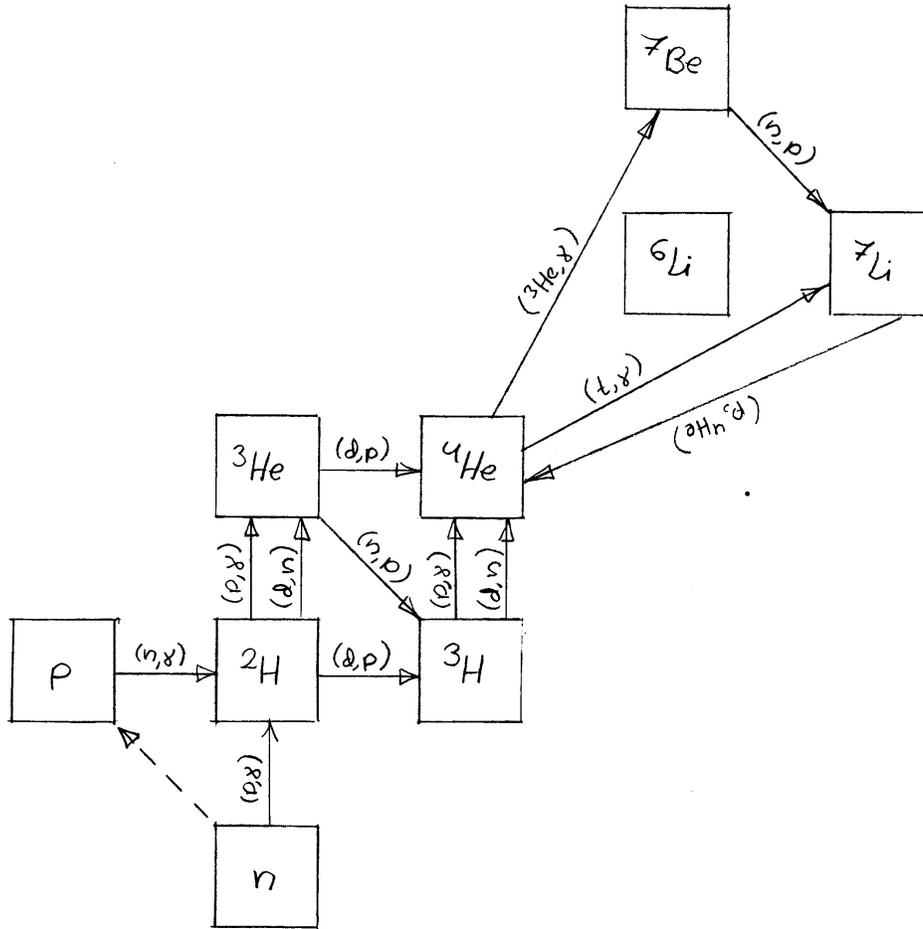
Figure 1: The 12 most important nuclear reactions in big bang nucleosynthesis.

one of the two incoming nuclei must be an isotope which is abundant during nucleosynthesis, i.e., n, p, ^{2}H or ^{4}He. The mass numbers $A = 5$ and $A = 8$ form bottlenecks, since they have no stable or long-lived isotopes. The $A = 5$ bottleneck is crossed with the reactions ^{4}He+^{3}He and ^{4}He+^{3}H, which form a small number of ^{7}Be and ^{7}Li. Their abundances remain so small that we can ignore the reactions (e.g., ^{7}Be + ^{4}He → ^{11}C + γ and ^{7}Li + ^{4}He → ^{11}B) which cross the $A = 8$ bottleneck. Numerical calculations also show that the production of the other stable lithium isotope, ^{6}Li is several orders of magnitude smaller than that of ^{7}Li.

Thus BBN produces the isotopes ^{2}H, ^{3}H, ^{3}He,^{4}He, ^{7}Li and ^{7}Be. Of these, ^{3}H (half life 12.3 a) and ^{7}Be (53 d) are unstable and decay after nucleosynthesis into ^{3}He and ^{7}Li. (^{7}Be actually becomes ^{7}Li through electron capture ^{7}Be + e$^-$ → ^{7}Li + $\nu_{\rm e}$.)

In the end BBN has produced cosmologically significant (compared to present abundances) amounts of the four isotopes, ^{2}H, ^{3}He, ^{4}He and ^{7}Li (the fifth isotope ^{1}H=p we had already before BBN). Their production in the BBN can be calculated,
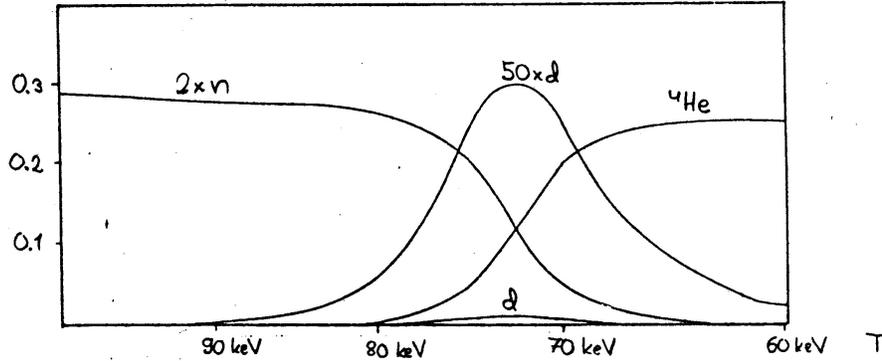
Figure 2: The time evolution of the n, ^{2}H (written as d) and ^{4}He abundances during BBN. Notice how the final ^{4}He abundance is determined by the n abundance before nuclear reactions begin. Only a small part of the neutrons decay or end up in other nuclei. Before becoming ^{4}He, all neutrons pass through ^{2}H. To improve the visibility of the deuterium curve, we have plotted it also as multiplied by a factor of 50. The other abundances (except p) remain so low, that to see them the figure should be redrawn in logarithmic scale (see Fig. 4.3 of Kolb&Turner. This Figure is for $\eta = 6 \times 10^{-10}$. The time at $T = (90, 80, 70, 60)$ keV is $(152, 199, 266, 367)$ s. Thus the action peaks at about $t = 4$ min.

and there is only one free parameter, the baryon/photon ratio

$$\eta \equiv \frac{n_{\mathrm{B}}}{n_\gamma} = \frac{\Omega_b \rho_c}{m_N n_\gamma} = \frac{\Omega_b}{m_n n_\gamma} \frac{3H_0^2}{8\pi G}$$

$$= 274 \times 10^{-10} \omega_b = 1.46 \times 10^{18} \left(\frac{\rho_{b0}}{\mathrm{kgm}^{-3}} \right). \tag{23}$$

Here ρ_{b0} is the average density of ordinary, or baryonic, matter today, $\Omega_b \equiv \rho_{b0}/\rho_c$ is the baryonic density parameter, and $\omega_b \equiv \Omega_b h^2$.

7.7   BBN as a function of time

Let us follow nucleosynthesis as a function of time (or temperature). See Fig. 4.3 in Kolb&Turner or Fig. 2 here.

^{2}H and ^{3}H are intermediate states, through which the reactions proceed towards ^{4}He. Therefore their abundance first rises, is highest at the time when ^{4}He production is fastest, and then falls as the baryonic matter ends in ^{4}He. ^{3}He is also an intermediate state, but the main channel from ^{3}He to ^{4}He is via ^{3}He+n→^{3}H+p , which is extinguished early as the free neutrons are used up. Therefore the abundance of ^{3}He does not fall the same way as ^{2}H and ^{3}H. The abundance of ^{7}Li also rises at first and then falls via ^{7}Li+p→^{4}He+^{4}He. Since ^{4}He has a higher binding energy per nucleon, B/A, than ^{7}Li and ^{7}Be have, these also want to return into ^{4}He. This does not happen to ^{7}Be, however, since, just like for ^{3}He, the free neutrons needed for the reaction ^{7}Be+n→^{4}He+^{4}He have almost disappeared near the end.

	B(MeV)	**B/A**
^{2}H	2.2245	1.11
^{3}H	8.4820	2.83
^{3}He	7.7186	2.57
^{4}He	28.2970	7.07
^{6}Li	31.9965	5.33
^{7}Li	39.2460	5.61
^{7}Be	37.6026	5.37
^{12}C	92.1631	7.68
^{56}Fe	492.2623	8.79

7.8   Primordial abundances as a function of the baryon-to-photon ratio

Let us then consider BBN as a function of η (see Fig. 3). The greater is η, the higher is the number density of nucleons. The reaction rates are faster and the nucleosynthesis can proceed further. This mean that a smaller fraction of "intermediate nuclei", ^{2}H, ^{3}H, and ^{7}Li are left over— the burning of nuclear matter into ^{4}He is "cleaner". Also the ^{3}He production falls with increasing η. However, ^{7}Be production increases with η. In the figure we have plotted the final BBN yields, so that ^{3}He is the sum of ^{3}He and ^{3}H, and ^{7}Li is the sum of ^{7}Li and ^{7}Be. The complicated shape of the ^{7}Li(η) curve is due to these two contributions: 1) For small η we get lots of "direct" ^{7}Li, whereas 2) for large η there is very little "direct" ^{7}Li left, but a lot of ^{7}Be is produced. In the middle, at $\eta \sim 3 \times 10^{-10}$, there is a minimum of ^{7}Li production where neither way is very effective.

The ^{4}He production increases with η, since with higher density nucleosynthesis begins earlier when there are more neutrons left.

7.9   Comparison with observations

The abundances of the various isotopes calculated from BBN can be compared to the observed abundances of these elements. This is one of the most important tests of the big bang theory. A good agreement is obtained for η in the range $\eta = 1.5$–6×10^{-10}. This has, in fact, been the best method to estimate the amount of ordinary matter in the universe, until the advent of the WMAP CMB data in 2003.[3]

The comparison of calculated abundances with observed abundances is complicated due to *chemical evolution*. The abundances produced in BBN are the *primordial* abundances of these isotopes. The first stars form with this composition. In stars, further fusion reactions take place and the composition of the star changes with time. Towards the end of its lifetime, the star ejects its outer parts into interstellar space, and this processed material mixes with primordial material. From this mixed material later generation stars form, and so on.

The observations of present abundances are based on spectra of interstellar clouds and stellar surfaces. To obtain the primordial abundances from the present abundances the effect of chemical evolution has to be estimated. Since ^{2}H is so fragile

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[3]Many cosmological parameters can be estimated from the CMB anisotropy, as will be discussed later. The WMAP estimate is $\eta = 6.1^{+0.3}_{-0.2} \times 10^{-10}$.
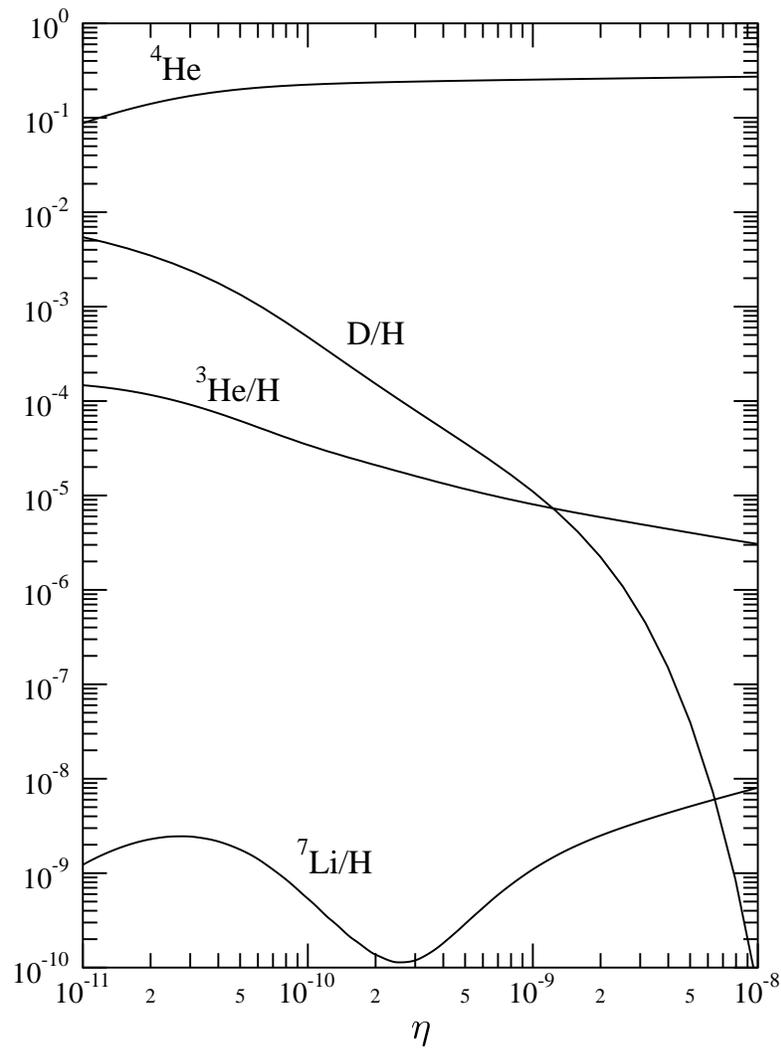
Figure 3: The primordial abundances of the light elements as a function of η. For ^{4}He we give the mass fraction, for D = ^{2}H, ^{3}He, and ^{7}Li the number ratio to H = ^{1}H, i.e., n_i/n_{H}.

(it's binding energy is so low), there is hardly any ^{2}H production in stars, rather any pre-existing ^{2}H is destroyed early on in stars. Therefore any interstellar ^{2}H is primordial. The smaller the fraction of processed material in an interstellar cloud, the higher its ^{2}H abundance should be. Thus all observed ^{2}H abundances are lower limits to the primordial ^{2}H abundance.[4] Conversely, stellar production increases the ^{4}He abundance. Thus all ^{4}He observations are upper limits to the primordial ^{4}He. Moreover, stellar processing produces heavier elements, e.g., C, N, O, which are not produced in the BBN. Their abundance varies a lot from place to place, giving a measure of how much chemical evolution has happened in various parts of the universe. Plotting ^{4}He vs. these heavier elements one can extrapolate the ^{4}He abundance to zero chemical evolution to obtain the primordial abundance. Since ^{3}He and ^{7}Li are both produced and destroyed in stellar processing, it is more difficult to make estimates of their primordial abundances based on observed present abundances.

Qualitatively, one can note two clear signatures of big bang in the present universe:

1. All stars and gas clouds observed contain at least 23% ^{4}He. If all ^{4}He had been produced in stars, we would see similar variations in the ^{4}He abundance as we see, e.g., for C, N, and O, with some regions containing just a few % or even less ^{4}He. This universal minimum amount of ^{4}He must signify a primordial abundance produced when matter in the universe was uniform.

2. The existence of significant amounts of ^{2}H in the universe is a sign of BBN, since there are no other known astrophysical sources of large amounts of ^{2}H.

Quantitatively, the observed abundances of all the BBN isotopes, ^{2}H, ^{3}He, ^{4}He and ^{7}Li point towards the range $\eta = 1.5$—6×10^{-10}. Since ^{2}H has the steepest dependence on η it provides potentially the best way to determine η more accurately. The most promising ^{2}H observations for this purpose have been from the absorption spectra of distant (high-z) quasars. This absorption is due to gas clouds that lie on the line-of-sight between us and the quasar. Some of these clouds lie also at a high redshift. Thus we observe them as they were when the universe was rather young, and therefore little chemical evolution had yet taken place. These absorption spectra tend to be complicated due to many intervening clouds, and therefore the deuterium abundance has been reliably determined only for a few such objects. These measurements point towards the higher end of the above range, to $\eta = 5$–6×10^{-10}. This is today the favored value for η, but there is enough controversy among the observers about the detailed interpretation of various measurements, so that it is best to take this tighter range as just tentative, as a BBN prediction.[5]

The wider range $\eta = 1.5$—6×10^{-10} corresponds to $\Omega_b h^2 = 3.65 \times 10^7 \eta = 0.0055$–0.022. With $h = 0.5$–0.85, this gives $\Omega_b = 0.008$–0.09, or roughly

$$\Omega_b = 10^{-2} \dots 10^{-1} \tag{24}$$

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[4]This does not apply to sites which have been enriched in ^{2}H due to a separation of ^{2}H from ^{1}H. Deuterium binds into molecules more easily than ordinary hydrogen. Since deuterium is heavier than ordinary hydrogen, deuterium and deuterated molecules have lower thermal velocities and do not escape from gravity as easily. Thus planets tend to have high deuterium-to-hydrogen ratios.

[5]However, it has now been confirmed by the WMAP results on CMB anisotropy, from which one can determine the baryon-to-photon ratio η at the time of recombination.

for a conservative range of the baryonic density parameter. The presently favored values, $\eta = 5\text{--}6 \times 10^{-10}$ and $h = 0.7 \pm 0.07$, give

$$\Omega_b = 0.031\text{--}0.055. \tag{25}$$

This is less than cosmological estimates for Ω_m. Therefore not all matter can be baryonic. In fact, most of the matter in the universe appears to be nonbaryonic dark matter. This is discussed in the next Chapter.