CMB PHYSICS Homework 1 Fall 2007

Due on Tuesday, September 11th, by 14:00 o'clock. Return your homework into the metal "mailbox" marked "CMB Physics" in hall A201.

1. **Scale factor and conformal time.** Find the scale factor and comoving Hubble parameter (of the flat FRW universe) as a function of conformal time, $a(\eta)$ and $\mathcal{H}(\eta)$, for

   (a) a matter-dominated universe, $w = 0$

   (b) a radiation-dominated universe, $w = \frac{1}{3}$

   (c) a vacuum-dominated universe, $w = -1$.

   What is the conformal age of the matter-dominated universe at the time when its Hubble constant has the value $H_0 = 70$ km/s/Mpc? What is the age in ordinary (cosmic) time?

2. **Matter+radiation background universe.** Find the scale factor and comoving Hubble parameter as a function of conformal time, $a(\eta)$ and $\mathcal{H}(\eta)$, for the flat FRW universe containing matter ρ_m and radiation ρ_r. Use the following notation: The scale factor and conformal time at the moment of matter-radiation equality, $\rho_m = \rho_r$, are denoted by $a_{\rm eq}$ and $\eta_{\rm eq}$. We then have $y \equiv a/a_{\rm eq} = \rho_m/\rho_r$, and $y_0 = \Omega_m/\Omega_r$. Give your answer as y and $\mathcal{H}$ in terms of η and $\eta_{\rm eq}$, and express $\eta_{\rm eq}$ in terms of the present values of cosmological parameters H_0, Ω_m and Ω_r. From the above follows that $\Omega_m + \Omega_r = 1$, but consider also the case where the universe has also vacuum energy, so that $\Omega_m + \Omega_r = 1 - \Omega_\Lambda$, but we are looking for $a(\eta)$ and $\mathcal{H}(\eta)$ only for the earlier times when vacuum energy can be ignored. Give the numerical value of $\eta_{\rm eq}$ for $H_0 = 70$ km/s/Mpc, $\Omega_m = 0.3$, and $y_0 = 3000$. Give the numerical value of $\eta_{\rm eq}$ and η_0 for $H_0 = 70$ km/s/Mpc, $\Omega_\Lambda = 0$, and $y_0 = 3000$.

3. **Inverse metric perturbation**. Show that, if the metric of the perturbed spacetime is

$$g_{\mu\nu} = a^2(\eta_{\mu\nu} + h_{\mu\nu})\,,$$

   then, to first order in $h_{\mu\nu}$,

$$g^{\mu\nu} = a^{-2}(\eta^{\mu\nu} - h^{\mu\nu})\,,$$

   where $h^{\mu\nu} \equiv \eta^{\mu\rho}\eta^{\nu\sigma}h_{\rho\sigma}$.