

$$\int_{\Omega} \left\{ \text{variable } (\mathcal{E}x^{p(\cdot)} \text{ onent}) \text{ Analysis} \right\} dx$$

University of Oulu
28 June – 2 July 2010


Organizing Committee

- Peter Hästö (University of Oulu) - Chairman
- Petteri Harjulehto (University of Helsinki)
- Matti Nuortio (University of Oulu)

Invited Speakers

- Lars Diening (University of Freiburg)
- Andrei Lerner (Bar-Ilan University)
- Teemu Lukkari (NTNU Trondheim)

Sponsors

- The Analysis and Applications-network 
- The Finnish National Graduate School in Mathematics and its Applications
- The Magnus Ehrnrooth Foundation

Program

All talks will be held in the lecture hall L10. A "conference lobby" will be organized in the room M242. Registration takes place in the conference lobby. The conference lobby is open to the participants at all times during and between the talks.

Monday 28th of June

Time	Description
14:00 – 15:00	Registration, room M242, in the second floor
17:00 – 19:00	
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19:00 – 20:30	Peter Hästö – Introductory lecture for researchers new to the field

Tuesday 29th of June

Time	Description
9:00–9:45	Lars Diening – Non-linear Calderon–Zygmund theory for the p -Laplacian including variable exponents
10:00–10:25	Tomasz Adamowicz – On strong $p(\cdot)$ -Laplacian
10:30–10:55	Qihu Zhang – The existence and nonexistence of boundary blow-up solutions for $p(x)$ -Laplacian equations
11:00–11:25	Svetlana Pastukhova – Compensated Compactness in PDEs with variable exponent of nonlinearity
Lunch break	
12:30–13:45	Excursion: Museum of Geology. Departure from room M242.
Coffee, in room M242	
14:00–14:45	Teemu Lukkari – Viscosity solutions of $p(x)$ -Laplace type equations
15:00–15:25	Visa Latvala – Strong minimum principle for quasisuperminimizers of $p(x)$ -growth
16:00–16:25	Mayte Pérez-Llanos – The limit as $p(x) \rightarrow \infty$ of solutions to the inhomog. Dirichlet problem of the $p(x)$ -Laplacian
16:30–16:55	Julio D. Rossi – Limits as $p(x) \rightarrow \infty$ of $p(x)$ -harmonic functions with non-homogen. Neumann boundary conditions
19:00–20:00	Introduction to Finnish sauna: women
20:00–21:00	Introduction to Finnish sauna: men

Wednesday 30th of June

Time	Description
9:00–9:45	Andrei Lerner – Maximal operator on variable L^p spaces
10:00–10:25	Ismail Aydin – Weighted Variable Sobolev Spaces and Capacity
10:30–10:55	Mihai Mihailescu – Γ -convergence for some power-law functionals involving nonstandard growth conditions
11:00–11:25	Marian Bocea – From Variable Exponent Power-Laws to Polycrystal Plasticity via Γ -convergence
Lunch break	
12:30–14:00	Excursion: Botanical Gardens. Departure from room M242
14:15–15:00	Teemu Lukkari – Viscosity solutions of $p(x)$ -Laplace type equations
15:05–15:30	Aleš Nekvinda – Best source and target spaces for average operator
Coffee, in room M242	
16:00–16:25	Henning Kempka – 2-microlocal Besov and Triebel-Lizorkin spaces of variable integrability

Thursday 1st of July

Time	Description
9:00–9:45	Lars Diening – Non-linear Calderon–Zygmund theory for the p -Laplacian including variable exponents
10:00–10:25	Yongqiang Fu – The principle of concentration compactness in variable exponent Lebesgue spaces and its application
10:30–10:55	Pankaj Jain – On grand Lebesgue and Lorentz spaces
11:00–11:25	Alexander Almeida – Variable exponent Besov spaces
Lunch break	
12:30–13:45	Excursion: Museum of Zoology. Departure from room M242.
Coffee, in room M242	
14:00–14:45	Andrei Lerner – Maximal operator on variable L^p spaces
15:00–15:25	Hans-Gerd Leopold – On function spaces of variable smoothness
16:00–16:25	Stanislas Ouaro – Entropy solution for elliptic problems with variable exponent and Neumann homogeneous boundary condition
16:30–16:55	Olli Toivanen – The fundamental convergence theorem for $p(\cdot)$ -superharmonic functions
17:00–17:25	Sergey Shmarev – Energy solutions of parabolic equations with non-standard growth: blow-up and formation of dead cores
17:30–	Pizza party, in room M234

Social Program

Museum of Geology, Tuesday 12:30 – 13:45

To enroll on the excursion, simply write your name on the list. The list will be available in M242 and at times it might be circulated in L10 during talks. At 12:30, we initially gather in the room M242. From there, we continue to the museum which is a really short walk away from the department — the museum is located inside the campus.

The excursion does not cost anything to the participants. Remember to enroll and remember to be on time!

About the museum: The geological museum of the University of Oulu was founded in 1969 with the aim of collecting, conserving and maintaining geological samples for university and general educational purposes and for research. The number of catalogued samples at the end of 2001 was 12296 of which about 3200 were exhibited. The collections are mainly from Fennoscandia but the mineral and rock collections in particular are of worldwide origin.

The mineral collections are exhibited according to the crystal chemical classification whereas geological processes form the basis of the rock and ore collections. A small paleontological collection provides an introduction to the development of life on Earth while Quaternary geology is presented by sediment samples and sedimentary structures being speciality of the museum.

Introduction to Finnish sauna, Tuesday 19:00 and 20:00

The sauna offers you both the chance of getting yourself thoroughly clean and chatting with your conference colleagues in a less formal atmosphere. In Finland, sauna evenings have an important aspect of social interaction.

The sauna evening will be organized in a location called "Koneen sauna", located right in the university campus close to the front doors 2S and 2T.

Separate turns in going to the actual sauna: women 19:00–20:00, men 20:00–21:00. Those not going to the sauna will still be able to enjoy the rest of the party. Refreshments will be available on behalf of the organizers. Also, a CD and DVD player should be available. You are welcome to also bring your own refreshments. Please, bring your own towel and shampoo!

Botanical Gardens, Wednesday 12:30 – 14:00

To enroll on the excursion, simply write your name on the list. The list will be available in M242 and at times it might be circulated in L10 during talks. At 12:30, we initially gather in the room M242. From there, we continue to the gardens which are a moderate walk away from the department — the gardens are located one street crossing away from the campus, to the north.

The excursion does not cost anything to the participants. Remember to enroll and remember to be on time!

About the gardens: The gardens lie on the shore of Lake Kuivasjärvi. They constitute an integral part of the Department of Biology at the University, and thus their main purpose is to provide living plant material, experimental plots and practical help for botanical teaching and research in the University. But at the same time they serve as an important educational resource for schools and a recreation area for the public.

Museum of Zoology, Thursday 12:30 – 13:45

To enroll on the excursion, simply write your name on the list. The list will be available in M242 and at times it might be circulated in L10 during talks. At 12:30, we initially gather in the room M242. From there, we continue to the museum which is a short walk away from the department — the museum is located inside the campus.

The excursion does not cost anything to the participants. Remember to enroll and remember to be on time!

About the museum: Zoological Museum of the University of Oulu focuses on northern nature and species. The main tasks are education and research. At the present there are about 50 000 vertebrate and about two million invertebrate specimens in the museum. Besides the Museum's own collecting activity, authorities and private citizens bring to the Museum wild animals that they have found dead. The material collected in students' thesis research also becomes part of the Museum's collections.

Pizza party, Thursday 17:30

Another social gathering in addition to the sauna evening. The pizza party will be organized in the room M234 at the department. M234 lies on the same second-floor corridor as the conference lobby M242.

Pizza and refreshments will be served. The party does not cost anything to the participants.

Local information

Arrival

The main arrival day is June 28. We will meet participants at the airport during this day and provide instructions and basic information and maps. If you arrive on June 27, you will need to make your own way to the university accommodation where you will be met by the organizers and given a key. **In either case, if you have requested university accommodation, please let us know by email (`peter.hasto(a)helsinki.fi`) your arrival time so that we know when to expect you.**

Busses

From the airport there is a direct bus connection to the university campus with bus 19. The journey takes 45–50 minutes and goes via the center of Oulu. The bus usually has a display of the current stop; to go to the accommodation exit at stop “Ylioppilaskylä P”, for the university exit at “Yliopisto P”. The order of the stops is

“Liikuntahalli P” → “Ylioppilaskylä P” → “Yliopisto P”.

From the center (railway station) also busses 4 and 6 go to the university.

Accommodation

University accommodation is at the addresses Yliopistokatu 16A and Yliopistokatu 18A. Apartment keys can be obtained from the organizers at the registration desks or during pickup at the airport (June 28). Please note that there is no reception desk in the accommodation buildings.

Food

Lunch you can buy in the campus restaurant; it should be quite cheap. The campus restaurant also has a café; you can buy morning and midday coffee there, if you wish. For grocery stores and other restaurants, please see *The Kaijonharju area: shops and restaurants* below.

The Kaijonharju area: shops and restaurants

Right next to the Linnanmaa campus and student housing area is the suburb of Kaijonharju. It is located to the east of the campus, starting from right behind the student housing towers. You will be later provided with a map of the Kaijonharju area. The area features several grocery stores, restaurants, and pubs; these will be displayed on the map.

Hall L10 and room M242

All the talks will be held in the lecture hall L10. The organizers will arrange guiding signs to L10 in the campus. Almost all other things happen in or start from M242, in the second floor of the department, which we have reserved as ‘conference lobby’. The organizers will arrange guiding signs to M242 in the campus.

Wireless local network

The campus area has a free local network titled PanOulu. No passwords or anything of the sort are required. Feel free to use the network! For more information, see <http://www.panoulu.net/index.shtml.en>.

What to do on Wednesday evening?

We have not planned a centralized social event for Wednesday evening. Feel free to come up with your own plans; however, below are some suggestions.

City centre. The city centre, some 5.5 km away from the campus, can be reached by bus or taxi. The Oulu Cathedral, the tallest building in Oulu, is located there. (Oulu is a low-lying city.) The cathedral was built in 1777 and renewed in 1832 and 1845. The remaining ruins of the previous Oulu Castle are located to the vicinity of the city centre.

The Oulu market square is located on the strands of the Oulu River delta; note that the market and the old Merchants' Hall might close early in the afternoon. Several parks are within the city centre, and a very big and popular park with a picturesque stream is located in the vicinity of the centre in a place called Hupisaaret.

Naturally, the city centre hosts several department stores, boutiques, cafés, restaurants, pubs, and night clubs.

Nallikari beach. The Nallikari beach, pompously called "The Northern Riviera," is located some 5 km away from the campus. It should be reachable by a bus, but this might prove a bit tricky due to less frequent summer and evening schedules. Going there on foot is another option, but again not for the faint of heart. Easiest way is by taxi. In the vicinity of the beach, there are trekking routes next to the sea strand. Next to the beach, there is a hotel and spa complex called Eden. Next to the beach, there is also a camping area, and further away there are indoor sports, miniature golf, and pony-riding facilities.

Web resources. There are English tourist resources conveniently available at <http://www.oulutourism.fi/en/etusivu.aspx>. There you will find e.g. the sections Oulu in a Nutshell, What to Do in Oulu, Staying in Oulu, and What's on in Oulu.

Invited talks

Non-linear Calderon–Zygmund theory for the p -Laplacian including variable exponents

Lars Diening (University of Freiburg)

Tuesday 9:00 - 9:45 & Thursday 9:00 - 9:45

It has been possible in the recent years to generalize parts of the linear Calderon-Zygmund theory to the non-linear setting of the p -Laplacian. We first discuss the basic principles and results of this method. After that we show how to generalize this to the context of variable exponents.

Maximal operator on variable L^p spaces

Andrei Lerner (Bar-Ilan University)

Wednesday 9:00 - 9:45 & Thursday 14:00 - 14:45

Let M be the Hardy-Littlewood maximal operator. Denote by \mathcal{M} the class of variable exponents $p(\cdot)$ for which M is bounded on $L^{p(\cdot)}$. There are a lot of works related to this class. We shall discuss only several directions concerning the relationship of \mathcal{M} with Muckenhoupt-type conditions and pointwise multipliers for BMO . A number of open questions will be posed.

Viscosity solutions of $p(x)$ -Laplace type equations

Teemu Lukkari (NTNU Trondheim)

Tuesday 14:00 - 14:45 & Wednesday 14:15 - 15:00

The modern concept of viscosity solutions, introduced by Crandall and Lions in the eighties, has turned out to be indispensable for, e.g., fully nonlinear equations. In particular, viscosity theory provides a stable notion of generalized solutions to the $p(x)$ -Laplacian in the case when $p(x) \rightarrow \infty$ in a suitable way.

We first discuss the viscosity solutions of

$$-\Delta_{p(x)}u := -\operatorname{div}(|Du|^{p(x)-2}Du) = 0.$$

It turns out that the viscosity and weak solutions are equivalent, and we show how this can be exploited to prove a removability theorem. Then we derive the equation

$$-\Delta_{\infty(x)}u := -(\langle D^2uDu, Du \rangle + |Du|^2 \ln |Du| \langle Du, Dp \rangle) = 0,$$

as the limit of

$$-\Delta_{kp(x)}u = 0$$

when $k \rightarrow \infty$. Finally, we sketch the proof of the comparison principle for viscosity solutions of the above equations. This is the core of the theory, and turns out to be tricky for both of the equations.

This is based on joint works with P. Juutinen, P. Lindqvist, and M. Parviainen.

Contributed talks

On strong $p(\cdot)$ -Laplacian

Tomasz Adamowicz (University of Cincinnati)

Tuesday 10:00 - 10:25

We generalize p -harmonic equation to the setting of variable exponent PDEs, obtaining a nonhomogeneous equation with many interesting properties not shared by the other existing nonstandard growth extensions of the p -Laplacian. Our discussion includes Harnack inequality, the relation between planar mappings of finite distortion and the gradient of solution and the global integrability of $p(x)$ -supersolutions.

The talk is based on joint work with Peter Hästö, Oulu University.

Variable exponent Besov spaces

Alexander Almeida (University of Aveiro)

Thursday 11:00 - 11:25

Weighted Variable Sobolev Spaces and Capacity

Ismail Aydin (Sinop University)

Wednesday 10:00 - 10:25

In this paper we define weighted variable Sobolev capacity and discuss properties of capacity in the space $W^{1,p(\cdot)}(\mathbb{R}^n, w)$. We investigate the role of capacity in the pointwise definition of functions in this space if the Hardy-Littlewood maximal operator is bounded on the space $W^{1,p(\cdot)}(\mathbb{R}^n, w)$. Also it is shown that the relation between the Sobolev capacity and Bessel capacity.

From Variable Exponent Power-Laws to Polycrystal Plasticity via Γ -convergence

Marian Bocea (North Dakota State University)

Wednesday 11:00 - 11:25

The asymptotic behavior of several classes of power-law functionals acting on fields belonging to variable exponent Lebesgue spaces and which are subject to constant rank differential constraints is studied via Γ -convergence. Several models of polycrystal plasticity are obtained as limiting cases of more flexible power-law models, and the effective yield set is characterized in each case by means of variational principles associated to the limiting supremal functionals. This is based on joint work with Mihai Mihailescu (University of Craiova, Romania, and Central European University, Hungary) and Cristina Popovici (North Dakota State University, U.S.A.)

The principle of concentration compactness in variable exponent Lebesgue spaces and its application

Yongqiang Fu (Harbin Institute of Technology)

Thursday 10:00 - 10:25

In this talk, we first establish a principle of concentration compactness in variable exponent Lebesgue spaces. Then, based on this concentration compactness principle, we study the existence of solutions for a class of $p(x)$ -Laplacian equations involving the critical exponent, especially we obtain a sequence of radially symmetric solutions associated with a sequence of positive energies going towards infinity.

On grand Lebesgue and Lorentz spaces

Pankaj Jain (University of Delhi)

Thursday 10:30 - 10:55

2-microlocal Besov and Triebel-Lizorkin spaces of variable integrability

Henning Kempka (Friedrich Schiller University Jena)

Wednesday 16:00 - 16:25

We introduce 2-microlocal Besov and Triebel-Lizorkin spaces as a generalization of the 2-microlocal spaces which were introduced by Bony. They are a good tool to measure the local regularity of functions. We give an introduction to the 2-microlocal formalism from Seuret & Levy Vehel, which connects regularity exponents to 2-microlocal function spaces.

Finally we present a further generalization of the 2-microlocal Besov and Triebel-Lizorkin spaces to the case of variable integrability and obtain a wavelet characterization for them. It turns out that these spaces are connected to spaces of variable smoothness and integrability which have recently been introduced by Diening, Hästö & Roudenko and Almeida & Hästö.

Strong minimum principle for quasisuperminimizers of $p(x)$ -growth

Visa Latvala (University of Eastern Finland)

Tuesday 15:00 - 15:25

The aim of the talk is to discuss the strong minimum principle for non-negative quasisuperminimizers of the variable exponent Dirichlet energy integral. The proof of the result is given under the assumption that the exponent p satisfies

$$|p(x) - p(y)| \leq c|x - y| \log \left(e + \frac{1}{|x-y|} \right). \quad (*)$$

In particular, the strong minimum principle holds for Lipschitz continuous exponents p .

As far as we know, the only known proof of the strong maximum principle in the variable exponent case is based on a direct method due to X.-L. Fan, Y. Zhao and Q. Zhang (2003) under the assumption that p satisfies $p \in C^1(\bar{\Omega})$ with $1 < p^- \leq p^+ < \infty$.

Our proof relies on a new version of the weak Harnack estimate which gives a precise control for the error term based on the modulus of continuity of the exponent p . If p satisfies (*), then the error term appears to be so small that the claim can be achieved by an iterative argument.

The talk is based on joint work with Peter Hästö, Petteri Harjulehto and Olli Toivanen.

On function spaces of variable smoothness

Hans-Gerd Leopold (Friedrich-Schiller-University Jena)

Thursday 15:00 - 15:25

Function spaces of variable or varying smoothness are supposed to classify the local smoothness behavior of a function or distribution in neighbourhoods of each point. We discuss different approaches, their relations and distinctions. Moreover we describe equivalent characterizations for some of these spaces, using pseudo-differential operators.

Γ -convergence for some power-law functionals involving nonstandard growth conditions

Mihai Mihailescu (University of Craiova)

Wednesday 10:30 - 10:55

This talk discusses some Γ -convergence results for power-law functionals with variable exponents. From the point of view of real world applications the main motivation of the results comes from the study of (first-failure) dielectric breakdown. Some connections with the generalization of the ∞ -Laplace equation to the variable exponent setting are also explored.

This talk is based on my joint work with M. Bocea.

Best source and target spaces for average operator

Aleš Nekvinda (Czech Technical University)

Wednesday 15:05 - 15:30

Let $Af(x) := \frac{1}{x} \int_0^x f(t)dt$ be the one-dimensional Hardy averaging operator. It is well known that A is bounded on L^p whenever $1 < p \leq \infty$. We improve this result in the following sense: we introduce a pair of new function spaces, the ‘source’ space S_p , which is strictly larger than L^p , and the ‘target’ space T_p , which is strictly smaller than L^p , and prove that A is bounded from S_p into T_p . Moreover, we show that this result cannot be improved within the environment of Banach function spaces. We present applications of this result to variable-exponent Lebesgue spaces $L^{p(x)}$. We also mean that the spaces S_p and T_p are associated and this fact gives us an information on a behavior of the dual operator.

Compensated Compactness in PDEs with variable exponent of nonlinearity

Svetlana Pastukhova

**(Moscow Institute of Radioengineering, Electronics and
Automation)**

Tuesday 11:00 - 11:25

Using appropriate regularization schemes, new variants of Compensated Compactness Principle (CCP), in combination with monotonicity arguments, allow to establish the weak solvability of various PDEs with nonstandard coerciveness and growth conditions, including equations with variable nonlinearity exponent. Below, we give only the variant of CCP, which is suitable to elliptic equations. There were obtained also hydromechanical and parabolic versions of CCP, applicable respectively to Navier – Stokes system for electrorheological fluids and to parabolic equations with variable nonlinearity exponent

$p(x, t)$, where p is an arbitrary measurable function with some restriction on its range of values.

In the following lemma, Ω is any domain in \mathbb{R}^d ($d \geq 2$) and α_* denotes the Sobolev exponent for the dimension $d-1$.

Lemma. *Assume: i) $\operatorname{div} w_n = 0$, $w_n \rightharpoonup w$ in $L^{\beta'}(\Omega)^d$, $\beta' = \frac{\beta}{\beta-1}$; ii) $u_n \rightharpoonup u$ in $W^{1,\alpha}(\Omega)$ and $u_n \in W^{1,\beta}(\Omega)$; iii) the sequence $w_n \cdot \nabla u_n$ is bounded in $L^1(\Omega)$; iv) the exponents α and β are such that $1 < \alpha \leq \beta < \alpha_*$.*

Then (up to the extraction of a subsequence) there is a weak convergence of measures on Ω

$$w_n \cdot \nabla u_n dx \rightharpoonup w \cdot \nabla u dx + d\mu^s,$$

where μ^s is the singular (with respect to dx) part of the limit measure.

In the case $\alpha = \beta > 1$ the assumptions i), ii) imply the above measure convergence with trivial μ^s , and that is the well-known div-curl lemma due to Murat and Tartar.

This talk is based on joint work with V.V. Zhikov.

The limit as $p(x) \rightarrow \infty$ of solutions to the inhomogeneous Dirichlet problem of the $p(x)$ -Laplacian

Mayte Pérez-Llanos (U. Técnica de Lisboa)

Tuesday 16:00 - 16:25

In this work we study the behaviour of the solutions to the following Dirichlet problem related to the $p(x)$ -Laplacian operator

$$\begin{cases} -\operatorname{div}(|\nabla u|^{p(x)-2} \nabla u) = f(x), & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

as $p(x) \rightarrow \infty$, for some suitable functions f . We consider a sequence of functions $p_n(x)$ that goes to infinity uniformly in $\bar{\Omega}$. Under adequate hypotheses on the sequence p_n , basically, that the following two limits exist,

$$\lim_{n \rightarrow \infty} \nabla \ln p_n(x) = \xi(x), \quad \text{and} \quad \limsup_{n \rightarrow \infty} \frac{\max_{x \in \bar{\Omega}} p_n}{\min_{x \in \bar{\Omega}} p_n} \leq k, \quad \text{for some } k > 0,$$

we prove that $u_{p_n} \rightarrow u_\infty$ uniformly in $\bar{\Omega}$. In addition, we find that u_∞ solves a certain PDE problem (that depends on f) in viscosity sense. In particular, when $f \equiv 1$ in Ω we get $u_\infty(x) = \operatorname{dist}(x, \partial\Omega)$ and it turns out that the limit equation is $|\nabla u| = 1$.

This talk is based on my joint work with Julio D. Rossi.

Limits as $p(x) \rightarrow \infty$ of $p(x)$ -harmonic functions with non-homogeneous Neumann boundary conditions

Julio D. Rossi (Universidad de Alicante)

Tuesday 16:30 - 16:55

In this talk we deal with the limit as $p(x) \rightarrow \infty$ of solutions to $-\Delta_{p(x)}u = 0$ in a domain Ω , with non-homogeneous Neumann boundary conditions, $|\nabla u|^{p(x)} \frac{\partial u}{\partial \eta} = g(x)$. Our approach consists on considering sequences of variable exponents converging uniformly to $+\infty$ and then determining the equation satisfied by a limit of the corresponding solutions.

This talk is based on my joint work with Mayte Perez-Llanos.

Energy solutions of parabolic equations with nonstandard growth: blow-up and formation of dead cores

Sergey Shmarev (University of Oviedo)

Thursday 17:00 - 17:25

We present recent results concerning the effects of blow-up and vanishing in solutions of the homogeneous Dirichlet problem for parabolic equations of the type "diffusion-absorption-reaction" with nonstandard growth conditions:

$$(1) \quad u_t = \sum_{i=1}^n D_i (a_i(x, t, u) |D_i u|^{p_i(x,t)-2} D_i u) + c(x, t) |s|^{\sigma(x,t)-2} u + f \quad \text{in } Q = \Omega \times (0, T],$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain with Lipschitz-continuous boundary, the exponents of nonlinearity $p_i(x, t)$, $\sigma(x, t)$ and the coefficients a_i, c are given functions; $D_i = \partial_{x_i}$.

We introduce the notion of *energy solution* of equation (1) and indicate sufficient conditions on the exponents p_i, σ and the problem data which guarantee existence and uniqueness of such solutions in a suitable function space of Sobolev-Orlicz type.

A special attention is paid to the study of the properties intrinsic for the solutions of equations with variable and anisotropic nonlinearity. The following issues are discussed:

- nonpropagation of disturbances from the data in solutions of the anisotropic equations without absorption terms, for example,

$$u_t = (|u_x|^{p_1-2} u_x)_x + (|u_y|^{p_2-2} u_y)_y, \quad \min\{p_1, p_2\} > 1 > \frac{1}{p_1} + \frac{1}{p_2}, \quad p_i \equiv p_i(x, t);$$

- blow-up and vanishing in a finite time of solutions of equation (1);
- blow-up and vanishing in the limit cases when equation (1) eventually becomes linear, for example,

$$u_t = \Delta u + u^{1+\epsilon(t)} \quad \text{with } \epsilon(t) \rightarrow 0 \text{ as } t \rightarrow \infty;$$

- simultaneous space and time localization.

The study of the localization properties of solutions is based on the analysis of the *local energy functions* which satisfy nonlinear ordinary differential inequalities with variable exponents of nonlinearity.

The results were obtained in collaboration with S. Antontsev.

Entropy solution for elliptic problems with variable exponent and Neumann homogeneous boundary condition

Stanislas Ouaro (University of Ouagadougou)

Thursday 16:00 - 16:25

In this work we study the following nonlinear homogeneous Neumann boundary value problem $b(u) - \operatorname{div} a(x, \nabla u) = f$ in Ω , $a(x, \nabla u) \cdot \eta = 0$ on $\partial\Omega$, where Ω is a smooth bounded open domain in \mathbb{R}^N , $N \geq 3$. We prove the existence and uniqueness of a weak solution for $f \in L^\infty(\Omega)$ and the existence and uniqueness of an entropy solution for L^1 -data f . The functional setting involves Lebesgue and Sobolev spaces with variable exponent.

The fundamental convergence theorem for $p(\cdot)$ -superharmonic functions

Olli Toivanen (University of Eastern Finland)

Thursday 16:30 - 16:55

I will review a manuscript by Visa Latvala, Teemu Lukkari and me. In it, we study the balayage related to the supersolutions of the variable exponent $p(\cdot)$ -Laplace equation. We prove the fundamental convergence theorem for the balayage and apply it for proving the Kellogg property, boundary regularity results for the balayage, and a removability theorem for $p(\cdot)$ -solutions.

The existence and nonexistence of boundary blow-up solutions for $p(x)$ -Laplacian equations

Qihu Zhang (Zhengzhou University of Light Industry)

Tuesday 10:30 - 10:55

In this talk, we consider the problem

$$\begin{cases} -\Delta_{p(x)} u + \rho(x)f(x, u) = 0, & \text{in } \Omega, \\ u(x) \rightarrow +\infty & \text{as } d(x, \partial\Omega) \rightarrow 0, \end{cases}$$

where $-\Delta_{p(x)} u := -\operatorname{div}(|\nabla u|^{p(x)-2} \nabla u)$, $\Omega \subset \mathbb{R}^N$ is a bounded domain, $\rho(x)$ is a singular coefficient, $u \in W_{loc}^{1,p(x)}(\Omega) \cap C(\Omega)$ and $u(x) \rightarrow +\infty$ as $d(x, \partial\Omega) \rightarrow 0$.

Our aim is to discuss the existence and nonexistence of boundary blow-up solutions for the problem, and give the boundary blow-up rate of solutions.

Participants

Qabil Abdelbaset, University of Hassan II, Morocco
Tomasz Adamowicz, University of Cincinnati, USA
Alexandre Almeida, University of Aveiro, Portugal
Kaouther Ammar, TU Berlin, Germany
Ismail Aydin, Sinop University, Turkey
Lauri Berkovits, University of Oulu, Finland
Marian Bocea, North Dakota State University, USA
Lars Diening, University of Freiburg, Germany
Said El Manouni, University of Al-Imam, Saudi Arabia
Yongqiang Fu, Harbin Institute of Technology, P. R. China
Heikki Hakkarainen, University of Oulu, Finland
Petteri Harjulehto, University of Helsinki, Finland
Peter Hästö, University of Oulu, Finland
Anne Heikkilä, University of Oulu, Finland
Åsa Hirvonen, University of Helsinki, Finland
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